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Statistical Fluctuations as the Origin of Nontopological Solitons

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Abstract

Nontopological solitons can be formed during a phase transition in the early universe as long as some net charge can be trapped in regions of false vacuum. It has been previously suggested that a particle-antiparticle asymmetry would provide a source for such trapped charge. We point out that, for the model and parameters considered, statistical fluctuations provide a much larger concentration of charge, and are therefore, the dominant source of charge fluctuations in solitogenesis.

I. INTRODUCTION

Nontopological soliton solutions in classical field theories have appeared in many forms since they were first introduced by Rosen[1] and by Friedberg, Lee, and Sirlin.[2] Examples include Q balls, [3] quark nuggets,[4] cosmic neutrino balls, [5] and soliton stars.[6]

The simplest nontopological soliton (NTS) solution involves a real scalar field, σ, and a complex scalar field, ϕ, with Lagrangian[7]

$$\begin{aligned} \mathcal{L} &= \partial_\mu\phi(\partial^\mu\phi)^\dagger + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(|\phi|,\sigma) \\ U(|\phi|,\sigma) &= \frac{\lambda_1}{8}(\sigma^2-\sigma_0^2)^2 + h|\phi|^2(\sigma-\sigma_0)^2 + \frac{\lambda_2}{3}(\sigma-\sigma_0)^3\sigma_0 + \Lambda, \end{aligned} \tag{1}$$

where the constant Λ is adjusted to give $U(0,\sigma)=0$ at the global minimum of the potential. The classical potential for σ has two local minima. At the global miminum ($\sigma = \sigma_- = -[(1+2\lambda_2/\lambda_1)/2 + [(1+2\lambda_2/\lambda_1)^2 + 8\lambda_2/\lambda_1]^{1/2}/2]\sigma_0$) the ϕ field has a mass $m_\phi^2 = h(\sigma_- - \sigma_0)^2$, while at the local minimum ($\sigma=\sigma_0$) the field ϕ is massless. The nontopological solition solution describes a configuration of massless ϕ particles trapped inside a region with $\sigma=\sigma_0$, separated from the true vacuum $\sigma=\sigma_-$ by a wall of thickness $\sim \sigma_0^{-1}$. The energy of the NTS configuration is[8]

$$E \approx \frac{\pi Q}{R} + \frac{4}{3}\pi\Lambda R^3 + \mathcal{O}(R^2\lambda_1^{1/2}\sigma_0), \tag{2}$$

where $Q = |N_\phi - N_{\bar{\phi}}|$ is the "charge" contained in the spherical NTS of radius R, and Λ is given by $\Lambda = -\lambda_1(\sigma_-^2-\sigma_0^2)^2/8 - \lambda_2(\sigma_- - \sigma_0)^3\sigma_0/3$. The three terms in Eq.(2) represent the kinetic energy of the confined massless ϕ field, the false vacuum energy of the NTS, and the surface energy of the wall separating the interior NTS region from the true vacuum. We will ignore the wall contribution in our analysis.

Minimizing the energy of the NTS configuration results in an NTS of mass and radius $M = (4\pi/3)\sqrt{2}Q^{3/4}\Lambda^{1/4}$ and $R = (Q/4\Lambda)^{1/4}$. This mass should be compared to the mass of Q free ϕ's in the true vacuum, $M_{\text{free}} = Qm_\phi = Qh^{1/2}|\sigma_- - \sigma_0|$. The NTS configuration will have a lower mass, and hence be stable, for charge Q greater than some minimum charge, given by

$$Q_{\text{MIN}} = \frac{1231}{h^2}\frac{\Lambda}{(\sigma_- - \sigma_0)^4}. \tag{3}$$

In this paper we will study in detail the case $\lambda_2/\lambda_1 = 0.15$. For this choice of λ_2/λ_1, $\Lambda = 0.6\lambda_1\sigma_0^4$, $Q_{\text{MIN}} = 18\lambda_1/h^2$, and $M_{\text{MIN}} = 46(\lambda_1/h^{3/2})\sigma_0$.

A scenario for the cosmological origin of NTS was proposed by Freeman, Gelmini, Gleiser, and Kolb[8] (hereafter, FGGK). In the FGGK scenario, there is a critical temperature, $T_C \simeq 2\sigma_0$, below which the Universe divides into domains of true ($\sigma = \sigma_-$) and false ($\sigma = \sigma_0$) vacuum. The characteristic size of these domains is determined by the correlation length, ξ, of the σ field at the transition. At high temperatures thermal fluctuations can cause a correlation volume to make the transition between the two minima. These fluctuations freeze out at the "Ginzburg" temperature, T_G. FGGK estimate T_G by the criterion that T_G is equal the maximum free energy of the correlation volume in the transition $F_M = U_M V_\xi$ (U_M is the maximum value of the potential in the region $\sigma_- \le \sigma \le \sigma_0$). For $\lambda_2/\lambda_1 = 0.15$, $T_G = 1.3\sigma_0/\lambda_1^{1/2}$. Of course T_G can never be larger than $T_C \simeq 2\sigma_0$.

At T_G, the probabilities of being in the false vacuum, $p(\sigma_0)$, and true vacuum, $p(\sigma_-)$, are Boltzmann distributed according to the difference in free energies of a correlation volume in the different minima

$$\frac{p(\sigma_0)}{p(\sigma_-)} = \exp[-\Delta F/T_G] = \exp[-\Lambda V_\xi/T_G] \tag{4}$$

(recall that $U(\sigma_-) \equiv 0$ by the addition of Λ). If $p(\sigma_0)/p(\sigma_-) \le 0.3$ then only finite regions of "false" vacuum will be populated. If the regions of false vacuum contain a net charge $Q \equiv |N_\phi - N_{\overline{\phi}}| > Q_{\rm MIN}$, true false vacuum regions can be stabilized and evolve to nontopological solitons.

The probability that a false vacuum region contains a charge $Q > Q_{\rm MIN}$ is the subject of this paper.

II. CHARGE FLUCTUATIONS

FGGK assumed that the net charge in a region was proportional to a cosmic asymmetry, like baryon number, between ϕ and $\overline{\phi}$. This cosmic asymmetry can be expressed in terms of an asymmetry parameter η:

$$\eta \equiv \frac{|n_\phi - n_{\overline{\phi}}|}{n_\phi + n_{\overline{\phi}}}. \tag{5}$$

In this paper we demonstrate that if $\eta \leq 0.5$, Poisson fluctuations will dominate the probability distribution, and the number density of NTS's produced will be independent of η, even in the limit $\eta \to 0$.

We will denote by $\overline{N}_\phi$, the *mean* number of ϕ's in some volume. The probability of finding the actual number N_ϕ of ϕ's is Poisson distributed: $P(N_\phi; \overline{N}_\phi) = e^{-\overline{N}_\phi} \overline{N}_\phi^{N_\phi} / N_\phi!$. In the limit of large $\overline{N}_\phi$, the distribution will be Gaussian, with mean and variance $\mu = \sigma^2 = N_\phi$:

$$P(N_\phi; \overline{N}_\phi) = \frac{1}{\sqrt{2\pi \overline{N}_\phi}} \exp\left[-(N_\phi - \overline{N}_\phi)^2 / 2\overline{N}_\phi\right]. \tag{6}$$

We will later discuss the validity of the Gaussian approximation. An expression similar to Eq.(6) obtains for the probability of finding a number $N_{\overline{\phi}}$ of $\overline{\phi}$'s if the mean is $\overline{N}_{\overline{\phi}}$. The total number, N, and charge, Q, defined as

$$N \equiv N_\phi + N_{\overline{\phi}} \qquad Q \equiv |N_\phi - N_{\overline{\phi}}| \tag{7}$$

will also be Gaussian distributed; with means $\overline{N} = \overline{N}_\phi + \overline{N}_{\overline{\phi}}$, $\overline{Q} = |\overline{N}_\phi - \overline{N}_{\overline{\phi}}| = \eta\overline{N}$; and variance $\sigma^2 = \overline{N}$. Therefore, the probability of finding a charge Q in a volume containing a mean number $\overline{N}$ of $(\phi + \overline{\phi})$'s is[9]

$$P(Q, \overline{N}) = \frac{1}{\sqrt{2\pi\overline{N}}} \exp\left[-(Q - \eta\overline{N})^2/2\overline{N}\right]. \tag{8}$$

As described by FGGK, below T_G the Universe divides into cells of correlation volume $V_\xi \simeq (4\pi/3)\xi^3$. Adjacent cells of false vacuum form "clusters" with density per unit cluster of

$$f(r) = br^{-1.5}e^{-cr} \tag{9}$$

for volume $V = rV_\xi$. The constants b and c are unknown. Scaling arguments imply that $c \to 0$ as $p(\sigma_0) \to p_c$ (where p_c is the critical probability for percolation, $p_c \sim 1/3$) and $b \to 0$ as $p(\sigma_0) \to 0$. It is expected that b and c are of order unity otherwise. The number density of r-clusters produced in the transition is simply $n(r) = f(r)V_\xi^{-1}$. In a volume $V = rV_\xi$, the mean number of $(\phi + \overline{\phi})$'s is $\overline{N} = r\overline{N}_\xi$, where $\overline{N}_\xi$ is the mean number of $(\phi + \overline{\phi})$'s in a correlation volume

The number density of false-vacuum domains with charge Q is simply given by $n_Q = \Sigma_{r=1}^{\infty} n(r)P(Q; \overline{N} = r\overline{N}_\xi)$, where $n(r) = f(r)V_\xi^{-1}$ as before, with $f(r)$ given by Eq.(9). Approximating the sum over r by an integral,[10] n_Q becomes

$$\begin{aligned} V_\xi n_Q &= \frac{b\exp(Q\eta)}{\sqrt{2\pi\overline{N}_\xi}} \int_0^\infty dr\; r^{-2} \exp\left[-(\eta^2\overline{N}_\xi/2 + c)r - Q^2/2\overline{N}_\xi r\right] \\ &= \frac{2b}{\sqrt{\pi}} \frac{\exp(Q\eta)}{Q} (\eta^2\overline{N}_\xi/2 + c)^{1/2} K_1\left[(\sqrt{2}Q/\overline{N}_\xi^{1/2})(\eta^2\overline{N}_\xi/2 + c)^{1/2}\right], \end{aligned} \tag{10}$$

where $K_1(z)$ is a modified Bessel function of the second kind of order one. For large argument, the expansion $K_1(z) \to e^{-z}\sqrt{\pi/2z}$ gives[10]

$$V_\xi n_Q = b\frac{\overline{N}_\xi^{1/2}}{Q^{3/2}}(\eta^2 + 2c/\overline{N}_\xi)^{1/4} \exp\left[Q\eta - Q(\eta^2 + 2c/\overline{N}_\xi)^{1/2}\right]. \tag{11}$$

The expression for $V_\xi n_Q$ can easily be converted into the ratio of the number density of Q's to the entropy density, $s = 2\pi^2 g_* T^3/45$. Using $\xi = \lambda_1^{-1} T_G^{-1}$, the correlation volume is $V_\xi = 4\pi\xi^3/3 = 4\pi/3\lambda_1^3 T_G^3$. Assuming the ϕ's are relativistic at $T = T_G$, $n_\phi \approx n_{\overline{\phi}} = \zeta(3)T_G^3/\pi^2$, and at $T = T_G$

$$\overline{N}_\xi = (n_\phi + n_{\overline{\phi}})V_\xi = \frac{8\zeta(3)}{3\pi\lambda_1^3} \approx \lambda_1^{-3}. \tag{12}$$

Since $V_\xi = 8\pi^3 g_*/135\lambda_1^3 s$,

$$\begin{aligned} Y_Q \;&\equiv\; \frac{n_Q}{s} = \frac{135\lambda_1^3 b}{8\pi^3 g_* Q^{3/2}}\left[\frac{8\zeta(3)}{3\pi\lambda_1^3}\right]^{1/2}\left(\eta^2 + \frac{3\pi\lambda_1^3 c}{4\zeta(3)}\right)^{1/4} \\ &\qquad \times \exp\left[Q\eta - Q\left(\eta^2 + 3\pi\lambda_1^3 c/4\zeta(3)\right)^{1/2}\right] \\ &= \frac{0.54 b\lambda_1^{3/2}}{g_* Q^{3/2}}(\eta^2 + 1.96\lambda_1^3)^{1/4}\exp\left[Q\eta - Q(\eta^2 + 1.96\lambda_1^3 c)^{1/2}\right]. \end{aligned} \tag{13}$$

There are two interesting limits of Eq.(13). In the limits $\eta^2 \ll 1.96\lambda_1^3 c$, and $\eta^2 \gg 1.96\lambda_1^3 c$, Y_Q becomes

$$Y_Q = \begin{cases} (0.64 b\lambda_1^{9/4}/g_* Q^{3/2})\exp(-1.4 Q\lambda_1^{3/2} c^{1/2}) & \eta^2 \ll 1.96\lambda_1^3 c \\ (0.54 b\lambda_1^{3/2}\eta^{1/2}/g_* Q^{3/2})\exp(-0.98\lambda_1^3 Q c\eta^{-1}) & \eta^2 \gg 1.96\lambda_1^3 c \end{cases} \tag{14}$$

Since Y_Q decreases exponentially with Q, the most abundant NTS will be the one with the smallest allowed charge, $Q = Q_{\rm MIN}$.

Note that in the "large" η limit, we essentially recover the results of FGGK. However, this case is only relevant for η *larger than of order unity.* A much more likely possibility is that the "small" η limit is the relevant one, and that $Y_Q \sim 10^{-3} Q_{\rm MIN}^{-3/2} e^{-Q_{\rm MIN}}$.

III. NUMERICAL RESULTS AND CONCLUSIONS

In the previous section, three approximations were used: 1) Gaussian rather than Poisson statistics, 2) the sum over r-clusters was replaced by an integral, and 3) the large-z expansion of the Bessel function $K_1(z)$ was used. In this conclusion section we present some numerical result and discuss the range of validity of the above approximations.

Clearly for "large" Q, $Q \geq$ 10–20, Gaussian statistics will be a good approximation. In Fig. 1 we compare an integration over r of Gaussian statistics, Eq.(10), to the more accurate sum over r of Poisson statistics. The Gaussian results are presented for η =0, 0.25 and 0.5, while the Poisson results are given for $\eta = 0$ only. It is clear that the Gaussian approximation is an adequate one. Integration over r rather than summing also introduces only a small error.

In Fig. 2 we present the large-z expansion of the Bessel function in Eq.(11). Comparison of Fig. 1 and Fig. 2 shows that for $\sqrt{2}Q\overline{N}_\xi^{-1/2}(\eta^2\overline{N}_\xi/2 + c)^{1/2} \geq 2$, the expansion is accurate. In Fig. 2 we also show for comparison the results of FGGK for $n_Q V_\xi$. Clearly it is a serious underestimate for n_Q unless $\eta^2 \gg 1.96\lambda_1^3 c$.

We conclude by illustrating the importance of the calculation of Y_Q. We use the example discussed in the introduction, $\lambda_2 = 0.15\lambda_1$, which gives $Q_{\rm MIN} = 18\lambda_1/h^2$, and $M(Q_{\rm MIN}) = 46\lambda_1\sigma_0/h^{3/2} = 2.5Q_{\rm MIN}h^{1/2}\sigma_0$. Assuming that the contribution to Ω from NTSs is dominated by those with $Q = Q_{\rm MIN}$, $Y_{NTS} \simeq 10^{-3}Q_{\rm MIN}^{-3/2}e^{-Q_{\rm MIN}}$, the present NTS energy density is $\rho_{NTS} = Y_{NTS}M_{NTS}s_0$, where s_0 is the present entropy density, s_0 =2800 cm^{-3}. Comparison of ρ_{NTS} to the critical density, ρ_C=1.88$\times10^{-29}h_0^2$g cm^{-3}, where h_0 is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}, gives

$$\Omega_{NTS}h_0^2 = 10^6 h^{1/2}(\frac{\sigma_0}{\rm GeV})\frac{e^{-Q_{\rm MIN}}}{Q_{\rm MIN}^{1/2}}. \tag{15}$$

For NTSs to be dynamically relevant today, $\Omega_{NTS}h_0^2$ should be in the range $10^{-2} \leq \Omega_{NTS}h_0^2 \leq 1$. Relevant values of $Q_{\rm MIN}$, or equivalently λ_1/h^2, are shown in Table I.

The conclusion of this paper is that statistical fluctuations are the dominant source of charge fluctuations in solitogenesis, not a cosmic asymmetry as assumed by FGGK.

The resulting Ω_{NTS} is independent of η, so long as $\eta \leq 0.1$. Finally, reasonable values of Q_{MIN} give Ω_{NTS} in a dynamically interesting range.

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TABLE I

	$\sigma_0 = 10^{19}$GeV		$\sigma_0 = 1$ GeV	
$\Omega_{NTS}h_0^2$	Q_{MIN}	λ_1/h^2	Q_{MIN}	λ_1/h^2
10^{-2}	60	3.3	17	0.94
1	55.5	3.1	12.5	0.7

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7. In general, one should include a term in the potential proportional to $|\phi|^4$. Inclusion of this term will not substantially alter our conclusions.

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9. When $\eta = 0$ the corresponding formula using Poisson statistics is $P(Q, \bar{N}) = e^{-\bar{N}} I_Q(\bar{N})$, where I_Q is a modified Bessel function of order Q. Turning the sum over r into an integral, $V_\xi n_Q \approx \bar{N}_\xi^{1/2}(z^2-1)^{1/4}\Gamma(Q-\frac{1}{2})P_{1/2}^{-Q}\left(z(z^2-1)^{-1/2}\right)$ where $P_{1/2}^{-Q}$ is an associated Legendre function of order 1/2, Γ is the gamma function, and $z = c + \bar{N}_\xi \approx c + \lambda_1^{-3}$.

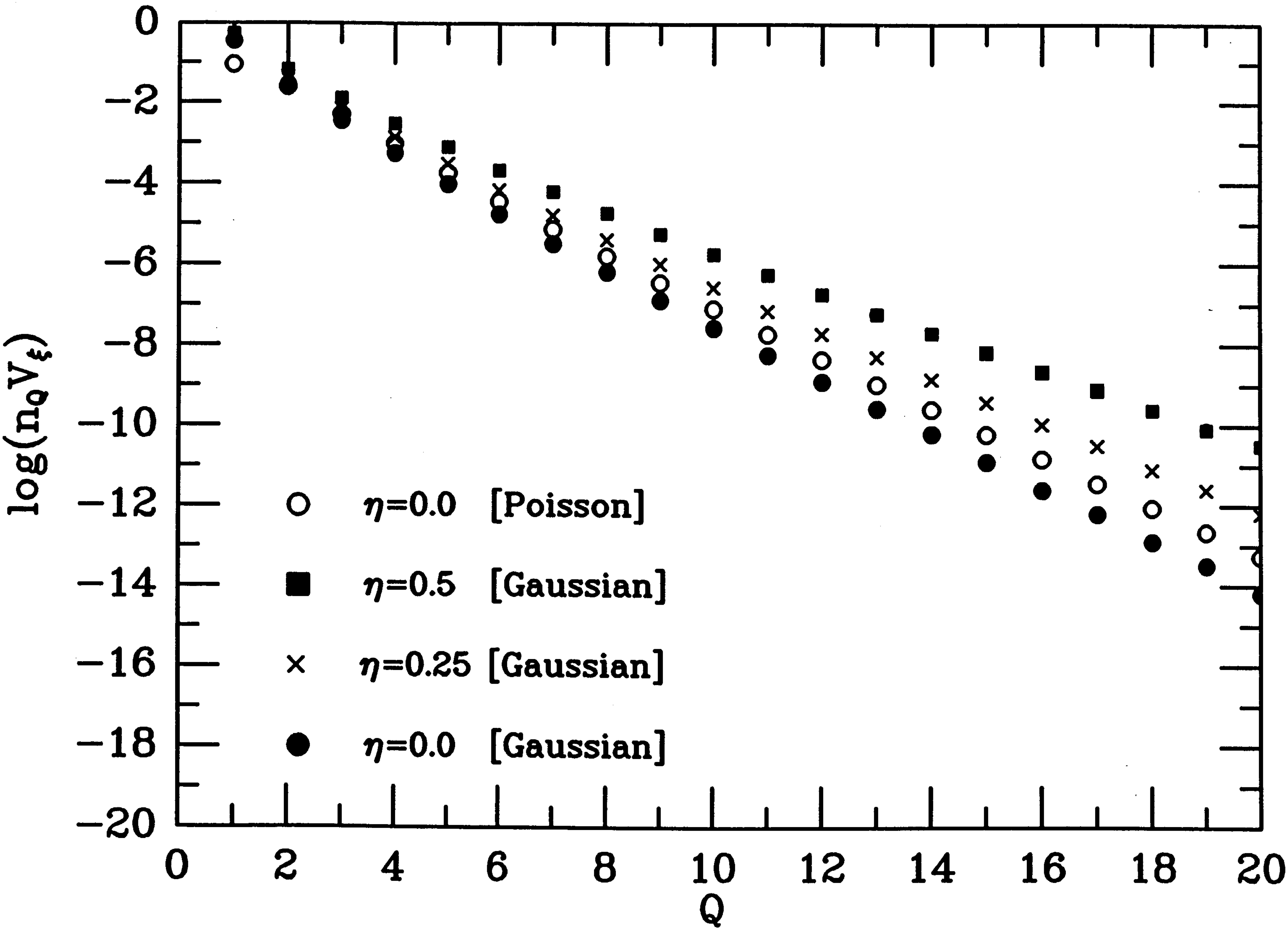
10. The validity of this approximation will be discussed in the final section.

11. We will only be interested in volumes much smaller than the horizon. The expression becomes more complicated for volumes larger than the horizon. See, e.g., P. J. E. Peebles, *The Large-Scale Structure of the Universe*, (Princeton University Press, Princeton, 1980).

FIGURE CAPTIONS

Figure 1: A comparison of Poisson (Footnote 9) and Gaussian probabilities (Eq. 10) as a function of Q. $\overline{N}_{\xi} = b = c = 1$ was assumed.

Figure 2: The result of the large-z expansion of the Bessel function in Eq. 11 is shown by the points marked *This Work*. Comparison of these points with the corresponding points in Fig. 1 shows that the large-z expansion is a good approximation. Also indicated by the points marked *FGGK* are the results of FGGK[8] which ignored statistical fluctuations.



0
−2
−4
−6
−8
−10
−12
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−16
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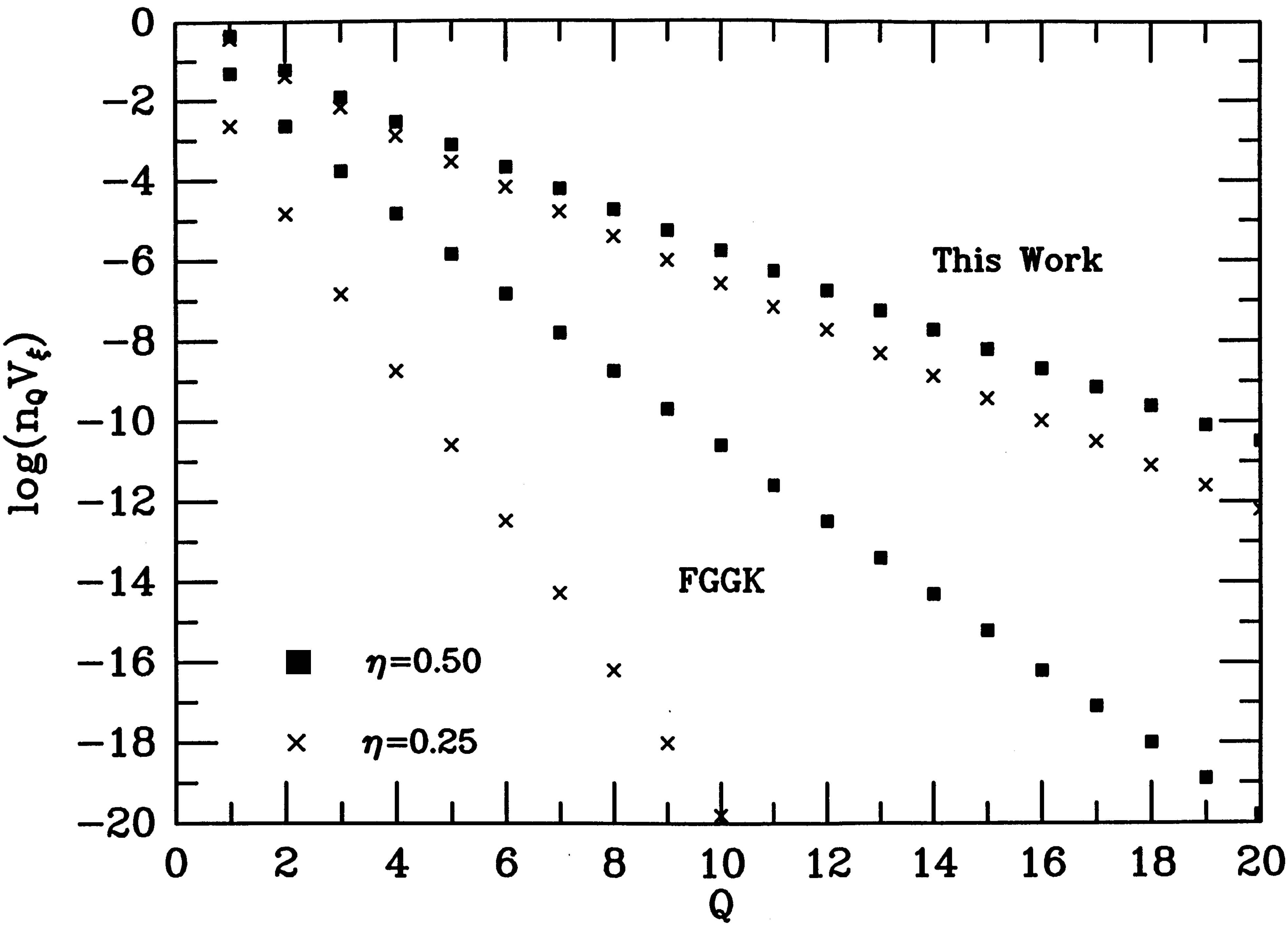
−20
$\log(n_Q V_\xi)$
0
2
4
6
8
10
12
14
16
18
20
Q
η=0.0 [Poisson]
η=0.5 [Gaussian]
η=0.25 [Gaussian]
η=0.0 [Gaussian]

FIG.1



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log(n$_Q$V$_\xi$)
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2
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18
20
Q
This Work
FGGK
η=0.50
η=0.25

FIG. 2