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Nonlinear Feedback Control of Multiple Robot Arms

T.J. Tarn and X. Yun
Washington University
St. Louis, MO 63130

A.K. Bejczy
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, CA 91109

1. Abstract

~~In this paper we model~~ multiple coordinated robot arms are modeled by considering the arms (1) as closed kinematic chains and (2) as a force constrained mechanical system working on the same object simultaneously. In both formulations a new dynamic control method is discussed. It is based on a feedback linearization and simultaneous output decoupling technique. Applying a nonli ear feedback and a nonlinear coordinate transformation, the complicated model of the multiple robot arms in either formulation is converted into a linear and output decoupled system. The linear system control theory and optimal control theory are used to design robust controllers in the task space. The first formulation has the advantage of automatically handling the coordination and load distribution among the robot arms. In the second formulation, by choosing a general output equation we could superimpose the position and velocity error feedback with the force-torque error feedback in the task space simultaneously.

2. Introduction

The notion of "multiple robot arms" originates from two everyday scenarios. The first scenario is an authropomorphic one by noting that humans have two arms and hands and everyday manual work is normally performed by two-handed humans. In fact, manual activities and tasks are normally perceived and designed such that they assume two-handed humans; a one-handed person is a handicapped person from that point of view. Thus, in order to replace humans with robots to perform normal manual activities it seems natural to visualize and design robots with two arms and hands. The second scenario is an industrial one by noting that production lines in industry assume an organized distribution of manipulative activities along the production line that can be carried out by a distributed set of robot arms in a proper arrangement.

Scenarios of multiple robot arms are also assumed and predicted for space applications in a natural way. Space station assembly, maintenance and servicing will require the in-site manual work of EVA astronauts in the initial operational configuration. This manual work also includes the simultaneous activities of two or more EVA astronauts in the handling or assembly of large structural elements in space. Most satellite servicing and maintenance operations also assume two-handed manual work of EVA astronauts. Thus, the objective of decreasing EVA activities in Earth orbit by introducing and increasing robot activities there requires the consideration and the design of the control of multiple robot arms.

The technically interesting and challengine problems in the control of multiple robot arms arise when (i) the work envelopes of two or more robot arms overlap and (ii) two or more robot arms simultaneously work on the same object in a presumably cooperative manner to perform a given task which cannot be performed by one arm only.

The Control problem of two or multiple robot arms has been studied by many investigators [1-12]. Although the control problem of two or multiple arms is complex, some examples of applications, such as a two-arm lathe loader, a two-arm robot press loader/unloader, and two single-arm robots working together to handle stamping press loading and unloading, are given by Chimes [1]. In these applications, the problem is solved specifically. The system design is based on a solid understanding of the problem.

Hemami and Wyman [2] investigated the problem of force control in closed chain dynamic systems. In their work, the dynamic system is linearized about an operating point and linear feedback is used to maintain the forces of constraints. The validity of the method is restricted to a rather small neighborhood of the operating point in which the dynamic system can be linearized. Orin and Oh [3] considered the control of force distribution in robotic mechanisms containing closed kinematic chains. The problem of solving for the input joint torques from a given trajectory is underspecified. The linear programming has been used to obtain a solution which optimizes a weighted combination of energy consumption and load balancing. The dynamic equations of the mechanisms are excluded from the control method. The stability of the control algorithm is in no way ensured. Ishida [4] developed a force control technique which uses a wrist force sensor to measure the interactive force between two arms. The parallel transfer task and the rotational transfer task are considered only. The control algorithm is derived for both master/slave mode and indistinguished mode (the same status mode). Fujii and Kurono [5] proposed the method of virtual reference. This method consists of the identification of the joint control mode required to perform a desired Cartesian motion. The control loop at each joint uses only position feedback and no compensation for the coupling between joints.

Alford and Belyeu [6] have designed a hierarchical computer control structure for two PUMA robot arms operating in a master/slave mode. The proposed coordinated control system has joint position predictors, a coordinate transformation, and a slave command modifier. An explicit control algorithm is derived and tested/implemented for an experimental path: a straight line in the vertical direction. However, the question on how to define the prediction function, the transformation, and the modification function is left open in the paper, and the dynamics of the arms is excluded from the algorithm.

When two robot arms work on an object certain constraints must be satisfied in order to carry out a smooth, coordinated operation. Zheng and Luh [7] have derived a set of holonomic constraints on positions and orientations of the end effectors for two robots in three specific working conditions, namely, handling a rigid-body object, handling a pair of pliers, and handling an object having a spherical joint. The result is extended to the constraints between joint velocities and accelerations of the two robots for the three above mentioned cases [8].

Considering tasks of transferring an object by holding it with two robot arms, Lim and Chyung [9] introduced a position control method using kinematic relations between the object and the two robot arms. By first specifying the trajectory of the object, the differential changes of each robot hand are computed from the differential changes of the planned path. The commands or differential changes of each joint of the two robot arms are generated by applying the inverse Jacobian matrix. The method is simple but applicable only when the involved motion is very slow. Freund and Hoyer [10-12] proposed a hierarchical control method for collision avoidance in multi-robot systems. The method adopts a hierarchical coordinator and is systematic. However, an algorithm is needed to design the couplings among robots. Vukobratovic and Potkonjak [13] described a method which can be used to obtain the closed chain dynamics of two coordinated robot arms. However, the reaction force and reaction moment between the two arms are retained in the final equations. Hayati [20] extended the idea of hybrid position/force control to the multi-arm case. Based on equations of motion for a multi-arm system, which are derived in a constrained coordinate frame located at the grasped object, a controller is designed to cooperate n robot arms such that the load is shared among the arms in a non-conflicting way. A minimization of the magnitude of forces and torques is performed to decide how much each robot arm should contribute. It appears that the existing coordinated control methods fall in lack of either systematic synthesis of the control system or full consideration of robot arm dynamics.

In this paper we concentrate on the application of nonlinear feedback to the control of multiple robot arms. Previously we derived a general algorithm for the control of a single rigid robot arm through nonlinear feedback and state transformation resulting exact system linearization and simultaneous output decoupling [15,16]. Our control design technique elevates the robot arm servo problem from the joint space to the task space with three important consequences. (i) On the joint level our scheme computes and commands drive forces or torques on their actuator-equivalent quantities (current, voltage, pressure). (ii) The robot arm system in the task space is considered as a linear system, and the powerful tools of linear control theory, including optimal control, are applicable to robot arm controller design in the task space. (iii) Our controller can directly respond to task space commands provided that these commands are formulated in form of closed time functions. The question discussed in this paper is: how can our control method be applied to the control of multiple robot arms.

We are discussing two modeling approaches. In the first approach, we model the multiple arm system as a single system, that is, as a closed loop kinematic chain. In the second approach we retain the single arm models, but we introduce task constraints and force-moment measurements in the control scheme. The paper concludes with a brief discussion of computational architectures that are needed to implement our control technique for the control of multiple robot arms.

3. Closed Chain Formulation

As the first approach to coordinated control of multiple robot arms, we consider the multiple robot arms as a single mechanical system consisting of kinematic closed chains. For tasks of lifting a heavy workpiece using robot arms, two or more robots are required if the workpiece is out of loading limit of any available robot arm. Suppose that m robot arms are used in such a task and that they all grasp on the same object (workpiece) in order to lift it, turn it, etc. Our primary concern is to obtain a dynamic model of these robots for the control purpose. Since they grasp on the same object, the dynamic behavior of one robot is not independent of the dynamic behavior of the other robots any more. A unity of mechnical system is rather formed by the robot arms involved and by the grasped object.

We will derive the Lagrange's equations of motion for this mechnical system. Those equations will serve as a model of the system to design control algorithms. For the m robots of consideration, we name them robot 1, robot 2, ..., and robot m, respectively. We assume that robot i has n_i links. We also assume that each robot firmly grasps the object so that there is no movement between its end effector and the object. Closed chains are formed in such a configuration by the m robot arms, the object, and the ground. Notice that the object and the last links of the robot arms become a single link. From the Kutzbach-Grubler criterion [17], the degrees of freedom of a spatial linkage structure connected by joints with each joint possessing one degree of freedom are given as follows

$$p = 6(i-1) - 5j \tag{1}$$

where i is the number of links and j is the number of joints. This formula reflects the fact that each moving link has six degrees of freedom and the fixed link (the ground) has none, and that each joint of one degree of freedom causes a loss of five degrees of freedom for a link. For our case of m robots, the degrees of freedom of this entire mechanical system is then

$$p = 6[\sum_{k=1}^{m} (n_k-1) + 1] - 5 \sum_{k=1}^{m} n_k = \sum_{k=1}^{m} n_k - 6m + 6 \tag{2}$$

where n_k is the number of links of robot k. If three robot arms are involved to perform a task, Table 1 shows 10 different combinations of three robot arms with five, six or seven degrees of freedom.

Before proceeding, let us define some notations that will be used in the rest of this section.

$\theta^i = [\theta_1^i \ \theta_2^i \ \dots \ \theta_{n_i}^i]'$: joint variables of robot i

$\theta = [(\theta^1)' \ (\theta^2)' \ \dots \ (\theta^m)']'$: joint variables of the mechanical system

$q = [q_1 \ q_2 \ \dots \ q_p]'$: generalized coordinates

$\tau = [\tau_1 \ \tau_2 \ \dots \ \tau_p]'$: generalized forces corresponding to q

$F^i = [F_1^i \ F_2^i \ \dots \ F_{n_i}^i]'$: joint force/torque of robot i

$F = [(F^1)' \ (F^2)' \ \dots \ (F^m)']'$: joint force/torque of the mechanical system

$n = n_1 + n_2 + \dots + n_m$.

The generalized coordinates q can be chosen arbitrarily as long as they are linearly independent of each other. They are functionally related to the joint variables θ. We denote the relation by

$$q = Q(\theta) \ . \tag{3}$$

Knowing the generalized coordinates q, the configuration of the mechanical system, thus the joint variable θ, is uniquely determined. We denote such inverse relation by

$$\theta = \Theta(q) \ . \tag{4}$$

With the above notations, the Lagrange's equations of motion for the mechanical system are described by

$$\left(\frac{\partial\Theta}{\partial q_i}\right)' \left(\frac{\partial^2 L}{\partial\dot\theta^2} \frac{\partial\Theta}{\partial q} q + \frac{\partial^2 L}{\partial\dot\theta^2} \begin{bmatrix} \dot q' \dfrac{\partial^2\Theta_1}{\partial q^2} \dot q' \\ \vdots \\ \dot q' \dfrac{\partial^2\Theta_n}{\partial q^2} \dot q \end{bmatrix} + \frac{\partial^2 L}{\partial\theta\partial\dot\theta} \frac{\partial\Theta}{\partial q} \dot q - \left(\frac{\partial L}{\partial\theta}\right)' \right) = \left(\frac{\partial\Theta}{\partial q_i}\right)' F$$

$$i = 1, 2, \dots, p \tag{5}$$

where L is the Lagrangian of the whole mechanical system. Equation (5) is a generalization of the equations of motion of two robot arms presented in [14].

We assign a coordinate frame to each link of every robot arm. We locate a world coordinate frame in the common work space of the m robots. In the process of expressing the kinetic and potential energies of the mechanical system, we divide the mass of the object into m parts. Each robot is responsible for one part of the object mass by adding it to the mass of the last link. After carrying out the derivations of the Lagrangian function, we obtain, the dynamic equations of the mechanical system

$$D(q)\ddot{q} + E(q, \dot{q}) + G(q) = J_\theta' F \tag{6}$$

where

$$D(q) = J_\theta' \tilde{D}(\theta(q)) J_\theta$$

$$J_\theta = \frac{\partial\theta}{\partial q}$$

$$\tilde{D}(\theta) = \begin{bmatrix} D^1 & & & \bigcirc \\ & D^2 & & \\ & & \ddots & \\ \bigcirc & & & D^m \end{bmatrix}$$

$D^r = (D^r_{ij})$ is the inertia matrix of robot r

$$D^r_{ij} = \sum_{k=\max(i,j)}^{n_r} \text{Trace}\left(\frac{\partial T^r_k}{\partial \theta^r_i} I^r_k \frac{\partial (T^r_k)'}{\partial \theta^r_j}\right)$$

$$E(q,\dot{q}) = J_\theta' \tilde{D}(\theta(q)) \begin{bmatrix} \dot{q}' \dfrac{\partial^2 \theta_1}{\partial q^2} \dot{q} \\ \vdots \\ \dot{q}' \dfrac{\partial^2 \theta_n}{\partial q^2} \dot{q} \end{bmatrix} + J_\theta' \begin{bmatrix} \dot{q}' J_\theta' E_1 \\ \vdots \\ \dot{q}' J_\theta' E_n \end{bmatrix} J_\theta \dot{q}$$

$$E_i = \begin{bmatrix} E_i^1 & & & \bigcirc \\ & E_i^2 & & \\ & & \ddots & \\ \bigcirc & & & E_i^m \end{bmatrix}, \quad i=1, \ldots, n$$

$E^r_i = (E^r_{ijk})$ is the coefficient of centripetal ($j=k$) or Coriolis ($j \neq k$) force of robot r

$$E^r_{ijk} = \sum_{s=\max(i,j,k)}^{n_r} \text{Trace}\left(\frac{\partial T^r_s}{\partial \theta^r_k \partial \theta^r_j} I^r_s \frac{\partial (T^r_s)'}{\partial \theta^r_i}\right)$$

$$G(q) = -J'_0 \begin{bmatrix} G^1 \\ G^2 \\ \cdot \\ \cdot \\ G^m \end{bmatrix}, \qquad G^r = \begin{bmatrix} G_1^r \\ \cdot \\ \cdot \\ \cdot \\ G_{n_r}^r \end{bmatrix}$$ is the gravity force of roLot r

$$G_i^r = \sum_{k=i}^{n_r} m_k^r \, g' \frac{\partial T_k^r}{\partial \theta_i^r} \bar{r}_k^r \; .$$

In the above definitions, $T_i^r = A_{01}^r A_{12}^r \cdots A_{(i-1)i}^r$, where A_{ij}^r is the Denavit-Hartenberg homogeneous transformation matrix from coordinate frame i to coordinate frame j of robot r; m_i^r is the mass of link i of robot r; $\bar{r}_i^r$ is the mass center of link i of robot r; I_i^r is the pseudoinertia matrix of link i of robot r; g is the acceleration of gravity, defined to be a 4x1 column vector with the last component being equal to zero.

Equation (6) characterizes the dynamic behavior of the whole mechanical system. However, this equation is nonlinear, coupled, and complicated. It poses great difficulty in controller designs. We propose to linearize and output decouple the system (6) using a nonlinear feedback and a nonlinear coordinate transformation. Let us introduce a state space variable x by setting

$$x_i = q_i \,, \quad x_{i+p} = \dot{q}_i \quad , \quad i=1, 2, \ldots, p$$

$$x^1 = [x_1 \; x_2 \; \ldots \; x_p]' \quad , \quad x^2 = [x_{p+1} \; \ldots \; x_{2p}]'$$

$$x = \begin{bmatrix} x^1 \\ x^2 \end{bmatrix}$$

The dynamic equation (6) can be written as

$$\dot{x} = \begin{bmatrix} x^2 \\ -D^{-1}(x^1)[E(x^1,x^2)+G(x^1)] \end{bmatrix} + \begin{bmatrix} 0 \\ D^{-1}(x^1) \; J'_0 \end{bmatrix} F$$
$$= f(x) + g(x)F \tag{7}$$

We take the position (orientation) of the object handled as the system output

$$y = h(x^1) = [h_1(x^1) \; h_2(x^1) \; \ldots \; h_p(x^1)]'. \tag{8}$$

For the nonlinear feedback, the so-called decoupling matrix is [15,16]

$$A(x) = J_h(x^1) \; D^{-1}(x^1) \; J'_0$$

where J_h is the Jacobian matrix of h. The nonlinear feedback has the form

$$F = \alpha(x) + \beta(x) \; u$$

where $\alpha(x)$ and $\beta(x)$ are determined from the following two algebraic equations [15,16]

$$A(x) \; \alpha(x) = -L_f^2 h \tag{9}$$

$$A(x) \; \beta(x) = \gamma. \tag{10}$$

In the above equations, $L_f^2 h$ is the second order Lie derivative of h along f,

$$\gamma = \begin{bmatrix} \gamma_1 & & & \\ & \gamma_2 & & 0 \\ & & \ddots & \\ 0 & & & \gamma_p \end{bmatrix},$$

$\gamma_i = [1\ 1\ \ldots\ 1]$ is a $1 \times m_i$ new vector with all entries equal to 1 and m_i, $i=1, \ldots p$, are chosen such that $m_i > 0$ and $m_1 + m_2 + \ldots + m_p = n$. The index m_i is associated with the fact that a total number of n independent actuators (inputs) are to be divided into p groups to control p outputs. The required nonlinear coordiante transformation is given by [15,16]

$$\phi(x) = [h_1\ L_f h_1\ \ldots\ h_p\ L_f h_p]' .$$

Since both equations (9) and (10) are underdetermined, there are infinite many solutions for them. Any solution serves the purpose of linearization and decoupling provided that $\beta(x)$ is invertible. A solution to equation (9) is given by [18]

$$\alpha(x) = -A^+(x)\ L_f^2 h(x) \tag{11}$$

where $A^+ = A'(AA')^{-1}$ is the generalized inverse of $A(x)$. The general solution to equation (10) is [18]

$$\beta(x) = A^+(x)\gamma + (I - A^+A)\ H \tag{12}$$

where H is an arbitrary matrix which is to be chosen to make $\beta(x)$ invertible.

After applying the nonlinear feedback and the nonlinear coordinate transformation, the original system (7) with output (8) is converted into the following linear and decoupled system

$$\dot{z} = Az + Bu \tag{13a}$$

$$y = Cz \tag{13b}$$

where

$$A = \begin{bmatrix} A_1 & & 0 \\ & \ddots & \\ 0 & & A_p \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & & 0 \\ & \ddots & \\ 0 & & B_p \end{bmatrix}, \quad C = \begin{bmatrix} C_1 & & 0 \\ & \ddots & \\ 0 & & C_p \end{bmatrix}$$

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ \gamma_i \end{bmatrix}, \quad C_i = [1\ 0], \quad i=1, \ldots, p.$$

Note that the obtained linear system (13) consists of p independent subsystems. The control problem of the whole mechanical system is then simplified to a design problem of individual subsystems. The ith subsystem is defined by

$$\begin{bmatrix} \dot{z}_{2i-1} \\ \dot{z}_{2i} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_{2i-1} \\ z_{2i} \end{bmatrix} + \begin{bmatrix} 0 \\ \gamma_i \end{bmatrix} u^i \tag{14a}$$

$$y_i = [1 \quad 0] \begin{bmatrix} z_{2i-1} \\ z_{2i} \end{bmatrix}, \quad i = 1, \ldots, p \tag{14b}$$

where u^i is the ith group input with m_i components. To stabilize the subsystem (14), we introduce a constant feedback $u^i = - K^i z^i + v^i$ with

$$K^i = \begin{bmatrix} 0 & 0 \\ k_{i1} & k_{i2} \end{bmatrix}$$

where $z^i = [z_{2i-1} \quad z_{2i}]'$, and v^i is the new reference input. With such a constant feedback, subsystem (14) becomes

$$\begin{bmatrix} \dot{z}_{2i-1} \\ \dot{z}_{2i} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_{i1} & -k_{i2} \end{bmatrix} \begin{bmatrix} z_{2i-1} \\ z_{2i} \end{bmatrix} + \begin{bmatrix} 0 \\ \gamma_i \end{bmatrix} v^i \tag{15a}$$

$$y_i = [1 \quad 0] \begin{bmatrix} z_{2i-1} \\ z_{2i} \end{bmatrix}, \qquad i = 1, \ldots, p, \tag{15b}$$

or in compact form

$$\dot{z}^i = \bar{A}_i z^i + B_i v^i$$

$$y_i = C_i z^i$$

where $\bar{A}_i$ can be easily identified from equation (15a). For the above system (15), the damping ratio ξ and the natural frequency ω_n are related with the feedback gains by

$$\omega_n^2 = k_{i1} \qquad\qquad 2 \xi \omega_n = k_{i2}.$$

We now consider equation (15) as the new mathematical model of the real system which is exactly linearized, output decoupled and stabilized. The desired (nominal) input to each subsystem can be derived from the following system

$$\begin{bmatrix} \dot{z}^d_{2i-1} \\ \dot{z}^d_{2i} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_{i1} & -k_{i2} \end{bmatrix} \begin{bmatrix} z^d_{2i-1} \\ z^d_{2i} \end{bmatrix} \begin{bmatrix} 0 \\ \gamma_i \end{bmatrix} (v^i)^d \tag{16a}$$

$$y^d_i = [1 \quad 0] \begin{bmatrix} z^d_{2i-1} \\ z^d_{2i} \end{bmatrix}, \qquad i = 1, \ldots, p \tag{16b}$$

where the superscript "d" indicates "desired" quantities. From equation (16), the desired input can be obtained in terms of the desired task space trajectory.

$$\gamma_i (v^i)^d = \ddot{y}^d_i + k_{i2} \dot{y}^d_i + k_{i1} y^d_i, \quad i = 1, \ldots, p. \tag{17}$$

It is observed that the left hand side of equation (17) is the sum of m_i inputs in task space computed from the planned trajectory. For a given planned trajectory, at any instant time the right hand side of equation (17) is a given value. Applying the generalized inverse, we obtain [18]

$$(v^i)^d = \gamma_i (\gamma \gamma_i^{\prime})^{-1} (\ddot{y}^d_i + k_{i2} \dot{y}^d_i + k_{i1} y^d_i). \tag{18}$$

Note that in our control design methodology the actual control vector is the task space command as formulated by equation (17). On the joint level, our methodology computes drive forces or torques for the individual actuators, and the servo design is on the task level.

Let the output error be defined as follows:

$$e_i = \begin{bmatrix} e_{i1} \\ e_{i2} \end{bmatrix} = \begin{bmatrix} y_i - y_i^d \\ \dot{y}_i - \dot{y}_i^d \end{bmatrix}$$

where y_i and $\dot{y}_i$ are the real (measured) values, and y_i^d and $\dot{y}_i^d$ are the desired values. To eliminate the output error e_i, we utilize an optimal error correcting control loop by minimizing the following cost functional

$$J = \int_0^T [(\Delta v^i)' R \ \Delta v^i + e_i(t)' Q \ e_i(t)] \ dt + e_i(T)' S \ e_i(T).$$

The optimal correction is given by

$$\Delta v^i = -R^{-1} B_i' P(t) \ e_i(t) \tag{19}$$

where P(t) is a positive definite solution of the Riccati equation

$$\dot{P}(t) = -P(t) \bar{A}_i - \bar{A}_i' P(t) + P(t) B_i R^{-1} B_i' P(t) - Q$$
$$P(t) = S,$$

with

$$\bar{A}_i = \begin{bmatrix} 0 & 1 \\ -k_{i1} & -k_{i2} \end{bmatrix}.$$

The overall structure of the controller design is depicted in Figure 1.

4. Force Control Approach

In this approach, we consider the dynamics of each robot separately, but we pose constraints on the dynamic equations by introducing the interactive force and interactive moment among the robot arms.

We have proposed a force control approach to the coordination of two robot arms performing a single task [19]. The coordination between two robot arms is achieved by monitoring the interactive force and moment at the end effectors. Now we extend this method to multi-arm case.

Suppose that m robot arms ($m \geq 2$) are working on an object, e.g., lifting or turning a heavy workpiece. The problem we are dealing with is to find a control algorithm for m robots such that the task is performed in a coordinated fashion. We assume that each robot has a force (torque) sensor installed at its end effector. Using force control approach, the coordination among m robot arms is realized by regulating the force and moment applied to the object by each robot. With the aid of proper task planning, m robot arms are able to move in a non-conflicting way.

The dynamic equations of a system of m robot arms are given as follows:

$$D_i(q^i) \ \ddot{q}^i + E_i(q^i, \dot{q}^i) + J_i'(q^i) \ F^i = \tau^i \quad , \quad i = 1, 2, \ldots, m$$

where q^i is an n_i-dimensional joint variable vector of robot i, n_i is the degrees of freedom of robot i, F^i is an n_i-dimensional vector of the force and moment measurements of robot i, τ^i is an n_i-dimensional joint torque (force) vector of robot i, and J_i is the Jacobian matrix of robot i.

Now we introduce a state variable x by letting

$$x^i = q^i, \qquad x^{m+i} = \dot{q}^i, \qquad i = 1, \ldots, m,$$

i.e.,

$$x^1 = [x_1 \ \ x_2 \ \ldots \ x_{n_1}]' = [q_1^1 \ \ q_2^1 \ \ldots \ q_{n_1}^1]' = q^1,$$

$$x^2 = [x_{n_1+1} \cdots x_{n_1+n_2}]' = [q_1^2 \cdots q_{n_2}^2]' = q^2,$$

$$\vdots$$

$$x^m = [x_{n_1+\ldots+n_{m-1}+1} \cdots x_n]' = [q_1^m \cdots q_{n_m}^m]' = q^m,$$

$$x^{m+1} = [x_{n+1} \cdots x_{n+n_1}]' = [\dot{q}_1^1 \cdots \dot{q}_{n_1}^1]' = \dot{q}^1$$

$$x^{m+2} = [x_{n+n_1+1} \cdots x_{n+n_1+n_2}]' = [\dot{q}_1^2 \cdots \dot{q}_{n_2}^2]' = \dot{q}^2$$

$$\vdots$$

$$x^{2m} = [x_{n+n_1+\ldots+n_{m-1}+1} \cdots x_{2n}]' = [\dot{q}_1^m \cdots \dot{q}_{n_m}^m]' = \dot{q}^m,$$

where $n = n_1 + n_2 + \ldots + n_m$. Then x is a 2n-dimensional vector partitioned into 2m blocks

$$x = [x_1 \;\; x_2 \cdots x_n \; x_{n+1} \cdots x_{2n}]' = \begin{bmatrix} x^1 \\ \vdots \\ x^m \\ x^{m+1} \\ \vdots \\ x^{2m} \end{bmatrix},$$

$$\tilde{x} = [x_1 \cdots x_n]$$

with the first m blocks (corresponding to the first n components $\tilde{x}$) representing the joint positions of m robots and with the last m blocks representing the joint velocities of m robots.

The dynamic equations of m robots can now be written in terms of state variable x as follows:

$$\dot{x} = \begin{bmatrix} \dot{x}^1 \\ \vdots \\ \dot{x}^m \\ \dot{x}^{m+1} \\ \vdots \\ \dot{x}^{2m} \end{bmatrix} = \begin{bmatrix} x^{m+1} \\ \vdots \\ x^{2m} \\ -D_1^{-1}(x^1)[E_1(x^1,x^{m+1})+J_1'(x^1)F^1] \\ \vdots \\ -D_m^{-1}(x^m)[E_m(x^m,x^{2m})+J_m'(x^m)F^m] \end{bmatrix} + \begin{bmatrix} O \\ \\ D_1^{-1}(x^1) \\ \vdots \\ D_m^{-1}(x^m) \end{bmatrix} \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_m \end{bmatrix}$$

or $$\dot{x} = f(x) + g(x)\tau \tag{20}$$
where f ang g can be easily identified from the above equation. We take the output equations of the form

$$
y = h(x) = \begin{bmatrix} h^1 \\ h^2 \\ \vdots \\ h^m \end{bmatrix} = \begin{bmatrix} W_p^1 P^1 + W_F^1 F^1 \\ W_p^2 P^2 + W_F^2 F^2 \\ \vdots \\ W_p^m P^m + W_F^m F^m \end{bmatrix} \tag{21}
$$

where W_p^i, W_F^i, $i = 1, \ldots, m$, are the weighting matrices, and P^i is the position and orientation vector of robot i in the world coordinate frame. The dimension of output vector y is n.

Equation (20) represents a nonlinear and coupled system with output (21). Using a nonlinear feedback $\tau = \alpha(x) + \beta(x)u$ and a nonlinear coordinate transformation T(x), we are able to linearize and output decouple the system (20). The $\alpha(x)$ and $\beta(x)$ in the nonlinear feedback are given by

$$
\alpha(x) = -A^{-1}(x)\, L_f^2 h \tag{22}
$$

$$
\beta(x) = A^{-1}(x) \tag{23}
$$

where

$$
A(x) = \frac{\partial h}{\partial x} \begin{bmatrix} D_1^{-1}(x^1) & & O \\ & \ddots & \\ O & & D_m^{-1}(x^m) \end{bmatrix} .
$$

The nonlinear transformation is given by

$$
T(x) = \begin{bmatrix} h_1 \\ L_f h_1 \\ \vdots \\ h_n \\ L_f h_n \end{bmatrix} \tag{24}
$$

Application of the nonlinear feedback and the nonlinear coordinate transformation converts the system (20) with the output (21) into the following linear and decoupled system

$$
\dot{z} = Az + Bu \tag{25a}
$$

$$
y = Cz \tag{25b}
$$

where

$$
z = [z_1 \ldots z_{2n}]' \quad , \quad u = [u_1 \ldots u_n]' \quad , \quad y = [y_1 \ldots y_n]' \quad ,
$$

$$
A = \begin{bmatrix} A_1 & & O \\ & \ddots & \\ O & & A_n \end{bmatrix} \quad , \quad B = \begin{bmatrix} B_1 & & O \\ & \ddots & \\ O & & B_n \end{bmatrix}, \quad C = \begin{bmatrix} C_1 & & O \\ & \ddots & \\ O & & C_n \end{bmatrix}
$$

$$
A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad , \quad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad , \quad C_i = [1 \;\; 0] \quad , \quad i=1, 2, \ldots, n.
$$

Note that system (25) consists of n independent subsystems. Likewise as in the closed chain formulation, for each subsystem we can design a constant feedback to stabilize it and design an optimal error-correcting loop to eliminate the output errors. The overall controller structure is shown in Figure 2.

5. Conclusion

Our approaches to the control problem of multiple robot arms are motivated by the desire of making rigorous use of the dynamics of robot arms involved in the task. The closed chain approach is initiated from the fact that the dynamic behaviors of the robot arms are not independent of each other any more if they grasp on a common object. In this approach, the multiple robot arms are modeled as a single mechanical system by choosing a set of generalized coordinates whose number equals the number of degrees of freedom of the whole system. Figure 1 shows the schematic structure of the controller for the closed chain approach as implemented on computers. From the initial physical task, the task planning of the upper left block in Figure 1 produces a trajectory in the task space expressed as a smooth function of time. The command generator block realizes equation (18) and yields the desired reference input. The lower left block is the implementation of the optimal error correction described by equation (19). It takes the task space error as its input, and produces the optimal correction as its output. The $Q(\theta)$ block to the right of the multiple robot arms establishes the generalized coordinates as well as their time derivatives from the measured joint positions and velocities of the robot arms. The bulk of the controller is the nonlinear feedback block which computes the joint driving torques or forces. Because the dynamic projection functions D^i, E_i, and G^i are derived in terms of the joint variables, it may be convenient to use the joint variables in addition to the generalized coordinates for computing the nonlinear feedback.

Different from the closed chain approach, the force control approach assumes that each robot arm has a force and moment sensor located at the end effector. The force and moment measurements are introduced into the dynamic equations and output (task) equations. This is schematically depicted in Figure 2. The measurements F^1, F^2, ..., F^m are transmitted to the nonlinear feedback block, the output h block, and the coordinate transformation T block. The three blocks to the left of the nonlinear feedback block in Figure 2 are structurally similar to those in Figure 1.

Using the results from differential geometric system theory, we are able to linearize and to decouple the complicated dynamic equations of multiple robot arms including the object held by the arms. Independent of the approach being taken, we eventually deal with a linear and decoupled system. Thus we can have a unified design technique for coordinated control of multiple robot arms.

It should be noted that both methods used in this paper are systematic and are robot arm independent. The most important feature is that the control algorithms are task independent, that is, there is no need to change the structure of the controller or even the parameters of the controller from task to task. As natural as would be, the change of tasks only causes the adjustment of the input command which is conveniently given in the task space rather than in the joint space. The two control methods can be used in slightly different situations. For example, if the robot arms are loosely connected through the object, the force control approach is preferable; if the robot arms are mechanically locked while transferring the object, the closed-chain approach is more likely a solution.

Each control scheme naturally leads itself for computational implementation using distributed computing system, possibly in multi-bus architecture. Figures 1 and 2 provide a high level structure of computational implementation requirements. The details of the implementation require a deeper analysis.

6. ACKNOWLEDGEMENT

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Table 1. Degrees of freedom of the closed chains formed by three robot arms

cases	n_1	n_2	n_3	i	j	$n_1+n_2+n_3$	p
1	7	7	7	20	21	21	9
2	6	7	7	19	20	20	8
3	6	6	7	18	19	19	7
4	5	7	7	18	19	19	7
5	5	6	7	17	18	18	6
6	6	6	6	17	18	18	6
7	5	6	6	16	17	17	5
8	5	5	7	16	17	17	5
9	5	5	6	15	16	16	4
10	5	5	5	14	15	15	3

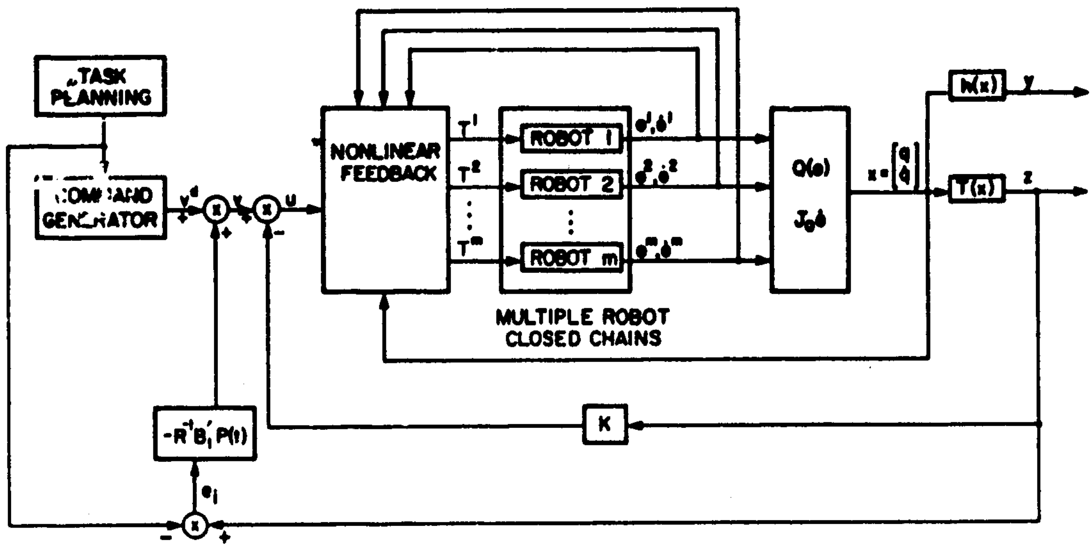

Fig. 1 Schematic Control Structure of the Closed Chain Approach

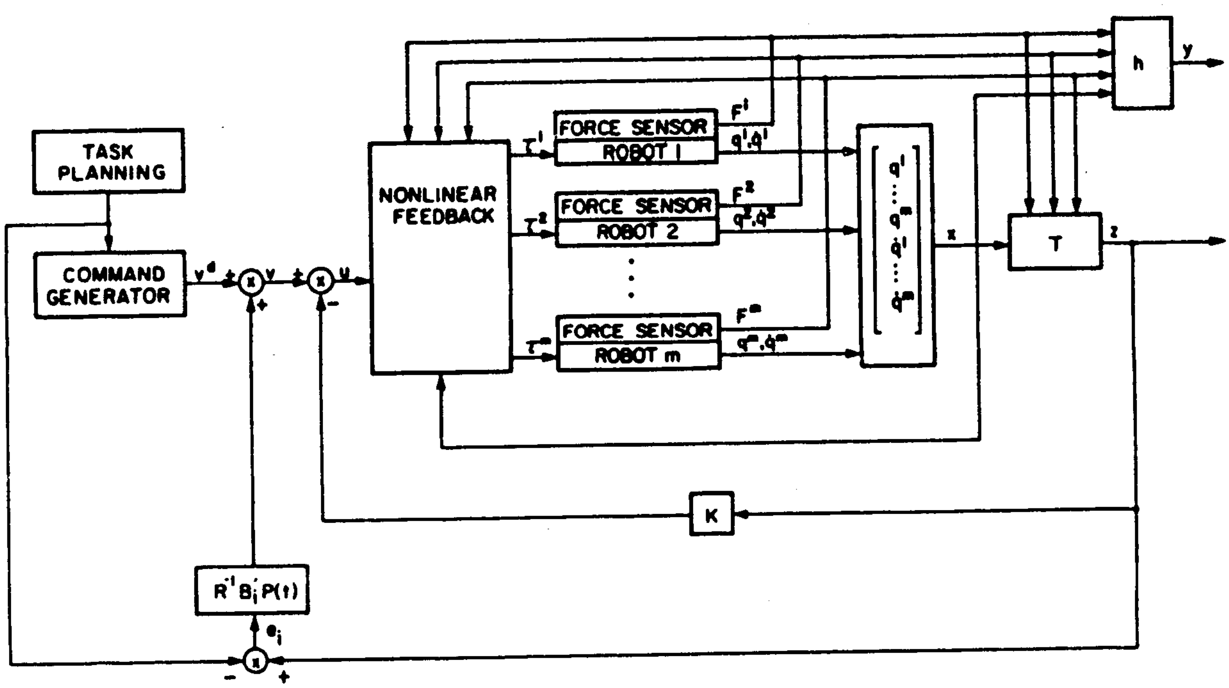

Fig. 2 Schematic Control Structure of the Force Control Approach