EFFECTS OF UNEQUAL PARTICLE NUMBER DENSITIES ON ALFVEN WAVES

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ABSTRACT

Space and astrophysical Alfvén waves usually exist in a plasma containing thermal and nonthermal particles, and generally the thermal electrons and ions have unequal number densities. Nevertheless, the linear properties of the waves are invariably assumed to be determined by a charge-neutral thermal plasma in the absence of the nonthermal particles, while the nonthermal particles cause growth or additional damping superposed onto the background model. Here, both analytic plasma theory and numerical solutions of the dispersion equation are used to show that this prescription is seriously flawed even for stable plasmas; even when the nonthermal particles do not contribute significantly to the dispersion equation, unequal thermal electron and ion number densities (due to the presence of the nonthermal particles) may cause fundamental low wavenumber modifications to the Alfvén modes, including the creation of a new resonance and severely modified dispersion. These results are found for both cold and warm plasmas. Previous work on Alfvén waves should be re-evaluated in view of these results.

Keywords: Alfvén Waves, Dispersion Relations, Kinetic Theory

1. INTRODUCTION

Alfvén and fast magnetoacoutic waves are the basic low frequency ($\omega \leq \Omega_i$) electromagnetic waves in a magnetized plasma. These waves are of great importance in many fields of space physics and plasma astrophysics, including particle acceleration at solar system and astrophysical shocks (Ref. 1), the production and maintenance of cosmic rays (Ref. 2), turbulence theory and nonlinear waves such as solitons. Recent research has generally been concerned with the nonlinear properties of, and detailed applications involving, Alfvén waves rather than on the linear properties of the waves such as mode structure and dispersion relations which have been assumed well established. In contrast the linear properties of Alfvén waves are explicitly considered in this paper, and important new results are found which imply a need for re-assessment of much work on detailed applications and nonlinear physics involving Alfvén waves.

Alfvén waves in space and astrophysical applications typically exist in plasmas containing both thermal and nonthermal particles, and generally the thermal electrons and ions will have unequal number densities due to a preponderance of one energetic species. The linear properties of the waves are, however, generally described using the following "conventional" prescription: dispersion properties (such as the mode structure and dispersion relations) are determined in a charge-neutral thermal plasma in the absence of the nonthermal particles, while the nonthermal particles merely cause growth or additional damping superposed onto the background plasma modes. This prescription is shown to be seriously flawed at low wavenumbers in this paper. Modifications to the linear properties of the waves, due to the presence of nonthermal particles are found to arise in two ways: (1) Unequal thermal electron and ion number densities cause fundamental low wavenumber modifications including the creation of a new resonance and severely modified dispersion (Ref. 3). (2) Direct contributions to the wave dispersion equation by the energetic particles may modify the Alfvén waves at low wavenumbers (as implied by the resonance condition $\omega \pm \Omega_\alpha - k_{\parallel} V_{\parallel}$ where α denotes the particle species).

In this paper I consider the effects of unequal thermal number densities in some detail, using firstly analytic plasma theory and then numerical solutions of the full wave dispersion equation. The direct effects of the energetic particles on the wave dispersion will be considered elsewhere (e.g., Ref. 3); note that others (Refs. 4, 5) have previously considered such effects.

2. MODIFICATIONS DUE TO UNEQUAL THERMAL NUMBER DENSITIES

The dispersion equation for parallel propagating electromagnetic waves in a magnetized plasma with both nonrelativistic thermal (Maxwellian, T) and

$$K^2 - W^2 - \sum_T \frac{\omega_{p\alpha}^2}{\Omega_i^2} \xi_\alpha Z(\xi_\alpha^\sigma) = \sum_{NT} \frac{\omega_{p\alpha}^2}{\Omega_i^2} F_\alpha(W,K), \quad (1)$$

with $\xi_\alpha = W/\sqrt{2}K\overline{V_\alpha}$, $\xi_\alpha^\sigma = (W + \sigma\epsilon_\alpha\Omega_\alpha/\Omega_i)/\sqrt{2}K\overline{V_\alpha}$, $\overline{V_\alpha}$ is the thermal speed divided by c, and $Z(\xi_\alpha^\sigma)$ is the Friede-Conte function. Here $W = \omega/\Omega_i$, $K = kc/\Omega_i$, σ is the handedness (± 1 or RH/LH) of the mode, and ϵ_α, Ω_α, $\omega_{p\alpha}$, and $F_\alpha(W,K)$ are the charge (unsigned) gyrofrequency, plasma frequency and plasma dispersion function,

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respectively of species α. Wave frequencies well above relevant collision frequencies have been assumed. For nonrelativistic particles with isotropic distribution function $f_\alpha(p)$, $F_\alpha(W,K)$ is proportional to

$$\int_0^\infty dp \int_{-1}^1 d\mu \frac{p^2(1-\mu^2)}{(W+\sigma\epsilon_\alpha\Omega_\alpha/\Omega_i - K\mu p)} \frac{\partial f_\alpha}{\partial p}, \quad (2)$$

where p is the particle momentum normalized by c and μ is the cosine of the pitch angle. Accordingly, each F_α depends on the distribution f_α of nonthermal particles and may differ considerably from the thermal plasma contribution to the first sum in (1) for the same number density of particles.

In cold or warm thermal plasmas where $\overline{V}_\alpha \lesssim 10^{-2}$, the large argument expansion of the Friede-Conte function is rigorously justified for Alfvén waves with $W \ll 1$ when the Alfvén speed V_A is much less than c, leading to the so-called "cold plasma" description of the terms in the thermal (T) sum in (1):

$$K^2 - W^2 + \sum_T \frac{\omega_{p\alpha}^2}{\Omega_i^2} \frac{W}{W+\sigma\epsilon_\alpha\Omega_\alpha/\Omega_i} = \sum_{NT} \frac{\omega_{p\alpha}^2}{\Omega_i^2} F_\alpha(W,K). \quad (3)$$

An analytic description of the low wavenumber modifications due to unequal thermal electron and ion number densities, and the circumstances under which these effects may be relevant, proceeds as follows. Consider a charge-neutral electron-proton plasma with cold or warm ($\overline{V}_\alpha \lesssim 10^{-2}$) thermal components and a single (for simplicity) nonthermal component, with total ion and electron number densities n_0. After minor algebra and the subsequent assumption $W \ll 1$ ($\omega \ll \Omega_i$) in the denominator of the thermal sum (T), (3) becomes

$$K^2 - W^2 - \frac{m_e}{m_i\Omega_i^2}\left[W^2(\omega_{pe}^2+\omega_{pi}^2) + \sigma W\left(\omega_{pe}^2 - \frac{m_i}{m_e}\omega_{pi}^2\right)\right] - F_{NT} = 0. \quad (4)$$

Conventional derivations assume that the thermal electron and ion number densities are equal, so that the W term in the square brackets vanishes, and that the nonthermal term may be ignored. Using the usual definition of the Alfvén speed $V_A(c^2/V_A^2 = \sum_T \omega_{p\alpha}^2/\Omega_\alpha^2)$ and writing $\omega_{po,i}^2 = n_0 e^2/m_i\epsilon_0$, one finds that $c^2/V_A^2 = 1/U_A^2 = \omega_{po,i}^2(1+m_e/m_i)/\Omega_i^2$ and the conventional derivation yields

$$K^2 - W^2 - W^2/U_A^2 = 0, \quad (5)$$

which for $U_A \ll 1$ (i.e., $V_A \ll c$) gives $W = KU_A$, or $\omega = kV_A$ as required.

Here, the electron and proton thermal number densities are permitted to differ from each other, as expected in a typical space or astrophysical plasma, so that the W term in the square brackets in (4) is retained: this term is responsible for the low wavenumber modifications to the Alfvén modes now described.

The effects of unequal thermal particle number densities on the wave dispersion are clearly separated from the effects of direct contributions (through F_{NT}) to the dispersion equation by the nonthermal particles when the direct contribution F_{NT} of the nonthermal particles is negligible, i.e., the form of the distribution f_{NT} is such that F is much less than the "cold plasma" contribution of the corresponding number density of thermal particles removed from the thermal sum in (4). Then, assuming that $\delta n = n_e - n_i$ (the difference in thermal number density) is small compared with n_0 and that $U_A \ll 1$, the dispersion equation (4) becomes

$$W^2 + \sigma\frac{\delta n}{n_0}W - K^2U_A^2 = 0. \quad (6)$$

Below a distribution function f_α suitable for describing nonthermal particles accelerated at shocks is shown to satisfy this condition that F_{NT} be ignorable.

For real W (i.e., neglecting damping) the solutions of (6) are

$$W = -\sigma\frac{\delta n}{2n_0} \pm \sqrt{K^2U_A^2 + \frac{1}{4}\left(\frac{\delta n}{2n_0}\right)^2}. \quad (7)$$

The analytic predictions therefore are: (1) At high wavenumbers K such that $KU_A > |\delta n|/n_0$, i.e., the third term in (6) dominates the second term, the ordinary Alfvén dispersion $W \sim KU_A$ is recovered. (2) Fundamental low wavenumber modifications occur when $KU_A < |\delta n|/n_0$. In particular, (3) the mode with $\sigma\delta n < 0$ has a new resonance at $W \sim |\delta n|/n_0$. (4) The other mode approaches a dispersion relation $W \propto K^2$. In summary, the effects of unequal thermal particle number densities may cause fundamental low wavenumber modifications to the Alfvén modes; the direct contributions F_{NT} of nonthermal particles may either aid or compete with these modifications due to unequal thermal particle number densities.

In space and astrophysical plasmas shocks are observed (Ref. 1) or inferred (Ref. 2) to accelerate nonthermal particles with distribution functions $f(p) \propto p^{-a}$ with $a \sim 4$. It is therefore natural to consider nonthermal particles with distribution function

$$f(p) \propto \frac{1}{(p^2+q^2)^2} \quad (8)$$

for $p \leq p_{\max} \ll 1$, and zero elsewhere, in this connection, since this distribution is analytic with $f(p) \propto p^{-4}$ at high momenta $p \gg q$. The plasma dispersion function F_{NT} for this distribution has been calculated analytically using the Plemelj formula in (2) [justified when the imaginary part of W is much less than the real part, as found in the following examples]. Numerical solutions to the dispersion equation (3) for a three-component electron-proton plasma comprising cold electrons and ions and nonthermal particles (with distribution function (8) for $p_{\max} = 0.1$ and $q = 10^{-2}$) are now presented.

Figure 1 shows the W-K dispersion curves for the above charge-neutral plasma with $\omega_{po,i}/\Omega_i = 300 \sim c/V_A$ containing various relative number densities of nonthermal electrons, i.e., deficits of thermal electrons $\delta n < 0$. The pure cold plasma results are also shown for comparison. The direct contribution $F_{NT}(W,K)$ of the nonthermal particles to (3) is found to be numerically ignorable, thereby fulfilling the requirements of the analytic theory above. Both the low and high wavenumber regimes are shown for the $\sigma = +1$ (full lines) and $\sigma = -1$ (dashed) modes. At high wavenumbers the curves closely follow the pure cold plasma solutions as the RH fast magnetoacoustic ($\sigma = +1$) and LH Alfvén ($\sigma = -1$) modes become the whistler and ion cyclotron modes, respectively. Deviations from the pure cold plasma curves become apparent at wavenumbers $KU_A \sim |\delta n|/n_0$. Once $K < |\delta n|/n_0 U_A$, the $\sigma = +1$ mode approaches a resonance at $W \sim |\delta n|/n_0$, while the $\sigma = -1$ mode approaches a dispersion relation $W \propto K^2$. The situation for nonthermal ions is shown in Figure 2; it is identical to Figure 1 except that the handedness of the modes approaching the new resonance and dispersion $W \propto K^2$ are reversed, consistent with (7). Thus, Figures 1 and 2 verify all the predictions of the analytic theory derived above, in a physically relevant setting. In addition, the full dispersion equation (1) has been solved for the nonthermal distribution (8) for a wide range of plasma temperatures: for $\overline{V}_\alpha < 10^{-2}$ the cold plasma description is found to be accurate to within 1 part in 10^8 for frequencies $W \lesssim 0.1$. Thus, the effects of unequal thermal electron and ion temperatures described above should exist and be potentially important in plasmas with either cold or warm thermal components.

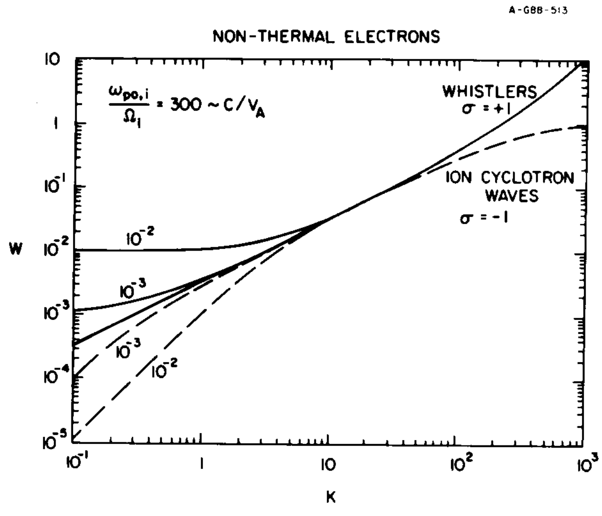

Figure 1. Dispersion curves for a cold plasma with various nonthermal electron densities for RH (full lines) and LH (dashed lines) waves verifying the analytic predictions. At low wavenumbers the pure cold plasma solutions are denoted by the lowest full line.

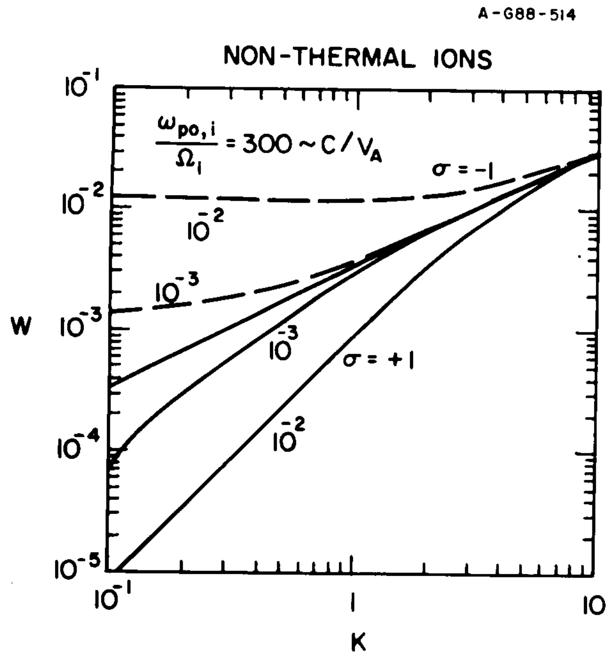

Figure 2. As for Figure 1, except for nonthermal ions.

3. SUMMARY AND FURTHER WORK

In summary, using analytic kinetic theory and numerical solutions of the wave dispersion equation, I have shown that: (1) Unequal thermal electron and ion number densities (due to the presence of nonthermal particles) in a charge-neutral plasma may lead to fundamental low wavenumber ($K < |\delta n|/n_0 U_A$) modifications of the Alfvén modes, including (2) the creation of a new resonance at $W \sim |\delta n|/n_0$ for the mode with $\sigma\delta n < 0$ and dispersion $W \propto K^2$ for the other mode. The nonthermal particles may also directly modify the Alfvén modes (e.g., Ref. 3) through the terms F_α in (1); these direct modifications aid or compete with the effects of unequal thermal number densities. The basic requirement for the effects of unequal thermal number densities to be significant in determining the wave dispersion is that the direct contributions F_α of the nonthermal particles be small (due to the form of the particle distributions f_α) compared to the "cold plasma" contribution of an equal number density of thermal particles. This requirement should not be considered too stringent: firstly, it is the conventional approximation made in basically all previous discussions of space and astrophysical Alfvén waves, and secondly, a natural, analytic distribution mimicing the (nonrelativistic) power-law distributions expected from shock acceleration in space and astrophysical plasmas obeys this requirement. Thus, in view of the expected ubiquity of physically interesting plasmas containing unequal thermal electron and ion densities, much previous work on detailed applications and nonlinearities involving Alfvén waves should be re-evaluated in view of these results.

One immediate implication for studies of Alfvén wave nonlinearities may be seen in Figures 1 and 2: conventionally Alfvén waves are considered modulationally unstable where $\partial^2\omega/\partial k^2 < 0$, as for the LH ($\sigma = -1$) mode but not the RH ($\sigma = +1$) mode in a pure cold plasma at high wavenumbers. Figures 1 and 2 indicate that the new low wavenumber resonance, due to the effects of unequal thermal particle number densities, is modulationally unstable according to this criterion. Moreover, in contrast to the conventional picture, these low wavenumber resonances are undamped even in warm thermal plasmas, and either the LH or RH mode may be modulationally unstable depending on the species with a deficit in thermal number density. Further research on this matter is needed, as well as on the detailed connection between this kinetic theory and MHD theory, the effects of additional ion species, and other distributions of nonthermal particles.

4. ACKNOWLEDGMENTS

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