Multigrid Calculation
of Three-Dimensional
Turbomachinery Flows

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Final Report

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I. Preface

This document serves as a Final Report, summarizing the work performed from September 1, 1985 through January 30, 1989 under Grant NAG 3-645 from the NASA Lewis Research Center. The Principal Investigator for the Grant was Professor David A. Caughey of the Sibley School of Mechanical and Aerospace Engineering of Cornell University; the Grant Technical Monitor was Dr. Rodrick V. Chima of the Computational Methods Branch of the NASA Lewis Research Center.

II. Accomplishments

Research was performed in the general area of Computational Aerodynamics, with particular emphasis on the development of efficient techniques for the solution of the Euler and Navier-Stokes equations for transonic flows through the complex blade passages associated with turbomachines. In particular, multigrid methods have been developed, using both explicit and implicit time-stepping schemes as smoothing algorithms. The specific accomplishments of the research supported by this grant have included:

(1) the development of an explicit multigrid method to solve the Euler equations for three-dimensional turbomachinery flows based upon the multigrid implementation of Jameson's explicit Runge-Kutta scheme [Jameson 1983];

(2) the development of an implicit multigrid scheme for the three-dimensional Euler equations based upon Lower-Upper factorization;

(3) the development of a multigrid scheme using a diagonalized ADI Implicit algorithm;

(4) the extension of the diagonalized ADI multigrid method to solve the Euler equations of inviscid flow for three-dimensional turbomachinery flows; and

(5) the extension of the diagonalized ADI multigrid scheme to solve the Reynolds-averaged Navier-Stokes equations for two-dimensional turbomachinery flows.

The work in each of these areas will now be described briefly. Copies of Smith & Caughey [1987], Yokota & Caughey [1988], Caughey [1988], and Caughey & Turkel [1988] are also included as Appendices to this report, and contain further details regarding this work.

The Grant has supported all or part of the Ph.D. Thesis research of Drs. Wayne A. Smith (Mechanical Enginering) and Jeffrey W. Yokota (Aerospace Engineering), who have completed their theses, and of Thomas Tysinger (Aerospace Engineering) and Culbert Laney (Applied Mathematics), both of whose thesis work is still in progress.

A. Explicit Runge-Kutta Multigrid Scheme

A fast and efficient three dimensional Euler solver for transonic flow through rotating blade passages has been developed and tested. The time dependent equations are discretized spatially using a finite volume approach, and are advanced temporally with a multiple stage time stepping scheme, as popularized by Jameson, Schmidt, & Turkel [1981]; see also Jameson [1982]. A dramatic increase in the rate of convergence for steady state solutions has been achieved with a multigrid algorithm (Brandt [1973]) which employs the multistage scheme as its smoothing procedure. The ability of the multistage scheme to damp high frequency error is enhanced by a carefully selected combination of the coefficients governing implicit residual smoothing, added dissipation, time steps, and the multistage scheme itself.

The Euler equations are formulated in terms of absolute flow variables in a Cartesian frame of reference rotating with constant angular velocity. This allows

the equations to be discretized in a way which ensures that a uniform stream satisfies the difference equations identically (Holmes & Tong [1985]). This property is not so important for the present turbomachinery application, but is critical for the solution of flows about propellers and helicopter blades.

The use of centered differences ensures that the scheme is second order accurate provided the mesh is sufficiently smooth, but also allows the solution to decouple at odd- and even-numbered points in the grid, necessitating the addition of suitable dissipative terms. These are introduced as an adaptive blend of second- and fourth-differences which maintain high accuracy while capturing shock waves with little or no overshoot.

A multiple stage time stepping scheme is used to discretize the remaining time derivative. The scheme is implemented within the context of the multigrid algorithm to accelerate convergence to the steady state. This is done following the procedure of Jameson [1983], but with the multigrid sequence defined recursively in a way which permits the definition of generalized "W-cycles," and allows greater amounts of smoothing to be done effectively on the coarser grids of the sequence.

The option to perform implicit residual smoothing was also included in the original code, but a numerical determination of the optimal values for use with multigrid of the stage coefficients and implicit smoothing parameter for a five-stage Runge-Kutta scheme showed that implicit smoothing should not be used (Smith & Caughey [1987], Smith [1987b]).

A number of results have been computed, for both compressor and turbine geometries; the values of the parameters determined from the simple one-dimensional optimization procedure have been used without modification to compute solutions to all the test cases. The grids used for these calculations were constructed using a modified version (Smith [1987a]) of the GRAPE code of Sorenson [1980].

Further details of the method and results of several calculations are included in Smith [1987b] and Smith & Caughey [1987]. These also include comparisons with the experimental and computational results of Chima & Strazisar [1982].

B. L-U Implicit Multigrid Scheme

A Lower-Upper (L-U) Factored Implicit scheme for solving the Euler equations of inviscid flow in three-dimensional, rotating internal cascade geometries has also been developed. The Euler equations are written in conservation form and are transformed to generalized coordinates. These are approximated using a finite-volume discretization, and suitable artificial dissipative terms are added so that

shocks are captured automatically. The spatial discretization is similar to that of Jameson *et al* [1981] and to that described above.

After a time-linearization of the nonlinear flux vectors, the implicit operator can be factored into lower- and upper- block bi-diagonal factors by using one-sided differences. This results in a scheme which requires the inversion only of the 5 × 5 blocks at each cell for each of the two effectively explicit sweeps through the domain. This requires significantly less computational labor than the block tridiagonal inversions of conventional Alternating Direction Implicit schemes, and incorporates the added efficiency that only two sweeps are required even for three-dimensional problems. The Jacobian matrices are split in a way which produces diagonally-dominant factors for each matrix inversion in each cell. This is ensured by constructing the splitting so that the eigenvalues of the forward-differenced matrices are non-positive, and those of the backward-differenced matrices are non-negative. The actual procedure follows closely that suggested by Jameson & Turkel [1981].

Artificial dissipation terms are added to the centered scheme to reduce odd-even decoupling of the solution, and to suppress overshoots in the vicinity of shock waves; these terms are constructed following the formulation proposed by Jameson *et al* [1981]. The explicit terms are constructed of blends of nonlinear second- and fourth-differences, while only the second-differences are treated implicitly.

The scheme is implemented within the context of the multigrid algorithm to accelerate convergence to the steady state. This is done following the procedure of Jameson [1983]. The calculations were performed on three-dimensional "H-type" grids for turbomachinery blade rows, which were generated using the version of the GRAPE code of Sorenson [1980] that was modified by Smith [1987a].

Further details of the method, as well as results illustrating its efficiency, are included in Yokota [1987] and Yokota & Caughey [1987].

C. Diagonal Alternating Direction Implicit Multigrid Scheme

The development of Alternating Direction Implicit multigrid methods is an attractive means to circumvent the problem of the slow convergence of explicit methods on grids containing cells of extremely high aspect ratio. These methods have also been studied recently by Jameson & Yoon [1985]. In order for the implicit method to be an effective smoothing algorithm when used in conjunction with the multigrid algorithm, it is important to include an accurate representation of the dissipative terms. Pulliam [1984] has also shown the importance of including the fourth-difference terms in the implicit operator even when multigrid is not used. Since, as described

in the preceding sections, these usually include fourth-differences to maintain high accuracy, their inclusion in the implicit operator requires the solution of block pentadiagonal systems for each one-dimensional factor. To avoid the high cost of solving block pentadiagonal systems, the equations in the present scheme are first diagonalized at each point using a local similarity transformation, following Chaussee & Pulliam [1981]. This has the effect of decoupling the equations, and requiring the solution of four (in two-dimensional problems) *scalar* pentadiagonal equations for each factor. The resulting method has good high-wavenumber damping, so is a good smoothing algorithm for use in conjunction with the multigrid method. It is also computationally efficient because of the need to solve only scalar systems. Additional computation is required to calculate the local similarity transformations, and to perform matrix multiplies of the residual, and of the intermediate and final corrections, but this is a small fraction of the work required to solve the block systems. In addition, on many computers these additional computations can be vectorized to further reduce the required CPU time.

In preparation for the incorporation of the viscous terms in the multigrid ADI scheme for turbomachinery problems (see subsection II.E below), a version of the implicit multigrid Euler code for two-dimensional cascades was developed. This code has also been used to perform experiments demonstrating the importance of treating the dissipation in a fully conservative manner, for flows containing shock waves, even for internal flow problems. Results are summarized here for a test case involving moderately strong shock waves in the flow through a simple cascade of NACA 0012 airfoils. The flow is computed through an unstaggered cascade of airfoils having a height-to-chord ratio of three for a ratio of downstream static to upstream total pressure of 0.631298 (corresponding to isentropic expansion to a Mach number of $M_\infty = 0.838$). The grid for this calculation, generated using Smith's extension (Smith [1987]) of the GRAPE Code (Sorenson [1980]), is shown in Figure 1. Blade surface distributions of surface pressure coefficient are shown in Figures 2(a) and 2(b) for cases in which the dissipative terms were differenced non-conservatively and conservatively, respectively. Contours of constant pressure for these two cases are shown in Figures 3(a) and 3(b). As is evident from these figures, the shocks are significantly weakened and shifted upstream by the non-conservative differencing.

The details of the new algorithm and results of several two-dimensional, transonic airfoil calculations are described in Caughey [1987b, 1988]. In addition, new results concerning the effects of numerical dissipation in finite-volume schemes have been presented by Caughey [1987a] and by Caughey & Turkel [1988].

D. Implicit Multigrid Scheme for the 3-D Euler Equations

Work has also begun on development of an implicit multigrid method to solve the three dimensional Euler equations for the transonic flow through rotating blade passages. The time dependent equations are discretized spatially using a finite volume approximation (see Jameson *et al* [1981]), and are advanced temporally using an implicit, approximate-factorization time stepping scheme, as developed by Caughey [1988].

The Euler equations are formulated in terms of absolute flow variables in a Cartesian frame of reference rotating with constant angular velocity. This allows the equations to be discretized in a way which ensures that a uniform stream satisfies the difference equations identically (Holmes & Tong [1985]). This property is not so important for the present turbomachinery application, but is critical for the solution of flows about propellers and helicopter blades.

The use of centered differences ensures that the scheme is second order accurate provided the mesh is sufficiently smooth, but also allows the solution to decouple at odd- and even-numbered points in the grid, necessitating the addition of suitable dissipative terms. These are introduced as an adaptive blend of second- and fourth-differences which maintain high accuracy while capturing shock waves with little or no overshoot.

An implicit Alternating Direction Implicit (ADI) time stepping scheme is used to discretize the remaining time derivatives. Each of the factors of the ADI scheme is diagonalized by a local similarity transformation to decouple the linear equations to be solved for the corrections at each time step. This results in a scheme which is significantly more efficient than the usual form, which requires block pentadiagonal solutions for each of the three factors for each time step. The scheme is implemented within the context of the multigrid algorithm to accelerate convergence to the steady state. This is done following the procedure of Jameson [1983], but with the multigrid sequence defined recursively in a way which permits larger amounts of smoothing to be done on the coarser grids of the sequence. Under separate support, three-dimensional solutions for the steady aerodynamic problem of transonic flow past a swept wing have been developed using this method by Yadlin & Caughey [1988].

The implicit scheme has been implemented using the steady-state portions of the explicit multigrid code developed by Smith [1987b]. Further details of the steady state approximation and the multigrid method are described by Smith [1987b] and by Smith & Caughey [1987].

Preliminary results have been computed for a simple test geometry consisting of a symmetric channel between two NACA 0012 airfoil sections. The sections are

located three chord lengths apart, with no stagger. Plane walls representing the hub and shroud are located one chord apart. The grid used for these calculations was constructed using a version of the GRAPE code of Sorenson [1980], modified to allow construction of H-type grids (Smith [1987a]); separate two-dimensional grids were generated in each spanwise plane, then stacked to form the three-dimensional grid. Contours of constant pressure are shown in Figure 4 on the mid-channel blade-to-blade surface for a sample calculation. The downstream static to upstream total pressure was chosen to correspond to an inlet Mach number of approximately 0.75. The calculation was performed on a grid containing 16 × 4 × 8 grid cells in the throughflow, radial, and blade-to-blade passage directions, respectively. A plot of the grid lines in the mid-channel plane is shown in the insert on Figure 4. Grids of higher density are, of course, necessary for meaningful results; this result is intended merely to show that the Diagonalized ADI multigrid solver is working for simple cases. The convergence history for this calculation is shown in Figure 5. Plotted is the root-mean-square (over the entire domain) of the residual of the continuity equation, which reaches machine zero in approximately 160 Work units using two levels of multigrid.

E. Implicit Multigrid Scheme for the 2-D Navier-Stokes Equations

The multigrid diagonal implicit algorithm developed by Caughey [1988] for the Euler equations of inviscid, compressible flow, has been extended to solve the compressible Navier-Stokes equations in two-dimensions. Because of its stability properties, the implicit scheme is particularly attractive for computing solutions on grids with highly-stretched cells, as is required for high Reynolds number flows. Even for flows at moderate Reynolds number, it is sometimes necessary to use grids with high aspect ratio cells to resolve boundary layer regions.

As for the Euler equation algorithm (Caughey [1988]), spatial derivatives are approximated using a finite volume formulation, and local time stepping is used to increase the convergence rate for steady problems. Artificial dissipation consisting of an adaptive blend of second and fourth differences is added to the scheme to insure convergence to a steady state and to allow accurate shock capturing for transonic flows. A recursive multigrid algorithm similar to that described by Smith & Caughey [1987] is implemented to accelerate convergence.

In the implicit Euler algorithm, the time linearization of the contribution of the convective flux vectors to the implicit operator gives rise to Jacobians. These Jacobians can be diagonalized to increase the efficiency of the algorithm; this can

always be done by a similarity transformation, since the system describing inviscid flows is hyperbolic. For the Navier-Stokes equations, however, it is not possible both to include the viscous terms in the implicit factor and to diagonalize the system, since the convective and viscous Jacobians are not simultaneously diagonalizable. It is desirable to maintain the efficiency of a diagonalized scheme, and so an alternate solution is required. There has been some success by implicitly including an approximation to the viscous Jacobian eigenvalues (Pulliam [1986]), but for the present scheme the viscous terms are neglected completely in the implicit factor, and thus contribute only to the explicit part of the equation. This can have no effect on a converged solution; it can affect only the stability of the iteration and rates of convergence. Converged solutions have been obtained using such a scheme, although recent experience suggests that implicit treatment of the viscous terms is necessary to remove low-frequency oscillations in the solution which can develop in some cases. With diagonalization, the computational efficiency of the algorithm is comparable to that of the explicit multi-stage schemes – i.e., the CPU time per time step is virtually the same.

The algorithm has been implemented in a computer code to calculate transonic flows past two-dimensional airfoils, and a number of test cases have been computed. The first case presented here is for the subsonic laminar flow ($Re = 5000$, $M_\infty = 0.5$) past a two-dimensional NACA 0012 symmetric airfoil at zero degrees angle of attack. The calculation is performed on a 192 × 48 cell "C"-grid generated using the GRAPE code elliptic mesh generator (Sorenson [1980]). The outer boundary of the mesh is located about 3 chords from the body. Care is taken to insure sufficient clustering in the region close to the body surface where viscous effects are significant. Approximately 10 mesh points are included within the boundary layer at the airfoil trailing edge, and the first point normal to the body surface is located at about .001 chords. The surface pressure distribution, presented in Figure 6, agrees well with that presented by Martinelli, Jameson, & Grasso [1986]. The flow separates at approximately 85% of the chord, as can be seen from the contour plot of the streamwise component of mass-flux density in Figure 7; this value is close to the values reported by both Swanson & Turkel [1985] and Jayaram & Jameson [1988] for this case.

The iterative process is begun by initializing the solution to free stream values. A plot of the convergence history is shown in Figure 8. Using six levels of multigrid and local time stepping, the solution has converged to a steady state in approximately 75 work units; this corresponds to 45 multigrid cycles. One work unit is defined as the amount of computational work required to advance the solution one

time step on the finest mesh level. It includes the work done on coarser levels during a multigrid cycle. Overall, the average residual is reduced by 7 orders of magnitude in about 300 work units or 180 multigrid cycles. This represents a substantial improvement over rates obtained by other researchers (e.g., Martinelli *et al* [1986] and Swanson & Turkel [1985].

The second case described here is for the transonic laminar flow ($Re = 500$, $M_\infty = 0.8$) past a NACA 0012 airfoil at ten degrees angle of incidence. A 192×48 cell "C"-grid is again used, with its outer boundary located about 3 chords from the body. A spacing of .01 chords is used for the first grid line normal to the body surface. The surface pressure distribution and contours of constant streamwise mass-flux density are shown in Figures 9 and 10, respectively. The solution appears to be in reasonable agreement with that obtained by Martinelli, *et al* [1986]. In this case, the flow separates from the upper surface of the airfoil at approximately 35% of the chord, as can be seen in Figure 10, and a large recirculating region forms on the upper surface of the airfoil.

The solution converges to a steady state from an initialized flow field of uniform free stream in less than 100 work units. The average residual is reduced by almost 10 orders of magnitude in 300 work units as shown in Figure 11; this can be compared to the results reported by Martinelli *et al* [1986], where a convergence of only 5 orders of magnitude in 1200 cycles was reported.

III. References

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4. Caughey, David A. [1988], *Diagonal Implicit Multigrid Algorithm for the Euler Equations*, **AIAA Journal**, Vol. 26, pp. 841-851.

5. Caughey, David A. & Turkel, Eli [1988], *Effects of Numerical Dissipation on Finite-Volume Solutions to Compressible Flow Problems*, **AIAA Paper 88-0621**, 26th Aerospace Sciences Meeting, Reno, Nevada.

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14. Jayaram, Mohan & Jameson, Antony [1988], *Multigrid Solution of the Navier-Stokes Equations for Flow Over Wings,* **AIAA Paper 88-0705,** 26th Aerospace Sciences Meeting, Reno, Nevada.

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18. Smith, Wayne A. [1987a], *Modified GRAPE Code,* Sibley School of Mechanical and Aerospace Engineering, Fluid Dynamics and Aerodynamics Program, **Report FDA 87-10,** Cornell University, Ithaca, New York, 14853.

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25. Yokota, Jeffrey W. [1987], *An L-U Implicit Multigrid Algorithm to Solve the Euler Equations for Transonic Flow in Rotating Turbomachinery Passages*, **Ph.D. Thesis**, Aerospace Engineering, Cornell University, Ithaca, New York, May 1987.

26. Yokota, Jeffrey W. & Caughey, David A. [1988], *L-U Implicit Multigrid Algorithm for the Three-Dimensional Euler Equations*, **AIAA Journal**, Vol. 26, pp. 1061-1069.

IV. Appendices

Copies of the abstracts of completed theses [T.1 - T.2], as well as Smith & Caughey [1987], Yokota & Caughey [1988], Caughey [1988], and Caughey & Turkel [1988] are attached as appendices to this report.

V. Acknowledgements

VI. Publications and Reports

P.1 *David A. Caughey, A Diagonal Implicit Multigrid Algorithm for Compressible Flow Calculations*, in **Advances in Computer Methods for Partial Differential Equations – VI**, R. Vichnevetsky and R. S. Stepleman, Eds., pp. 270-277, IMACS, New Brunswick, N.J., 1987.

P.2 *David A. Caughey, An Efficient Implicit Multigrid Algorithm for the Euler Equations of Compressible Flow*, **Proc. of the International Conference of Fluid Mechanics** (Beijing 1987), pp. 392-397, Peking University Press, Beijing, China, July 1-4, 1987.

P.3 *David A. Caughey, Diagonal Implicit Multigrid Algorithm for the Euler Equations*, **AIAA Journal**, Vol. 26, pp. 841-851, July 1988.

P.4 *David A. Caughey & Eli Turkel, Effects of Numerical Dissipation on Finite-Volume Solutions to Compressible Flow Problems*, **AIAA Paper 88-0621**, 26th Aerospace Sciences Meeting, Reno, Nevada, January 11-14, 1988.

P.5 *Wayne A. Smith, Modified GRAPE Code*, Sibley School of Mechanical and Aerospace Engineering, Fluid Dynamics and Aerodynamics Program, **Report FDA 87-10**, Cornell University, Ithaca, New York, 14853, June 1987.

P.6 *Wayne A. Smith & D. A. Caughey, Multigrid Solution of Inviscid Transonic Flow through Rotating Blade Passages*, **AIAA Paper 87-0608**, 25th Aerospace Sciences Meeting, Reno, Nevada, January 1987.

P.7 *Thomas L. Tysinger & David A. Caughey, Multigrid Diagonal Implicit Algorithm for the Compressible Navier-Stokes Equations*, Submitted for presentation at the AIAA 28th Aerospace Sciences Meeting, Reno, Nevada, January 1990.

P.8 *Jeffrey W. Yokota & D. A. Caughey, L-U Implicit Multigrid Algorithm for the Three-Dimensional Euler Equations*, **AIAA Journal**,Vol. 26, pp. 1061-1069, September 1988.

VII. Theses

T.1 *Jeffrey W. Yokota*, **Ph.D.** Thesis, Aerospace Engineering, May 1987, *An L-U Implicit Multigrid Algorithm to Solve the Euler Equations for Transonic Flow in Rotating Turbomachinery Passages*.

T.2 *Wayne A. Smith*, **Ph.D.** Thesis, Mechanical Engineering, August 1987, *Multigrid Solution of the Euler Equations*.
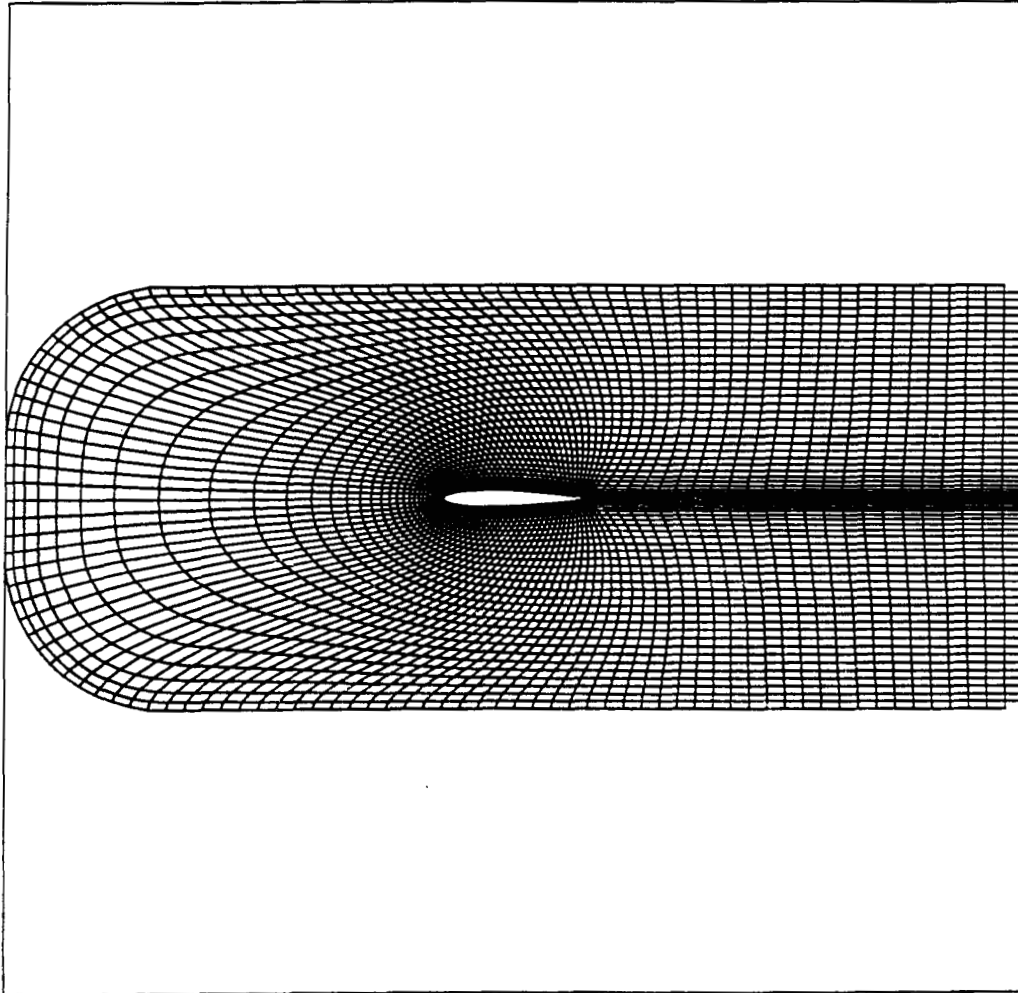
NACA 0012 Airfoil (C-Grid Cascade)

Grid 128 x 32

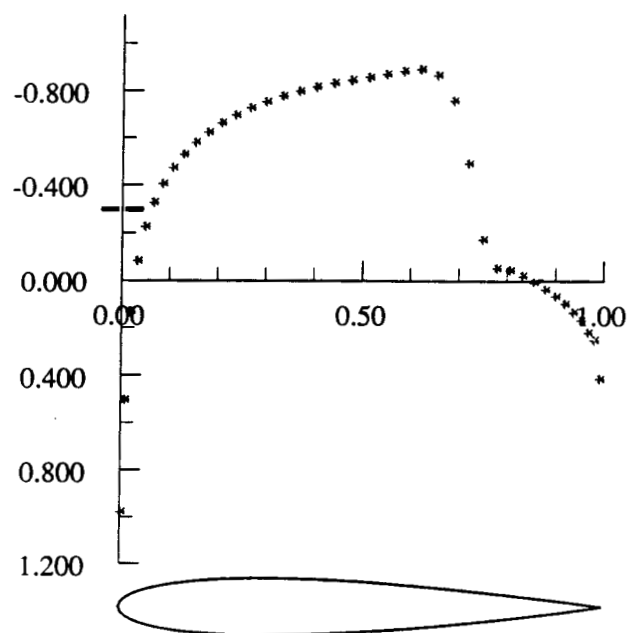Figure 1. Computational grid for cascade test of non-conservative numerical dissipation.

Figure 2(a). Blade surface pressure distribution for transonic flow through cascade; non-conservative numerical dissipation.
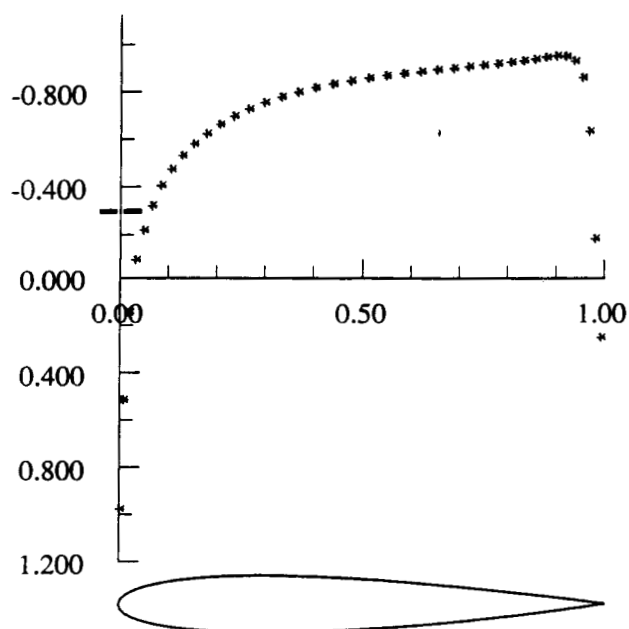
Figure 2(b). Blade surface pressure distribution for transonic flow through cascade; fully conservative numerical dissipation.
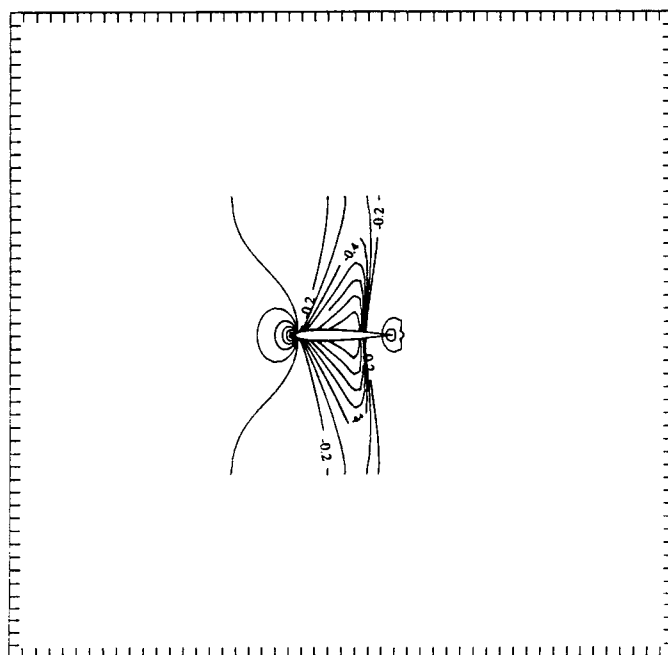
Figure 3(a). Contours of constant pressure for transonic flow through cascade; non-conservative numerical dissipation.
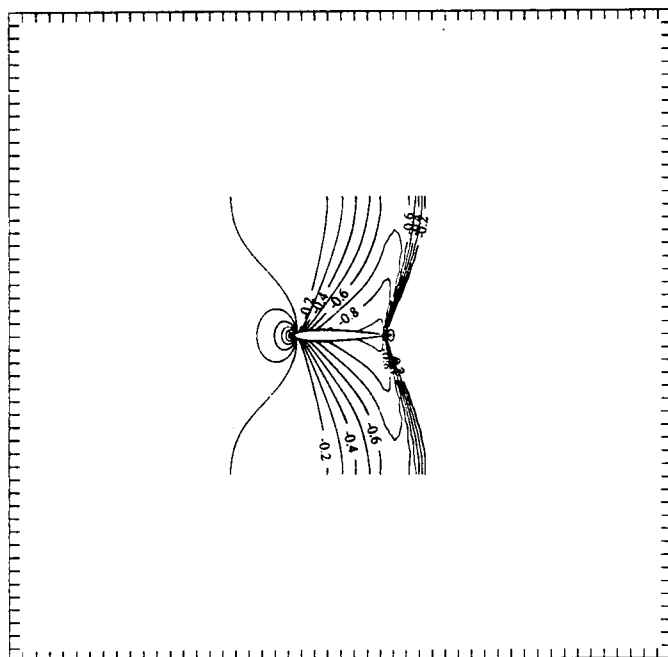
Figure 3(b). Contours of constant pressure for transonic flow through cascade; fully conservative numerical dissipation.

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res   = 0.2025E-06
min   = 0.5132E+00
max   = 0.8636E+00
k   = 3
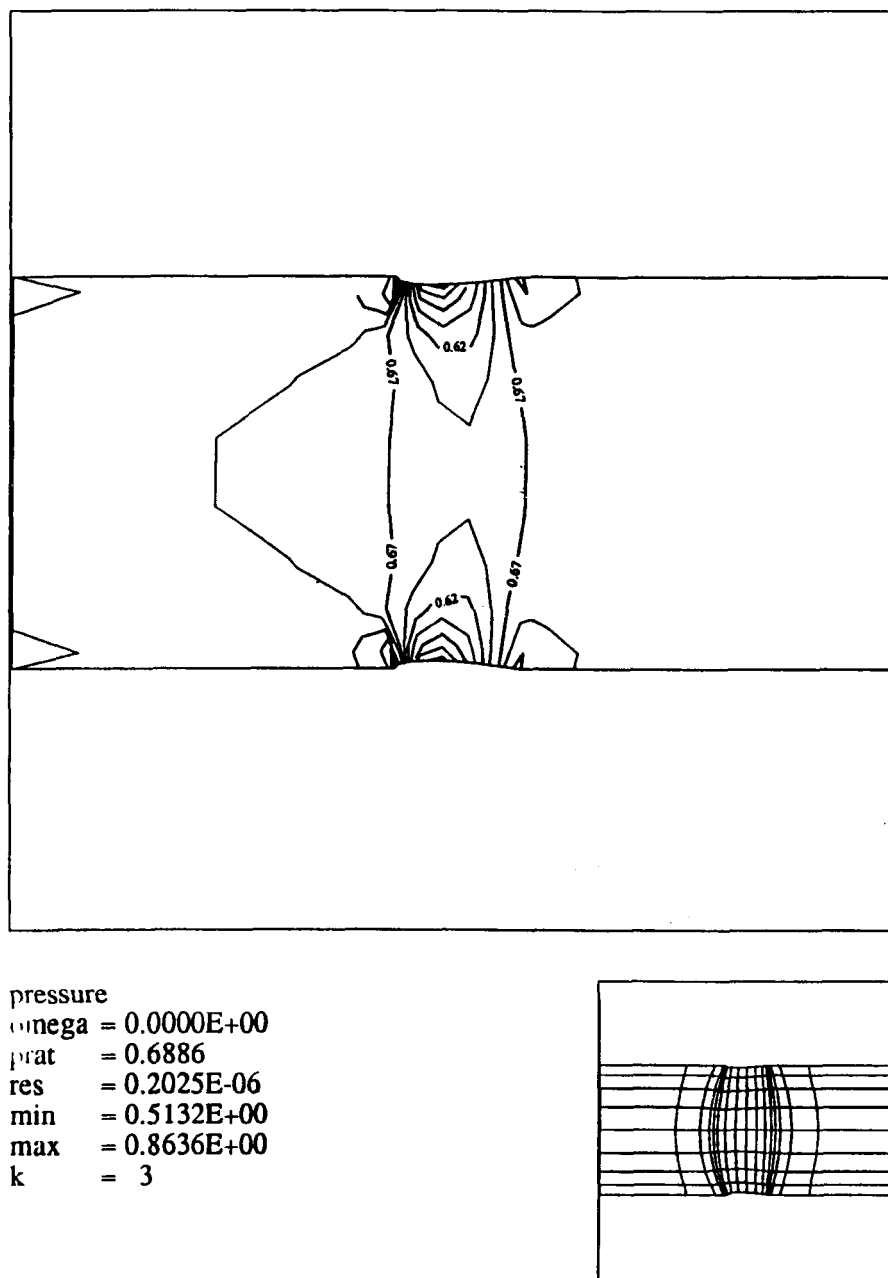
Figure 4. Contours of constant pressure at mid-plane of three-dimensional test channel.
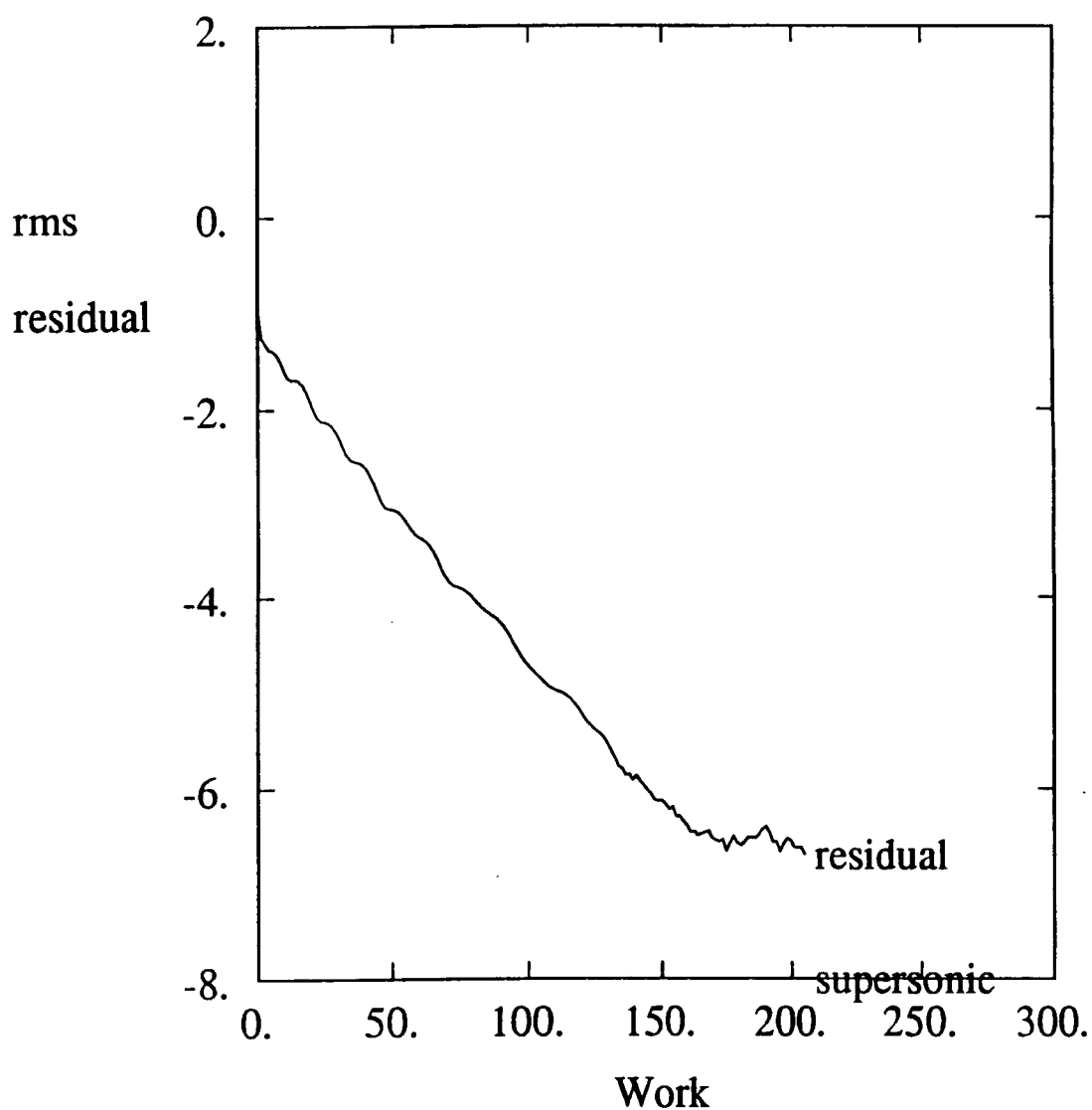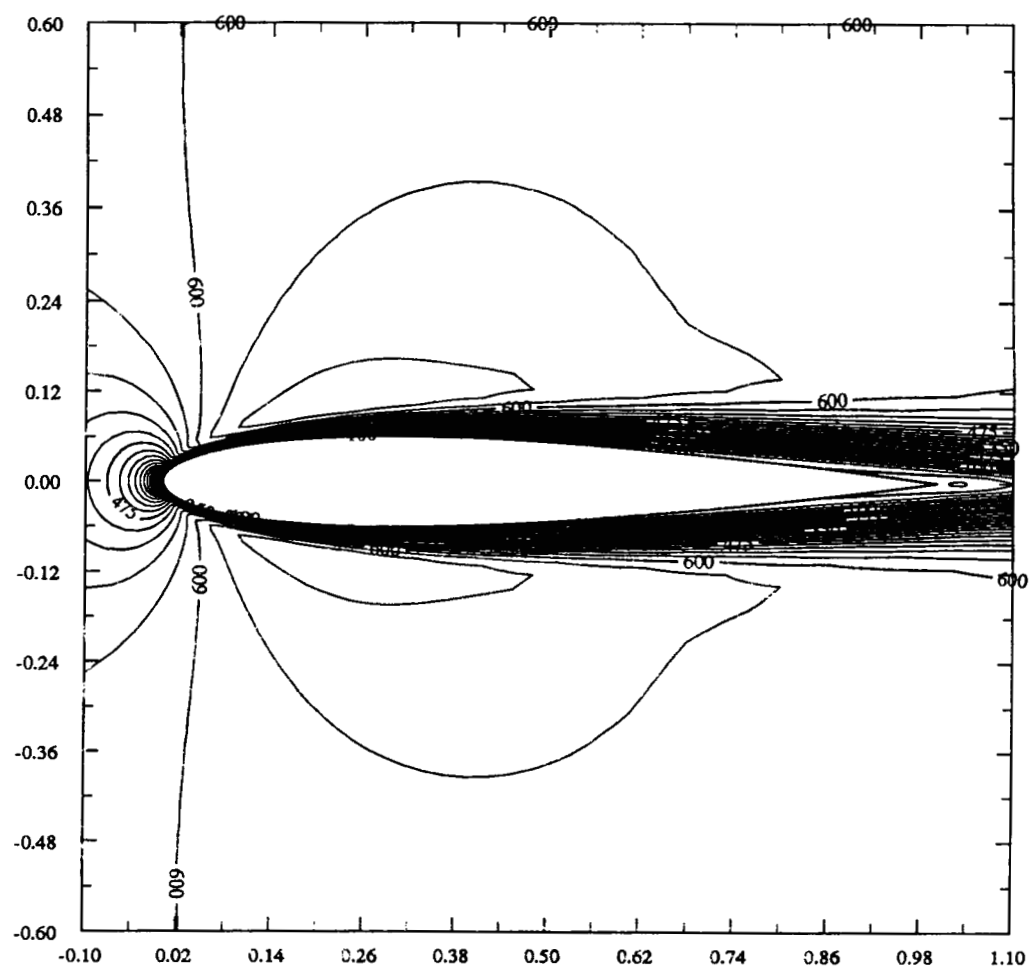
Figure 5. Residual convergence history for three-dimensional diagonal ADI multigrid solution.
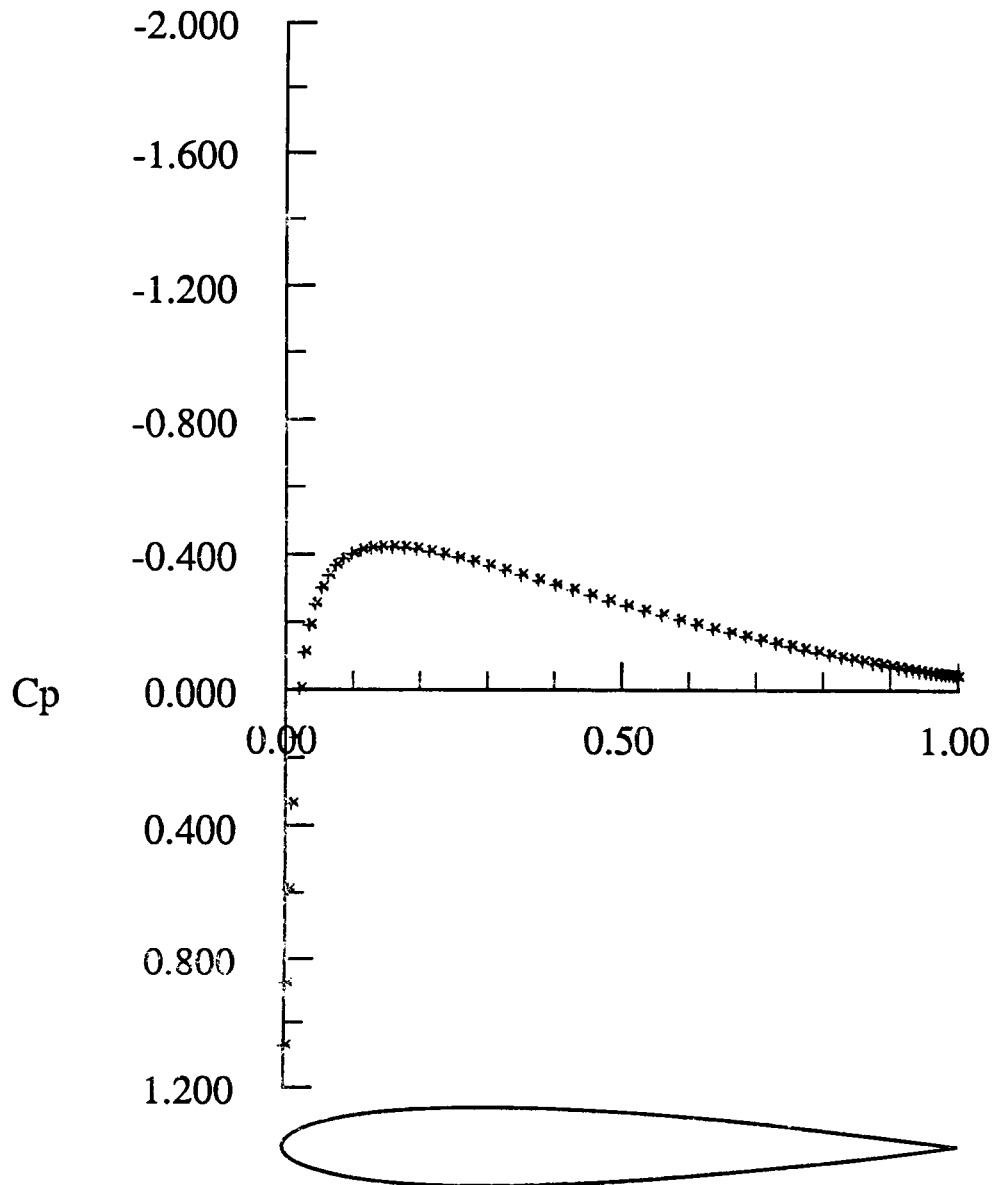
FLO53C: NACA 0012 FREEAIR SUBSONIC

Mach 0.500	Alpha 0.000	Re 5000.0
Cl 0.0000	Cd 0.0495	Cm 0.0000
Grid 192x48	Work 300.88	Res 0.601E-10

Figure 6. Surface pressure distribution for symmetrical, laminar flow past NACA 0012 Airfoil; $Re = 5000.0$, $M_\infty = 0.50$, $\alpha = 0.0$

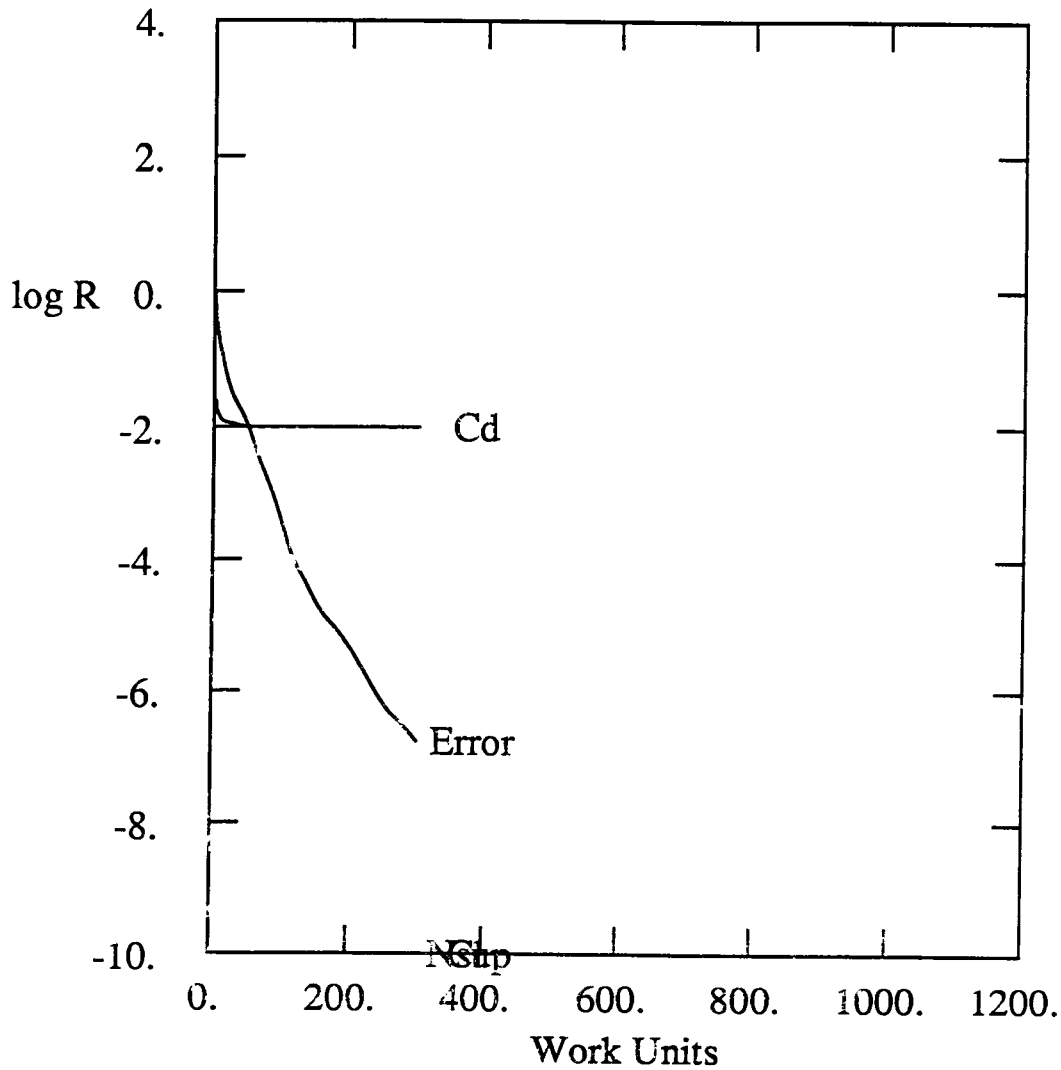FLO53C: NACA 0012 FREEAIR SUBSONIC

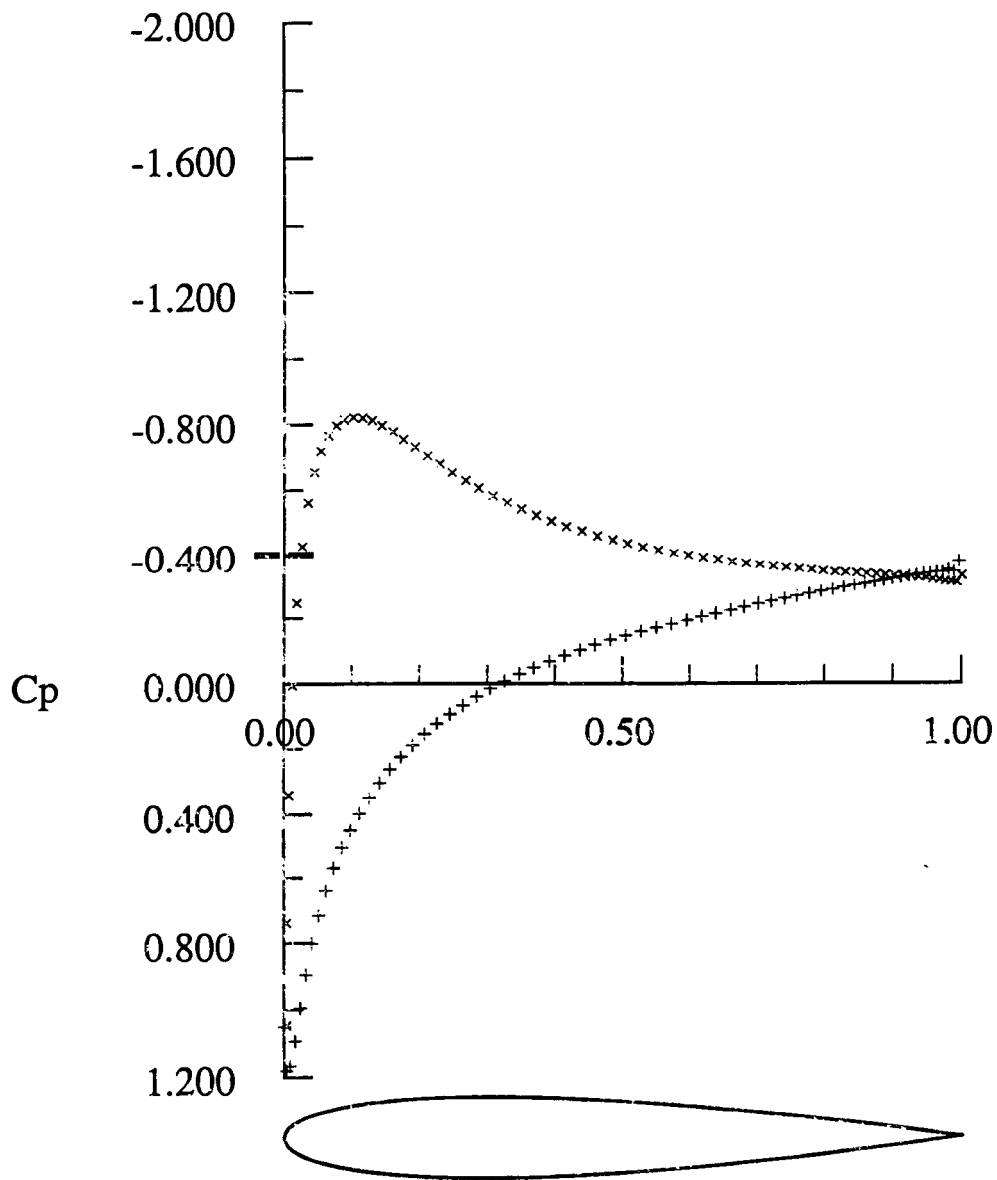| Minimum = -.1000E+00 | | Incrmnt = 0.2500E-01 |
| Maximum = 0.7000E+00 | w(2,i,j) contours | Scale = 0.1000E+05 |

Figure 7. Streamwise mass-flux density for symmetrical, laminar flow past NACA 0012 Airfoil; $Re = 5000.0$, $M_\infty = 0.50$, $\alpha = 0.0$

Figure 8. Residual convergence history for symmetrical, laminar flow past NACA 0012 Airfoil; $Re = 5000.0$, $M_\infty = 0.50$, $\alpha = 0.0$

FLO53C: NACA 0012 FREEAIR T.L.F.

Mach 0.800	Alpha 10.000	Re 500.0
Cl 0.4277	Cd 0.2407	Cm 0.0000
Grid 192x48	Work 300.88	Res 0.517E-12

Figure 9. Surface pressure distribution for transonic, separated flow past NACA 0012 Airfoil; $Re = 500.0$, $M_\infty = 0.80$, $\alpha = 10.0$

FLO53C: NACA 0012 FREEAIR T.L.F.

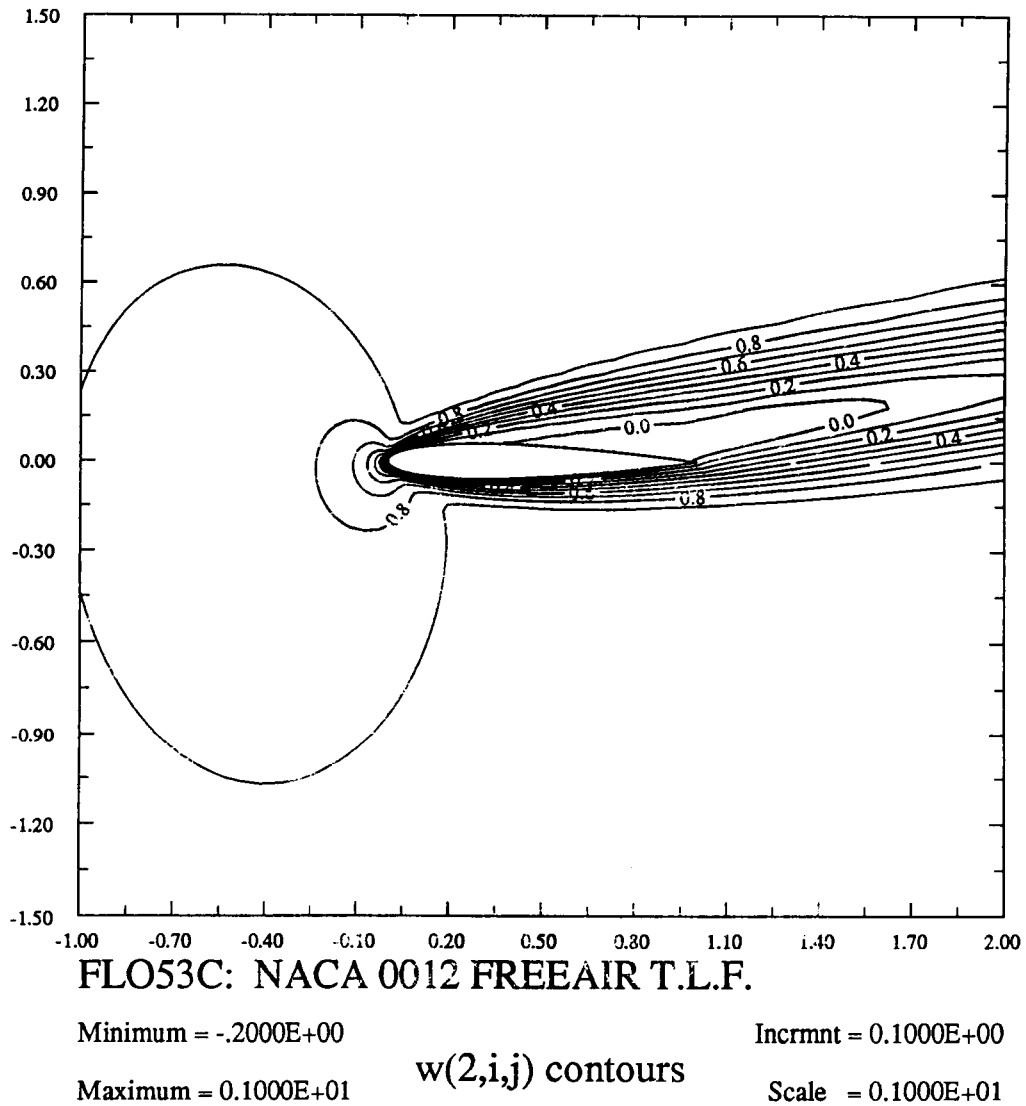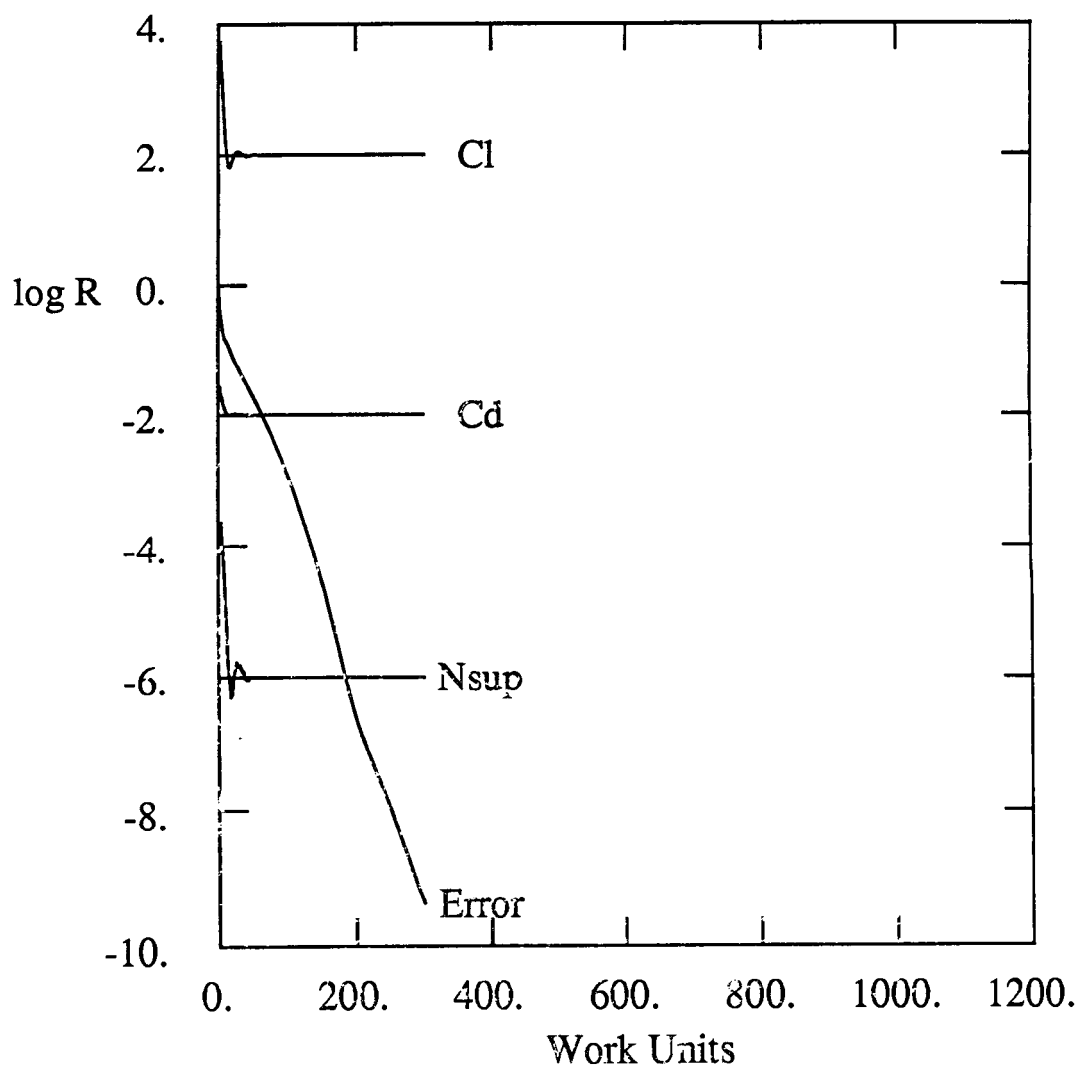| Minimum = -.2000E+00	w(2,i,j) contours	Incrmnt = 0.1000E+00
Maximum = 0.1000E+01		Scale = 0.1000E+01

Figure 10. Streamwise mass-flux density for transonic, separated flow past NACA 0012 Airfoil; $Re = 500.0$, $M_\infty = 0.80$, $\alpha = 10.0$

FLO53C: NACA 0012 FREEAIR T.L.F.

Mach 0.800	Alpha 10.000	Prat -1.000
Res1 0.120E-02	Re 500.	CFL 4.00
Res2 0.517E-12	Fbc 1.	Grid 192x48
Work 300.88	Rate 0.9308	MG levels 6

Figure 11. Residual convergence history for transonic, separated flow past NACA 0012 Airfoil; $Re = 500.0$, $M_\infty = 0.80$, $\alpha = 10.0$

Multigrid Solution of the Euler Equations
Wayne Allen Smith, Ph.D.
Cornell University 1987

A fast solver for three-dimensional, inviscid, transonic flow in rotating domains is presented. The scheme is designed to be to be extended to the numerical simulation of viscous transonic flows about realistic geometries. The techniques employed are therefore chosen for their extensibility as well as for their efficiency. Techniques that rely on simplifications made to more general flow equations by assumptions of inviscid flow of an ideal gas are ruled out, as are those whose analytic manipulations of the governing equations would become unwieldy as the complexity of the equations increases. Even with this generality, the resulting scheme provides the most efficient solution of the Euler equations for internal flows available to date.

Spatial discretization of the governing equations is performed with a finite volume scheme to provide geometric generality and to facilitate the construction of a conservative scheme. Temporal discretization is performed with a multiple-stage time-stepping scheme to allow the calculation of both time-accurate and steady solutions. The use of multigrid plays a critical role in making the explicit scheme efficient. The effects of residual smoothing for convergence acceleration are examined and residual smoothing is found to be detrimental when used in conjunction with multigrid and an efficient high wavenumber damping multistage scheme. Multistage parameters that

provide efficient high wavenumber schemes are determined through an optimization procedure that uses a simple hyperbolic model equation. Boundary conditions employing simple extrapolations based solely on assumptions of the hyperbolic character of the governing equations are shown to be sufficient. The efficiency of the scheme is demonstrated with representative computational results taken from a high speed compressor. Finally, the great potential of the scheme to exploit computational parallelism is demonstrated by implementing the scheme on a hypercube, a local-memory multi-processor computer.