RESULTS OF AN INTEGRATED STRUCTURE/CONTROL LAW DESIGN SENSITIVITY ANALYSIS

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ABSTRACT

Next generation air and space vehicle designs are being driven by increased performance requirements, demanding a high level of design integration between traditionally separate design disciplines. Interdisciplinary analysis capabilities have been developed, for aeroservoelastic aircraft and large flexible spacecraft control for instance, but the requisite integrated design methods are only beginning to be developed. One integrated design method which has received attention is based on hierarchical problem decompositions, optimization, and design sensitivity analyses. This paper highlights a design sensitivity analysis method for Linear, Quadratic Cost, Gaussian (LQG) optimal control laws, which predicts the change in the optimal control law due to changes in fixed problem parameters using analytical sensitivity equations. Numerical results of a design sensitivity analysis for a realistic aeroservoelastic aircraft example are presented. In this example, the sensitivity of the optimally controlled aircraft's response to various problem formulation and physical aircraft parameters is determined. These results are used to predict the aircraft's new optimally controlled response if the parameter was to have some other nominal value during the control law design process. The sensitivity results are validated by recomputing the optimal control law for discrete variations in parameters, computing the new actual aircraft response, and comparing with the predicted response. These results show an improvement in sensitivity accuracy for integrated design purposes over methods which do not include changes in the optimal control law. Use of the analytical LQG sensitivity expressions is also shown to be more efficient than finite difference methods for the computation of the equivalent sensitivity information.

INTRODUCTION

The design of new generation air and space vehicles is increasingly becoming subject to extensive requirements for design integration, that is, close coordination in the design of the various systems of the vehicle. For example, many modern fighter aircraft require integration of the flight control system and the propulsion system so that sufficient power is available at all flight conditions possible with the flight control system. To meet the challenge of the integrated system design requirements, design methods which tie together existing system design methods are needed.

One such integrated design methodology currently under development at NASA Langley Research Center is based on hierarchical problem decompositions, multilevel optimization methods, and design sensitivity analyses [1]. This methodology depends on the decomposition of the integrated design problem into vehicle requirements, system requirements, and subsystem requirements. Optimization methods are used to satisfy all levels of the design requirements, subject to the constraints that any previously satisfied design requirements remain satisfied. The continued satisfaction of previous design requirements is achieved through the use of design sensitivity information which relates the change in the previous design to the current design variables. This sensitivity information is used as gradient information in the current optimization to make sure the constraints are satisfied.

One application of the multilevel integrated design methodology is to the aeroservoelastic design of aircraft, which is the simultaneous consideration of aircraft aerodynamics, control laws, and structural dynamics. This application requires the incorporation of dynamic response design requirements and a control law design method which uses the available feedback signals, both of which required development and validation of appropriate design sensitivity information. Linear Quadratic Gaussian (LQG) control law design methods were selected. The sensitivity developments have recently been completed [2] and the application and validation of the sensitivity expressions is described here. Initially, aerodynamic design would not be attempted, although aerodynamic effects must be included in the calculation of dynamic responses.

Integrated Interdisciplinary Methods Are Needed for Advanced Air and Space Vehicle Design

One Approach Is Hierarchically Decomposed, Optimization and Sensitivity Analysis Based Methods

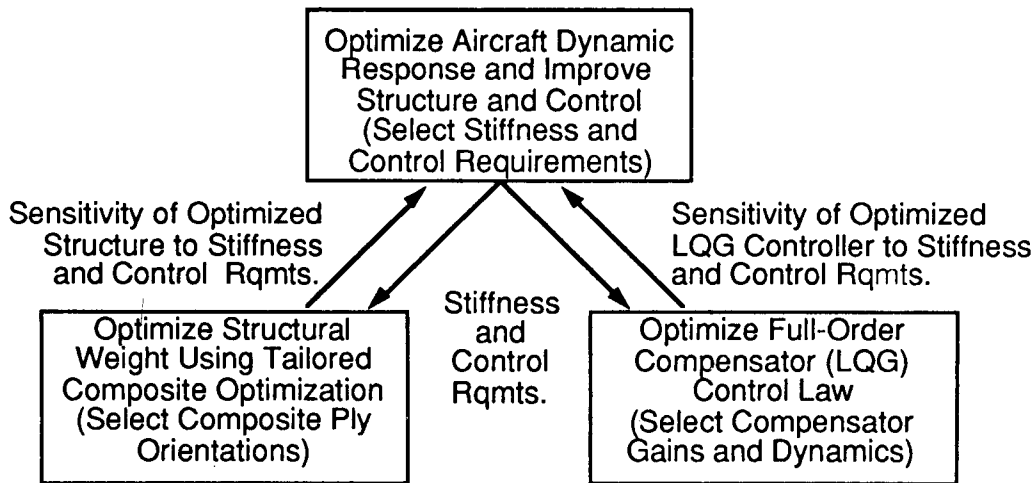Criteria for Initial Aeroservoelastic Design Method:

- Include Dynamic Response, Stability, and Robustness Requirements In Problem Formulation

- Control Law Design Method Must Use Measured Feedback Signals

- Use Existing Multilevel Structural Optimization Methods

Emphasis Here is on Sensitivity Analysis and Validation Results

MULTILEVEL STRUCTURE/CONTROL LAW DESIGN

A multilevel, integrated structure/control law design problem for an aeroelastic aircraft can be formulated conceptually as shown. In this formulation, the structural design problem is to minimize the weight of the structure subject to stiffness and stress requirements, and also to control law design requirements. Since the aircraft is aeroelastic, steady-state control actions (control surface deflections) can change structural deflections under given loads, and so must be considered in the structural design. The control law design problem is to minimize a quadratic performance index in aircraft responses and control inputs. Since the structural design defines the structural dynamic properties of the aircraft, the control law design problem is also dependent on the structural design requirements. The multilevel optimization approach to integrated design then treats the structural and control law design requirements as design variables, selecting those requirements so that the dynamic response of the vehicle is improved, and so that the structural design and the control law design are also improved. It requires the sensitivity of the optimized structure and control law designs to stiffness and control design requirements as gradient information at the upper level.

Analytical expressions for the sensitivity of optimized LQG control laws have previously been developed directly from the necessary conditions of optimality for the LQG problem. These results will be described following a statement of the LQG problem formulation.

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                    ┌──────────────────────┐
                    │ Optimize Aircraft Dynamic │
                    │ Response and Improve  │
                    │ Structure and Control │
                    │ (Select Stiffness and │
                    │ Control Requirements) │
                    └──────────────────────┘
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Sensitivity of Optimized Structure to Stiffness and Control Rqmts.

Sensitivity of Optimized LQG Controller to Stiffness and Control Rqmts.

Stiffness and Control Rqmts.

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┌──────────────────────┐         ┌──────────────────────┐
│ Optimize Structural  │         │ Optimize Full-Order  │
│ Weight Using Tailored│         │ Compensator (LQG)    │
│ Composite Optimization│        │ Control Law          │
│ (Select Composite Ply│         │ (Select Compensator  │
│ Orientations)        │         │ Gains and Dynamics)  │
└──────────────────────┘         └──────────────────────┘
```

Analytical Expressions for the Sensitivity of the Optimized LQG Control Law Have Previously Been Developed

Sensitivity Expressions Were Derived Directly From the Necessary Conditions of Optimality for the LQG Problem

LQG CONTROL LAW FORMULATION

The Linear, Quadratic Cost, Gaussian (LQG) optimal control law problem formulation is shown below, where x is the system state vector, u is the vector of control inputs, y is the vector of pertinent system responses, z is the vector of measured outputs to be used for feedback, and w and v are uncorrelated, zero mean, Gaussian distributed "white" noise disturbance vectors. The matrices A, B, C, D, and M are appropriately dimensioned coefficient matrices, and W and V are intensity matrices of the white noise disturbance vectors. It is assumed that each of these matrices is a known continuous differentiable function of one or more scalar parameters p which have some known nominal value. The LQG problem is to find the control u(t) such that the cost function J is a minimum, where the weighting matrices Q and R are also assumed to be known continuous differentiable functions of p. The solution of this problem is well known and is the interconnection of the optimal Linear Quadratic Regulator (LQR) and the optimal Kalman Filter (KF) state estimator as shown below, where the matrices G and F are the regulator and state estimator gain matrices respectively [3]. Clearly the gain matrices G and F are functions of the parameter p, and it is desired to know the change in G and F due to variations in the nominal value of the parameter p. Analytical expressions for the change (sensitivity) of G and F with respect to p have been derived from the LQG necessary conditions of optimality, and are summarized on the next page.

$$\dot{x} = A(p)x + B(p)u + D(p)w \qquad E(w) = 0; \; E(w(t)w^T(\tau)) = W(p)\delta(t\text{-}\tau)$$

$$y = C(p)x \qquad\qquad\qquad E(w(t)v^T(\tau)) = 0$$

$$z = M(p)x + v \qquad\qquad E(v) = 0 \,; E(v(t)v^T(\tau)) = V(p)\delta(t\text{-}\tau)$$

Problem is to find u(t) such that J is minimized for a given p:

$$J = \lim_{\tau \to \infty} E\left\{\frac{1}{\tau}\int_0^\tau \left[y^T Q(p)y + u^T R(p)u\right]dt\right\}$$

Solution is the Interconnection of the Optimal Regulator and Kalman Filter

$$u = -G\hat{x}$$

$$\dot{\hat{x}} = A\hat{x} + Bu - F(z - M\hat{x})$$

Want Analytical Expressions for the Sensitivity of the Solution to Changes in Fixed Parameter p

LQG CONTROL LAW SENSITIVITY

The optimal LQR and KF gain matrices G and F are computed as shown below, where S and T are the steady-state solutions of the appropriate nonlinear matrix Riccati equations. Also shown are expressions for the partial derivatives of G and F with respect to p. Under the assumptions regarding the functional dependence of B, M, R, and V on p, the only unknowns in these expressions are the partial derivatives of the Riccati equation solutions S and T with respect to p. Analytical expressions for these partial derivatives can be derived from the necessary conditions of optimality [2] and the final results are shown below. These expressions are valid only when the necessary conditions of optimality are satisfied, that is when G and F are the gain matrices which make the cost function J be a minimum. They are themselves linear Lyapunov equations in the unknown derivatives (sensitivities) S_p and T_p, and have coefficient matrices which are asymptotically stable by the properties of the LQR and KF solutions. The asymptotic stability properties of the coefficient matrices guarantees that the Lyapunov equations have solutions which exist and are unique. Additionally, the coefficient matrices are the same for every parameter p, with only the known term in the {} brackets changing. This affords considerable computational savings, since the coefficient matrices need only be decomposed once for the initial solution of the Lyapunov equations, stored, and reused for the remaining parameter sensitivity calculations.

(Note: Subscript p denotes partial derivative w.r.t. parameter p)

LQG Solution Given by:

$$G = R^{-1}B^T S \quad ; \quad 0 = A^T S + SA - SBR^{-1}B^T S + C^T QC$$

$$F = TM^T V^{-1} \quad ; \quad 0 = AT + TA^T - TM^T V^{-1}MT + DWD^T$$

Sensitivity of G and F with Respect to p is:

$$G_p = -R^{-1}R_p R^{-1}B^T S + R^{-1}B_p^T S + R^{-1}B^T S_p$$

$$0 = S_p(A-BG) + (A-BG)^T S_p + \{SA_p + A_p^T S + (C^T QC)_p - S(BR^{-1}B^T)_p S\}$$

$$F_p = T_p M^T V^{-1} + TM_p^T V^{-1} - TM^T V^{-1}V_p V^{-1}$$

$$0 = (A-FM)T_p + T_p(A-FM)^T + \{A_p T + TA_p^T + (DWD^T)_p - T(M^T V^{-1}M)_p T\}$$

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OPTIMAL COST SENSITIVITY

Several equivalent expressions for the optimized value of the LQG cost function in terms of the LQR and KF gain matrices G and F and the Riccati equation solutions S and T are shown below, where J^* denotes the optimized cost function value and tr{} denotes the trace of a matrix. By the chain rule of differentiation, the partial derivative of the optimized cost is the derivative of the optimized cost with respect to the gain matrix (G or F) times the derivative of the gain matrix with respect to p, plus the partial derivative products of all the other matrices in the cost function expressions. However, since the cost function J has been optimized with respect to the gain matrices (i.e. G and F satisfy the necessary conditions of optimality), the derivatives J^*_G and J^*_F are identically equal to zero, which means that the sensitivity of the optimized cost J^*_p is independent of changes in the optimal gain matrices G and F [2]. This makes the optimized value of the cost function J^* unattractive for use in the integrated structure/control law design algorithm as a measure of control law performance, since the sensitivity J^*_p does not reflect the actual changes in the optimal control law. For this reason, other measures of the optimally controlled systems performance, such as time and frequency responses, system eigenvalues, and covariance responses must be used in the integrated design methodology even though these responses have not been optimized with respect to G and F. The sensitivities of these other performance measures do reflect the effects of the change in the optimal gain matrices G and F due to changes in the parameter p. Analytical expressions for the sensitivities of these other controlled system performance measures also exist and are summarized on the next pages.

Optimized Cost Function Value

$$J^* = tr\{SFVF^T + TC^TQC\} = tr\{SDWD^T + TG^TRG\}$$

Consider That

$$J^*_p = J^*_F F_p + J^*_S S_p + ... = J^*_G G_p + J^*_T T_p + ...$$

But J^* is Optimal With Respect to F and G, i.e.

$$J^*_F = J^*_G = 0$$

So the Sensitivity of the Optimized Cost is Independent of the
Sensitivity (Changes) in the Optimal Gain Matrices

DYNAMIC RESPONSE SENSITIVITY

Once the optimal LQG control law is computed, the regulator and Kalman Filter equations can be interconnected to form a set of state-space equations which represent the controlled system. This is represented below where the vector x is the controlled system state vector, y is the controlled system outputs, and W is the combined vector of disturbance inputs. Taking the partial derivative of the state equations with respect to the parameter p and interchanging the order of differentiation leads to the system sensitivity equations shown. When integrated over time for a known input time history $w(t)$ these equations give the sensitivity of the controlled system state vector and output vector time histories as a function of both the input and state vector time histories. These equations can also be used to determine the sensitivity of the frequency response of a single input/output pair by transformation of the system and sensitivity equations into the Laplace domain and replacing the Laplace transform variable s by the complex frequency $j\omega$ for zero initial conditions. Denoting the complex response of one input/output pair at a given frequency ω by h and the corresponding complex sensitivity result by h_p, the sensitivity of magnitude and phase of the response are computed as shown. If the interest is in more than one input/output pair, the singular values of the complex transfer function matrix H relating the input vector W and the output vector y are often calculated at discrete frequencies ω as a means of determining the response magnitude in all loops simultaneously. Assuming that none of the singular values is repeated, the sensitivity of the singular values at a given frequency is calculated from the complex transfer function sensitivity matrix using the same unitary transformation pair as determined in the singular value calculation [4].

$$\dot{x} = Ax + Dw \qquad \dot{x}_p = A_p x + A x_p + D_p w$$

$$y = Cx \qquad y_p = C_p x + C x_p$$

Sensitivity Equations Depend on System Response - Can Be Solved in Either Time or Frequency Domain

Frequency Response Sensitivities -

For a Complex Response $h = a + jb$ and Sensitivity $h_p = a_p + jb_p$

$$|h| = \sqrt{a^2 + b^2} \quad ; \quad |h|_p = \frac{1}{|h|}(aa_p + bb_p)$$

$$\phi = \tan^{-1}\frac{b}{a} \quad ; \quad \phi_p = \frac{1}{|h|^2}(ab_p - ba_p)$$

Singular Values of Complex Transfer Function Matrix H:

$$\Sigma = U^* H V \qquad ; \quad \Sigma_p = U^* H_p V$$

(* here denotes complex conjugate transpose)

DYNAMIC RESPONSE SENSITIVITY (CONC.)

The eigenvalues of the system dynamics matrix A of a linear state-space system are often used as a measure of stability and performance. If the change in the matrix A with respect to a parameter p is known and there are no repeated eigenvalues, then the sensitivity of the eigenvalues due to a change in the parameter p can be calculated in terms of the derivative matrix A_p and the matrix E, whose columns are the right eigenvectors of the matrix A [5].

The response of a linear system to Gaussian distributed, "white" noise random inputs is measured in terms of covariance or mean square quantities. These are computed using the (steady-state) covariance equations shown below, where the matrix W is the intensity matrix of the random noise input and X is the state vector covariance to be calculated. Once X is known, other response quantities of interest are easily computed. Differentiation of the covariance equation with respect to the parameter p results in an equation for the sensitivity of the state vector covariance X_p in terms of the state vector covariance X [6]. The sensitivity of the other response quantities of interest are also easily computed.

System Eigenvalue Sensitivity

$$\Lambda = E^{-1}AE \quad ; \quad \text{diag}(\Lambda_p) = \text{diag}(E^{-1}A_p E)$$

Covariance Response Sensitivity

$$0 = AX + XA^T + DWD^T \; ; \; Y = CXC^T \; ; \; U = GXG^T$$

$$0 = AX_p + X_p A^T + A_p X + XA_p^T + (DWD^T)_p \; ; \; Y_p = (CXC^T)_p \; ; \; U_p = (GXG^T)_p$$

NUMERICAL SENSITIVITY STUDY

The previously described analytical sensitivity expressions for the change in optimal LQG control law designs and the optimally controlled linear system responses have been exercised on a real aeroservoelastic aircraft example. This problem considered the DAST ARW-II (Drones for Aerodynamic and Structural Testing, Advanced Research Wing II) aircraft, which was a Firebee drone vehicle modified for high risk aeroelastic and aeroservoelastic stability testing [7]. A mathematical model of the longitudinal dynamics of this vehicle including rigid-body pitch and plunge motions, three symmetric vibration modes, and elevon and symmetric aileron control surfaces was used. This model included unsteady aerodynamic effects for each mode. Vehicle pitch rate and vertical acceleration at the center-of-gravity, and outboard vertical wing acceleration measurements were available as feedback signals. An LQG optimal control law problem was formulated for this example to stabilize a nominally unstable short period mode. The sensitivity of the optimal control law and the dynamic responses of the controlled aircraft were computed for twelve different problem formulation and physical parameters. The response sensitivities computed included the sensitivity of the covariance response of the vehicle subjected to Dryden random vertical gust environment, the sensitivity of the vehicle time response to a discrete 1-Cosine vertical gust, and the sensitivity of the frequency response in the elevon loop of the aircraft.

25th Order State-Space Model of DAST ARW-II
- Rigid Body Plunge, Pitch, 3 Symmetric Elastic Modes, Unsteady GAF's

- Elevon and Symmetric Aileron Control Surfaces

- Pitch Rate and Acceleration at C.G., Outboard Wing Acceleration Sensors

Sensitivity Information Calculated For Twelve Design Parameters:
- Response to Random Gust Environment (Covariance)

- Time Response to Discrete 1-Cos Gust

- Frequency Response of Open Elevon Loop (Aileron Loop Closed)
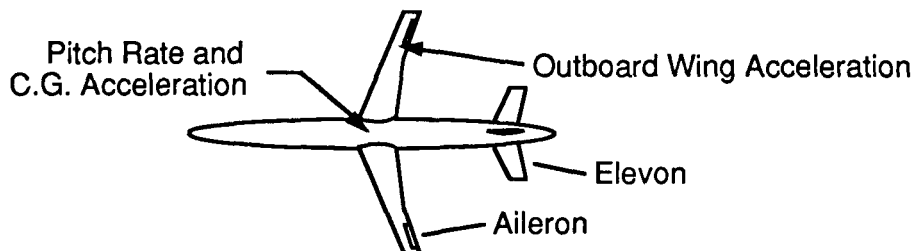
SENSITIVITY PARAMETERS

Shown below are the nominal values and descriptions of the twelve parameters for which the sensitivity of the DAST ARW-II control law and dynamic responses were computed. All twelve parameters influence the dynamic responses of the controlled system. The first four parameters are elements in the weighting matrices of the cost function for the LQG problem and directly affect the LQR regulator gain matrix G discussed previously. Parameters 5 through 8 are elements of the noise intensity matrices in the LQG formulation and directly affect the KF gain matrix F. The final four parameters represent physical quantities or characteristics of the vehicle and affect the LQR gain G, the KF gain F, and the basic dynamics of the vehicle. Parameter 9 is a wing bending stiffness related parameter which was used to uniformly increase or decrease the natural frequencies of the two wing bending modes. Parameter 10 is a wing torsion stiffness parameter similar to parameter 9 that was used to scale the wing torsion mode natural frequency. Parameters 11 and 12 were used to locate the wing accelerometer used for feedback longitudinally and laterally on the wing. The results to be presented in the next several figures emphasize the sensitivity of the aircraft responses to the four physical related parameters 9 through 12.

Parameter	Nominal Value	Description
1	0.01	Q Matrix Weight on Pitch Rate
2	0.01	Q Matrix Weight on Fwd. Wing. Acc.
3	1.00	R Matrix Weight on Elevon Com.
4	1.00	R Matrix Weight on Aileron Com.
5	2.00×10^{-3}	Pitch Rate Sensor Noise Intensity
6	6.00×10^{-3}	Aft Wing Acc. Sensor Noise Intensity
7	1.00×10^{-6}	Injected Elevon Noise Intensity
8	1.00×10^{-6}	Injected Aileron Noise Intensity
9	1.00	Wing Bending Stiffness Parameter
10	1.00	Wing Torsion Stiffness Parameter
11	7.58	Aft Wing Acc. Longitudinal Location
12	2.00	Aft Wing Acc. Lateral Location

OPTIMAL COST SENSITIVITY

   The optimal LQG control law for the DAST ARW-II example problem was computed and analyzed for sensitivity to the twelve sensitivity parameters. Shown below are the value of the optimized cost function (J^*) and the semi-relative sensitivities of the cost function value to the sensitivity parameters. (Semi-relative sensitivity results are normalized such that the results are directly comparable for equal percent changes in the nominal parameter values.) Two sets of results are shown. Under the heading Design Sensitivity is the sensitivity of the optimized cost function to the twelve parameters computed using the analytical LQG sensitivity expressions discussed earlier. Under the heading Alternate Sensitivity is the sensitivity of the optimized cost to the four physical parameters 9 through 12 when the change in the optimized control law (gain matrices G and F) is ignored. These sensitivity results show only the effect of a change in basic system dynamics and do not include the effects of a change in the control law. The results are identical, verifying the previous assertion that the cost function sensitivity does not reflect changes in the optimized control law. Furthermore, the current method provides sensitivity information for a wider range of parameters than the alternate sensitivity information, since the first eight parameters affect only the gain matrices G and F.

(Optimal Cost = 1.222, Semi-Relative Sensitivity)

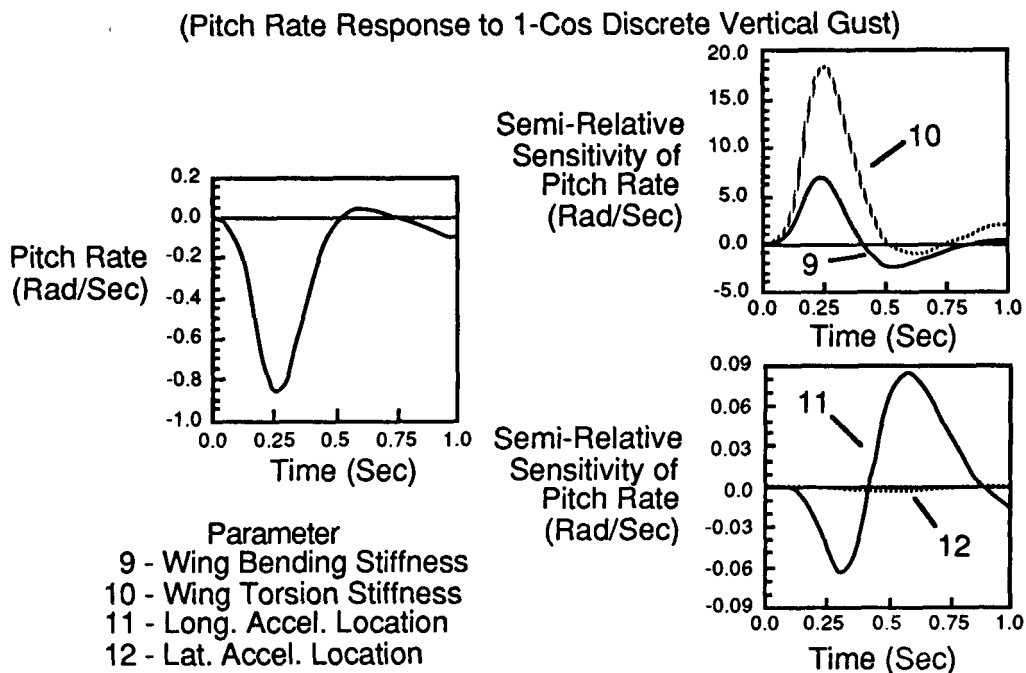| Parameter	Design Sensitivity	Alternate Sensitivity
1	5.17×10^{-4}	
2	2.35×10^{-1}	
3	7.58×10^{-3}	
4	4.90×10^{-1}	
5	1.35×10^{-3}	
6	2.53×10^{-2}	
7	5.90×10^{-6}	
8	1.48×10^{-10}	
9	-1.57×10^{1}	-1.57×10^{1}
10	-4.44×10^{1}	-4.44×10^{1}
11	9.84×10^{-3}	9.84×10^{-3}
12	-2.14×10^{-3}	-2.14×10^{-3}

COVARIANCE RESPONSE SENSITIVITY

The covariance response of the optimally controlled DAST ARW-II aircraft was computed for a 12 ft./sec. RMS vertical gust input using a Dryden gust spectrum. The sensitivity of the RMS vehicle pitch rate and center-of-gravity acceleration and vertical wing acceleration were computed for the twelve sensitivity parameters as shown. The wing acceleration result was measured at a constant point independent of the wing acceleration feedback signal so that the sensitivity results for parameters 11 and 12, which actually locate the feedback sensor, are consistent with the results for all the other parameters.

The results shown are best interpreted in terms of their sign and the magnitude of the exponents. For example the sensitivities of the three responses to parameter 9, the wing bending stiffness parameter, are all negative with the largest effect on wing acceleration. This means an increase in the wing bending stiffness would largely decrease the wing acceleration while also decreasing the pitch rate and c.g. acceleration. A positive change in parameter 10, the wing torsional stiffness parameter, would yield a larger decrease in the wing acceleration than the bending stiffness but would increase the pitch rate and c.g. acceleration results. A negative change in parameter 11, which locates the wing acceleration feedback sensor longitudinally on the wing, would decrease all three responses, while a change in parameter 12, the lateral wing feedback sensor locating parameter, would have a negligible effect compared to parameter 11.

(12 ft/sec RMS Vertical Gust Input, Semi-Relative Sensitivity)

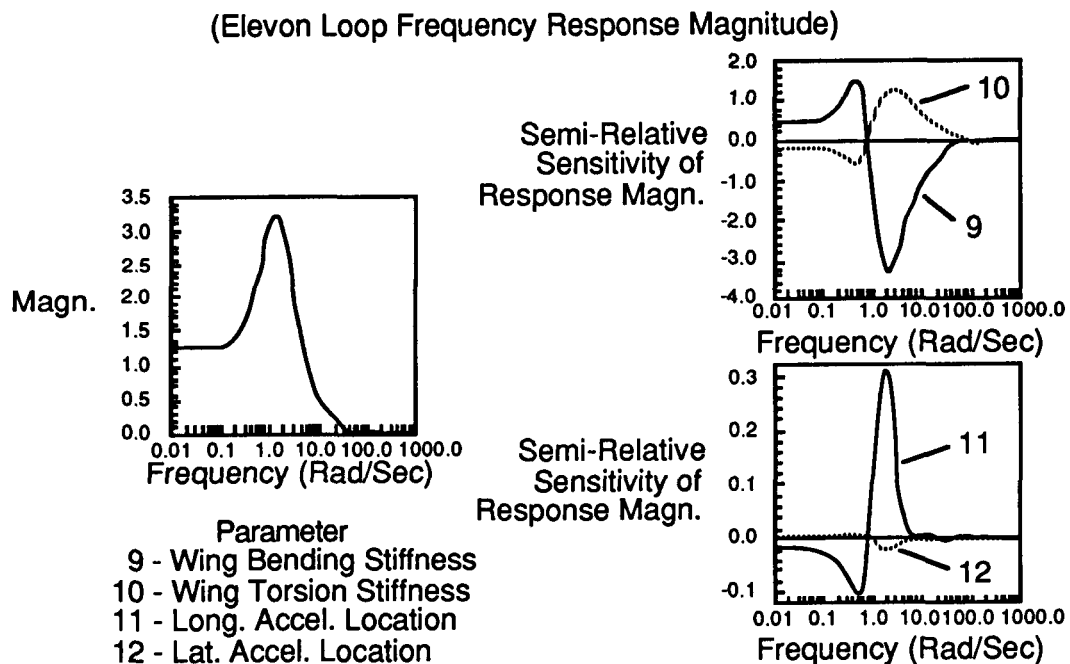| Parameter	Pitch Rate $(5.15 \times 10^{-2} \text{deg/sec})$	C. G. Acceleration $(2.65 \times 10^{-2} g)$	Wing Acceleration $(2.35 \times 10^{1} g)$
1	-6.18×10^{-3}	2.24×10^{-4}	6.24×10^{-5}
2	6.18×10^{-5}	-6.62×10^{-4}	-2.69×10^{0}
3	-1.60×10^{-2}	2.08×10^{-3}	1.00×10^{-1}
4	4.04×10^{-4}	1.93×10^{-3}	8.21×10^{0}
5	5.91×10^{-3}	9.91×10^{-4}	5.21×10^{-3}
6	1.22×10^{-3}	9.04×10^{-4}	7.98×10^{-1}
7	-1.77×10^{-4}	-4.77×10^{-6}	3.75×10^{-6}
8	-1.74×10^{-9}	1.55×10^{-8}	4.36×10^{-5}
9	-9.84×10^{-2}	-3.38×10^{-3}	-5.32×10^{1}
10	7.00×10^{-2}	4.04×10^{-2}	-1.22×10^{2}
11	1.35×10^{-3}	1.16×10^{-3}	2.94×10^{-1}
12	-3.43×10^{-5}	-6.64×10^{-5}	-6.80×10^{-2}

TIME RESPONSE SENSITIVITY

The time response of the optimally controlled DAST ARW-II aircraft was computed for a 1-cosine discrete vertical gust input with a maximum amplitude of 5 ft./sec. and a duration of 0.25 seconds. Shown below is the pitch rate response of the vehicle over one second and the sensitivity of that pitch rate response to the four physical parameters 9 through 12. The pitch rate response is more sensitive to the wing bending and torsion stiffness parameters than the wing acceleration feedback sensor location parameters. Increasing either the wing bending or torsion stiffness would tend to alleviate the peak negative pitch rate response at about 0.25 seconds. A negative change in the wing acceleration longitudinal position would also tend to reduce the peak negative pitch rate response at 0.25 seconds, but would increase oscillation in the response by adding an additional peak at about 0.65 seconds. The lateral location of the wing acceleration sensor would have a negligible effect on the pitch rate response.

(Pitch Rate Response to 1-Cos Discrete Vertical Gust)

Pitch Rate (Rad/Sec)

Semi-Relative Sensitivity of Pitch Rate (Rad/Sec)

Semi-Relative Sensitivity of Pitch Rate (Rad/Sec)

Time (Sec)

Parameter
9 - Wing Bending Stiffness
10 - Wing Torsion Stiffness
11 - Long. Accel. Location
12 - Lat. Accel. Location

FREQUENCY RESPONSE SENSITIVITY

The magnitude of the elevon loop frequency response, computed with the aileron loop closed, is shown below as is the sensitivity of frequency response magnitude to the four physical parameters 9 through 12. Any one of the three parameters 9 through 11 could be used to reduce the peak magnitude of the response at about 2.0 rad./sec., or to decrease the bandwidth of the control loop by reducing the response magnitude above 2.0 rad./sec. Both actions could not be achieved using a single parameter, since the sensitivity results show that any parameter change used to decrease the peak response at 2.0 rad./sec. would tend to increase the bandwidth by increasing the response magnitude at higher frequencies.

(Elevon Loop Frequency Response Magnitude)

Magn.

Frequency (Rad/Sec)

Parameter
9 - Wing Bending Stiffness
10 - Wing Torsion Stiffness
11 - Long. Accel. Location
12 - Lat. Accel. Location

Semi-Relative Sensitivity of Response Magn.

Frequency (Rad/Sec)

Semi-Relative Sensitivity of Response Magn.

Frequency (Rad/Sec)

SENSITIVITY VALIDATION

The covariance, time, and frequency response sensitivity results presented in the previous figures were validated against actual response changes, including the optimal LQG control law change effects, due to variations in various nominal parameter values. This was accomplished by computing the new optimal LQG control law for up to ±25% changes in nominal parameter values in ±5% increments, and then computing the actual covariance, time, or frequency response for that parameter value. These results were then compared with predictions of the new responses obtained by a first-order Taylor series expansion about the nominal response using the available sensitivity data and the magnitude of the parameter change. In addition, a second set of sensitivity data for the four physical parameters 9 through 12 was generated which ignored the changes in the optimal LQG control law (changes in the G and F matrices). This second set of sensitivity data was also used to predict changes in the optimally controlled response of the system due to changes in the nominal parameter values. Percent error comparisons for the two sets of response predictions are shown below for variations in the wing bending stiffness parameter (parameter 9).

For the covariance response predictions, the percent error in prediction of the mean square pitch rate response to the random gust environment is shown on the left. For the current design sensitivity method, which includes the effects of the optimal control law change, the percent error passes through zero with no slope, indicating an exact derivative result. The alternate sensitivity method, which does not include the control law change effects, has a nonzero slope in the error at the nominal parameter value. For variations in the wing bending stiffness of up to +15%, the design sensitivity method gives more accurate predictions (smaller errors) of the actual pitch rate response. Similar types of results are shown on the right for elevon loop frequency response magnitude predictions and the pitch rate time response predictions for the discrete vertical gust input. In the case of these results, the percent error calculations were integrated over the frequency range or time interval to obtain a single error number for each varied value of wing bending stiffness parameter. As was the case for the covariance response predictions, the design sensitivity method gives more accurate results about the nominal parameter value, up to +15% variation in the wing bending stiffness parameter.
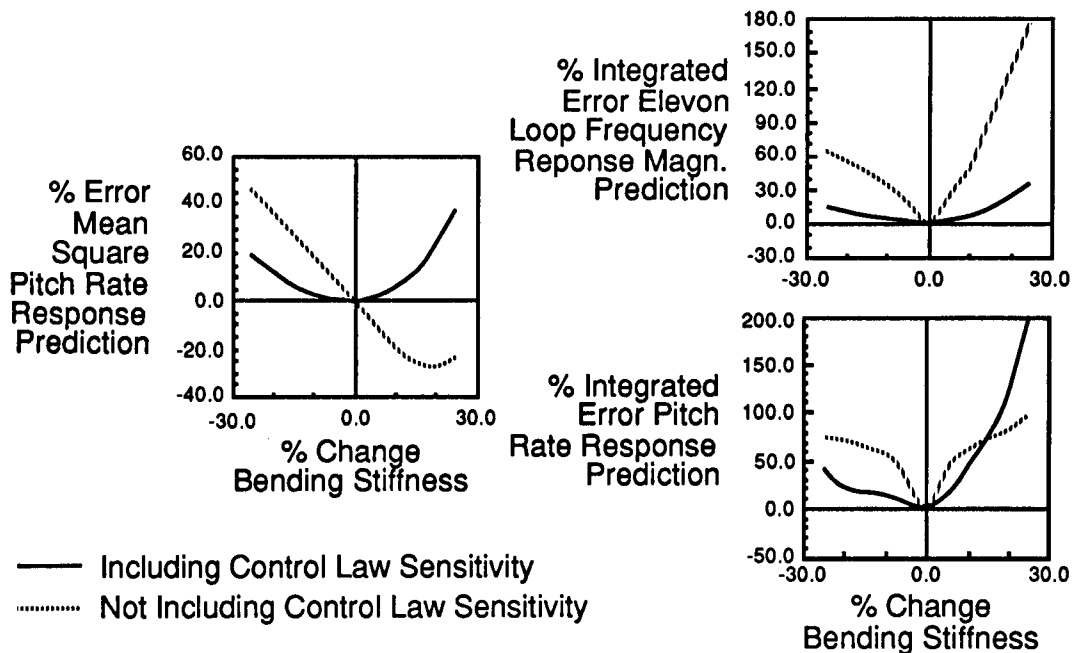
COMPARISON OF COMPUTATIONAL TIMES

The analytical expressions for the sensitivity of the optimal LQG control law involve the solution of two linear Lyapunov equations for each parameter of interest. In order to assess the computational burden associated with these calculations, a comparison of computational times to compute the derivative information using the analytical expressions and by finite difference methods was made. Four results are shown. The first is the CPU time required for the original LQG optimal control law solution using a DEC MicroVax II computer and a commercially available control analysis and design software package. The second result is the CPU time required for the original LQG solution and the solution of the two Lyapunov equations for the sensitivity of the gain matrices G and F and the Riccati matrices S and T for a single parameter. The third result is the CPU time required for the original LQG solution and a second LQG solution for a perturbed parameter value, as would be required for a one-step finite difference calculation of the change in the gain matrices G and F. The actual finite difference calculation is not included in the CPU time. The final result is similar to the third except two perturbed LQG solutions are computed, as would be required for a two-step finite difference calculation. Again the actual finite difference calculation is not included in the CPU time result. These results show that it is significantly faster to use the analytical expressions rather than finite difference calculations for the equivalent derivative information for a single parameter. As discussed earlier, the coefficients of the Lyapunov equations for the Riccati sensitivities are the same for every parameter, which can lead to additional computational savings by eliminating expensive decomposition of the coefficient matrices for each parameter. This means the computational efficiency of the analytical approach will be even better than shown here for the multiple parameter case.

Calculation	CPU Time (Sec.)
Original LQG Solution	100.68
LQG and Analytical Sensitivity of G, F, S, T	133.55
LQG and One Perturbed LQG For Numerical Sensitivity of G, F (Not Including Difference Calculation)	196.44
LQG and Two Perturbed LQG For Numerical Sensitivity of G, F (Not Including Difference Calculation)	287.39

CONCLUDING REMARKS

This paper has highlighted a method for computing the sensitivity of optimal LQG control laws to various problem parameters using analytical sensitivity expressions. The LQG sensitivity results are used in conjunction with the sensitivity of dynamic systems responses, also calculated using analytical expressions, to predict the changes in optimally controlled system responses due to changes in the nominal values of the problem parameters of interest. These sensitivity results are shown to be useful for integrated structure/control law design problems through a large aeroservoelastic aircraft example. Sensitivities of covariance, time, and frequency responses of the aircraft to twelve parameters were computed, and the results for four physical parameters were emphasized. The sensitivity results were validated against actual response changes due to changes in the nominal values of various parameters and found to be more accurate than alternate sensitivity calculations. It was also found that it is cheaper to evaluate the analytical expressions than to calculate the equivalent sensitivity derivatives by finite difference means.

A Control Law and Dynamic Response Sensitivity Analysis Capability Has Been Developed

Exercised on a Large Aeroservoelastic Mathematical Model Example

- Sensitivities to Twelve Control Law and Physical Design Parameters Calculated

- Validated Against Actual Response Changes Due to Changes in Design Parameters

- More Accurate For Integrated Design Purposes Than Standard Sensitivity Analysis Methods

- Analytical Expressions Cheaper To Evaluate Than Equivalent Finite Difference Calculations

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