N89-25200

OPTIMIZING FOR MINIMUM WEIGHT WHEN TWO DIFFERENT FINITE ELEMENT MODELS AND ANALYSES ARE REQUIRED

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INTRODUCTION

Whether designing an automobile, aircraft, building, or ship, the structural engineer must consider many loading conditions and meet multiple design criteria. Arriving at a minimum weight structure which satisfies all of the design constraints requires the integration of the results from all analyses and loading conditions. This is a relatively straightforward process if all of the analyses use the same analysis model. However, if each analysis requires a separate model, each model must still vary by the same amount when the design variables change. Typically, this optimization process is further complicated when constraints from the different analyses drive the design variables in opposite directions. For example, the stress constraint from a static analysis may cause a decrease in a design variable. However, the minimum frequency constraint from a vibration analysis may cause an increase in the same design variable.

This paper discusses the FESOP (Finite Element Structural Optimization Program) program's ability to perform minimum weight optimization using two different finite element analyses and models. FESOP uses the ADS optimizer developed by Dr. Garret Vanderplaats to solve the nonlinear constrained optimization problem. The design optimization problem in the paper requires a response spectrum analysis and model to evaluate the stress and displacement constraints. However, the problem needs a frequency analysis and model to calculate the natural frequencies used to evaluate the frequency range constraints. The paper summarizes the results of both the successful and unsuccessful approaches used to solve this difficult weight minimization problem. The results show that no one ADS optimization algorithm worked in all cases. However, the Sequential Convex Programming and Modified Method of Feasible Directions algorithms were the most successful (Figure 1).

MININUM WEIGHT STRUCTURAL DESIGN

***Multiple Analysis Types and Models**

- ***Static, Vibration, Response Spectrum***

***Multiple Loading Conditions**

***Conflicting Design Constraints**

- ***Stress, Displacement, Frequency***

***Different Functional Design Groups**

- ***Static, Vibration***

FIGURE 1

PROBLEM

The engineer faces many conflicting requirements when designing equipment foundations. The design requirements are conflicting because a minimum weight response spectrum (stress) design will tend to decrease the structural stiffness, while a minimum weight natural frequency avoidance design will tend to increase the structural stiffness. Another problem arises from the fact that separate response spectrum and natural frequency analysis models may be required. A much finer finite element mesh may be needed in the vibration analysis to accurately determine the natural frequencies of vibration. This paper presents a multidisciplinary optimization procedure and program which has successfully integrated these analysis methods to

(1) solve both the natural frequency and response spectrum finite element foundation models at the same time;
(2) optimize these foundations for minimum weight while meeting both frequency avoidance and response spectrum design criteria;
(3) arrives at producible equipment foundation designs (Figure 2).

Thus, instead of a time consuming trial and error approach to performing combined response spectrum and natural frequency avoidance foundation design, an automated process, using the FESOP computer program, now exists to arrive quickly and efficiently at producible and weight effective equipment foundations designs. The following paragraphs describe how FESOP was used to develop producible minimum weight designs.

FESOP
(Finite Element Structural Optimization Program)

*Solves Both Natural Frequency and Response Spectrum Analyses in Same Execution

*Permits Different Finite Element Models for Each Analysis

*Optimizes For Minimum Weight Using ADS

*Satisfies Stress, Displacement, and Frequency Avoidance Constraints

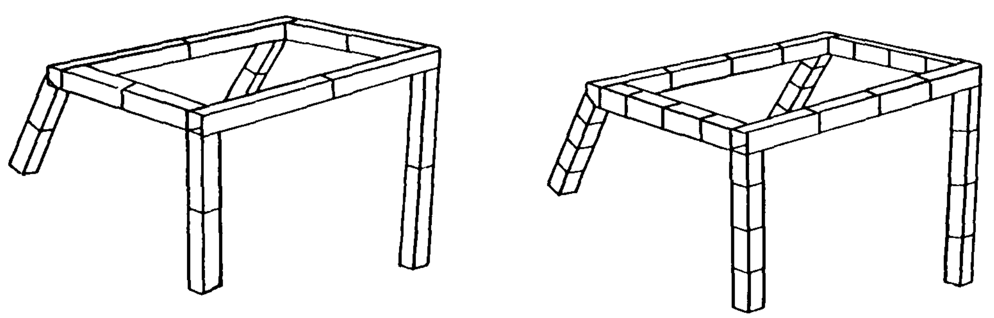
*Arrives at Producible Equipment Foundations

FIGURE 2

SAMPLE PROBLEM

Figure 3a is the response spectrum (stress) finite element model and Figure 3b is the vibration frequency finite element model. The response spectrum model employs the minimum number of finite elements needed to accurately assess the structure's performance, with only the equipment mass plus enough lumped masses to accurately model the foundation mass. However, the vibration model requires a much finer finite element discretization with an element mass definition but no equipment mass to accurately determine the structure's natural frequencies of vibration. Each math model, therefore, requires a separate analysis. In the normal design situation the engineer would set up the two models, run both analyses, evaluate two sets of results, change both models, rerun both models, and continue this process until the "optimum" design was established. At best this is a very time consuming and very imprecise procedure since the engineer relies only on his experience and intuition to modify the structure. In FESOP, an automated procedure exists: to read in both models, to perform both analyses, to evaluate the results of both analyses, to modify the math models as dictated by the numerical optimization program ADS, and to arrive at a producible true minimum weight foundation design while meeting all criteria.

SAMPLE EQUIPMENT FOUNDATIONS



a - Response Spectrum Model

b - Vibration Model

FIGURE 3

ENGINEER'S FUNCTION

While FESOP improves and automates the normal design process, the engineer's knowledge is still required to achieve acceptable results. Usually two or more attempts with FESOP are required to arrive at an optimum weight equipment foundation due to the highly complex nature of the frequency avoidance problem. However, FESOP does provide an efficient means to arrive at this desirable result quickly and with little or no guesswork. In addition, by properly specifying the design constraints and variables, a truly producible structure will result.

While at first glance this would seem to be a very expensive process, in the long run the costs will be cheaper because the engineer will spend considerably less time making alterations to the design and rerunning the required analyses. He will be able to devote more cogitative effort to solving his design problem, and the design will be far superior in all aspects (Figure 4).

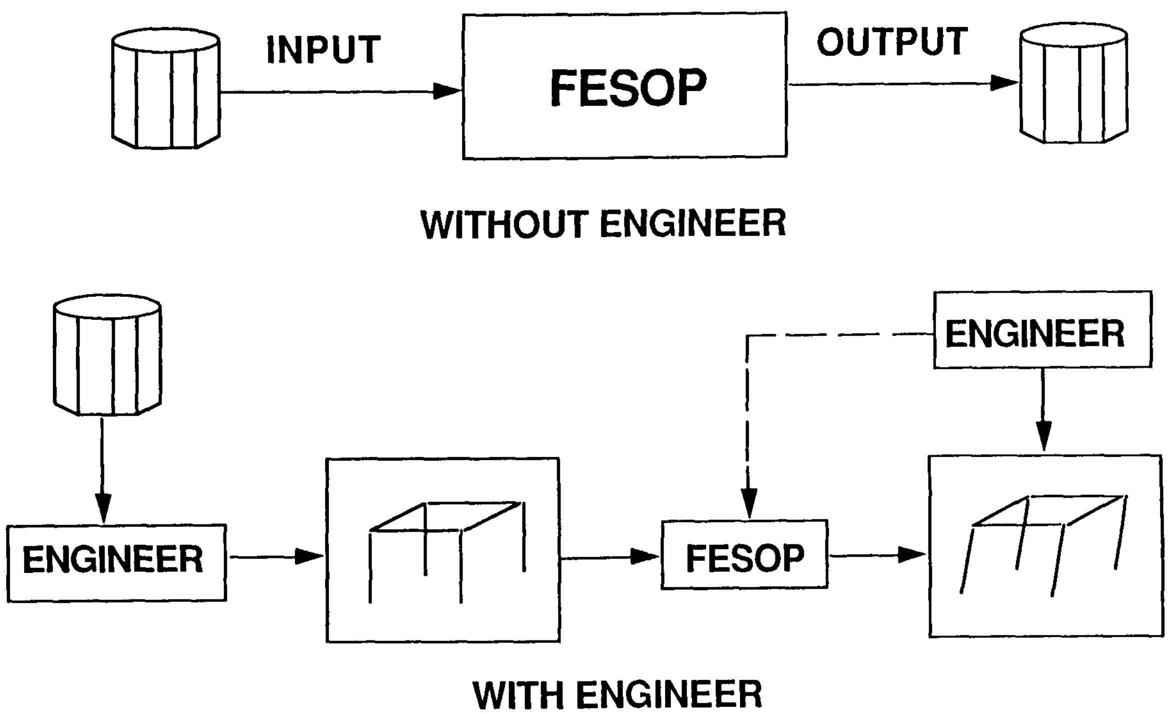


FIGURE 4

REQUIREMENTS

A combined response spectrum and vibration minimum weight design can be accomplished using many different approaches with FESOP. Some of the more important considerations for successful completion are the fineness of the finite element model; the choice and number of design variables; the choice of the optimization algorithm; the initial design of the FESOP analysis; and the producibility of the resulting structure. A number of recommended procedures have been developed to help ensure the best minimum weight design in the quickest manner possible, (Figure 5). In the following sections these important considerations are addressed, with samples of both good and bad applications to emphasize the point. Finally, a summary section discusses the successful combination of all of these features.

Fineness of Finite Element Model

Selection and Number of Design Variables

Optimization Algorithm

*** - MFD, MMFD, SLP, SQP, SCP**

Starting Point

- **Upper or Lower Bound**
- **Feasible or Infeasible**

Procedure

- **One Step (All Constraints)**
- **Multiple Steps (Selected Constraints Then All)**

FIGURE 5

* Defined in Figure 9.

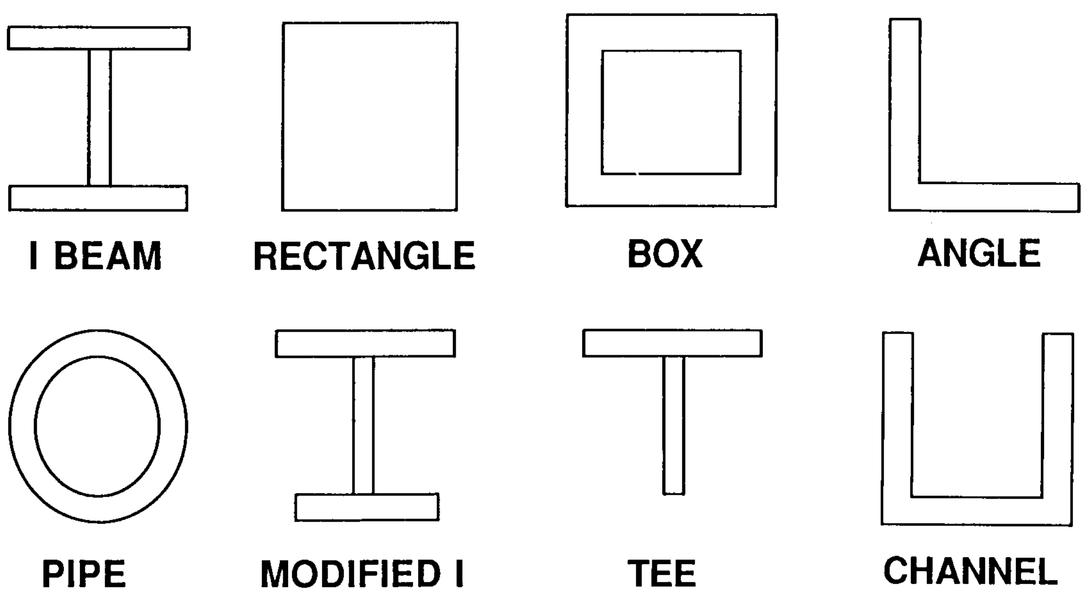
REQUIRED FINITE ELEMENT MODELS

In the sample problem, the vibration finite element model, Figure 3b, has approximately two times the number of finite elements as the response spectrum finite element model, Figure 3a. The vibration model is sufficiently complex to demonstrate a combined response spectrum and vibration foundation design optimization with FESOP. Although the discretization of the frequency model is different from that of the stress model, all of the reference data (material properties, cross-sections, eccentricities, thicknesses, etc.) must be and are identical in the two models. The reason for this is that as a design variable for one model's changes, it must change identically for the other model. The important thing to stress is that the major differences between the two models are the number of finite elements, the number of node points, and the definition of the mass associated with each model.

THE CHOICE AND NUMBER OF DESIGN VARIABLES

The choice and number of design variables affects the computer time it takes to arrive at an optimum solution; the ability of FESOP to give a true minimum weight solution; and the ability of FESOP to arrive at a producible structure. The greater the number of design variables, the more finite element solutions are required to determine the constraint gradients needed for the ADS optimizer, and consequently the longer and more costly the FESOP analysis. For example, in the sample problem every beam element box cross-section has five shape parameters: the depth, the width, the top thickness, the bottom thickness, and the side thickness (Figure 6). Thus, with the response spectrum model, there could be 16 different cross-sections (16 beam elements), with 5 design variables for each cross-section, or a total of 80 design variables. However, specfying such a large number of design variable would be ridiculous for two reasons: (1) more than 800 gradient evaluations would be required for both the models to obtain an optimum design, and (2) the resulting structure would clearly not be very producible. A more reasonable scheme would be to specifiy all of the horizontal members as having the same cross-section and all of the vertical or nearly vertical members having another cross- section. This would leave a total of ten design variables and only 100 gradient evaluations for a normal FESOP run. However, even in this case the structure could be very unproducible with mismatched cross-sections at the joints.

ALL WIDTHS AND THICKNESSES CAN BE DESIGN VARIABLES



Typical Beam Cross-Sections

FIGURE 6

SELECTED CROSS-SECTIONS

A better solution would be to allow only five design variables: the depth, width, and top thickness of the horizontal members; the top thickness of the vertical members; and the top thickness of the inclined members (Figure 7). The bottom and side thickness of the horizontal members; the depth, width, and bottom and side thickness of the vertical members; and the depth, width, and bottom and side thickness of the inclined members would all be dependent design variables. In this case the bottom and side thicknesses of each cross-section would equal the top thickness of the same cross-section. This would mean each box section would have a uniform thickness. The depth and width of the inclined members would be equal to the depth and width of the horizontal members, and the depth and width of the vertical members would equal each other and the width of the horizontal members. Figure 7 shows the five design variables for this case. In addition to such design variable linking for the sample foundation, the eccentricities at the joints are also linked to changes in the depth and width of the members. By doing this these eccentricities which are dependent upon the shape of the cross-section will change as the design variables change. With such limitations, the resulting optimized foundation will be very producible.

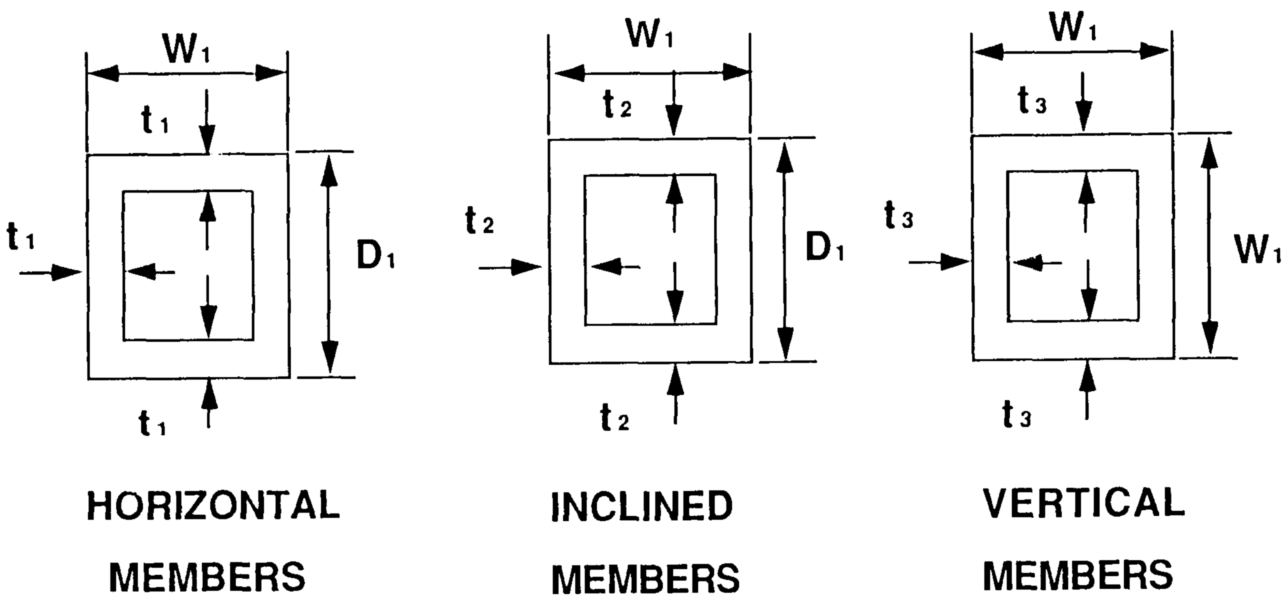


FIGURE 7

UNPRODUCIBLE OPTIMIZED STRUCTURE

Figure 8 is an example of a structure which was optimized without consideration of its producibility. The lack of linking created an impossible structure to build.

If futher restrictions were made to the sample problem by making all widths and depth of each cross-section equal to the depth of the horizontal members, there would be only four design variables. And if the inclined and vertical members had the same thicknesses, the number of design variables would be three. However, because three design variables would allow very little variation in the structure, obtaining a minimum weight foundation could be very difficult. Experience has shown that with too few design variables an optimum weight foundation which satisfies all constraints frequently cannot be obtained. Therefore, it is simply too restrictive to make all of the box sections square with the same width and depth, but allowing the depth and widths to vary independently allows sufficient leeway to permit an optimum to be found. So having too many design variables or having too few design variables will both produce poor results. The best results are obtained by the judicious blend of design variables, as in this case where there are five design variables.

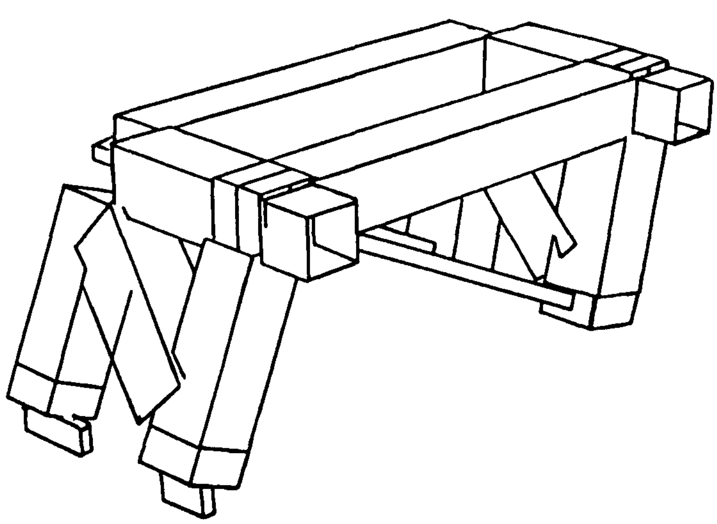


FIGURE 8

THE CHOICE OF THE OPTIMIZATION ALGORITHM

The ADS optimizer in FESOP offers the analyst many different optimization algorithms to choose from: the method of feasible directions (MFD), the modified method of feasible directions (MMFD), sequential linear programming (SLP), sequential quadratic programming (SQP), or sequential convex programming (SCP). (See Figure 9.) In problems with stress and/or displacement constraints, all of these methods will arrive at nearly the same minimum weight solution, with the only difference being the time it takes to arrive at the minimum weight solutions. However, with the combined response spectrum and vibration foundation design problem, which includes frequency avoidance constraints, the choice of the optimizer can make a significant difference. As will be shown, starting at the same point, two different optimization algorithms can produce two different optimum structures. In addition, both methods many not be able to produce an optimum weight foundation which satisfies all the constraints. Thus, no one algorithm will produce the best optimum all of the time. Therefore, in general, at least two of the optimizers should be used to insure the best chance of finding an optimum.

MFD - METHOD OF FEASIBLE DIRECTIONS

MMFD - MODIFIED METHOD OF FEASIBLE DIRECTIONS

SLP - SEQUENTIAL LINEAR PROGRAMMING

SQP - SEQUENTIAL QUADRATIC PROGRAMMING

SCP - SEQUENTIAL CONVEX PROGRAMMING

FIGURE 9

For the sample problem shown in Figures 3a and 3b, two different optimizers were selected to optimize for minimum weight and avoid frequencies from 80 to 120. The starting point for the optimization process was choosen as the upper limit of all design variables. Figure 10 shows the results using both the MMFD and SCP algorithms. In each case the process was started with only frequency avoidance constraints and no stress or displacement constraints. At points A and B the frequency only analysis was stopped and all other constraints were added. This is only one of the many ways to approach the problem. The SCP method arrived at a valid solution, but the MMFD method had two frequency constraint violations. With the MMFD method ADS simply could not find a way to change the design variables to eliminate the frequencies (82.0 and 114.1) within the range 80 to 120. However, Figure 10 also shows the results of using the MMFD method to avoid the frequency range of 48 to 72. In this case, the MMFD method was successful. Therefore, the analyst should always attempt more than one method when trying to avoid frequency ranges. Because the SCP method is the least expensive, I would recommend using it to start and then running the same problem with the MMFD method.

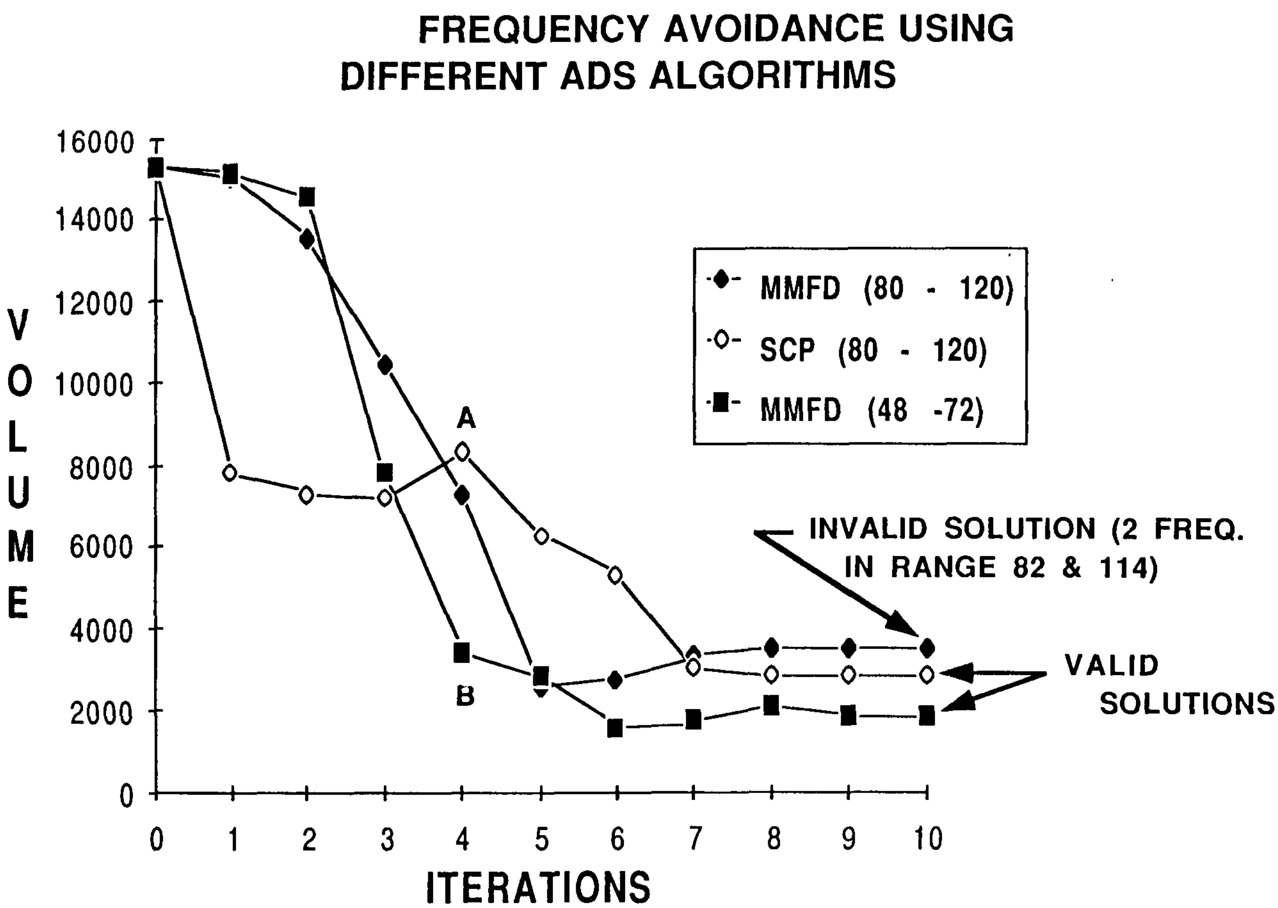


FIGURE 10

STARTING POINT FOR RESPONSE SPECTRUM AND VIBRATION FOUNDATION DESIGN

The shock and vibration foundation design optimization process can begin in any number of ways:

(1) By applying all stress, displacement, b/t ratio, and frequency avoidance constraints from the start
(2) By applying all constraints except the frequency avoidance constraints to obtain a fully stressed design, and by then optimizing with all constraints
(3) By applying only the frequency avoidance constraints until a minimum weight foundation is found, and by then including the rest of the constraints
(4) By using a minimum frequency constraint instead of a frequency avoidance constraint, and by then applying the frequency avoidance constraint
(5) By starting with either a feasible stress design, an understressed design or an overstressed design in combination with one of the above (Figure 11).

Based upon this sample problem, no one starting procedure works best all the time, and some methods for starting almost never work and, therefore, should be avoided. In Figure 10 an understressed design was chosen for the starting point with all design variables at the upper limits. A frequency avoidance only starting procedure for the range 80 to 120 was initiated with two optimization algorithms, MMFD and SCP. Similarly, one was started to avoid the range 48 to 72 using only the MMFD algorithm. In the first case, the SCP algorithm worked and the MMFD did not, however, in the second case the MMFD algorithm worked. Looking at Figures 12 and 13 where other starting point procedures were tried, potentially better optimum solutions exist.

(1) Apply all stress, displacement, b/t ratio, and frequency avoidance constraints from the start

(2) Apply all constraints except frequency avoidance (fully stressed design), and then optimize with all constraints

(3) Apply frequency avoidance only constraint, and then optimize with all constraints

(4) Use minimum frequency only start, and then all constraints

(5) Vary the inital design (lower or bound, feasible or infeasible) in conjunction with the first four procedures

FIGURE 11

In Figure 12, the SCP algorithm was used in conjunction with three different starting procedures in an attempt to arrive at an optimum weight foundation. This foundation was to avoid the natural frequencies of vibration from 48 to 72 and satisfy all stress constraints. The three approaches were

(1) to first optimize with a minimum frequency constraint of either 48 or 72

(2) to first optimize with only a frequency avoidance constraint (i.e. no stress or displacement constraints at the start)

(3) to first optimize with no frequency constraints of any type (i.e. ignoring frequencies)

The first and second approaches were successful in producing an optimum weight structure; however, the optimum volumes differed significantly. In the first case, the final structure had no frequencies of vibration below 72 and a volume of 3300. In the second case (frequency avoidance only), a minimum weight structure with frequencies above and below the range was obtained, with a smaller volume of 2250. Attempting to first optimize with a minimum frequency of 48 and trying to first optimize by ignoring frequencies, both resulted in invalid solutions. For both of these cases, the final structures had unallowable frequencies within the range of 48 to 72. In these unsuccessful cases, the ADS optimizer simply could not find a way to change the design variables so as to move away from an invalid structure. This inability to move to a valid solution clearly demonstrates the need to attempt more than one approach when trying to obtain a minimum weight foundation with frequency constraints.

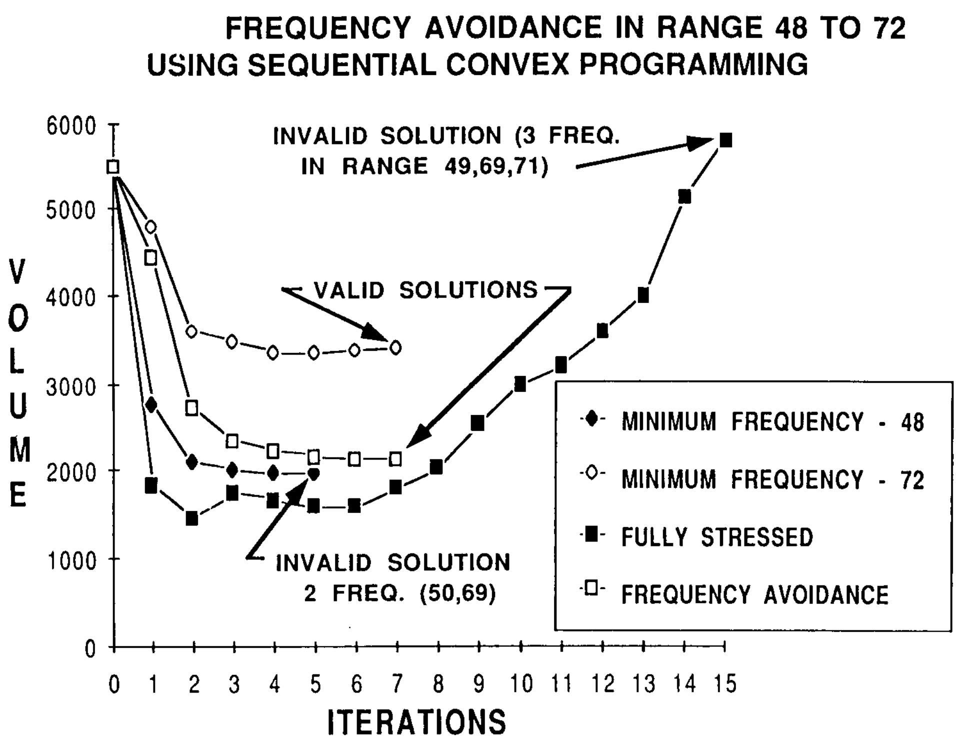


FIGURE 12

In Figure 13, two of the starting procedures employed in Figure 12 were used when trying to avoid frequencies in the range of 80 to 120. First, the SCP algorithm and a frequency avoidance only procedure was attempted. While this combination successfully obtained a minimum weight foundation with no natural frequencies in the range of 48 to 72 (Figure 12), the method was a complete failure when seeking to avoid the frequencies of 80 to 120. Similarly, optimizing with only stress constraints to start was a total failure in Figure 12, but provided two valid solutions in Figure 13, one for the SCP algorithm and one for the MMFD algorithm. The significance of this is that one starting procedure does not work all the time.

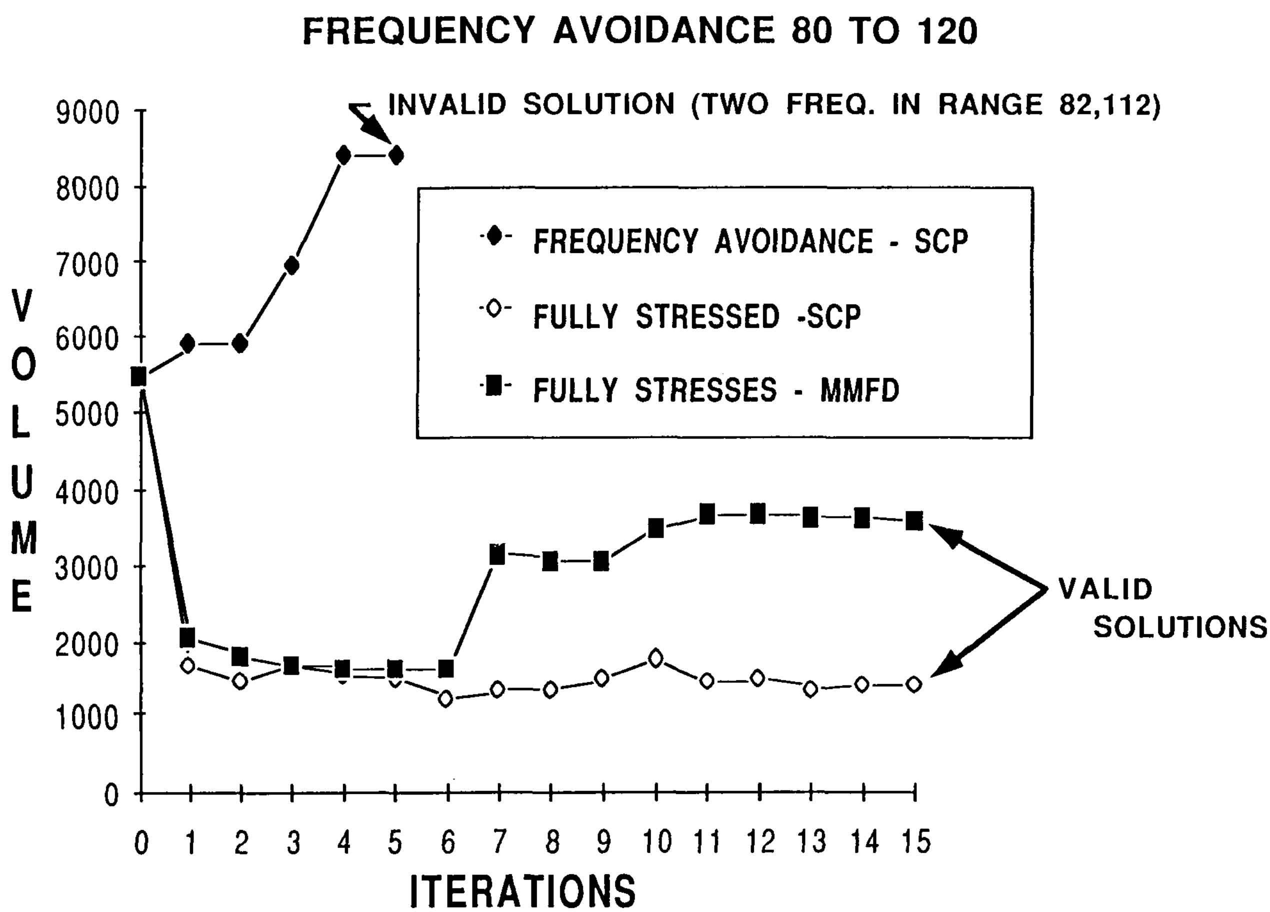


FIGURE 13

By reviewing the results in Figures 11 through 13, a number of conclusions can be drawn.

(1) An optimum weight foundation which avoids certain natural frequencies of vibration can be found using FESOP's combined response and vibration capability.

(2) Because of the highly complex nature of the frequency avoidance problem, a number of attempts with both different starting points and different optimization algorithms should be used to find the best optimum.

(3) A procedure which should give a feasible optimum is to start with all design variables at their upper bound limits and perform a frequency avoidance only analysis with both the SCP and MMFD optimizers.

(4) Next, an attempt from a reasonable design with either a frequency avoidance or maximum stress only starting point should be tried to see if a better optimum is obtained.

(5) If no valid solution is obtained, a minimum frequency constraint for the upper bound of the allowable range should be tried. This should be the last resort because this will always result in the heaviest foundation.

Following these guidelines will help to insure the optimum equipment foundation in terms of producibility and weight.

TABLE - COMPARISON OF RESULTS

Frequency Range	Optimizer	Starting Point	Initial Constraint	Success	Objective
48 TO 72	SCP	*FEASIBLE STRESS DESIGN (FSD)*	$F_{min} > 48$	NO	2000
			$F_{min} > 72$	YES	3300
			$F < 48$, $F > 72$	YES	2250
			Stress Only	NO	5500
	MMFD	*UPPER BOUND*	$F < 48$, $F > 72$	YES	1800*
80 TO 120	SCP	*UPPER BOUND*	$F < 80$, $F > 120$	YES	2800
		FSD	$F < 80$, $F > 120$	NO	8300
			Stress Only	YES	1700*
	MMFD	*UPPER BOUND*	$F < 80$, $F > 120$	NO	3500
		FSD	Stress Only	YES	3300

** Best Optimum For Given Range*

PRODUCIBILITY CONSIDERATIONS

Producibility (Figure 14) is a factor which must be considered at all stages of the optimization process. The definition of the finite element model and, more importantly, the design variables must be made with producibility in mind. Otherwise, a foundation that is clearly unproducible, like the one shown in Figure 8, will result. The first step toward insuring a producible structure is to set limits on design variables which will be both reasonable and producible. However, this alone is not always enough because, during the optimization process, combinations of design variables which were not anticipated will probably result. Therefore, limits on the relationships between design variables should be made. In the sample problem the thickness of the box beams was limited to 25 percent of the cross-section width to insure that unreasonably thick box beams would not result. FESOP allows limits to be specified on the relationship between any two cross-section design variables, thus helping to insure a producible structure. In addition, as was mentioned on the choice of design variables, many design variables should be linked so as to guarantee that the changes in the structure will be uniform. This is important because it means that with design variable linking, radical size and shape changes will not take place.

*A Primary Consideration at All Stages of the Optimization Process

*Vital to Definition of the Design Variables

*Must Also Limit Relations Between Design Variables

*Avoids Unproducible Structures

FIGURE 14

RESULTS/CONCLUSIONS

Based upon the results presented, the following conclusions can be drawn:

(1) One can successfully optimize two different finite element models and analyses with FESOP.

(2) No one ADS optimizer works best all of the time.

(3) Many starting procedures are possible, and each can produce different "optimums".

(4) Producibility is a vital consideration.

(5) The engineer's active participation is essential.

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