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AN ANALYTICAL SENSITIVITY METHOD FOR USE IN INTEGRATED AEROSERVOELASTIC AIRCRAFT DESIGN

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ABSTRACT

Next generation air and space vehicle designs are being driven by increased performance requirements, demanding a high level of design integration between traditionally separate design disciplines. Interdisciplinary analysis capabilities have been developed for aeroservoelastic aircraft and large flexible spacecraft, but the requisite integrated design methods are only beginning to be developed. One integrated design method which has received attention is based on hierarchal problem decompositions, optimization, and design sensitivity analyses. This paper highlights a design sensitivity analysis method for Linear Quadratic Gaussian (LQG) optimal control laws, enabling the use of LQG techniques in the hierarchal design methodology. The LQG sensitivity analysis method calculates the change in the optimal control law and resulting controlled system responses due to changes in fixed design integration parameters using analytical sensitivity equations. Numerical results of a LQG design sensitivity analysis for a realistic aeroservoelastic aircraft example are presented. In this example, the sensitivity of the optimal control law and aircraft response for various parameters such as wing bending natural frequency is determined. The sensitivity results computed from the analytical expressions are used to estimate changes in response resulting from changes in the parameters. Comparisons of the estimates with exact calculated responses show they are reasonably accurate for ± 15% changes in the parameters. It is also shown that evaluation of the analytical expressions is computationally faster than equivalent finite difference calculations.

INTRODUCTION

The design of new generation air and space vehicles is increasingly becoming subject to extensive requirements for design integration, that is, the close coordination of the design of the various parts of the vehicle. For example, many modern fighter aircraft require integration of the flight controls and engines so that sufficient power is available at all flight conditions. Similarly, the aircraft flight control and structural designs must be integrated to avoid potential aeroservoelastic instabilities. To meet the challenge of integrated aircraft design requirements, methods which tie together existing aerodynamic, structure, control, and propulsion design methods are needed.

One such integrated design methodology currently under development at the NASA Langley Research Center is based on hierarchical problem decompositions, multilevel optimization methods, and design sensitivity analyses.[1] This methodology depends on the decomposition of the integrated vehicle design problem into vehicle requirements and separate aerodynamic, structure, control, and/or propulsion subsystem requirements. The subsystem designs are obtained independently subject to a set of fixed design integration parameters, using existing design methods and tools. An iterative optimization method is used to satisfy the integrated vehicle design requirements through modification of the design integration parameters and repeated subsystem designs. Subsystem design sensitivity data relative to the design integration parameters are used as the gradient information for the optimization procedure.

An application of the hierarchal integrated design methodology is to the aeroservoelastic design of aircraft control laws and structure, including the effects of unsteady aerodynamic forces due to structural and control surface motions. This application requires the use of dynamic response requirements as the integrated design objective and a control law design method that uses the feedback signals actually available from the aircraft sensors. Both of these requirements necessitated the development and validation of appropriate design sensitivity expressions. Linear Quadratic Gaussian (LQG) optimal control law methods were selected for the control law design. Dynamic response criteria considered include time responses to control surface motions and discrete aerodynamic gusts, stochastic responses to random gust environments, closed-loop system eigenvalues, and open- and closed-loop frequency responses.

The sensitivity developments have recently been completed.[2] A summary of these results and the application and validation of the sensitivity expressions to an aeroservoelastic aircraft example are described in this paper. Sensitivity results have been computed and are shown for design integration parameters related to aircraft wing bending stiffness, feedback accelerometer location, and control law design specifications. These parameters are typical of those which would be used to obtain an integrated structure/control law design of an aeroservoelastic aircraft by the hierarchal design method. The sensitivity results were also used to validate the analytical sensitivity expressions. This was accomplished by comparing the sensitivity result with changes in responses due to control laws designed for different fixed values of the design integration parameters. Finally, the relative computational effort of computing the sensitivity information using analytical expressions versus numerical finite difference methods was investigated.

INTEGRATED DESIGN METHOD

A general integrated structure/control law design formulation based on hierarchal decompositions and multilevel optimization is shown in Figure 1. In Figure 1, the structural design and the control law design are independent lower level design problems. These lower level designs are coordinated using a set of design integration parameters. The upper level design optimization problem reflects the desired objectives of the integrated aircraft structure/control law design. As a hypothetical example, the upper level objective might be to reduce peak transient responses of the aircraft due to a gust encounter and to reduce the weight of the structure. The actual peak transient responses of the aircraft would come from analysis of the control law design at the lower level, while the actual structural weight would come from the lower level structural optimization. These might then be combined as a weighted sum of square errors between the actual and desired values to form a single upper level optimization performance index. The upper level design variables, which are the design integration parameters, would then be selected to optimize the integrated design.

The values of the design integration parameters at any time are treated as fixed for the lower level designs. The sensitivities of the lower level designs to these fixed parameters are computed and used in turn to compute the gradient of the related part of the upper level performance index. That is, these sensitivities are the gradients necessary to perform the top level optimization. In the present hypothetical example, one of the design integration parameters may be a local structural stiffness requirement, which appears as an equality constraint in the lower level structural design. The sensitivity of the optimized structural weight to this parameter is computed at the lower level and returned for use in computing the part of the gradient of the upper level performance index that is related to structural weight. Another of the design integration parameters might be a maximum allowable control surface deflection limit. The sensitivity of the optimal control law design with respect to this parameter would then be used to compute the sensitivity of the peak transient gust response of the controlled aircraft, as required to perform the upper level design integration optimization.

In many cases, existing nonlinear programming-based structural optimization and design sensitivity analysis methods can be used for the lower level structural design. These methods may themselves be hierarchal, multilevel optimization algorithms.[3-4]

In the rest of this paper, the use of Linear Quadratic Gaussian optimal control law design methods in hierarchal integrated aircraft structure/control law design is examined in detail. Expressions for the sensitivity of controlled system time, frequency, and stochastic responses in terms of state-space coefficient sensitivity matrices are discussed below. The sensitivity of optimized LQG control law to fixed parameters must be known to compute the needed state-space coefficient sensitivity matrices. Analytical expressions for the sensitivities of the LQG gain matrices to fixed problem parameters are discussed next, followed by the controlled system response sensitivity expressions.

LQG CONTROL LAW SENSITIVITY

The Linear Quadratic Gaussian (LQG) optimal control law problem[5-7] is to find the control u(t) for the system

$$\dot{x}_s = A_s x_s + B_s u + D_s w_s \tag{1a}$$

$$y = C_s x_s \tag{1b}$$

$$z = M_s x_s + v \tag{1c}$$

such that the cost function

$$J = \lim_{\tau \to \infty} \frac{1}{\tau} E \int_0^{\tau} \left(y^T Q y + u^T R u \right) dt \tag{2}$$

is minimized, where E denotes expected value and τ is the final time. In equations (1), x_s is the system state vector of dimension ($n_x \times 1$), y is vector of system responses of dimension ($n_y \times 1$), and z is a vector of measured system outputs of dimension ($n_z \times 1$). The vectors w_s ($n_w \times 1$) and v ($n_v \times 1$, $n_v = n_z$) are zero mean, Gaussian distributed, white noise disturbances with intensity matrices W_s and V respectively, and the matrices A_s, B_s, C_s, D_s, and M_s are real valued coefficient matrices of appropriate dimensions.

It is assumed that the matrix pair (A_s, B_s) is stabilizable, the pair (M_s, A_s) is detectable, and the pair (Q_o, A_s) is detectable, where Q_o is defined by $Q_o^T Q_o = C_s^T Q C_s$ and the matrices Q and R are positive semi-definite and positive definite respectively. The solution of the LQG optimal control law problem is then the interconnection of the optimal Linear Quadratic Regulator and the optimal linear state estimator or Kalman Filter.[5-7] In the Kalman Filter, the measured outputs z are used to create estimates for the actual system states x_s. Thus, the optimal control law is

$$u = - G_s x_f \tag{3a}$$

$$\dot{x}_f = A_s x_f + B_s u + F_f (z - M_s x_f) \tag{3b}$$

where the gain matrices F_f and G_s are given by

$$F_f = T M_s^T V^{-1} \tag{4a}$$

$$G_s = R^{-1} B_s^T S \tag{4b}$$

and the matrices S and T are the positive definite solutions of the steady-state nonlinear matrix Riccati equations

$$0 = A_s^T S + S A_s - S B_s R^{-1} B_s^T S + C_s^T Q C_s \tag{5a}$$

$$0 = A_s T + T A_s^T - T M_s^T V^{-1} M_s T + D_s W_s D_s^T \tag{5b}$$

It is assumed that the state-space model coefficient matrices A_s, B_s, C_s, D_s, and M_s, the noise intensity matrices W_s, and V, and the cost function weighting matrices Q and R are time-invariant, continuous, differentiable functions of a number of parameters p_i, $i = 1,...,n_p$, whose nominal values are fixed during the solution of the LQG optimal control law problem. It is further assumed that the functional dependence of the above matrices on the parameters is known so that the partial derivative of each matrix with respect to each parameter is [illegible] known.

Analytical expressions for the sensitivity of the optimal LQG problem solution above to the parameters p_i can be obtained by differentiation of the necessary conditions of optimality. Since the solution of the LQG optimal control problem is the interconnection of the optimal Linear Quadratic Regulator (LQR) and Kalman Filter (KF), the necessary conditions for the LQG problem are the necessary conditions for the LQR and KF problems. Detailed derivations for the sensitivity expressions are presented in reference 2. The results are summarized here.

Regulator Sensitivity - The sensitivity of the optimal LQR gain matrix G_s with respect to the i^{th} parameter p_i is

$$\frac{\partial G_s}{\partial p} = - R^{-1} \frac{\partial R}{\partial p} R^{-1} B_s^T S + R^{-1} \frac{\partial B_s^T}{\partial p} S + R^{-1} B_s^T \frac{\partial S}{\partial p} \tag{6}$$

where the subscript i is dropped for convenience throughout the remainder of the paper except where necessary to avoid confusion. The unknown sensitivity of the steady-state LQR Ricatti solution S is obtained from

$$0 = \frac{\partial S}{\partial p}\left(A_s - B_s G_s\right) + \left(A_s - B_s G_s\right)^T \frac{\partial S}{\partial p} + \left\{ S\frac{\partial A_s}{\partial p} + \frac{\partial A_s^T}{\partial p} S + \frac{\partial Q}{\partial p} - S\left(\frac{\partial B_s}{\partial p} R^{-1} B_s^T - B_s R^{-1} \frac{\partial R}{\partial p} R^{-1} B_s^T + B_s R^{-1} \frac{\partial B_s^T}{\partial p} \right) S \right\} \tag{7}$$

Equation (7) is a linear Lyapunov equation which has a unique solution [reference 5, pg. 103, Lemma 1.5] since the coefficient matrix $A_s - B_s G_s$ is asymptotically stable by the properties of the LQR solution [reference 5, pg. 237, Theorem 3.7]. This equation must be solved for each different design parameter p_i, however the coefficient matrix ($A_s - B_s G_s$) is the same for every parameter. This can be used to advantage in developing a numerical algorithm for solving equation (7) for large numbers of parameters.

Kalman Filter Sensitivity - The sensitivity of the Kalman Filter gain matrix F_f with respect to p is

$$\frac{\partial F_f}{\partial p} = \frac{\partial T}{\partial p} M_s^T V^{-1} + T \frac{\partial M_s^T}{\partial p} V^{-1} - T M_s^T V^{-1} \frac{\partial V}{\partial p} V^{-1} \tag{8}$$

The unknown sensitivity of steady-state KF Ricatti solution T is obtained as the solution of

$$0 = \left(A_s - F_f M_s\right) \frac{\partial T}{\partial p} + \frac{\partial T}{\partial p}\left(A_s - F_f M_s\right)^T + \left\{ \frac{\partial A_s}{\partial p} T + T \frac{\partial A_s^T}{\partial p} + \frac{\partial D_s}{\partial p} W_s D_s^T + D_s \frac{\partial W_s}{\partial p} D_s^T + D_s W_s \frac{\partial D_s^T}{\partial p} + T\left(\frac{\partial M_s^T}{\partial p} V^{-1} M_s - M_s^T V^{-1} \frac{\partial V}{\partial p} V^{-1} M_s + M_s^T V^{-1} \frac{\partial M_s}{\partial p} \right) T \right\} \tag{9}$$

which is also a linear Lyapunov equation with a unique solution since the coefficient matrix $A_s - F_f M_s$ is asymptotically stable by the properties of the Kalman Filter solution. The coefficient matrix is the same for every parameter p_i for this Lyapunov equation as well.

Optimal Controlled System Sensitivity - Defining a state estimate error vector ε, an augmented state vector x, and an augmented noise vector w as

$$\varepsilon = x_s - x_f \tag{10a}$$

$$x = \begin{Bmatrix} x_s \\ \varepsilon \end{Bmatrix} \tag{10b}$$

$$w = \begin{Bmatrix} w_s \\ v \end{Bmatrix} \tag{10c}$$

the open-loop combined system and state estimator can be written in state-space form as

$$\dot{x} = A_o x + Bu + Dw \tag{11a}$$

$$y = Cx \tag{11b}$$

$$u = Gx \tag{11c}$$

where the matrices A_o, B, C, D, and G are defined as

$$A_o = \begin{bmatrix} A_s & 0 \\ B_s G_s & A_s - B_s G_s - F_f M_s \end{bmatrix}$$

$$B = \begin{bmatrix} B_s \\ B_s \end{bmatrix}$$

$$C = \begin{bmatrix} C_s & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} D_s & 0 \\ D_s & -F_f \end{bmatrix}$$

$$G = \begin{bmatrix} -G_s & G_s \end{bmatrix}$$

The closed-loop system state-space equation is then

$$\dot{x} = A_c x + Dw \qquad (12)$$

where the matrix A_c is

$$A_c = \begin{bmatrix} A_s - B_s G_s & B_s G_s \\ 0 & A_s - F_f M_s \end{bmatrix}$$

The derivative matrices of A_o, B, C, D, and G with respect to p are

$$\frac{\partial A_o}{\partial p} = \begin{bmatrix} \frac{\partial A_s}{\partial p} & 0 \\ \frac{\partial B_s}{\partial p} G_s + B_s \frac{\partial G_s}{\partial p} & \frac{\partial A_s}{\partial p} - \frac{\partial B_s}{\partial p} G_s - B_s \frac{\partial G_s}{\partial p} - \frac{\partial F_f}{\partial p} M_s - F_f \frac{\partial M_s}{\partial p} \end{bmatrix}$$

$$\frac{\partial B}{\partial p} = \begin{bmatrix} \frac{\partial B_s}{\partial p} \\ \frac{\partial B_s}{\partial p} \end{bmatrix}$$

$$\frac{\partial C}{\partial p} = \begin{bmatrix} \frac{\partial C_s}{\partial p} & 0 \end{bmatrix}$$

$$\frac{\partial D}{\partial p} = \begin{bmatrix} \frac{\partial D_s}{\partial p} & 0 \\ \frac{\partial D_s}{\partial p} & \frac{\partial F_f}{\partial p} \end{bmatrix}$$

$$\frac{\partial G}{\partial p} = \begin{bmatrix} -\frac{\partial G_s}{\partial p} & \frac{\partial G_s}{\partial p} \end{bmatrix}$$

where the derivative matrices of the optimal gain matrices G_s and F_f are given by equations (6) and (8) respectively. The derivative of the closed-loop state-space dynamics matrix A_c with respect to p is

$$\frac{\partial A_c}{\partial p} = \begin{bmatrix} \frac{\partial A_s}{\partial p} - \frac{\partial B_s}{\partial p} G_s - B_s \frac{\partial G_s}{\partial p} & \frac{\partial B_s}{\partial p} G_s - B_s \frac{\partial G_s}{\partial p} \\ 0 & \frac{\partial A_s}{\partial p} - \frac{\partial F_f}{\partial p} M_s - F_f \frac{\partial M_s}{\partial p} \end{bmatrix}$$

DYNAMIC RESPONSE SENSITIVITY

Equations for the sensitivities of a given linear state-space dynamic system to variations in parameters which define that system can be obtained by partial differentiation of the state-space equations with respect to the desired parameter. For example, consider the linear, time-invariant state-space system

$$\dot{x} = Ax + Bw \qquad (13a)$$

$$y = Cx \qquad (13b)$$

where x is the system state vector of dimension (n_x x 1), w is the system input vector of dimension (n_w x 1), and y is the system output vector of dimension (n_y x 1). The matrices A, B, and C are appropriately dimensioned coefficient matrices. Note that equations (13) can describe either an open or a closed-loop system and that the control input vector w is taken here to be a general input or reference command.

The sensitivity of the system state and output response[8] can be obtained from the equations

$$\frac{\partial \dot{x}}{\partial p} = \frac{\partial A}{\partial p}x + A\frac{\partial x}{\partial p} + \frac{\partial B}{\partial p}w \qquad (14a)$$

$$\frac{\partial y}{\partial p} = \frac{\partial C}{\partial p}x + C\frac{\partial x}{\partial p} \qquad (14b)$$

where the order of differentiation with respect to time t and the parameter p has been interchanged in (14a).

Equations (13) and (14) can be combined into a single system of equations as

$$\begin{Bmatrix} \dot{x} \\ \frac{\partial \dot{x}}{\partial p} \end{Bmatrix} = \begin{bmatrix} A & 0 \\ \frac{\partial A}{\partial p} & A \end{bmatrix} \begin{Bmatrix} x \\ \frac{\partial x}{\partial p} \end{Bmatrix} + \begin{bmatrix} B \\ \frac{\partial B}{\partial p} \end{bmatrix} w \qquad (15a)$$

$$\begin{Bmatrix} y \\ \frac{\partial y}{\partial p} \end{Bmatrix} = \begin{bmatrix} C & 0 \\ \frac{\partial C}{\partial p} & C \end{bmatrix} \begin{Bmatrix} x \\ \frac{\partial x}{\partial p} \end{Bmatrix} \qquad (15b)$$

or more compactly as

$$\dot{x}_\rho = A_\rho x_\rho + B_\rho w \qquad (16a)$$

$$y_\rho = C_\rho x_\rho \qquad (16b)$$

where the subscript ρ refers to sensitivity equations, and the definitions of the vectors and coefficient matrices follow from equations (15).

Time Response - The sensitivity of the time response of a state-space system to known parameter variations is obtained by solving equations (16) as a function of time for a given input w(t). The theoretical solution for systems of equations of this type is well known[9] and is given by

$$x_\rho(t) = e^{A_\rho t} x_\rho(0) + \int_0^t e^{A_\rho (t-\tau)} B_\rho w(\tau) d\tau \qquad (17a)$$

$$y_\rho(t) = C_\rho x_\rho(t) \qquad (17b)$$

where $x_\rho(0)$ is the initial condition of the system defined by equations (16).

Frequency Response - The frequency response of a linear time-invariant state-space system can be obtained by Laplace transformation of the state-space equations and replacement of the Laplace transform variable s with the complex frequency s = jω (for zero system initial conditions)[5,9]. The sensitivity of the frequency response can be obtained using the same technique on the system sensitivity equations (16). The result is

$$y_\rho(j\omega) = C_\rho \left(j\omega I - A_\rho \right)^{-1} B_\rho w(j\omega) \qquad (18)$$

The frequency response h(jω) and frequency response sensitivity $h_\rho(j\omega)$ of a given input/output pair calculated from equation (18) are complex quantities expressed in terms of real and imaginary components as a function of frequency ω. These quantities are usually more conveniently expressed in terms of magnitude and phase, and sensitivity of the magnitude and phase, than in their real and imaginary components.

The magnitude |h| and phase ϕ of a complex quantity h(jω) = a + jb can be calculated from the real and imaginary components as

$$|h| = \sqrt{a^2 + b^2} \qquad (19a)$$

$$\phi = \tan^{-1}\frac{b}{a} \qquad (19b)$$

The sensitivities of the magnitude and phase are obtained by differentiating equations (19) with respect to p

$$\frac{\partial |h|}{\partial p} = \frac{1}{|h|}\left(a\frac{\partial a}{\partial p} + b\frac{\partial b}{\partial p} \right) \qquad (20a)$$

$$\frac{\partial \phi}{\partial p} = \frac{1}{|h|^2}\left(a\frac{\partial b}{\partial p} - b\frac{\partial a}{\partial p} \right) \qquad (20b)$$

where ∂a/∂p and ∂b/∂p are the real and imaginary components of ∂h/∂p respectively.

Singular Values - The output vector y(s) is related to w(s) by a transfer function matrix of dimension $n_y \times n_u$ in the multi-input, multi-output case. When the transfer function matrix is some $n_u \times n_u$ or $n_y \times n_y$ matrix H(s) of a controlled system, it is often desirable to compute the minimum and maximum singular values of H(s) as a function of the complex frequency s = jω, since these singular value quantities have been related to various control system design criterion[10]. The singular values and the sensitivity of the singular values to parameters as a function of frequency ω can be obtained from H(jω) and $\partial H(j\omega)/\partial p$ using the definition of the singular value decomposition of a complex matrix.

For the square complex matrix H of dimension $n_h \times n_h$, the singular value decomposition of H is defined by

$$H = U\Sigma V^* \quad (21)$$

where (*) denotes complex conjugate transpose, U and V are unitary transformation matrices, and Σ is

$$\Sigma = \mathrm{diag}([\sigma_1\ \sigma_2\ \ldots\ \sigma_{n_h}]) \quad (22)$$

where σ_i is the i^{th} real scalar singular value of H. For the i^{th} singular value, equation (21) can be written as either

$$Hv_i = u_i\sigma_i \quad (23a)$$

or

$$H^* u_i = v_i\sigma_i \quad (23b)$$

where u_i and v_i are the i^{th} columns of U and V respectively.

For the case when H is a known continuous function of the parameter p_j for which the derivative $\partial H/\partial p_j$ is also known and the σ_i are distinct (no repeated singular values), then the sensitivity of the i^{th} singular value σ_i to the parameter p is[11-13]

$$\frac{\partial \sigma_i}{\partial p} = \mathrm{Real}\left(u_i^* \frac{\partial H}{\partial p} v_i \right) \quad (24)$$

Eigenvalues - The eigenvalues and right eigenvectors of a real $n_x \times n_x$ matrix A are defined by the equation[9]

$$AE = E\Lambda \quad (25)$$

where E is the matrix whose columns are the right eigenvectors and Λ is a diagonal matrix with the i^{th} diagonal element being the i^{th} eigenvalue, assuming A has no repeated eigenvalues and a full set of linearly independent eigenvectors. The left eigenvectors are just the rows of the inverse modal matrix E^{-1}. For the i^{th} eigenvalue λ_i, equation (25) can be written as either

$$Ae_i = \lambda_i e_i \quad (26a)$$

$$l_i^T A = \lambda_i l_i^T \quad (26b)$$

where e_i and l_i are the i^{th} right and left eigenvectors respectively.

The sensitivity of the i^{th} eigenvalue λ_i with respect to the parameter p is[14-16]

$$\frac{\partial \lambda_i}{\partial p} = l_i^T \frac{\partial A}{\partial p} e_i \quad (27)$$

Covariance Response - Often the response of an asymptotically stable linear system to random disturbances or inputs which can only be described in statistical terms is desired. In this case, the response of the system is computed using the covariance equation and the noise intensity matrices which model the random disturbance or noise[5-6]. Equations for the sensitivity of the covariance response have been developed by differentiation of the covariance equation with respect to the parameter variation of interest.[17]

The input w(t) of the linear system given by equations (13) is assumed to be a zero mean, Gaussian distributed, "white" noise with noise intensity matrix W. The steady-state covariance response of the (asymptotically stable) system is given by solution of the steady-state covariance equation

$$0 = AX + XA^T + BWB^T \quad (28)$$

and the steady-state output covariance is

$$Y = C^T XC \quad (29)$$

Assuming the derivative matrices of A, B, C, and W with respect to a parameter p_i are known, equation (28) can be differentiated with respect to p_i to obtain

$$0 = A\frac{\partial X}{\partial p} + \frac{\partial X}{\partial p}A^T + \left\{ \frac{\partial A}{\partial p}X + X\frac{\partial A^T}{\partial p} + \frac{\partial B}{\partial p}WB^T + B\frac{\partial W}{\partial p}B^T + BW\frac{\partial B^T}{\partial p} \right\} \quad (30)$$

Equation (30) is linear Lyapunov equation which has a unique solution by virtue of strictly stable eigenvalues of the A matrix. The sensitivity of the output covariance is

$$\frac{\partial Y}{\partial p} = \frac{\partial C}{\partial p}XC^T + C\frac{\partial X}{\partial p}C^T + CX\frac{\partial C^T}{\partial p} \quad (31)$$

AEROSERVOELASTIC AIRCRAFT SENSITIVITY STUDY

Description - An aeroservoelastic control law design sensitivity problem was formulated and analyzed for various control law and structural parameter variations. This was done to numerically demonstrate the application of the sensitivity developments of previous sections to a realistic aircraft structure/control law design problem.

A mathematical model of the longitudinal dynamics of the Drone for Aerodynamic and Structural Testing, Advanced Research Wing-II (DAST ARW-II) Firebee aircraft, Figure 2, was developed for this example.[18-19] The open-loop state-space model is of 25^{th} - order incorporating rigid body plunge and pitch modes, three elastic vibration modes, elevon and aileron control surfaces with actuators, and a Dryden[20] second-order vertical gust input model. It has elevon and aileron commanded deflections as inputs, pitch rate and normal acceleration at the c.g., wing acceleration at two locations, and actual control surface deflections and rates as outputs. At Mach 0.75 and 15,000 feet altitude, the open-loop aircraft model has two real short period roots, one stable at -3.625/sec. and one unstable at 1.101/sec., and a lightly damped aeroelastic flutter mode with eigenvalues at $-0.162 \pm j118.3$/sec.

A control law design problem was formulated to stabilize the unstable short period root of the aircraft while maintaining or augmenting the stability of the aeroelastic mode using reasonable control surface deflections and rates. Center-of-gravity pitch rate and acceleration, and wing acceleration from the aft wing accelerometer were selected as feedback measurements. These measurements were assumed to be noisy. A random gust environment of 5 ft./sec. (60 in./sec.) root mean square vertical gust velocity was selected for the control law design.

The original eight mathematical model outputs were weighted in the LQG cost function by the matrix Q. The diagonal elements of Q were selected using the "Bryson" rule [reference 6, pg. 169] as one over the square of the desired maximum output. The weighting matrix R on commanded control surface deflections was selected as an identity matrix. The Dryden gust input noise intensity was selected to give a 5 ft./sec. RMS gust input, and the measurement noises were selected to be approximately 10% of the expected output due to the gust input. Table 1 summarizes the numerical values for the weighting matrices and noise statistics.

Numerous parameters were selected to exercise the sensitivity analysis techniques described in previous chapters. Sensitivity results for four of these parameters are presented here. The four parameters and their nominal values are listed in Table 2. Parameter 1 is an element of the cost function weighting matrix Q, affecting the regulator portion of the optimal LQG control law solution. Parameters 2 is an element of the noise intensity matrix V, which affects the Kalman Filter portion of the LQG solution. Parameter 3 is a factor that simulates the effects of structural wing bending stiffness changes by uniformly scaling the two wing bending mode natural frequencies. Parameter 4 locates the aft wing accelerometer relative to the forward accelerometer through a scaling of the nominal longitudinal separation distance between the accelerometers. Sensitivity matrices of the open-loop state-space model and LQG matrices to the four parameters were also generated.

Sensitivity - A numerical sensitivity analysis of the aeroservoelastic example problem was conducted. This numerical analysis consisted of the following: 1) solution of the optimal LQG control law problem for nominal parameter values, 2) calculation of the sensitivity of the LQG solution to the four parameters, and 3) computation of the nominal controlled system eigenvalues, covariance response, and time and frequency responses, and the sensitivity of those responses, to the four parameters using the optimal control law sensitivity information.

Sensitivity results presented throughout this paper are multiplied by the nominal value of the parameter of interest, such that the (semi-relative) sensitivity results for every parameter can be directly compared on a percent parameter change basis. This type of semi-relative sensitivity data has the same units as the response of interest in all cases.

Closed-loop system eigenvalues and their sensitivities to each of the parameters were computed using equations (26a) and (27). The closed-loop short period and flutter mode eigenvalues and their sensitivities are given in Table 3. Note in Table 3 that the sign order of the sensitivity of the imaginary part is significant. The notation ± means a positive change in the parameter will increase the magnitude of the imaginary part of the eigenvalue, whereas the notation -/+ means a positive parameter change will decrease the magnitude. Neither the short period nor flutter mode eigenvalues are affected at all by parameters 2 and 4, since these parameters affect only the Kalman Filter portion of the LQG solution. The wing bending stiffness parameter, while having an expected large effect on the flutter mode eigenvalues, also has a significant effect on the aircraft short period mode eigenvalues since the sensitivity results are of the same order of magnitude.

Covariance responses and sensitivities of the aircraft model to a 5 ft./sec. RMS random vertical gust environment were computed using equations (28) - (31). Mean-square responses and sensitivities derived from the covariance results for aircraft pitch rate, normal c.g. acceleration, and wing acceleration at the forward wing accelerometer are given in Table 4. Note that an increase in wing bending stiffness would tend to decrease the aircraft pitch rate, c.g. acceleration, and the wing tip acceleration in the random gust environment, as would moving the aft wing accelerometer forward (a negative change in parameter 4). Parameter 1 could be used to tradeoff pitch rate response with c.g. and wing acceleration since the sensitivity derivatives have opposite signs.

Output time responses and sensitivities were computed for the closed-loop aircraft subjected to the 1 - cosine discrete gust

$$w(t) = \begin{cases} 60 \times (1.0 - \text{cosine}\,(\pi t / .25)) & 0.0 \le t \le .25 \\ 0.0 & .25 < t \le 1.0 \end{cases} \tag{32}$$

using equations (17). Time histories of aircraft pitch rate and c.g. acceleration are shown in Figures 3 and 4. Also shown in these figures are the sensitivities of the responses to parameters 3 and 4. Note here that of the two parameters, the pitch rate response is most sensitive to the wing bending stiffness. The sensitivity of the c.g. acceleration is largest with respect to parameter 3, indicating that wing bending stiffness is a significant factor in normal accelerations due to gust encounters.

The complex frequency response and sensitivities of the elevon open-loop transfer function with the aileron loop closed were calculated using equation (18). The complex (real and imaginary) results were converted to magnitude and sensitivity of the magnitude using equations (20). The magnitude result is shown in Figure 5, as are the sensitivities of the magnitude to parameters 3 and 4. The magnitude is most sensitive to the wing bending stiffness at about .6 rad/sec., although the peak sensitivity for parameter 4 coincides with the peak of the magnitude at 1.1 rad/sec.

Sensitivity Validation - The sensitivity analysis results were evaluated for accuracy by comparing predicted covariance responses with covariance responses computed for variations in the nominal values of the parameters. The four parameters were varied ± 25% from their nominal values in 5% increments, and the new optimal control law and controlled aircraft covariance response were computed for each parameter variation. These computed responses were compared with sensitivity derivative-based first-order predictions of the response computed by

$$f_p = f_n + \frac{\partial f}{\partial p} \times \Delta p \tag{33}$$

where f refers to the response of interest, Δp is the parameter change, $\partial f/\partial p$ is the appropriate sensitivity derivative, and the subscripts $_p$ and $_n$ refer to predicted and nominal responses respectively. Percentage errors in the covariance response predictions were calculated as

$$\%E = \frac{f_c - f_p}{f_c} \times 100 \tag{34}$$

where the subscript $_c$ refers to the computed response.

Validation results for the prediction of the aircraft pitch rate, c.g. acceleration, and aft wing accleration due to variations in parameter 3, the wing bending stiffness parameter, are shown in Figure 6. The percent errors in predicting pitch rate and c.g. acceleration are reasonable even for large variations in the value of the parameter. Further, the slope of the error curve is zero near the zero parameter change(nominal value) point, where the sensitivity derivative used for the prediction was originally calculated. This indicates that the sensitivity derivative is exact at this point, verifying the derivation of the analytical sensitivity expressions. The percent error results for the aft wing acceleration prediction are larger than for the other two response predictions, however the error is less than about -30% for ± 10% variations in the parameter. In an actual application of these sensitivity methods, parameter variation magnitudes would normally be restricted by good engineering practice to be relatively small values, closer to the region where the sensitivity results are nearly exact.

COMPUTATIONAL COSTS

The computational burden associated with the numerical evaluation of the analytical LQG problem sensitivity equations can appear to be substantial, since solution of two matrix Lyapunov equations (equations (7) and (9)) is required to obtain the sensitivities of the linear quadratic regulator and Kalman Filter gain matrices to a single parameter. For this reason, a comparison of the analytical sensitivity evaluation versus one- and two-step finite difference calculations for the equivalent sensitivity information was made. The measure of comparison for the three calculations was central processing unit time (CPU seconds) on a Digital Equipment Corporation MicroVAX II computer, where the LQG sensitivity equations were programmed as user functions to a commercially available linear systems analysis computer code.

The one-step regulator gain matrix G finite difference sensitivity was calculated as

$$\frac{\partial G}{\partial p} \cong \frac{G_1 - G_n}{.025} \tag{35}$$

where G_1 is the perturbed LQR gain for the wing bending stiffness parameter (parameter 3) perturbed positively by 2.5% (1.025 times the nominal value). The KF gain matrix F finite difference sensitivity was calculated similiarly.

The two-step regulator gain matrix G finite difference sensitivity was calculated as

$$\frac{\partial G}{\partial p} \cong \frac{G_2 - G_1}{.05} \tag{36}$$

and the wing bending stiffness parameter was perturbed by ± 2.5%. G_2 refers the LQR gain matrix obtained for parameter 3 perturbed to 1.025 times nominal, and G_1 refers to the gain matrix for parameter 3 perturbed to .975 times nominal. The Kalman Filter gain matrix F sensitivity was again calculated in the same manner.

The results of the CPU time comparisons are shown in Table 5, as is the CPU time required for solution of the LQG problem without sensitivity calculations. These results show that the analytical sensitivity expressions require substantially less CPU time than either the one- or two-step numerical finite difference approaches for a single parameter sensitivity analysis, and require only a 33% increase in CPU time over the nominal LQG problem solution. Furthermore, the computational advantage of the analytical approach is likely to increase when sensitivity calculations for more than one parameter are involved, since additional computational efficiency can be achieved by storage of the decomposed coefficient matrices of the sensitivity equations (7) and (9). Similar computational efficiencies are not possible with the finite difference approaches, since they require a solution of the LQG problem for each perturbation of the parameter of interest.

CONCLUSION

This paper has highlighted a method for computing the sensitivity of optimal LQG control laws to various parameters using analytical sensitivity expressions. The LQG sensitivity results are used to predict changes in closed-loop aircraft responses due to changes in the nominal values of the parameters of interest. These sensitivity results are shown to be useable for hierarchal integrated structure/control law design problems through a large aeroservoelastic aircraft example. Sensitivities of covariance, time, and frequency responses of the aircraft to various parameters were computed. The sensitivity results were validated against computed response changes due to changes in the nominal values of various parameters and found to be accurate for ± 15% changes in the parameter values. It was also found that it is cheaper to evaluate the analytical LQG sensitivity expressions than to calculate the equivalent sensitivity information by finite difference means.

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TABLES

Table 1 LQG Control Law Problem Data

Symbol	Value	Description
Q(1,1)	0.01	Pitch Rate Weight
Q(2,2)	1.00	C.G. Acceleration Weight
Q(3,3)	0.01	Fwd. Wing Acceleration Weight
Q(4,4)	0.01	Aft Wing Acceleration Weight
Q(5,5)	2.04×10^{-2}	Elevon Deflection Weight
Q(6,6)	4.40×10^{-3}	Aileron Deflection Weight
Q(7,7)	1.56×10^{-4}	Elevon Rate Weight
Q(8,8)	1.83×10^{-6}	Aileron Rate Weight
R(1,1)	1.00	Commanded Elevon Weight
R(2,2)	1.00	Commanded Aileron Weight
V(1,1)	2.00×10^{-3}	Pitch Rate Noise Intensity
V(2,2)	1.50×10^{-3}	C.G. Accl. Noise Intensity
V(3,3)	6.00×10^{-3}	Aft Wing Accl. Noise Intensity.
U(1,1)	1.00×10^{-6}	Input Noise Intensity (Elevon Loop)
U(2,2)	1.00×10^{-6}	Input Noise Intensity (Aileron Loop)
W	3.60×10^{3}	Dryden Gust Model Noise Intensity

Table 2 Integrated Aircraft Design Problem Parameters.

Number	Nominal Value	Description
1	0.01	Q(1,1) Pitch Rate Weight
2	2.00×10^{-3}	V(1,1) Pitch Rate Noise Intensity
3	1.00	Wing Bending Stiffness Factor
4	7.58	Aft Wing Accl. Location

Table 3 Semi-Relative Closed-Loop Short Period and Flutter Mode Eigenvalue Sensitivities to Parameters.

Param.	Short Period Mode (1/sec.) $(-5.136 \pm j2.742)$	Flutter Mode (1/sec.) $(-5.046 \pm j1.178 \times 10^{2})$
1	$-4.74 \times 10^{-1} -/+ j7.28 \times 10^{-1}$	$4.69 \times 10^{-5} -/+ j1.50 \times 10^{-5}$
2	0.00	0.00
3	$4.72 \times 10^{-1} \pm j4.04$	$3.70 \pm j2.83 \times 10^{1}$
4	0.00	0.00

Table 4 Semi-Relative Closed-Loop Mean-Square Response Sensitivities to Parameters (5 ft./sec. RMS Gust).

Param.	Pitch Rate (deg./sec.) (5.15×10^{-2})	C.G. Accl. (g's) (2.65×10^{-2})	Wing Accl. (g's) (2.35×10^{1})
1	-6.18×10^{-3}	2.24×10^{-4}	6.24×10^{-5}
2	5.91×10^{-3}	9.91×10^{-4}	5.21×10^{-3}
3	-9.84×10^{-2}	-3.38×10^{-3}	-5.32×10^{1}
4	1.35×10^{-3}	1.16×10^{-3}	2.94×10^{-1}

Table 5 CPU Time Comparisons of Analytical Sensitivity Expressions versus Finite Difference Calculations.

Method	Time (Sec.)
LQG Solution Only	100.68
Analytical Sensitivity Expressions	133.55
One-Step Finite Difference	196.44
Two-Step Finite Difference	287.39

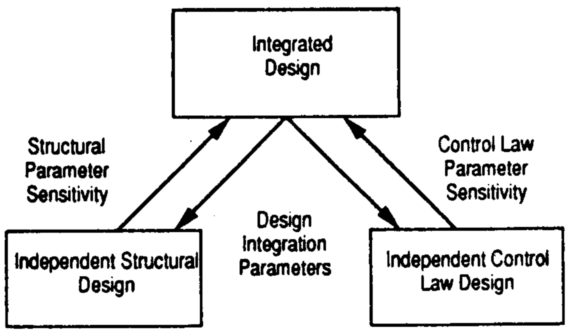

Figure 1. General integrated structure/control law design problem formulation.

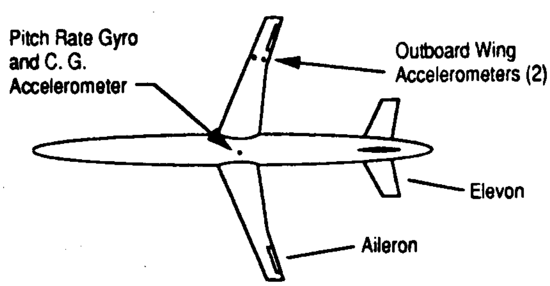

Figure 2. Aeroservoelastic aircraft example problem configuration.

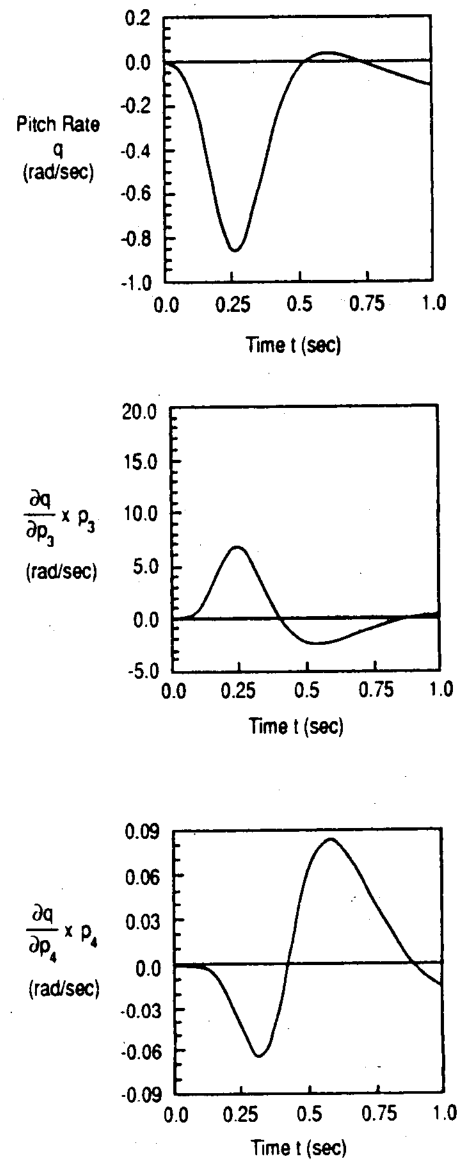

Figure 3. Pitch rate response to 1 - cosine discrete gust and sensitivity to parameters 3 and 4.

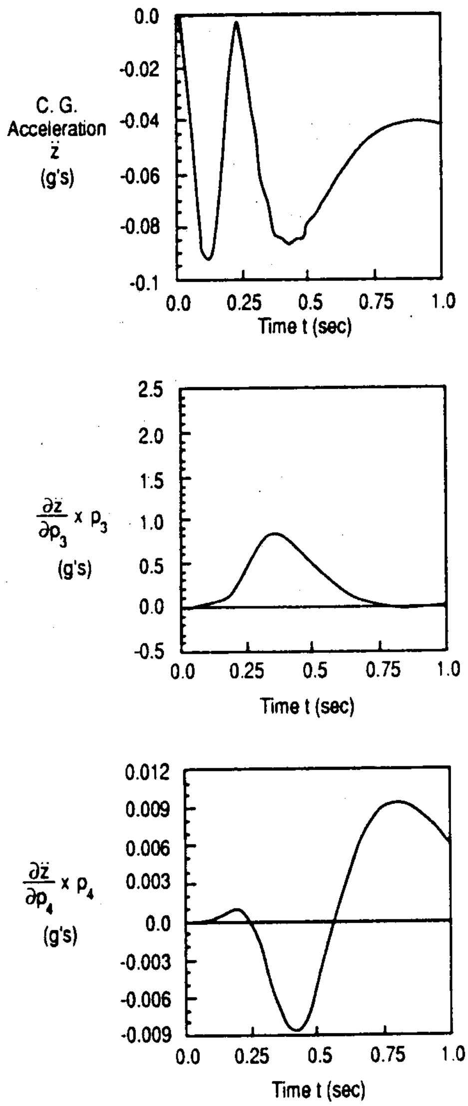

Figure 4. C. G. acceleration response to 1 - cosine discrete gust and sensitivity to parameters 3 and 4.

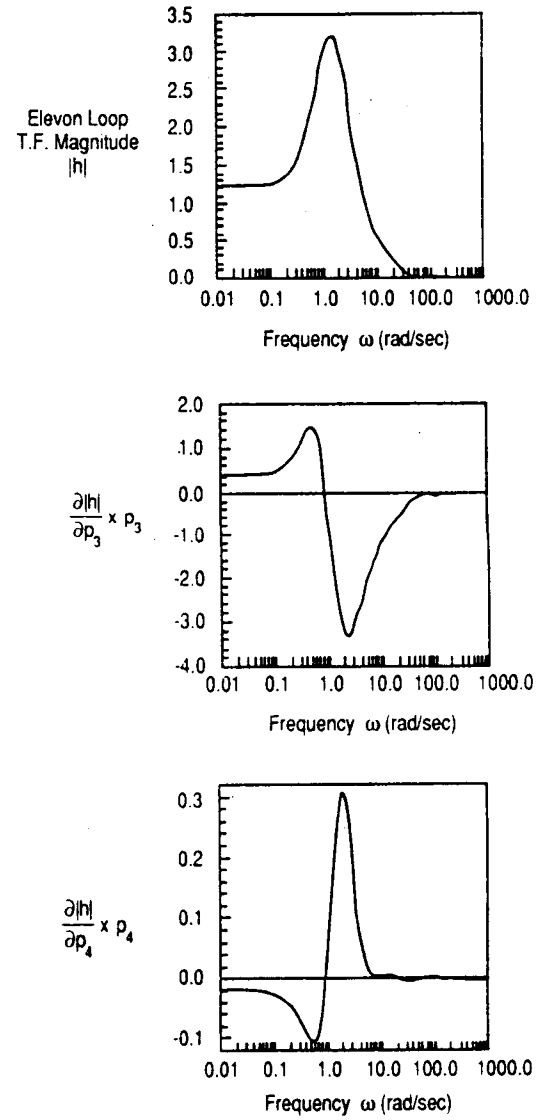

Figure 5. Open elevon loop transfer function magnitude and sensitivity to parameters 3 and 4 (aileron loop closed).

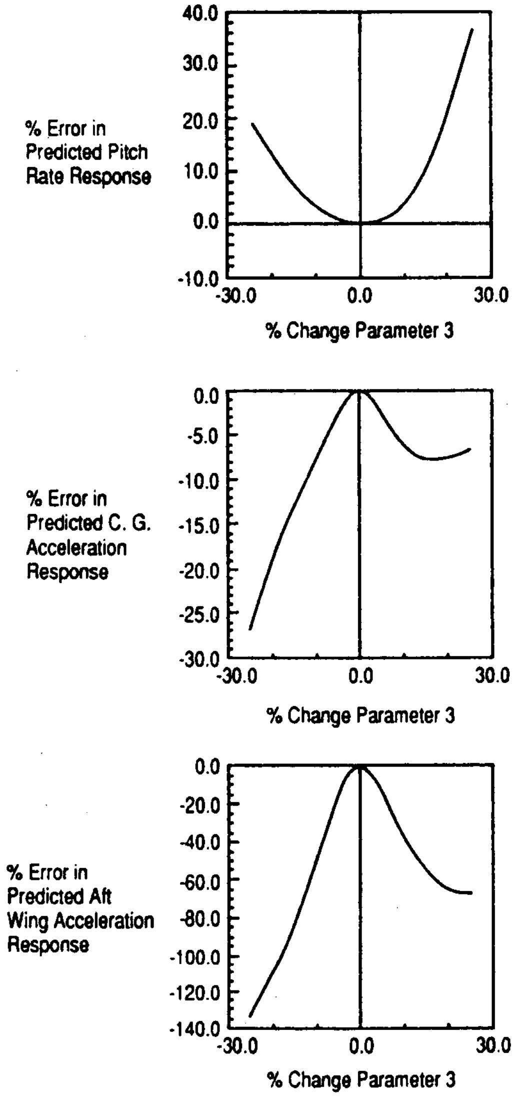

Figure 6. Percent prediction errors in covariance responses for ± 25% changes in parameter 3 nominal value.

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16. Abstract

Next generation air and space vehicle designs are being driven by increased performance requirements, demanding a high level of design integration between traditionally separate design disciplines. Interdisciplinary analysis capabilities have been developed for aeroservoelastic aircraft and large flexible spacecraft, but the requisite integrated design methods are only beginning to be developed. One integrated design method which has received attention is based on hierarchal problem decompositions, optimization, and design sensitivity analyses. This paper highlights a design sensitivity analysis method for Linear Quadratic Gaussian (LQG) optimal control laws, enabling the use of LQG techniques in the hierarchal design methodology. The LQG sensitivity analysis method calculates the change in the optimal control law and resulting controlled system responses due to changes in fixed design integration parameters using analytical sensitivity equations. Numerical results of a LQG design sensitivity analysis for a realistic aeroservoelastic aircraft example are presented. In this example, the sensitivity of the optimal control law and aircraft response for various parameters such as wing bending natural frequency is determined. The sensitivity results computed from the analytical expressions are used to estimate changes in response resulting from changes in the parameters. Comparisons of the estimates with exact calculated responses show they are reasonably accurate for ±15% changes in the parameters. It is also shown that evaluation of the analytical expressions is computationally faster than equivalent finite difference calculations.

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