

ON COMPUTATIONAL SCHEMES FOR GLOBAL-LOCAL
STRESS ANALYSIS

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1. INTRODUCTION

This paper primarily deals with an overview of global-local stress analysis methods and associated difficulties and recommendations for future research. The phrase global-local analysis is understood to be an analysis in which some parts of the domain or structure are identified, for reasons of accurate determination of stresses and displacements or for more refined analysis than in the remaining parts. The parts of refined analysis are termed local and the remaining parts are called global. Typically local regions are small in size compared to global regions, while the computational effort can be larger in local regions than in global regions.

2. CONTENTS

This paper is divided into the following parts:

- Motivation-Problems (problems that motivated global-local analysis)
- Common Features
- Focus Problem
- Analysis Methods
- Global-Local Approaches
- Example Problem
- Conclusions and Recommendations

3. MOTIVATION PROBLEMS

The following stress analysis problems, among many others, motivate us to use global-local approaches:

- Free-Edge Stress Concentration in Laminates
- Contact Stress Problems
- Impact
- Fracture Mechanics
- Unbounded-Domain Problems

Almost all laminated composite structural elements have free edges at the boundaries (including holes) of the elements. It is well known that the transverse normal and shear stresses are very large at the edges (more precisely, the stresses are large within a distance of the order of thickness of the laminate from the free edge). While the classical laminate theory is adequate to describe the behavior of the laminate everywhere except in the "boundary layer" in which the transverse normal and shear stresses are large, a refined theory, often the 3-D elasticity theory, is needed to describe the state of stress near the edges.

Contact stress problems (for example, bolted and bonded joints, tire contact, and metal-forming problems) require the use of a special theory that accounts for appropriate constitutive laws and friction and allows for slip, slide, and separation of the mating parts in the contact regions. Elsewhere, appropriate elasticity theory can be used.

Impact of two solid bodies can be modelled by the use of one theory in the vicinity of contact and by another theory elsewhere. Since the stresses are much larger in the contact region than elsewhere, more refined theory and analysis are required in contact regions. Of course, the theory and analysis used depend on the type of structure, loading and deformation.

Structures containing cracks, whether formed during manufacturing or service, require special treatment of stress fields around cracks, often using 3-D stress analyses and/or nonlinear fracture mechanics theories, while the linear elastic fracture mechanics theory is adequate away from the cracks.

Problems involving unbounded regions (for example, soil mechanics and earthquake engineering) are by their nature divided into local and global regions. Global regions, in theory, can be infinite but in practice they are finitely large, and less refined theory and/or analysis is used to determine the stress field and other pertinent information.

4. COMMON FEATURES

The motivating problems listed previously share certain common physical features that are significant from the modelling and analysis points of view. The following list provides some of these features:

- Stress Concentration (large local gradients)
- Three-Dimensional State of Stress
- Large Rotations/Strains
- Local Discontinuities (holes, discontinuous fibers, etc.)
- Material Nonlinearities (nonlinear elasticity, plasticity, etc.)

A global-local stress analysis should account for all features that are present in the problem. Of course, some of those features are not to be included in the global model.

5. FOCUS PROBLEM

The focus problem identified by NASA Langley Research Center is a blade-stiffened panel with a discontinuous stiffener. The problem has the following physical features:

- Geometric Discontinuities
- Local Stress Gradients
- Eccentric Loading
- Large Displacements
- Free Edges

We shall return to this problem later to discuss the global-local analysis approach.

6. ANALYSIS METHODS

6.1 COMMON APPROACHES

The commonly used analysis methods for structural problems are

- Classical (or Analytical) Methods
- Classical Variational Methods
- Finite-Difference Method
- Finite-Element Method
- Boundary Element Method

Some noted advantages and disadvantages of these methods are outlined next.

6.2 CLASSICAL AND VARIATIONAL METHODS

The classical method of solving problems exactly is the best there is. However, most practical problems (which have irregular geometries, anisotropic materials, discontinuities, geometric and/or material nonlinearities, etc.) do not admit exact solutions by the classical approach.

The classical variational (e.g., Ritz, Galerkin and weighted-residual) methods yield continuous solutions throughout the domain, giving high resolution of displacements and stresses. They are computationally efficient for a given problem. For a given order of approximation, previously computed (for lower order approximation) matrix coefficients can be used. These methods, however, have two major shortcomings: (i) the approximation functions are not easy and are often impossible to

construct for most practical structural problems; and (ii) the variational methods cannot be implemented on a computer for the analysis of a class of problems because the resulting algebraic equations depend on the approximation functions, which in turn depend on a specific problem.

6.3 FINITE-DIFFERENCE METHOD (FDM)

The finite difference method is simple in formulation (based on the representation of derivatives of a function in terms of a finite Taylor's series expansion) and easy to implement on the computer. The method dominated the field of numerical methods until the sixties, when the finite-element method gained popularity, especially in solid and structural mechanics. The disadvantages of the finite-difference method include: difficulty in representing complex geometries, inexact representation of boundary conditions on non-straight boundaries, and difficulty in developing higher-order approximations. Because of these difficulties the method does not lend itself for general-purpose code development. The method is seldom used for spatial approximations in structural mechanics problems.

6.4 FINITE-ELEMENT METHOD

The finite-element method overcomes the shortcomings of the classical variational methods. This approach is systematic (modular) and natural and allows an accurate representation of complex geometries. Higher-order approximations are easy to use without changing the modular structure of the approach. The method is ideally suited for general purpose and computer program development. The disadvantages, compared to other computing methods, are the large formulative and computational efforts. The finite-element method is the most frequently used numerical method in structural analysis. It is now a key component of any mechanical CAD/CAM system.

6.5 BOUNDARY ELEMENT METHOD (BEM)

The finite-difference and finite-element methods can be classified as domain methods because they involve approximations of the entire domain. The boundary element method, also known as the boundary integral method (BIE), seeks approximations only on the boundary of the domain by converting the governing differential equations to integrals over the boundary of the domain. The dimensionality of the problem is thereby effectively reduced by one. Because the interior of the domain is not approximated, the computational time involved is less ($BEM/FEM \approx \frac{1}{n}$, where $n \times n$ is the finite-element mesh). The BEM offers continuous interior modelling within the solution domain, giving high resolution of displacements and stresses. The method is unsuitable for problems requiring information at a large number of internal points. Application of BEM to nonlinear problems and problems with discontinuities is not fully established.

6.6 CONCLUSIONS

In conclusion, the general application of the finite-element method to structural problems is unmatched to date. Structural problems, because of the modular nature of FEM, are better modelled by FEM. A suitable combination of FEM and BEM can be advantageous in some problems (e.g., unbounded-domain problems).

7. GLOBAL-LOCAL APPROACHES

7.1 MODELS AND METHODS

In formulating a given problem, either the same theory is used throughout or a refined theory is used locally and a less refined theory globally.

The global-local analysis methods can be FEM throughout, FEM and classical solution, or BEM and FEM. When the same theory and FEM are used throughout, it is understood that locally special elements are used (e.g., friction element, interface element with sliding, or elements which allow opening and closing along element interfaces). Classical solutions are available, for example, for infinite plates with holes. The solution is not valid far away from the hole if the plate is long but not infinite. In such cases, the FEM can be used globally and the classical solution can be used locally. For soil mechanics and earthquake engineering problems, a combination of BEM and FEM proves computationally efficient. In some situations experimental methods globally and computational methods locally are recommended.

7.2 SOME EXAMPLES

Some example problems that require global-local analysis are listed here.

- Free-edge stress analysis of laminates
- Contact stress problems
- Stress analysis of structures with discontinuities
- A blade-stiffened panel with a discontinuous stiffener - the focus problem

As mentioned earlier, free-edge stress problem requires a refined theory near free edges. For example, the classical laminate theory globally and either quasi-3D or full 3-D theory locally (depending on the lamination scheme, geometry and loading) can be used to analyze the problem. The problem will be discussed in more detail later.

In bolted joint problems, an experimental technique such as the Moiré interferometry can be used to determine the surface displacements (and hence strains and stresses) and the finite-element method can be used to determine the interior displacement and stress fields.

In the stress analysis of plates with holes, one can use a laminate theory away from holes and 3-D elasticity theory around the hole, and use the finite-element method to model the entire problem.

In the case of the focus problem, which has all features that are present in the examples discussed above, the global theory should be a shear deformation plate theory (2D) with the Von Karman geometric non-linearity and the local theory can be the fully 3D laminate theory. The finite-element method should be used throughout. When FEM is used locally, the fully 3D elements or 3D degenerate elements can be used.

7.3 DIFFICULTIES

In using global-local approaches, we face some difficulties. Some of these are

- Interfacing between regions
- Interfacing between methods
- Selection of regions
- Changing regions and interfaces

When the finite-element method is used, the elements used globally and locally can be different. Then it is important to have compatibility of the nodal degrees of freedom at the interface of the elements. A special interface element might be needed in some situations. When two different methods are used, the unknowns in the two methods should be the same. Selection of the local and global regions depends on the physical features and accuracy desired. In some cases, the regions might have to be determined only after a preliminary analysis. The global and local regions can change during the history of deformation/ loading. For example, in elastic-plastic analysis, the plastic zones are unknown a priori and they change with loading.

8. EXAMPLE PROBLEM

Here we briefly discuss the free-edge stress problem in symmetric laminates. Figure 1 shows the laminate geometry, loading, and the domain modelled. Because of the assumed symmetry of the lamination about the midplane and the constant straining along the x-axis, the displacement field can be approximated (ref. 1) as

$$u = U_0 x + U(y, z)$$

$$v = V(y, z)$$

$$w = W(y, z)$$

where U_0 is a constant, K.

The displacement field is three dimensional but it leads, when substituted into the Navier equations of equilibrium, to three partial differential equations in two independent variables, y and z .

It is well known that the transverse normal stress, σ_z , is very large (unbounded) near the free edge. To reduce its magnitude a cap is used on the free edge. The effect of the cap on the stress distribution σ_z is investigated. The finite-element method with the four-node bilinear rectangular element is used to model the computational domain. A refined mesh is used near the free edge and in the cap.

Figures 2 and 3 show the distribution of the transverse normal stress σ_z along the width of the laminate for $[0^\circ/90^\circ]_5$ and $[45^\circ/-45^\circ]_5$ laminates, respectively, ($E_1 = 137.89$ GPa, $E_2 = E_3 = 14.48$ GPa, $G_{12} = G_{13} = G_{23} = 5.86$ GPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.21$). Results for both capped and uncapped laminates are presented (for $k = 0.001$, $b = 25.4$ cm, $h = 2.54$ cm and thickness of the cap, $t = 0.08$ cm). We observe that the stress is essentially zero inside the laminate but has quite a large magnitude within a distance of $y/b = 0.1$ (one-tenth of the width) from the free edge. Hence a laminate theory is sufficient to model the interior, while the quasi-3D can be used to model the free-edge stress field. The effect of the free-edge reinforcement (i.e., cap) on the stress magnitude is significant; the magnitude is reduced to less than one-third of that without cap.

For a more detailed and complete stress distribution near the free edge of a more general laminate (e.g., without symmetry about the midplane), a three-dimensional model is needed.

9. CONCLUSIONS AND RECOMMENDATIONS

9.1 AREAS NEEDING SUPPORT

A review of the literature shows there are very few cases of global-local analyses of structural problems involving the "physical features" discussed earlier. It is recommended that the following areas of global-local approaches be investigated:

- Global-local analysis of problems with "common features" outlined earlier
- Investigation and development of interface elements
- Feasibility of BEM as a computational tool for nonlinear problems and its interface with FEM
- Development of adaptive mesh refinements and time-stepping algorithms
- Exploitation of the vector and parallel processor computers for efficient structural analysis

- Finite-element calculations
- Solution of equations
- Eigenvalue computations

The use of parallel processors can dictate the solution procedures, for example, iterative methods over one-step methods.

9.2 NASA'S INVOLVEMENT

NASA (CSM) should be involved in the global-local analysis development because of the tremendous impact this field has on computational mechanics applied to space structures. In particular, NASA should undertake the following tasks in the global-local analysis area:

- Support individual grants (as opposed to large group grants)
- Collaborate with university faculty and graduate students by identifying specific problem areas and providing computational time and scientific advice
- Give graduate student residencies, during which students spend a few weeks (perhaps the summers) at NASA
- Conduct workshops (say, once in two years) to bring the latest developments for critical evaluation and to set future directions.

ACKNOWLEDGMENTS

This paper was prepared via the support of the Structural Mechanics Branch of NASA Langley Research Center (NAG-1-459).

REFERENCE

Heyliger, P. R. and Reddy, J. N., "Reduction of Free-Edge Stress Concentration in Symmetric Laminates," Virginia Polytechnic Institute and State University, Blacksburg, VA, CCMS Report 85-01, January 1985.

BIBLIOGRAPHY

Arnold, D. N., Babuska, I. and Osborn, J., "Finite Element Methods: Principles for Their Selection," Computer Methods in Appl. Mech. & Engng., Vol. 45, pp. 57-96, 1984.

Atluri, S. N. GALLAGHER, R. H., and Zienkiewicz, O. C., Hybrid and Mixed Finite Element Methods, Wiley, Chichester (U.K.), 1983.

- Banerjee, P. K. and Butterfield, R., Boundary Element Methods in Engineering Science, McGraw-Hill, London, 1981.
- Brebbia, C. A., The Boundary Element Method for Engineers, Pentech Press, London, 1980.
- Buzbee, B. L. and McKnight, A. L., "Impact of Vector Processors on Algorithms," Los Alamos Scientific Laboratory, Los Alamos, NM 87545, LA-8123-MS, 1979.
- Calahan, D. A., "Exploiting Parallelism in Designing Algorithms," Basic Quest. Des. Theory, (Symposium, New York, 1974), pp. 285-302, 1974.
- Chen, S. C., Kuck, D. J. and Sameh, A. H., "Practical Parallel Band Triangular System Solvers," ACM Trans. Math. Software, Vol. 4, pp. 270-77, 1978.
- Conrad, V. and Wallach, Y., "Iterative Solution of Linear Equations on a Parallel Processor System," IEEE Trans. Computers, Vol. 26, pp. 838-47, 1977.
- Csanky, L., "Fast Parallel Matrix Inversion Algorithms," SIAM J. Comput., Vol. 5, pp. 618-23, 1976.
- Durnier, A., "A Theory of Friction," Int. J. Solids & Struct., Vol. 20, No. 7, pp. 637-647, 1984.
- Fenner, R. T., "The Boundary Integral Equation (Boundary Element) Method in Engineering Stress Analysis," J. Strain Analysis, Vol. 18, No. 4, 1983.
- Feng, T., (ed.), Parallel Processing, Springer-Verlag, New York, 1975.
- Gadala, M. S. and Oyavas, G. AE., "Numerical Solutions of Nonlinear Problems of Continua-I (Survey of Formulation Methods and Solution Techniques)," Computers and Structures, Vol. 19, No. 5/6, pp. 865-877, 1984.
- Heller, D., "A Survey of Parallel Algorithms in Numerical Linear Algebra," SIAM Rev., Vol. 20, pp. 740-77, 1978.
- Knight, Jr., N. F. and Starnes, Jr., J. H., "Postbuckling Behavior of Selected Curved Stiffened Graphite-Epoxy Panels Loaded in Axial Compression," 26th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, AIAA Paper No. 85-0768-CP, Orlando, FL., 1985.
- Matthies, H. and Strang, G., "The Solution of Nonlinear Finite Element Equations," Int. J. Numer. Meth. in Engrg., Vol. 14, pp. 1613-1626, 1979.
- Ortega, J. M. and Voigt, R. G., "Solution of Partial Differential Equations on Vector and Parallel Computers," NASA CR-172500, Jan. 1985.
- Pagano, N. J. and Soni, S. R., "Global-Local Laminate Variational Model," Int. J. Solids and Struct., Vol. 19, No. 1, pp. 207-228, 1981.
- Pipes, R. B. and Pagano, N. J., "Interlaminar Stresses Composite Laminates Under Uniform Axial Extension," J. Comp. Materials, Vol. 4, pp. 538-548, 1970.

- Reddy, J. N., An Introduction to the Finite Element Method, McGraw-Hill, New York, 1984.
- Reddy, J. N., Energy and Variational Methods in Applied Mechanics, Wiley, New York, 1984.
- Rybicki, E. F., "Approximate Three-Dimensional Solutions for Symmetric Laminates Under Inplane Loading," J. Comp. Materials, Vol. 5, pp. 354-360, 1971.
- Traub, J. F. (ed.), Complexity of Sequential and Parallel Numerical Algorithms, Academic Press, New York, 1973.
- Whitcomb, J. D., Raju, L. S. and Goree, J. G., "Reliability of the Finite Element Method for Calculating Free-Edge Stresses in Composite Laminates," Computers and Structures, Vol. 15, No. 1, pp. 23-37, 1982.
- Zienkiewicz, O. C., Gago, J. P., De, S. R., and Kelly, D. W., "The Hierarchical Concept in Finite Element Analysis," Computers and Structures, Vol. 16, No. 1-4, pp. 53-65, 1983.

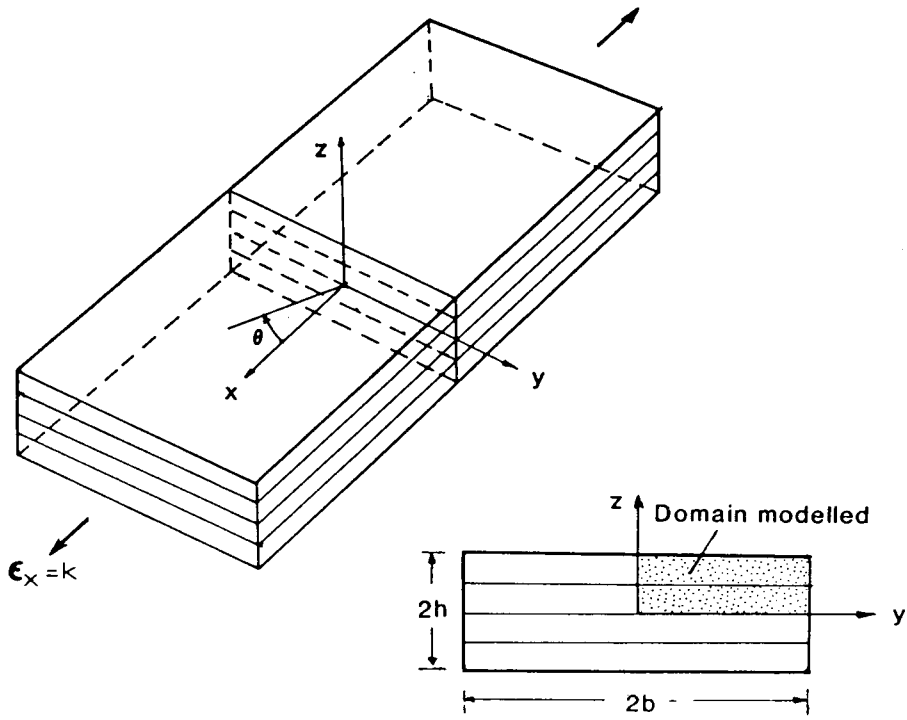


Figure 1. Laminate geometry and loading (cap is not shown).

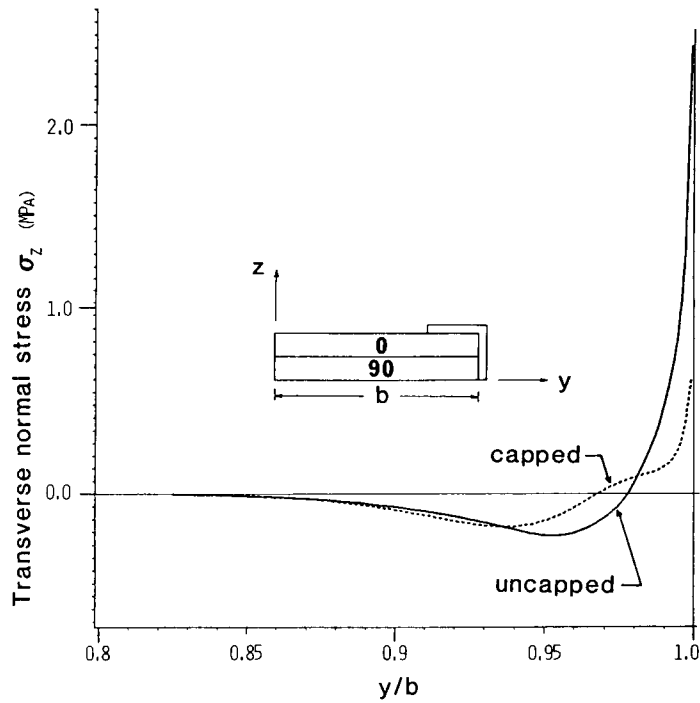


Figure 2. Distribution of the transverse normal stress along the width of a cross-ply laminate $[0^\circ/90^\circ]_5$.

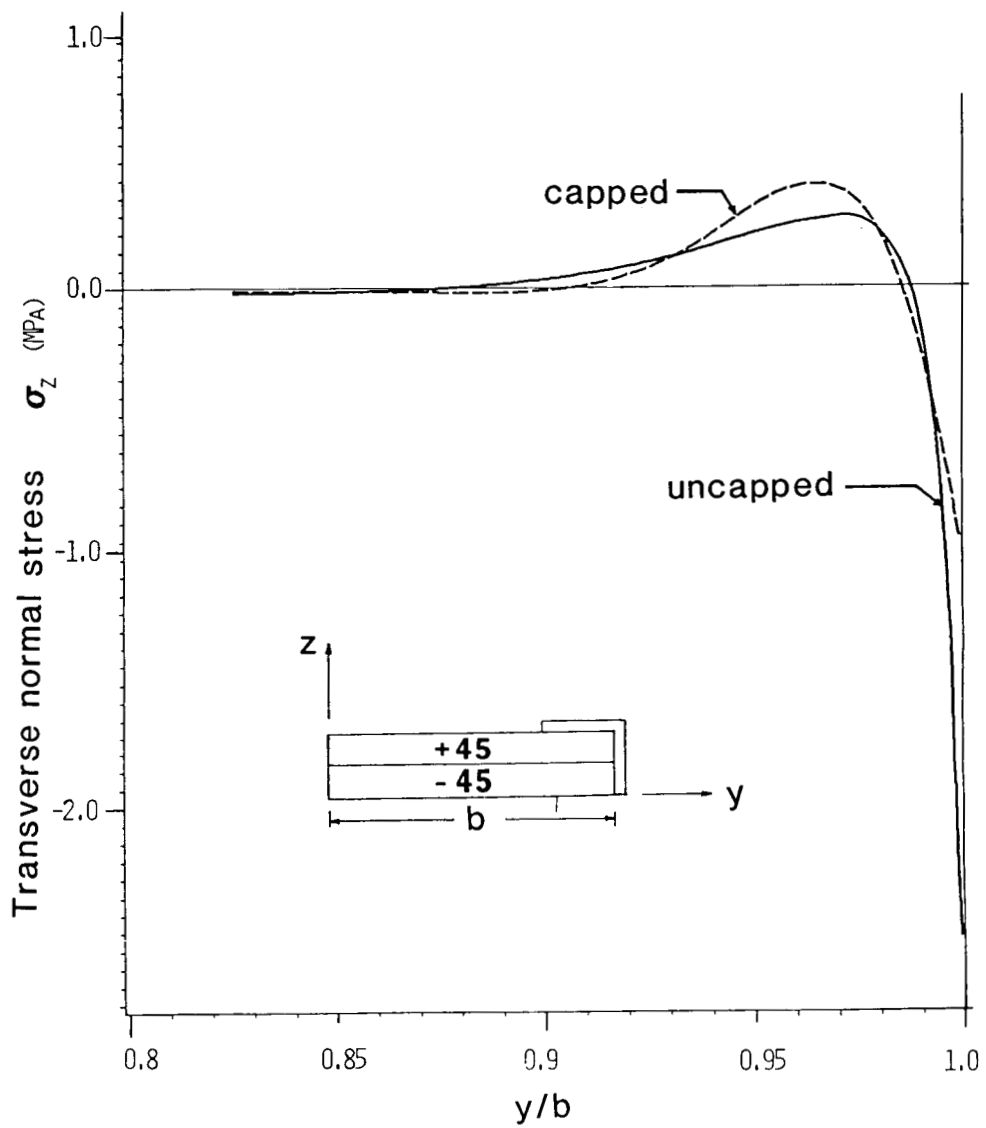


Figure 3. Distribution of the transverse normal stress (σ_z) along the width (y/b) of the laminate $[45^\circ/-45^\circ]_s$.