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SECOND ORDER OBSERVERS

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# On the State Estimation of Structures with Second Order Observers

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## Abstract

The use of Linear Quadratic Regulator (LQR) control synthesis techniques implies the availability of full state feedback. For vibration control of structures, usually only a limited number of states are measured from which an observer model reconstructs the full state. This paper shows that using second order observers is a viable technique for reconstructing the unmeasured states of structures under mildly restrictive conditions. Moreover, the computational advantages of the second order observer as compared to a first order observer indicate that significantly larger observer models may be utilized. Numerical examples are used to demonstrate the performance of second order observers. The implications of second order observers in the development of Control/Structures Interaction (CSI) technology is discussed.

## 1. Introduction

Recent emphasis on the optimal interdisciplinary design of actively controlled spacecraft has led to the formation of a focused research activity within NASA called Control/Structures Interaction (CSI). The aim of the CSI activity is to coordinate University, Industry and Government research to develop the technology required to make active control of flexible spacecraft routine. To this end, several investigators have begun to exploit the second order form of the differential equations which describe structural dynamics. The controllability and observability of linear second order models has been studied by Laub and Arnold<sup>1</sup>, Hughes and Skelton<sup>2</sup> and by Bender and Laub.<sup>3</sup>

Park and Belvin<sup>4,5</sup> demonstrated the computational advantages of simulating the coupled CSI equations in second order form. Unlike classical first order simulation of the CSI equations, simulation in second order form enables the symmetry and sparsity of the structural equations to be exploited. Significant computational efficiency is gained by treating the CSI equations in second order form. Moreover, a natural partitioning of controls and structures is indicated by maintaining the structural equations in second order form.<sup>6</sup>

The linear quadratic regulator (LQR) control synthesis technique has excellent robustness properties. Safonov and

Athens<sup>6</sup> have proven, when the full state is available, that the LQR technique provides 60° of phase margin and infinite gain margin. However, for space structures the full state is not available and an observer must be used in conjunction with LQR. No guarantees can be made on the robustness properties for the observer based LQR controller.<sup>7</sup> Thus the observer plays a key role in the study of CSI when full state feedback is employed.

Observer state reconstruction suffers from model errors, measurement noise and unmodeled dynamics. Unmodeled dynamics result from the use of reduced order models.<sup>8</sup> To reduce the level of unmodeled dynamics, the size of the observer could be increased, provided the observer solution can still be performed in real-time. The computational advantages associated with the second order observer may permit larger observers with no penalty in solution time as compared to the first order observer.

Second order observer models have received recent attention in the literature. Hashemipour and Laub<sup>9</sup> utilized an optimal observer known as the Kalman Filter for discretized (time and space) second order structures models. In addition, robust computational procedures for solution of the Kalman Filter estimation error covariance matrices have been developed for second order models in Ref. 10. Unfortunately, the continuous time Kalman Filter has only been successfully developed in first order form.

It seems paradoxical that the optimal way to estimate the states of a second order system is with a first order observer model. Although there is more design freedom when using a first order model, the disadvantage of numerically simulating the CSI equations in first order form may prove that sub-optimal second order observers are most practical. It is from this perspective that some analytical and numerical results are presented herein to highlight the advantages and disadvantages of using second order observers to estimate the states of realistic spacecraft.

In section 2, the governing CSI equations are presented using nomenclature similar to Refs. 9 and 10. It includes the classical first order observer and a logical form for the second order observer. Stability and performance of the observers are discussed in Section 3. Practical computational aspects of the observers are examined in Section 4. Section 5 presents numerical results which demonstrate the viability of second order observers. In addition, implications of the second order observer on CSI technology are discussed.

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## 2. Controlled Structure Simulation Equations

A control-structure interaction system can be represented as shown in Fig. 1. The linear time-invariant discretized equations of motion for the structure (plant) may be described by the following set of continuous-time equations:

$$\begin{aligned} M\ddot{q} + D\dot{q} + Kq &= f + Bu + Gw \\ q(0) &= q_0, \quad \dot{q}(0) = \dot{q}_0 \\ z &= H_d q + H_v \dot{q} + v \\ u &= F_1 q + F_2 \dot{q} \end{aligned} \quad (2.1)$$

where  $M$ ,  $D$ ,  $K$  are the mass, damping and stiffness matrices, respectively. These matrices, typically derived using finite element modeling, are symmetric and sparse. The vector  $f$  represents known, state independent, applied forces.  $B$  is the actuator location matrix and  $u$  is the state dependent control force. The matrix  $G$  describes the disturbance location and  $w$  is the disturbance vector. The displacement and velocity initial conditions are given by  $q_0$  and  $\dot{q}_0$ , respectively. The measured output of the system is  $z$  where  $v$  is a vector of measurement noise. The matrices  $H_d$  and  $H_v$  represent the displacement and velocity sensor locations and  $F_1$  and  $F_2$  are feedback gain matrices of the controller.

The conventional technique for dealing with multi-variable control problems is to convert the system to first order state-space form.

$$\begin{aligned} \dot{x} &= Ax + Ef + Bu + Gw \\ z &= Hx + v, \quad x(0) = x_0 \\ u &= Fx \end{aligned} \quad (2.2)$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix} \\ E &= \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ M^{-1}G \end{bmatrix} \\ H &= [H_d \ H_v], \quad F = [F_1 \ F_2] \\ x &= \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \end{aligned}$$

The control gain  $F$  may be synthesized by a number of techniques as given in standard texts such as Ref. 11. When full state feedback is used, and the full state cannot be measured, an observer model must be developed to estimate the states of the entire structure from the measured states. The form of the first order and second order observer models is discussed below.

### 2.1 First Order Observer Model

The observer model is usually constructed by augmenting (2.2) by a state correction term such that

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{E}f + \tilde{B}u + L(z - \tilde{H}\tilde{x}) \\ \tilde{x}(0) &= 0 \end{aligned} \quad (2.3)$$

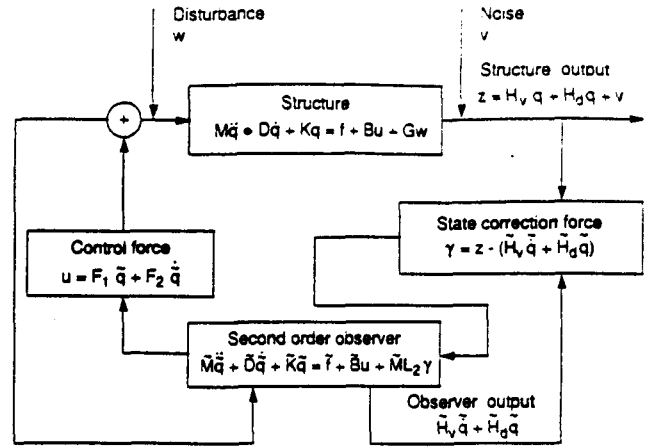


Figure 1. Typical Control/Structure Interaction System

where

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \quad \tilde{x} = \begin{bmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{bmatrix}$$

and  $\tilde{A}$ ,  $\tilde{E}$ ,  $\tilde{B}$  and  $\tilde{H}$  are a reduced order description of the structural system, actuator and sensor locations. The vector  $\tilde{x}$  is the estimated state. Adding the state correction term  $L(z - \tilde{H}\tilde{x})$  forces the observer model to track the measured states of the structure. Since the disturbance is unknown, it is not included in the observer.

The observer gain matrix  $L$  can be synthesized by a number of techniques. If covariance matrices of the noise and disturbance processes in (2.1) are available, the Kalman Filter can be constructed.<sup>9-11</sup> The Kalman Filter provides an optimal balance of observer performance and noise rejection; however, a first order observer is required. Since a structure is represented with second order models, it seems natural to choose a second order observer model for estimation of structural states. The second order observer is described below.

### 2.2 Second Order Observer Model

A linear observer model could be derived by adding a state correction term to a reduced order form of (2.1), such that the estimated states  $\tilde{q}$  and  $\dot{\tilde{q}}$  are computed from

$$\begin{aligned} \tilde{M}\ddot{\tilde{q}} + \tilde{D}\dot{\tilde{q}} + \tilde{K}\tilde{q} &= \tilde{E}f + \tilde{B}u + \tilde{M}L_2\gamma \\ \tilde{q}(0) &= 0, \quad \dot{\tilde{q}}(0) = 0 \end{aligned} \quad (2.4)$$

where  $\tilde{M}$ ,  $\tilde{D}$  and  $\tilde{K}$  represent a reduced order model of the structure and  $\gamma$  is a state correction term of the form

$$\gamma = z - (\tilde{H}_d \tilde{q} + \tilde{H}_v \dot{\tilde{q}})$$

The second order observer form given by (2.4) may be solved efficiently by keeping the control and state dependent state correction terms on the right-hand-side (RHS) of the equation. This permits the symmetry and sparsity of the observer mass, damping and stiffness matrices to be exploited by the

computational algorithms. References 4 and 5 present a partitioned solution procedure for this type of equation.

From (2.4) one observes the absence of  $L_1$ , which must be null for a second order observer to be constructed. The absence of  $L_1$  is needed to establish the identity relation  $\dot{\tilde{q}} = \dot{q}$ . A second order system is written in first order form by using the relation that the time derivative of position is equal to the velocity as shown in the top row of equation (2.2). To transform a first order system to a second order one, the identity relation must exist. The effects of  $L_1 = 0$  are described in the next section.

### 3. Stability and Performance of Observers

To study the ability of the observer to predict the states of the structure, the equation for the error between the actual states and the predicted states,  $e = (q - \tilde{q})$  is examined. To simplify the discussion, we consider an unreduced observer  $\tilde{A} = A$ ,  $\tilde{B} = B$ ,  $\tilde{H} = H$ . The deterministic error is given by

$$\dot{e} = \begin{bmatrix} -L_1 H_d & I - L_1 H_v \\ -M^{-1}K - L_2 H_d & -M^{-1}D - L_2 H_v \end{bmatrix} e \quad (3.1)$$

From (3.1), it is seen that the error asymptotically goes to zero, provided the matrix  $[A - LH]$  is stable. The next section describes the stability limitation of second order observers using a simple single-degree-of-freedom model.

#### 3.1 Observer Stability for a SDOF Model

A deterministic single-degree-of-freedom (SDOF) model consisting of a spring ( $k$ ), a damper ( $d$ ) and a mass ( $m$ ) is used in this section to highlight differences in stability between the first order and second order observer models. The system equations in first order form may be written for the structure as

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -k/m & -d/m \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} f + \begin{bmatrix} 0 \\ b/m \end{bmatrix} u \quad (3.2)$$

and for the observer as

$$\dot{\tilde{x}} = \begin{bmatrix} -L_1 H_d & 1 - L_1 H_v \\ -k/m - L_2 H_d & -d/m - L_2 H_v \end{bmatrix} \tilde{x} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} f + \begin{bmatrix} 0 \\ b/m \end{bmatrix} u + \begin{bmatrix} L_1 H_d & L_1 H_v \\ L_2 H_d & L_2 H_v \end{bmatrix} z \quad (3.3)$$

The stability of the observer can be determined from the poles of the error equation

$$\dot{e} = \begin{bmatrix} -L_1 H_d & 1 - L_1 H_v \\ -k/m - L_2 H_d & -d/m - L_2 H_v \end{bmatrix} e \quad (3.4)$$

The poles of  $(A - LH)$  in (3.4) are

$$\lambda_{1,2} = \sigma \pm j\omega \quad (3.5)$$

where

$$\sigma = \frac{-(d/m + L_2 H_v + L_1 H_d)}{2}$$

$$\omega^2 = [4k + 2L_1 H_d + 4mL_2 H_d - m(L_1 H_d + L_2 H_v)^2 - 2L_2 H_v d - 4L_1 H_v k - d^2/m]/4m$$

The pole locations given by (3.5) show the effect of the observer gains on the error between the actual and predicted states. The  $L_1$  gain increases the rate of decay of the error when position measurements are used. The  $L_1$  gain also effects the frequency of the error response when velocity measurements are used. The  $L_2$  gain uses the velocity measurement to increase the rate of decay and the  $L_2$  gain uses position measurement to effect the frequency of the error response.

From the above considerations, distinct differences are seen in the first order and second order observers ( $L_1 = 0$ ). The first order observer can produce a stable reconstruction error utilizing position and/or velocity measurements. The second order observer poles are given by

$$\sigma = \frac{-(d/m + L_2 H_v)}{2}$$

$$\omega^2 = \frac{4k + 4mL_2 H_d - m(L_2 H_v)^2 - 2L_2 H_v d - d^2/m}{4m}$$

which show velocity measurements must be available to augment the error decay rate. For example, if the structure was undamped, ( $d = 0$ ), and no velocity measurements were made ( $H_v = 0$ ) then the first order observer would have the real part of the poles located at

$$\sigma = \frac{-L_1 H_d}{2}$$

Thus the first order observer would be stable. However, the second order observer would be unstable,  $\sigma = 0$ . Even though stability is gained by a small amount of damping, ( $d \neq 0$ ), the second order observer would have poor performance without augmenting the damping through velocity feedback. Observer performance is discussed in the next section.

#### 3.2 Performance Considerations

The performance of the observer depends on the eigenvalues and eigenvectors of the matrix  $[A - LH]$ . The eigenvalues determine the speed and rate of observer convergence. Rule of thumb choices for the eigenvalues should be somewhat larger than the closed loop system  $[A + BF]$ , but not exceedingly so, to prevent problems due to high frequency noise. The eigenvalues and eigenvectors of the observer matrix can be placed to maximize noise and disturbance rejection. Because the second order observer restricts the design,  $L_1 = 0$ , some of the freedom in placing the eigenvalues and eigenvectors is lost.

Robust methods for designing second order observer gains remains a fruitful area of research. The results herein have synthesized Kalman Filter gains and subsequently ignored the  $L_1$  gain. This approach may or may not lead to a stable observer. If  $L_2$  is not stabilizing it must be augmented. Even if  $L_2$  is stabilizing some loss of performance is expected, par-

ticularly in terms of noise rejection. However, this may be a reasonable penalty for the increased computational speed gained by using a second order observer model which could be translated into reducing the level of unmodeled dynamics. The model error (uncertainty) would be the same for the first or second order observer. Future research is needed to determine the robustness characteristics of second order observers.

#### 4. Computational Requirements for Simulation

Solution of the CSI equations is typically performed using digital computations. It is this consideration that leads to the study of second order observers for CSI technology. As shown in the following paragraphs, the second order observer provides the potential for larger observers and/or faster computational solutions than can be realized with the first order observer

##### 4.1 First Order Observer Solution

The first order observer given by (2.3) may be solved by several methods. One popular method is to compute the matrix exponential of

$$\mathbf{A}_o = [\bar{\mathbf{A}} + \bar{\mathbf{B}}\mathbf{F} - \mathbf{L}\bar{\mathbf{H}}] \quad (4.1)$$

so that

$$\tilde{\mathbf{x}}^{n+1} = \Phi(h)\tilde{\mathbf{x}}^n + \Gamma(h)(\mathbf{E}\mathbf{f}^n + \mathbf{L}\mathbf{z}^n) \quad (4.2)$$

where

$$\Phi(h) = e^{\mathbf{A}_o h}, \quad \Gamma(h) = \int_0^h e^{\mathbf{A}_o s} ds$$

and  $h$  is the temporal step size.

Defining  $N$  as the number of structural degrees of freedom, then  $\Phi(h)$  is a  $2N$  by  $2N$  matrix.

Because  $\Phi(h)$  is computed once for time invariant systems, the matrix exponential computation does not significantly impact the total time required for a long simulation. However, computing the matrix exponential for large systems does cause accuracy and storage problems.  $\Phi(h)$  occupies  $4N^2$  storage locations since the matrix is typically not symmetric or sparse. Independent of the number of sensors, (4.2) requires, as a minimum, multiplication of  $\Phi(h)$  by a vector at each time step. This represents  $4N^2$  operations at each step of the simulation.

An alternative approach to the matrix exponential route is to discretize the equations of motion in time to derive difference equations. This approach has been used by the present authors in Refs. 4 and 5. Midpoint implicit integration formulas were used to discretize (2.3) to derive a linear equation

$$\begin{aligned} \mathbf{S}\tilde{\mathbf{x}}^{n+1/2} &= \mathbf{g}^{n+1/2} \\ \tilde{\mathbf{x}}^{n+1} &= 2\tilde{\mathbf{x}}^{n+1/2} - \tilde{\mathbf{x}}^n \end{aligned} \quad (4.3)$$

where

$$\mathbf{S} = \mathbf{I} - \frac{h}{2}\mathbf{A}_o, \quad \mathbf{g} = \frac{h}{2}(\mathbf{E}\mathbf{f}^{n+1/2} + \mathbf{L}\mathbf{z}^{n+1/2}) + \tilde{\mathbf{x}}^n$$

To solve (4.3) an L-U decomposition is performed once for time invariant systems and subsequently a lower and upper triangular system is solved at each time step. Since the L-U decomposition of  $\mathbf{S}$  is not sparse or symmetric,  $4N^2$  storage locations are required. In addition, the two triangular system solutions require  $4N^2$  operations. Thus, the first order observer requires the same computational resources for both the matrix exponential approach and the time discretization approach.

##### 4.2 Second Order Observer Solution

The second order observer model given in (2.4) can be solved using the time discretization approach as presented in Refs. 4 and 5. A summary of the procedure is presented herein.

We express a set of the mid-point implicit formulas with the step size  $h$ :

$$\begin{cases} \tilde{\mathbf{q}}^{n+1/2} = \tilde{\mathbf{q}}^n + \delta \dot{\tilde{\mathbf{q}}}^{n+1/2}, & \delta = h/2 \\ \dot{\tilde{\mathbf{q}}}^{n+1/2} = \dot{\tilde{\mathbf{q}}}^n + \delta \ddot{\tilde{\mathbf{q}}}^{n+1/2} \\ \tilde{\mathbf{q}}^{n+1} = 2\tilde{\mathbf{q}}^{n+1/2} - \tilde{\mathbf{q}}^n \end{cases} \quad (4.4)$$

The selection of the mid-point implicit formula (or the trapezoidal rule) for controlled structures is due to its minimal frequency distortion and no numerical damping characteristics.

Implicit time discretization of (2.4) using (4.4) yields the following difference equation:

$$\begin{cases} \tilde{\mathbf{S}}\tilde{\mathbf{q}}^{n+1/2} = \tilde{\mathbf{g}}^{n+1/2} \\ \tilde{\mathbf{S}} = \tilde{\mathbf{M}} + \delta \tilde{\mathbf{D}} + \delta^2 \tilde{\mathbf{K}} \\ \tilde{\mathbf{g}}^{n+1/2} = \delta^2 (\mathbf{f}^{n+1/2} + \tilde{\mathbf{B}}\mathbf{u}^{n+1/2} + \tilde{\mathbf{M}}\mathbf{L}_2\gamma^{n+1/2}) \\ \quad + \tilde{\mathbf{M}}(\tilde{\mathbf{q}}^n + \delta \dot{\tilde{\mathbf{q}}}^n) + \delta \tilde{\mathbf{D}}\dot{\tilde{\mathbf{q}}}^n \\ \tilde{\mathbf{q}}^{n+1} = 2\tilde{\mathbf{q}}^{n+1/2} - \tilde{\mathbf{q}}^n \\ \dot{\tilde{\mathbf{q}}}^{n+1/2} = (\tilde{\mathbf{q}}^{n+1/2} - \tilde{\mathbf{q}}^n)/\delta \end{cases} \quad (4.5)$$

From (4.5) it can be seen that  $\mathbf{u}^{n+1/2}$  and  $\gamma^{n+1/2}$  are required to numerically solve for  $\tilde{\mathbf{q}}^{n+1/2}$ . The partitioned solution procedure<sup>5,6</sup> uses equation augmentation to predict  $\mathbf{u}^{n+1/2}$  and  $\gamma^{n+1/2}$  thereby maintaining the symmetry and sparsity of the  $\tilde{\mathbf{S}}$  matrix even after L-U decomposition. For the second order observer,  $\tilde{\mathbf{S}}$  is an  $N$  by  $N$  matrix. If the bandwidth of  $\tilde{\mathbf{S}}$  is proportional to  $N$  by a constant  $\alpha$ :

$$\text{Bandwidth} = \alpha N$$

then, only  $\alpha N^2$  storage location are needed. Most importantly, only  $2N(\alpha N + 1)$  operations are needed for the two triangular system solutions at each time step.

Comparing the second order observer minimum solution time to the first order observer minimum solution time yields

$$\frac{(2\alpha N^2 + 2N)}{4N^2} = \frac{\alpha + \frac{1}{N}}{2}$$

Note this is an approximate count of the number of operations as the right-hand-side of (4.2) and (4.5) require additional operations. Nevertheless, for a typical bandwidth of 10 percent, and  $N \gg 1$ , the second order observer requires approximately 5 percent of the number of operations as does the first order observer. (Reduced order observer

models based on a modal description of the structure yields  $\alpha N = 11$ ) Such a drastic reduction in computations may lead to new applications of observers in CSI technology.

## 5. Results and Discussion

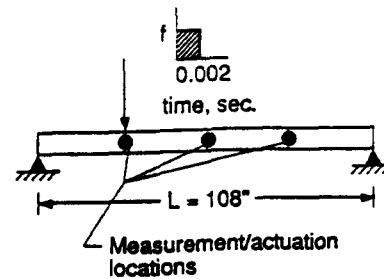
To evaluate the performance of the second order observers numerically, several examples are presented. The first example is relatively simple, a beam with an unreduced observer model. The second example is more realistic in that a large truth model is used in conjunction with a reduced order observer. The encouraging results from these examples prompt us to consider serious use of second order observers in CSI technology.

### 5.1 Beam Example

To study the effects of approximating a first order observer by a second order observer, the beam model shown in Fig. 2 has been used for numerical studies. The beam is subjected to a step load for 0.002 seconds and a regulator control system is activated at  $t=0.01$  sec. A finite element model was constructed with 8 planer beam elements to form a 27 degree of freedom representation of the beam. A three mode modal space control law was used with the performance constraint that the vibration amplitude must be less than 0.025 in. within 0.1 sec. after the control system is activated. Lateral position and velocity measurements were made at each of the three locations indicated in Fig. 2 for a total of 6 measurements. A Kalman Filter was constructed to predict the unmeasured states of the system. Results for the unreduced structure and observer are shown in Fig. 3. The observer, shown by the dashed line, rapidly converges to the true position of the beam lateral deflection (measured at the point of loading).

To determine the viability of a second order observer,  $L_1$  was set equal to zero and the observer equation was solved in second order form. The response of the second order observer is shown in Fig. 4. The second order observer is slower to converge to the actual beam deflection than the first order observer. Nevertheless, the second order observer did converge, thus, the observer stability was maintained when  $L_1$  was neglected.

The beam response with the control law based on a perfect observer (all states are reconstructed with no error), the first order observer and the second order observer is presented in Fig. 5. Although the second order observer produced noticeable performance degradation in the initial time after the control system was activated, it achieved nearly the same level of vibration attenuation after  $t=0.1$  sec. as did the perfect observer. Thus, the second order observer is quite viable for this example where both position and velocity measurements were available.



Properties	Open loop frequencies (Hz)
$E = 40E + 6 \text{ lb/in.}^2$	$f_1 = 56.06$
$G = 2.4E + 6 \text{ lb/in.}^2$	$f_2 = 233.22$
$p = 1.132E - 4 \text{ lb-s}^2/\text{in.}$	$f_3 = 556.73$
	$f_4 = 2693.6$

Figure 2. Pinned-Pinned Beam Structure

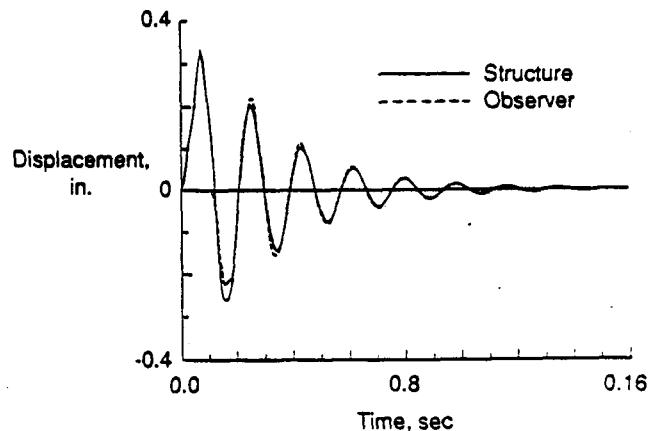


Figure 3. Beam and First Order Observer Response

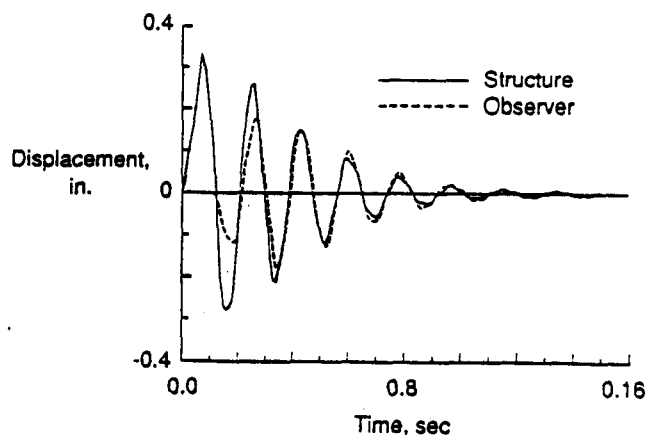


Figure 4. Beam and Second Order Observer Response

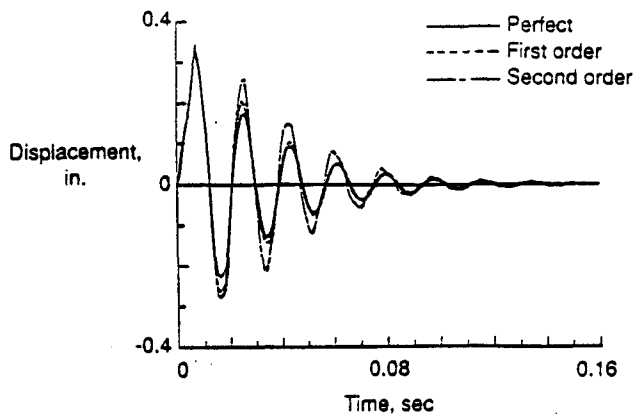


Figure 5. Comparison of Beam Response with Different Observers

### 5.2 Earth Pointing Satellite Example

A more realistic example of spacecraft being studied for active vibration control is the Earth Pointing Satellite (EPS) shown in Fig. 6. This model is a derivative of a geostationary platform designed for study of Earth Observation Sciences (EOS). Two flexible antennas are attached to a truss (bus). Typical missions involve pointing one antenna to earth, while tracking with the other antenna.

The truss has a three meter square cross section. It consists of 51 mm diameter by 1.59 mm wall thickness graphite epoxy tubes. The modulus of elasticity and mass density of the tubes are  $275.9 \text{ E}+09 \text{ N/M}^2$  and  $3250.0 \text{ Kg/M}^3$ , respectively. Although the antennas have the same tube dimensions as the truss, nonstructural mass make the effective tube density  $= 9750.0 \text{ Kg/M}^3$  for the two antennas. The finite element representation of the EPS consisted of 570 degrees of freedom. Table 1. lists the first 20 frequencies of the EPS structure.

Table 1. EPS Vibration Frequencies (Hz.)

1. 0.000,	2. 0.000
3. 0.000,	4. 0.000
5. 0.000,	6. 0.000
7. 0.242,	8. 0.406
9. 0.565,	10. 0.656
11. 0.888,	12. 0.888
13. 1.438,	14. 1.536
15. 1.776,	16. 1.776
17. 3.026,	18. 3.026
19. 3.513,	20. 3.531

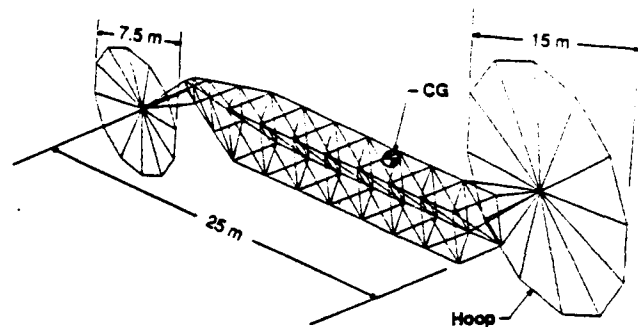


Figure 6. Earth Pointing Satellite Structure

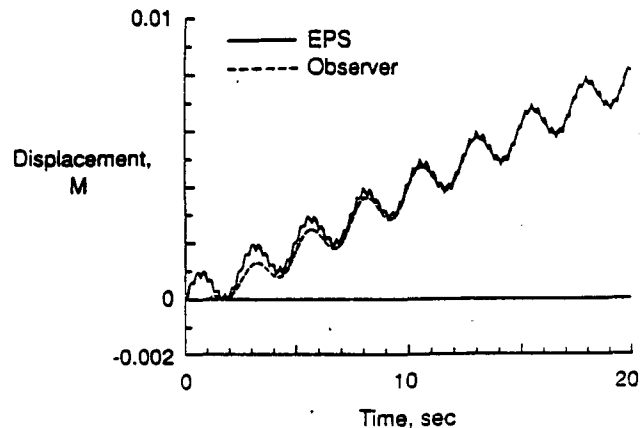


Figure 7. EPS Antenna and Second Order Observer Response

To study the viability of second order observers for this structure, sensors were located on the 15 M antenna hoop shown in Fig. 6 to measure the position and velocity of the antenna hoop. In addition, sensors were located at the center of gravity (CG) to measure the spacecraft attitude and attitude rate. The number of measurements used in the construction of Kalman Filter gains varied from 10 to 20 to create three different reduced order observer models. Again, the second order observer was derived by neglecting the  $L_1$  gain which can result in an unstable observer for some problems. The observer was studied independent of a feedback control law by simply applying an impulsive loading to the open loop structure. An impulse was applied as an initial angular velocity about the z-axis of  $100.0 \text{ rad/sec}$  between the 15 M antenna and the truss. Note that this excitation will excite both rigid body and flexible vibrations of the EPS.

Figure 7 shows the structure and observer displacement response for 15 modes in the observer model, and a full 570 degree of freedom model of the EPS. Although the observer is stable, considerable error exists at the higher frequencies. Figure 8 shows the error between the EPS states and the observer states as the number of modes in the observer model was increased from 10 to 20. These results show the importance of larger observers in state estimation of realistic spacecraft.

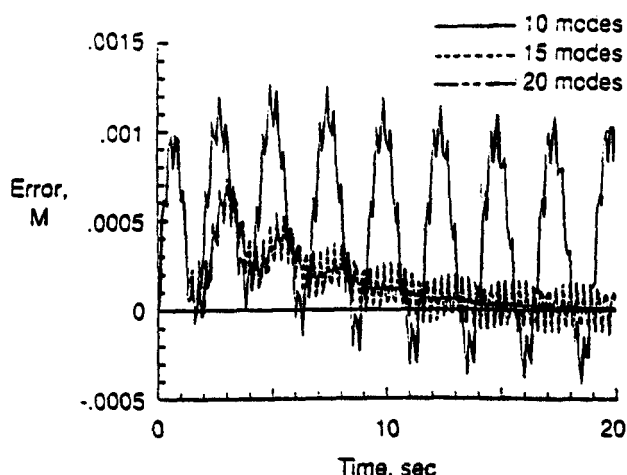


Figure 8. Reconstruction Error Convergence for EPS Observers

### 6. Concluding Remarks

Classical multivariable control observer gain synthesis algorithms produce first order observer models. Results have been presented which show that approximating the first order observers by a second order observer can be successfully carried out if velocity measurements are available.

This paper, in conjunction with previous works by the authors, has shown that second order observer models are computationally advantageous. The increased computational speed can be translated to larger observers and/or faster sampling rates. Larger observers allow the designer to control more 'modes' of the structure which is increasingly important for future spacecraft missions. On the other hand, if the designer chooses, he may translate the increased computational speed into faster measurement/control updates. This may prevent phase distortion in some applications and may actually enable observer based controllers when large bandwidths are necessary.

Advances in CSI technology will occur by exploiting the physical attributes of structures and structural models. This paper should serve to stimulate interest in the use of second order observers for state estimation of structures. Second order observers may prove sub-optimal as compared to the first order observer, nevertheless, practical considerations such as computational speed may be the decisive factor in developing CSI technology. More research is needed to determine if full state feedback is the control law of preference for controlled spacecraft. If the answer is affirmative, the second order observer promises to play an important role. However, much research is still needed in the areas of directly synthesizing second order observer gains and on determining robustness of the second order observer.

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16. Abstract  The use of Linear Quadratic Regulator (LQR) control synthesis techniques implies the availability of full state feedback. For vibration control of structures, usually only a limited number of states are measured from which an observer model reconstructs the full state. This paper shows that using second order observers is a viable technique for reconstructing the unmeasured states of structures under mildly restrictive conditions. Moreover, the computational advantages of the second order observer as compared to a first order observer indicate that significantly larger observer models may be utilized. Numerical examples are used to demonstrate the performance of second order observers. The implications of second order observers in the development of Controls-Structures Interaction (CSI) technology is discussed.					
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