

MODAL REDUCTION STRATEGIES FOR INTERCONNECTED FLEXIBLE BODIES SIMULATION

F. O. Eke and G. K. Man  
 Jet Propulsion Laboratory  
 Pasadena, CA

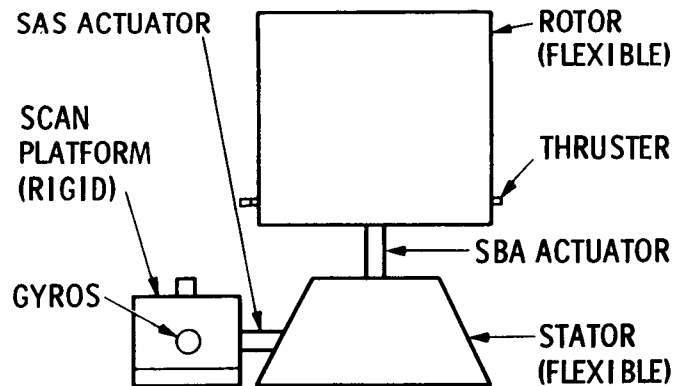
**INTRODUCTION**

MULTI-BODY DYNAMICS PROGRAMS REQUIRE CHARACTERIZATION OF EACH BODY

- RIGID BODY: GEOMETRY AND MASS PROPERTIES
- FLEXIBLE BDOY
  - EXACT TYPE OF INPUT DEPENDS ON PROGRAM
  - ALL INVOLVE MODAL CHARACTERISTICS IN SOME FORM
  - ALWAYS NEED FOR MODAL TRUNCATION
  - SYSTEMATIZE TRUNCATION PROCEDURE

**GALILEO SPACECRAFT**

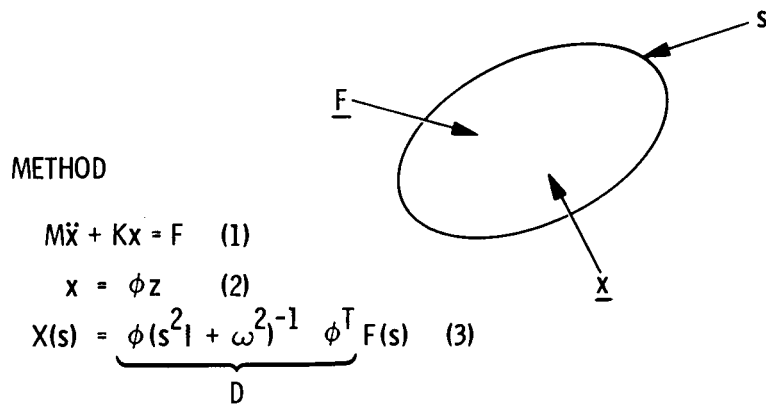
- ACTUATORS: SBA, SAS, THRUSTERS
- SENSORS: GYROS, CLOCK AND CONE ENCODERS, SUN SENSOR, STAR SCANNER
- CLOCK (SBA) CONTROL LOOP IS ACTIVE DURING ALL ATTITUDE CONTROL MANEUVERS
  - CLOCK CONTROLLER BANDWITCH  $\cong 0.5$  Hz
  - GYRO ROLLOFF FREQUENCY  $\cong 15$  Hz
- NEED "ADEQUATE" MODEL OF PLANT FOR DESIGN AND SIMULATION



# TRUNCATION CRITERIA

- CONTROL SYSTEM SPECIFICATIONS CAN SET TRUNCATION CRITERIA AT SYSTEM LEVEL ONLY
  - SYSTEM MODE WITH FREQUENCY ABOVE 15Hz CAN BE DROPPED
  - ELIMINATE MODES THAT DO NOT INTERACT "STRONGLY" WITH THE CONTROL SYSTEM

## SYSTEM LEVEL TRUNCATION



FOR RESPONSE AT  $i$  LOCATION DUE TO STEP INPUT AT  $j$  LOCATION,

$$X_i(s) = D_{ij} F_j(s) = \sum_{k=1}^m \left\{ \phi_{ik} \phi_{jk} A / \left[ s (s^2 + \omega_k^2) \right] \right\} \quad (4)$$

## SYSTEM LEVEL TRUNCATION (CONT'D)

CONTRIBUTION OF kth MODE TO RESPONSE:

$$X_i^k(s) = \phi_{ik} \phi_{jk} A / [s(s^2 + \omega_k^2)] \quad (5)$$

OR

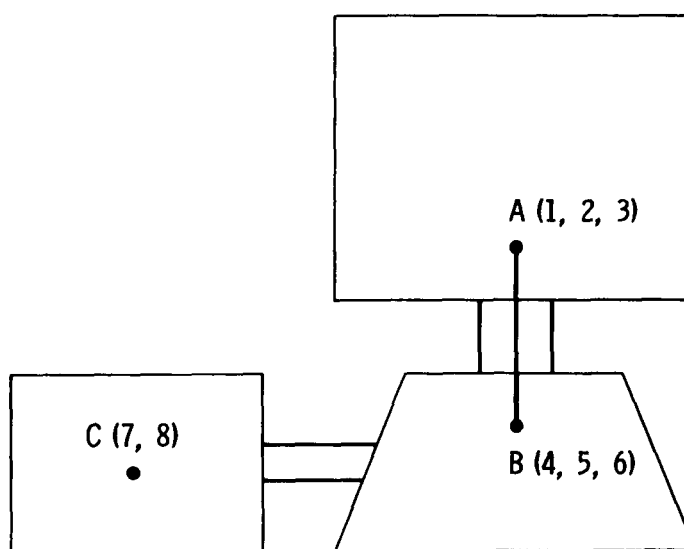
$$X_i^k(t) = (\phi_{ij} \phi_{jk} A / \omega_k^2) [1 - \cos(\omega_k t)] \quad (6)$$

SINUSOIDAL RESPONSE WITH PEAK-TO-PEAK AMPLITUDE TO

$$X_i^k = 2 \phi_{ik} \phi_{jk} A / \omega_k^2 \quad (7)$$

A MEASURE OF IMPORTANCE OF MODE K

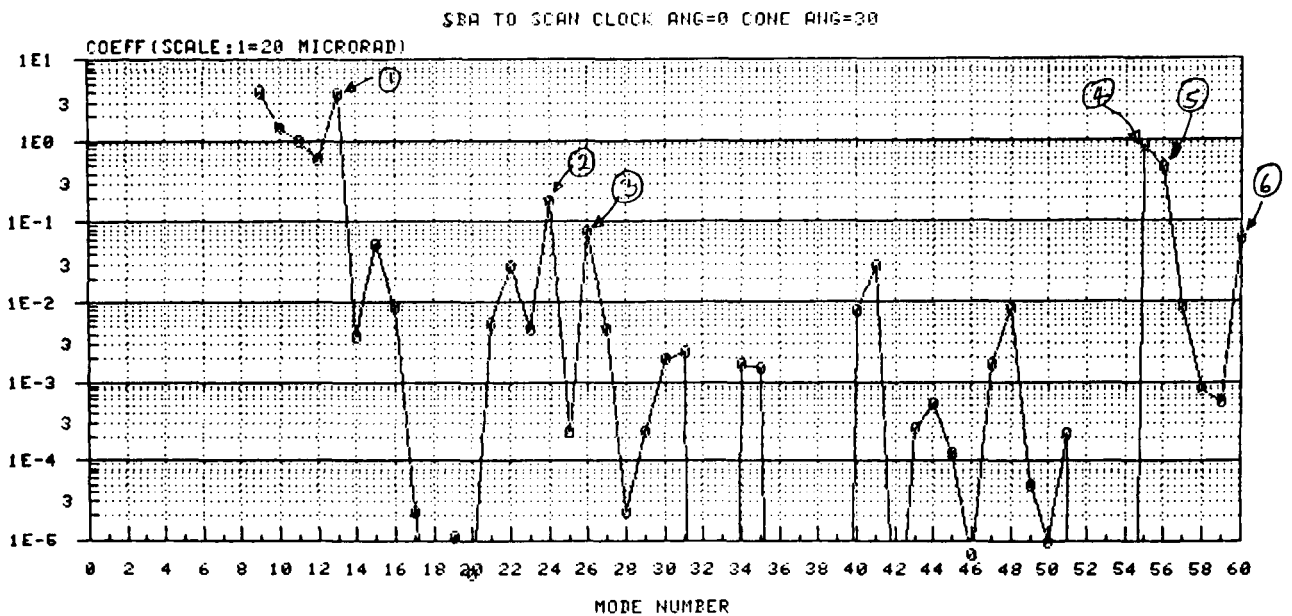
## APPLICATION TO GALILEO



# APPLICATION TO GALILEO (CONT'D)

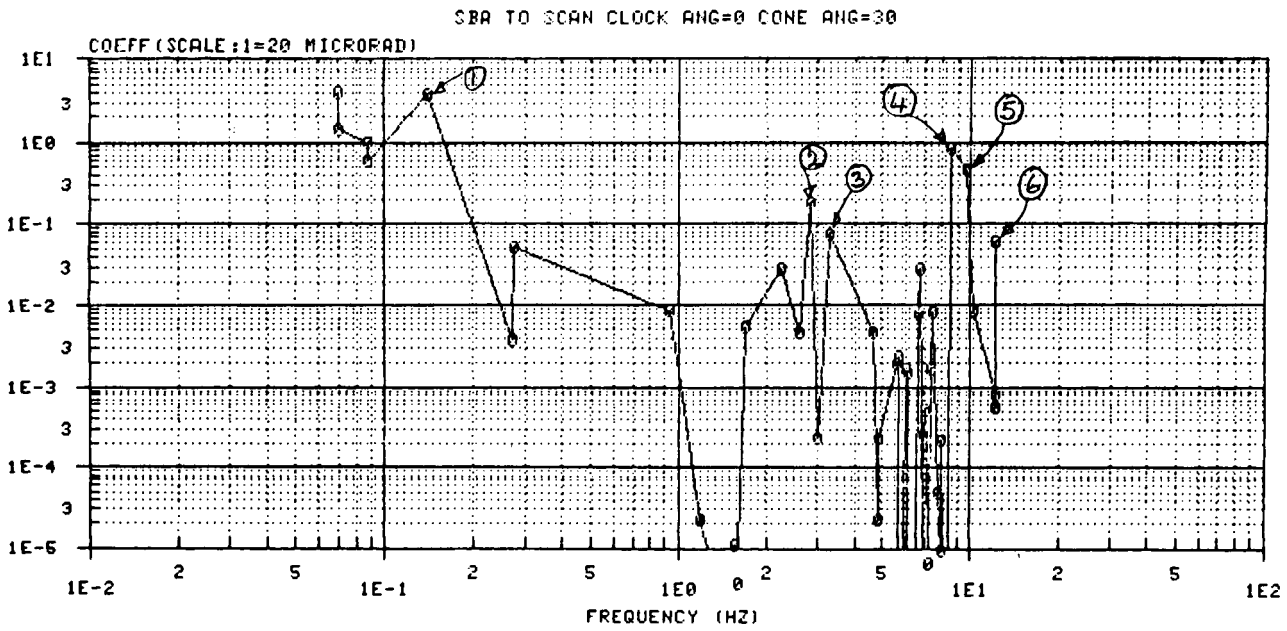
- AVAILABLE DATA
  - EIGENVALUES, EIGENVECTORS FOR UP TO 60 MODES
- PLOT MODAL INFLUENCE COEFFICIENTS
- DISCARD MODES WITH "LOW" COEFFICIENTS
- USE BODE PLOT TO CHECK RESULTS

## MODAL INFLUENCE COEFFICIENTS

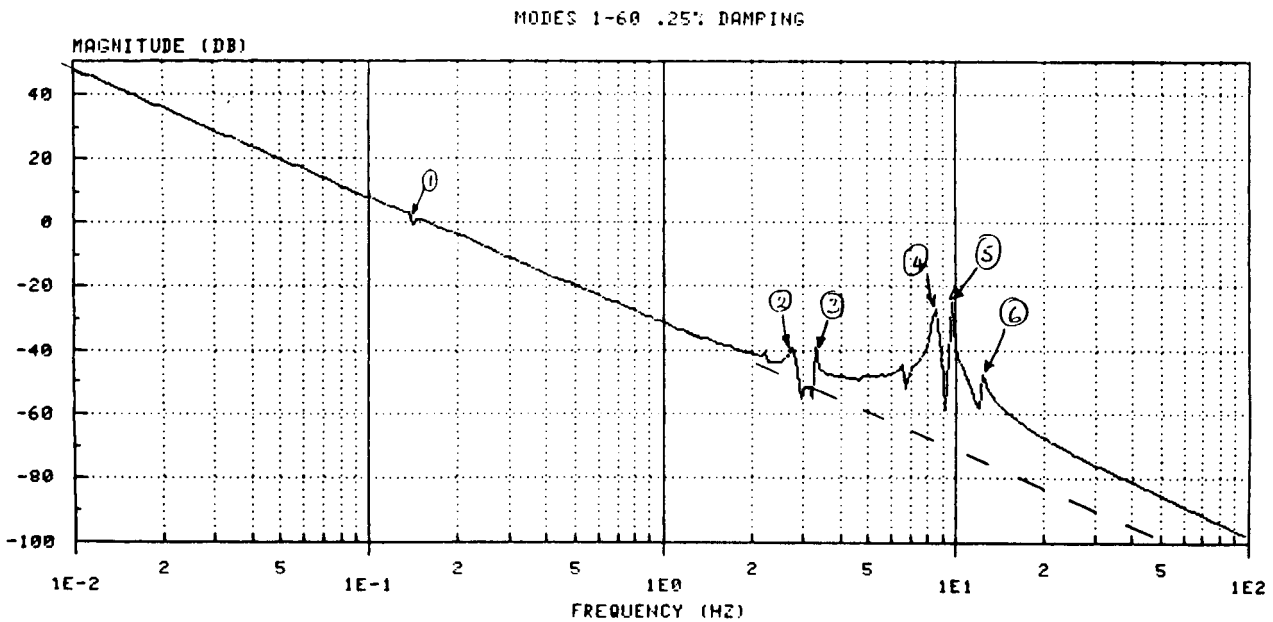


# MODAL INFLUENCE COEFFICIENTS

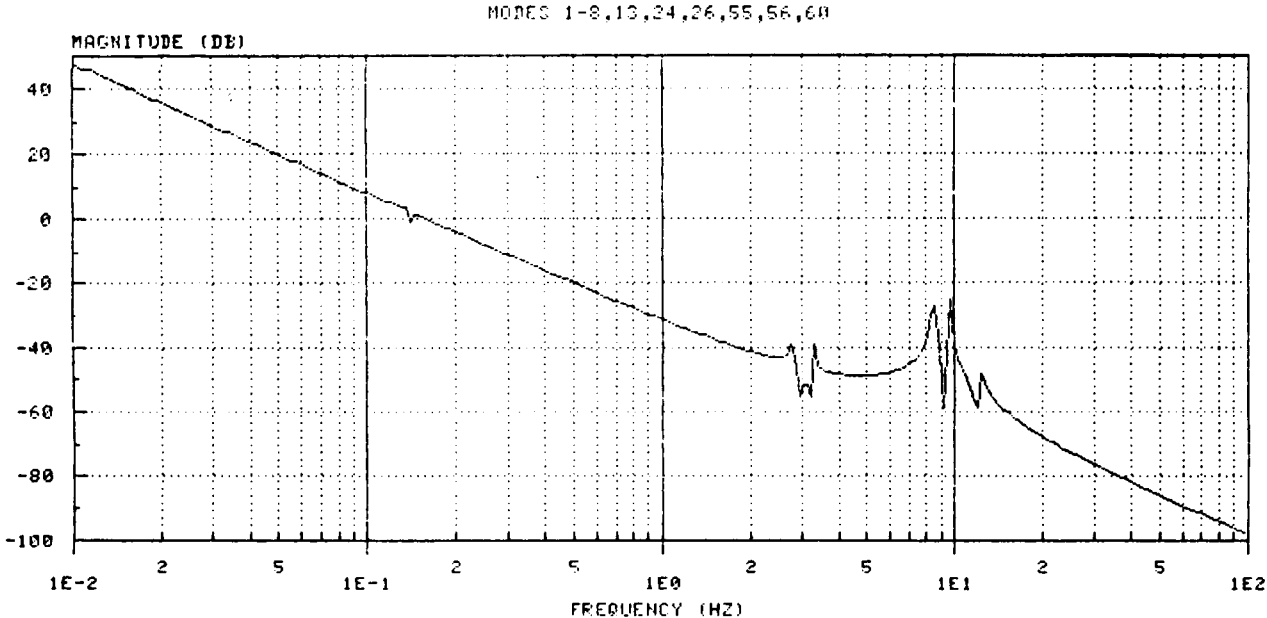
ORIGINAL PAGE IS  
OF POOR QUALITY



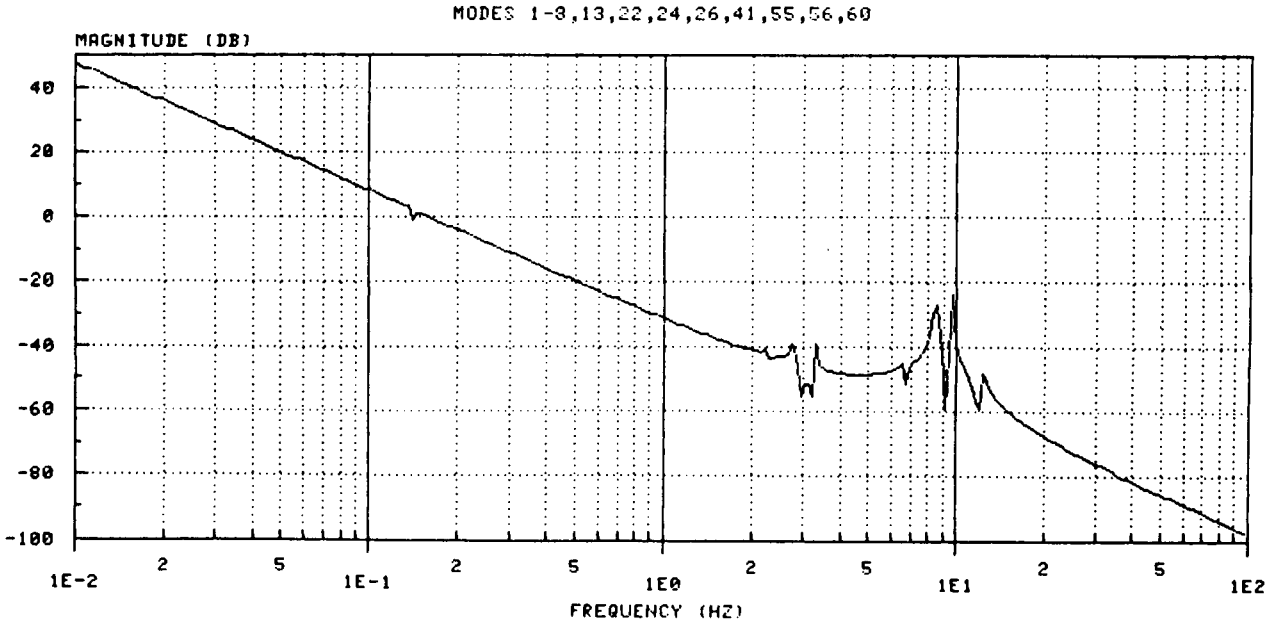
# BODE PLOT OF PLANT ALPHA = 0 BETA = 30



# BODE PLOT OF PLANT CLOCK = 0 CONE = 30



# BODE PLOT OF PLANT ALPHA = 0 BETA = 30



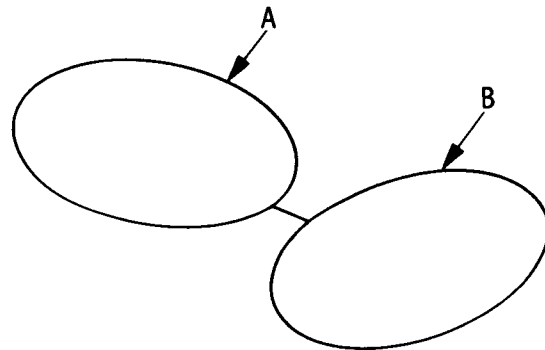
# TRUNCATION AT COMPONENT LEVEL

- AVAILABLE
  - COMPONENT "FREE-FREE" MODES
  - SYSTEM MODES TO BE RETAINED
- PROBLEM
  - DETERMINATION OF "IMPORTANT" COMPONENT FREE-FREE MODES FROM KNOWLEDGE OF SYSTEM MODES
- SOLUTION
  - RETAIN THOSE COMPONENT MODES THAT "CONTRIBUTE SUBSTANTIALLY" TO IMPORTANT SYSTEM MODES

## COMPONENT LEVEL TRUNCATION (CONT'D)

$$\left. \begin{aligned} M_A \ddot{x}_A + K_A x_A &= F_A \\ x_A &= \phi_A q_A \\ I \ddot{q}_A + \omega^2 q_A &= \phi_A^T F_A \end{aligned} \right\} \begin{array}{l} \text{BODY A} \\ \text{D.O.F.} = n_A \end{array} \quad (8)$$

$$\left. \begin{aligned} M_B \ddot{x}_B + K_B x_B &= F_B \\ x_B &= \phi_B q_B \\ I \ddot{q}_B + \omega^2 q_B &= \phi_B^T F_B \end{aligned} \right\} \begin{array}{l} \text{BODY B} \\ \text{D.O.F.} = n_B \end{array} \quad (9)$$



$$\left. \begin{aligned} M \ddot{x} + Kx &= F \\ x &= \phi q \\ I \ddot{q} + \omega^2 q &= \phi^T F \end{aligned} \right\} \begin{array}{l} \text{COMBINED} \\ \text{SYSTEM} \\ \text{D.O.F.} = n \leq (n_A + n_B) \end{array} \quad (10)$$

## COMPONENT LEVEL TRUNCATION (CONT'D)

- SYSTEM AUGMENTED  $\phi$  MATRIX =  $\bar{\phi}$

- SYSTEM MATRIX WITH SOME ROWS REPEATED

$$\begin{array}{c}
 \left[ \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_{n_A} \\ \hline x_{n_A} \\ \vdots \\ x_{n_A+n_B} \end{array} \right] = \left[ \begin{array}{cccc} \phi_{11} & \phi_{12} & \cdots & \phi_{1m} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n_A 1} & \phi_{n_A 2} & \cdots & \phi_{n_A m} \\ \hline \phi_{n_{A+1} 1} & \cdots & \cdots & \phi_{n_{A+1} m} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n'_1} & \phi_{n'_2} & \cdots & \phi_{n'_m} \end{array} \right] \left[ \begin{array}{c} q_1 \\ q_2 \\ \vdots \\ q_{n_A} \\ \hline q_{n_{A+1}} \\ \vdots \\ q_{n'} \end{array} \right] \quad (11)
 \end{array}$$

$\bar{\phi}_A$  (points to top part of matrix)  
 $\bar{\phi}_B$  (points to bottom part of matrix)

- PARTITION  $\bar{\phi}$  INTO  $\bar{\phi}_A$  AND  $\bar{\phi}_B$

## COMPONENT LEVEL TRUNCATION (CONT'D)

- DELETE COLUMNS OF  $\bar{\phi}$  THAT CORRESPOND TO SYSTEM MODES THAT WERE DROPPED

- REDUCED  $\phi$  MATRICES:  $\hat{\phi}_A$  AND  $\hat{\phi}_B$

- USE  $\hat{\phi}_A$  AND  $\hat{\phi}_B$  AS TRANSFORMATION MATRICES FOR BODIES A AND B RESPECTIVELY

$$\bullet \hat{\phi}_A^T M_A \hat{\phi}_A \ddot{q}_A + \hat{\phi}_A^T K_A \hat{\phi}_A q_A = \hat{\phi}_A^T F_A \quad (12)$$

$$\text{OR } \bullet \hat{M}_A \ddot{q}_A + \hat{K}_A q_A = \hat{\phi}_A^T F_A \quad (13)$$

$$\bullet \hat{M}_B \ddot{q}_B + \hat{K}_B q_B = \hat{\phi}_B^T F_B \quad (14)$$



## COMPONENT LEVEL TRUNCATION (CONT'D)

- $\hat{M}_A, \hat{K}_A, \hat{M}_B, \hat{K}_B$  NOT NECESSARILY DIAGONAL

- DIAGONALIZE VIA ANOTHER MODAL ANALYSIS

- $\hat{q}_A = \psi_A \bar{q}_A$  (15)

- $\hat{q}_B = \psi_B \hat{q}_B$  (16)

→  $I\ddot{\bar{q}}_A + \bar{\omega}_A^2 \bar{q}_A = \psi_A^T \hat{\phi}_A^T F$  (17)

$I\ddot{\bar{q}}_B + \bar{\omega}_B^2 \bar{q}_B = \psi_B^T \hat{\phi}_B^T F$  (18)

## COMPONENT LEVEL TRUNCATION (CONT'D)

- $\bar{\omega}_A, \bar{\omega}_B$  ARE DIAGONAL; THEY ARE ALSO SUB-MATRICES OF  $\omega_A, \omega_B$  RESPECTIVELY, AND CONTAIN FREQUENCIES OF COMPONENT MODES TO BE RETAINED
- SIMILARLY  $\Phi_A = \hat{\phi}_A^T \Psi_A$  AND  $\Phi_B = \hat{\phi}_B^T \Psi_B$  ARE SUBMATRICES OF  $\phi_A$  AND  $\phi_B$ , AND CONTAIN THE EIGENVECTORS OF COMPONENT MODES TO BE RETAINED

## **SUMMARY AND CONCLUSION**

- DETERMINE SYSTEM MODES TO BE RETAINED USING
  - AVAILABLE CRITERIA
  - MODAL INFLUENCE COEFFICIENTS
  - BODE
- DESCEND TO COMPONENT LEVEL VIA A TWO-PHASE DIAGONALIZATION PROCESS STARTING WITH SUBMATRICES OF TRUNCATED AUGMENTED SYSTEM MODAL MATRIX

## **FUTURE WORK**

- STREAMLINE SIMULATION CODES – ESPECIALLY DYNAMICS FORMULATION METHOD
- DEVELOP VERY EFFICIENT AND EASILY IMPLEMENTABLE MODEL REDUCTION STRATEGY