THE ELECTRON BEAM INSTABILITY REVISITED: GROWTH ABOVE AND BELOW f_p

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ABSTRACT

The growth of electrostatic waves near the plasma frequency f_p due to an unstable electron beam is investigated by solving the unmagnetized electrostatic dispersion equation numerically. These numerical solutions are compared with analytic theories for reactive (or fluid-type) and kinetic versions of the beam instability, and for the O'Neil/Malmberg connection of the beam and Langmuir modes. Conditions for growth significantly above or below f_p are given. Three general results given are found: (1) The unstable waves do not grow on a mode with Langmuir dispersion except in the limit of a very dilute beam with growth on O'Neil/Malmberg's connected mode. (2) The properties of the unstable mode depend strongly on beam parameters such as beam density, speed and temperature. (3) The frequency of maximum growth frequently lies significantly above or below f_p, and differs significantly from that predicted by the Langmuir dispersion relation. These results imply important consequences for theories of strong turbulence and non-linear wave-wave processes, and observational identification of f_p from observed wave frequencies.

1. INTRODUCTION

Electrostatic waves excited near the electron plasma frequency f_p by electron beams are of widespread interest in plasma physics; specific applications include planetary foreshocks (Ref. 1), strong Langmuir turbulence (Ref. 2) and pulsar magnetospheres (papers this volume). Theoretical concepts for these electron "beam instabilities" have changed relatively little since their initial development (Refs. 3-5), in part because most research effort has been concentrated on the non-linear and strong turbulence evolution of the waves. This paper is, however, specifically concerned with the linear properties of the waves and the instabilities themselves.

The two primary characterizations of the beam instability are due to Bohm and Gross (Ref. 3) and Briggs (Ref. 4), and to O'Neil and Malmberg (Ref. 5). This first characterization corresponds to the physical growth mechanism of the waves: viz. the reactive (or non-resonant or fluid-like) and kinetic (or resonant or resistive) instabilities in which growth occurs due to particle bunching or by inverse Landau damping, respectively. These different growth mechanisms imply different wave spectra for the two instabilities (e.g, Ref. 6). The second characterization (Ref. 5) corresponds to the dispersive character of the growing mode: the waves might be "beam modes" with $\omega_r \sim k\, v_b$, or might lie on a mode with approximately Langmuir dispersion formed after connection of the ordinary Langmuir mode and the beam mode. Although analytic theory (Ref. 7) implies that the reactive instability occurs on a beam mode, the detailed connection between these two characterizations of the beam instability is not clear. One aim of this paper is to clarify the connection between these two characterizations of the beam instability. This is done by solving the exact electrostatic unmagnetized dispersion equation numerically, and comparing the numerical solutions with analytic solutions. It is shown that the reactive instability does tend to occur on the beam mode, but that the instability becomes kinetic on a modified extension of the beam mode (the 'modified- beam mode') long before the O'Neil and Malmberg connection between the Langmuir mode and the beam mode takes place. The unstable mode is shown to be intrinsically beam-associated (i.e., dependent on beam parameters, at least) unless the beam is extremely dilute, i.e., relative beam densities $n_b/n_o \lesssim 10^{-4}$; that is, growth does not usually occur on a normal mode of the plasma in the absence of the beam, such as the Langmuir mode. Consequently, there is no a priori reason for growth to occur above f_p at Langmuir-like frequencies. Conditions for reactive or kinetic growth well above and well below f_p, as observed in the Earth's foreshock (Refs. 8, 9) are therefore investigated here.

Previous numerical investigations of this dispersion equation (e.g., Ref. 10) have paid relatively little attention to the dispersive characteristics of the growing waves. Yet the detailed dispersion relation of the waves is vital in theoretical descriptions of non-linear wave processes and strong turbulence phenomena (e.g., Ref. 2): differences between the actual dispersion relation and the assumed dispersion relation imply differences in the governing equations for the wave fields and possible kinematic difficulties for the wave processes, as discussed in Section 4 below. This paper proceeds as follows: Section 2 contains a brief description of previous analytic theory, while Section 3 contains the results of the paper. A more complete version of this work is described in Ref. 6.

2. RELEVANT ANALYTIC THEORY

Given the longitudinal part of the unmagnetized, spatially homogeneous dielectric tensor $K^L = 1+K_i+K_e+K_b$, where the subscripts, i, e, and b refer to the contributions of ions,

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background electrons and beam electrons, respectively, the dispersion equation is $K^L = 0$. Assuming that all species have Maxwellian distributions, K_α is of the form

$$K_\alpha = -\frac{\omega_{p\alpha}^2}{k^2 V_\alpha^2}\Big(\Phi\big(\frac{\omega - \underline{k}\cdot\underline{v}_\alpha}{\sqrt{2}\,k\,V_\alpha}\big) - 1\Big) \qquad (1)$$

where Φ is related to the Friede-Conte function, and $\omega_{p\alpha}$, V_α and $\underline{v}_\alpha$ are the angular plasma frequency, thermal speed and drift velocity of species α, respectively. Writing $\omega = \omega_r + i\gamma$, a wave is said to be resonant with species α if $\xi_\alpha = (\omega_r - \underline{k}\cdot\underline{v}_\alpha)/\sqrt{2}\,k\,V_\alpha \sim 1$. The beam instability is said to be resonant if $\xi_b < 2$, corresponding to the distribution function of species α at the phase velocity of the wave being 2% of its value at $v_\phi = v_\alpha$.

The reactive instability is derived by ignoring $Im\ K_\alpha$ (i.e., $Im\ K_\alpha = 0$), whence the dispersion equation is a real equation for a complex variable ω, thereby implying the presence of complex conjugate solutions (Ref. 7). Ignoring $Im\ K_\alpha$ corresponds to ignoring Landau (and inverse Landau) damping; growth is instead due to particle bunching. Assuming $|\xi_b| \gg 1$ (non-resonant) one finds complex conjugate roots with maximum growth with $\omega_p = \underline{k}\cdot\underline{v}_b$.

$$\omega_r = \underline{k}\cdot\underline{v}_b\Big(1 - \frac{1}{2}\big(\frac{n_b}{2n_o}\big)^{\frac{1}{3}}\Big)\ ,\quad \gamma_r = \frac{\sqrt{3}}{2}\big(\frac{n_b}{2n_o}\big)^{\frac{1}{3}}\,\omega_p \quad . \qquad (2)$$

The kinetic instability is usually derived by ignoring $Re(K_b, K_i)$, assuming $|\xi_b| \ll |$ (resonant), and taking

$$\gamma = \frac{Im\ K_b}{\frac{\partial}{\partial\omega_r}(Re\ K_e)} \qquad (3)$$

with ω_r determined by the equation $Re\ K_e = 0$. Growth is then due intrinsically to inverse Landau damping, with maximum growth rate

$$\gamma_k = \sqrt{\frac{\pi}{2e}}\ \frac{n_b}{n_o}\big(\frac{v_b}{V_b}\big)^2\omega_p \qquad (4)$$

where $e = \exp(1)$, on the Langmuir mode. A commonly used criterion (e.g., Ref. 7) for determining whether the instability is reactive or kinetic is

$$P = \big(\frac{n_b}{n_o}\big)^{\frac{1}{3}}\ \frac{v_b}{V_b} > 1 \qquad (5)$$

or $P < 1$, respectively. A more accurate empirical criterion is given below.

O'Neil and Malmberg's (Ref. 5) criterion for whether the instability is on the beam-mode or on the connected Langmuir-beam mode is

$$s = 2^{\frac{1}{3}}/P < 1.47 \qquad (6)$$

or $s > 1.47$, respectively. Gary's results (Ref. 10) and the results below verify this criterion empirically.

3. SUMMARY OF RESULTS

3.1 Reactive/kinetic, beam/Langmuir-beam, and dispersion relations

Figure 1 shows the evolution of the dispersion curves for a beam with fixed relative density $n_b/n_o = 10^{-3}$ and drift speed $v_b/V_e = 10$ as the beam temperature T_b/T_e is varied ($T_e/T_i = 3$ is fixed). The two complex conjugate roots corresponding to the reactive instability are clearly visible in Figure 1(a), together with their beam dispersion $\omega_r \sim kv_b$ and non-resonant character. It is clear that the reactive/kinetic transition takes place long before the O'Neil and Malmberg (Ref. 5) connection of the Langmuir and beam mode takes place in Figure 1(f); furthermore, the instability does not take place on the pure Langmuir mode, or on a mode with Langmuir-like dispersion until greater beam temperatures than those in Figure 1(f) are reached. The following changes occur as T_b increases in Figures 1(b) through (d): (1) the complex conjugacy of the beam roots disappears, (2) the dispersion curve of the growing mode consists of a portion with beam-type dispersion $\omega_r \sim k\ V_b$ for wavenumbers smaller than some k_* and slowly-varying dispersion $\omega_r \sim A$ for $k > k_*$, where A is of order ω_p. (3) The wavenumber k_m and real frequency ω_m corresponding to maximum growth increase with T_b, corresponding to the peak in the growth rate curve (as a function of wavenumber) moving continuously towards and along the $\omega_r \sim A$ portion of the dispersion curve. The instability is marginally resonant in Figure 1(b) and strongly resonant by Figure 1(c), corresponding to a kinetic instability. By Figure 1(d) the portion of the growing mode with beam dispersion (i.e., small wavenumbers) is heavily damped, as is the Langmuir mode for higher wavenumbers. In contrast, the Langmuir mode at small wavenumbers, and the modified beam mode at larger wavenumbers are lightly damped or growing. As T_b increases further, the two lightly damped and two heavily damped portions of these modes connect, leading to O'Neil and Malmberg's 'Langmuir-beam' mode (Gary's terminology) upon which the instability is found, and a heavily damped mode of no further interest. This connection is shown in Figures 1(e) and (f). For completeness note that a progression of figures analogous to Figure 1 could also be constructed by either decreasing the beam density or decreasing the beam speed, keeping other parameters constant.

The strong modification of the beam root's dispersive characteristics as the instability becomes resonant (and so kinetic rather than reactive) necessitates a clarification of terminology: (1) Portions of dispersion relations with $\omega_r \sim kv_o$ are called 'beam' type. (2) Due to its origin with the beam the strongly modified mode (Figures 1(b)–(e)), on which growth occurs following modification of the complex conjugate beam mode, is called the "modified-beam" mode. (3) The connected O'Neil and Malmberg mode is called the "Langmuir-beam" mode following Gary (Ref. 10). With this terminology the instability is resonant (kinetic) on the modified-beam mode and Langmuir-beam mode, and non-resonant (reactive) on the beam mode. These identifications of the reactive and kinetic instabilities are consistent with the variations in the maximum growth rate γ_m as a function of relative beam density n_b/n_o in equations (2) and (4). Finally, values of P corresponding to the reactive instability are $P \gtrsim 6$, while the kinetic instability is strongly resonant when $P \lesssim 2$. Reactive growth occurs on a beam mode which strongly obeys beam-type dispersion. Qualitatively, the modified-beam mode's dispersion curve consists of a beam-type portion at low wavenumbers and a portion for which ω_r varies slowly; it is inappropriate to approximate the modified-beam mode with the Langmuir dispersion relation, particularly as the maximum growth rate usually occurs below f_p.

The dispersion relation, near where the growth rate is maximum, of the Langmuir-beam mode is also often considerably different from the Langmuir dispersion relation (Figures 1(f) and 2). Figure 2 shows the dispersion relation near maximum growth for beams with various beam densities n_b/n_o but constant beam speed $v_b/V_e = 10$ and temperature $T_b/T_e = 1$; the Langmuir-beam connection takes place near $n_b/n_o = 5 \times 10^{-4}$. Stringent conditions on n_b/n_o are apparent for the dispersion relation to be closely Langmuir-like: n_b/n_o should be at least a factor of 10 less than that beam density satisfying the O'Neil and Malmberg criterion (6), i.e., $n_b/n_o < 5 \times 10^{-5}$. This rule should be of great practical importance: for the above warm, relatively small velocity beam one requires $n_b/n_o < 5 \times 10^{-5}$, and for faster, colder beams one requires even more dilute beams; however, in most laboratory and space science applications (e.g., the Earth's foreshock, but not Type III solar radio bursts) one expects $n_b/n_o \gtrsim 10^{-3}$.

This sub-section may be summarized as follows: (1) The unstable waves do not have the Langmuir dispersion relation, except in the limit of a very dilute beam with growth on the Langmuir-beam mode: $n_b/n_o < 5 \times 10^{-5}$ for $v_b/V_e \gtrsim 10$ and $T_b/T_e \lesssim 1$. (2) In most applications growth should occur on the beam mode (reactive) or modified-beam mode (kinetic). (3) The properties of the unstable waves depend strongly on the beam parameters, such as beam density, speed, and temperature. (4) The frequency of maximum growth frequently lies either significantly below or above f_p, and is significantly different from the frequency predicted by the Langmuir dispersion relation.

3.2 Growth significantly above and below f_p

Figures 1 and 2 show that the frequency of maximum growth for the beam instability often lies below f_p, despite the conventional belief (due to the idea that growth occurs on the unmodified Langmuir mode) that growth occurs above f_p. Effective growth well above or below f_p is due to the strong dependence on beam parameters of the dispersion relations of the beam, modified-beam and Langmuir-beam (unless the beam is very dilute) modes, and the non-Langmuir nature of these modes' dispersion relations. Figure 3 illustrates the roles of beam speed, temperature and density in determining the real frequency at maximum growth ω_m (Ref. 9). Growth significantly below ω_p generally (except as in Figure 2) requires growth to occur on the beam (reactive) or modified-beam (kinetic) modes. For high beam speeds ($v_b/V_e > 20$) ω_m becomes approximately constant, equal to ω_n say, and dependent only on n_b/n_o. For fixed beam density and temperature, decreasing the beam speed causes ω_m to increase above ω_n once $v_b/V_e \lesssim 10$, and subsequently to decrease towards zero frequency once $v_b/V_e \lesssim 2$, provided the beam temperature is sufficiently small. This increase in ω_m above ω_n increases with decreasing n_b/n_o until $n_b/n_o \sim 10^{-5}$. Increasing the beam density corresponds to moving the entire curve in Figure 3, especially ω_n, to lower frequencies: note that for $n_b/n_o \gtrsim 10^{-3}$, $\omega_n \lesssim 0.99\omega_p$. For warm beams (e.g., the dashed curves) damping overcomes growth before ω_m can increase substantially above ω_n, and certainly before ω_m decreases towards zero frequency. This Figure provides a natural explanation for waves observed at frequencies significantly above and below ω_p in the Earth's foreshock (see Refs. 8, 9).

There are therefore two primary ways to obtain growth significantly below ω_p: dense beams or cold, slow, relatively dilute beams. Growth significantly above ω_p requires relatively slow ($2 \lesssim v_b/V_e \lesssim 10$) and dilute (since otherwise ω_n is well below ω_p) beams which may be either cold or warm. Detailed conditions for reactive or kinetic growth well above or below ω_p are given in Ref. 9.

4. DISCUSSION

Due to length restrictions the discussion is restricted to the following two points: (A) The results of this paper indicate that, due to the assumption of the Langmuir dispersion relation in derivations of the Zakharov equations (e.g., Ref. 2), and the neglect of particle bunching effects therein, the Zakharov equations are valid only under stringent circumstances: $n_b/n_o \lesssim 10^{-4}$ for $v_b/V_e > 10$ and $T_b/T_e \lesssim 10$. (B) The generally non-Langmuir dispersion relations, and tendancy for maximum growth to occur below ω_p, of waves growing by the beam instability imply kinematic difficulties for non-linear wave processes. For example, frequency and wavevector conservation for the Langmuir wave decay $L \rightarrow L' + S$ (L denotes the electron plasma wave, while S is an ion acoustic wave) is impossible when $n_b/n_o \gtrsim 10^{-3}$ since ω_m is below ω_p, yet ω'_L (the backward propagating wave) is greater than ω_p (Ref. 6). A more detailed discussion is given in Ref. 6.

6. ACKNOWLEDGEMENTS

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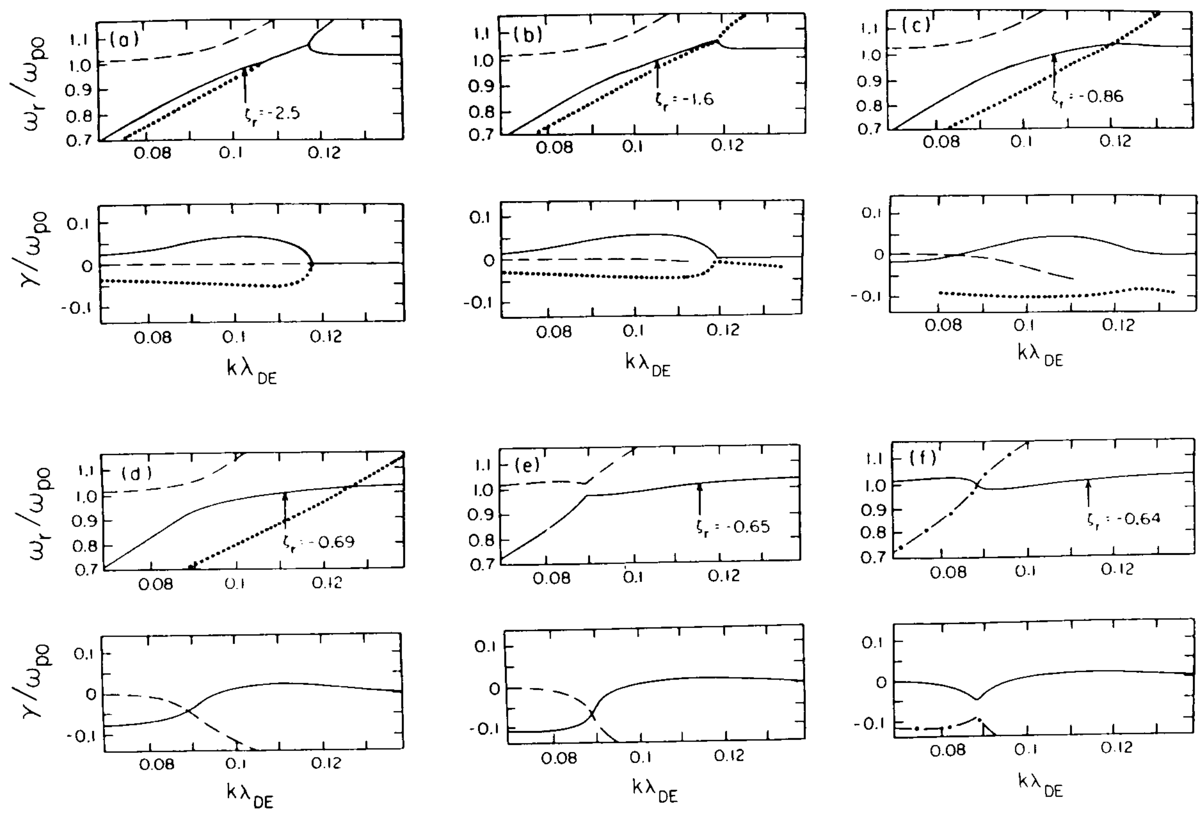

Figure 1. Plots of the dispersion relation and growth rate for the growing mode (full line), damped beam mode (dashed line), Langmuir mode (dotted line), and Langmuir-beam mode (dash-dot line) for various beam temperatures. For parts (a) to (f) T_b/T_e equals 0.017, 0.067, 0.33, 1.0, 1.5, and 1.67, respectively. The fixed beam parameters are $n_b/n_o = 10^{-3}$, $v_b/V_e = 10$, and $T_e/T_i = 3$.

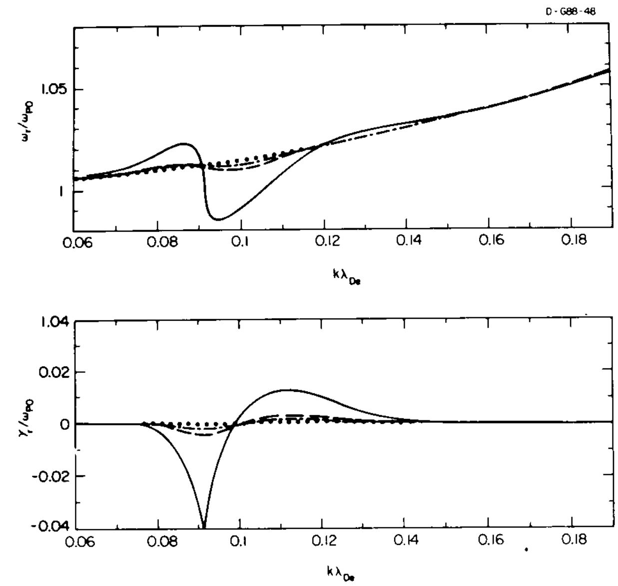

Figure 2. The dispersion relation of the Langmuir-beam mode as a function of n_b/n_o: 5×10^{-4} (full line), 10^{-4} (dashed), 5×10^{-5} (dash-dot), and 10^{-5} (dotted). Here $v_b/V_e = 10$, $T_b/T_e = 1$, and $T_e/T_i = 3$. Connection of the Langmuir and beam modes occurs near $n_b/n_o = 5 \times 10^{-4}$. The mode has the dispersive characteristics of the Langmuir mode for $n_b/n_o < 10^{-5}$.

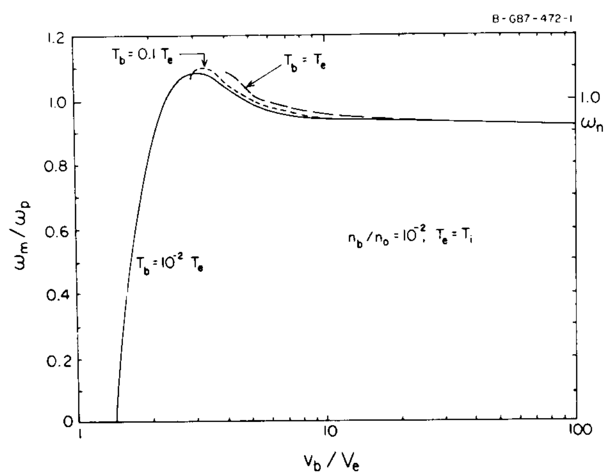

Figure 3. The frequency of maximum growth, ω_m, as a function of v_b/V_e for $n_b/n_o = 10^{-2}$ and various beam temperatures. For high beam speeds ω_m is approximately constant, equal to ω_n, and primarily dependent only on the beam density. If the beam is sufficiently cold, ω_m first increases above and then decreases below ω_n as v_b/V_e decreases.