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Dynamics and Control of Coordinated Multiple Manipulators

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ABSTRACT

~~In this paper we present~~ a technique is presented for controlling multiple manipulators which are holding a single object and therefore form a closed kinematic chain. The object, which may or may not be in contact with a rigid environment, is assumed to be held rigidly by n robot end-effectors. The derivation is based on setting up constraint equations which reduce the 6xn degrees of freedom of n manipulators each having six joints. Additional constraint equations are considered when one or more of the degrees of freedom of the object is reduced due to external constraints. Utilizing the operational space dynamics equations, a decoupling controller is designed to control both the position and the interaction forces of the object with the environment. Finally, ~~computer~~ simulation results for the control of a pair of two-link manipulators are presented.

1. INTRODUCTION

The topic of multiple robot control is relatively new in robotics research. The extension of robot control techniques to the case of multiple manipulators is necessitated by realities encountered both for manipulating small objects and for handling large workpieces. The manipulation of objects normally requires at least two hands to simultaneously position and reorient the object so that either one or both hands can perform their respective tasks. Here, the employment of an extra arm is required not by limitations on force/torque output of an arm but by the fact that in complex task execution, the workpiece must frequently be reoriented to expose the hard-to-reach areas. In fact, some experts have suggested that robots will not truly be integrated in the future factory environment until multiple robot task planning and control have reached a certain level of maturity. In this paper we will only address the control problem associated with multiple manipulators handling a commonly held object. Other problems, such as several manipulators working in the same workcell without manipulating the same objects simultaneously, or two arms working on the same objects but having relative motion with respect to the workpiece, are important research topics not addressed in this paper.

Early work in this area [1, 2] has mostly dealt with master/slave control techniques. This approach is conceptually simple. One manipulator is position servoed while the other manipulator is force servoed. The force servoed arm is controlled in compliant mode to follow the master (position servoed) arm. Additional feed forward forces (torques) are added to the force controlled arm to realize the required interaction forces between the arms and the object which is being manipulated. More recently, among others, Luh and Zheng have reported their work in dual arm coordination. We will mention only some of their work here. In reference [3] they discuss the kinematic issues related to the coordination of dual manipulator robots. Their approach is based on position servoing of one of the arms and the use of kinematic constraint equations to modify the trajectory of the second arm to realize cooperation between the arms. In reference [4] they extend their analysis to include the constraint equations for the joint acceleration. Then, using the arm Jacobian, joint torques are computed to drive the arms.

In this paper we derive the equations of motion in the so-called Operational Space (or Cartesian state space) [5]. We assume a general case of n cooperating robots which are holding an object rigidly. This object may also be constrained from motion in one or more dimensions by an external environment. Equations of motion are derived using the Lagrange multiplier technique. References [6] and [7] provide excellent background for this formulation. It is assumed that each manipulator is equipped with a force/torque sensor capable of measuring three orthogonal forces and torques in a given coordinate frame. The aim is to control the position of the object and its interaction forces with the environment in the sense of hybrid control [8]. This paper extends the previously reported results in [9] by a more compact formulation and by explicitly computing the interaction forces between the robots. Finally, a simulation study for a pair of two-link cooperating manipulators is presented to validate the analysis.

2. DYNAMICS OF MULTIPLE COOPERATING ROBOT MANIPULATORS

In this section, we will derive the dynamics equations for a closed kinematic chain formed by n manipulators rigidly grasping an object. We will assume that each manipulator has six joints and that none of the manipulators experience singularity (i.e., degenerate manipulator Jacobian) as they perform a task. The grasped object may be in contact with a rigid environment.

In deriving the equations of motion, we will assume that there exists a coordinate frame which is attached to the object. In fact, in the next section, we will use this coordinate frame to specify the desired motion/force of the object. Figure 2.1 shows a schematic drawing of this multiple manipulator system.

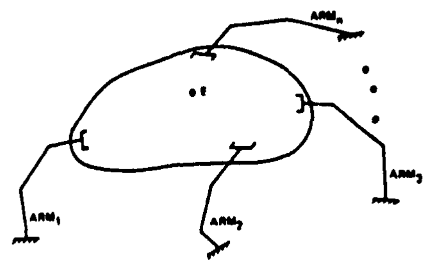


Figure 2.1.

The derivations of the equations of motion are considerably simplified when a) we use the Cartesian state dynamics equations [5,10], and b) we lump the object mass/inertia into that of the sixth link of the arms.

Let us begin with the well known equation of motion for a single multi-link arm [9,10]

$$\underline{M}(\underline{q})\,\underline{\ddot{q}} + \underline{V}(\underline{q},\underline{\dot{q}}) + \underline{G}(\underline{q}) = \underline{\tau} \qquad (2.1)$$

where $\underline{q} \in R^n$ denotes the joint state variable, $\underline{M}(\underline{q}) \in R^{n \times n}$ is the inertia matrix which is symmetric and positive definite, $\underline{V}(\underline{q},\underline{\dot{q}}) \in R^n$ is the centrifugal and Coriolis force/torque vector, $\underline{G}(\underline{q}) \in R^n$ is the gravity vector, and $\underline{\tau} \in R^n$ is the joint force/torque vector. In order to simplify the wording of the paper, we will assume that all the joints are revolute and hence use the term "torque" when referring to generalized joint force/torque vectors. The above equation applies only to idealized frictionless rigid arms.

In the first part of the derivation, we will consider an object-fixed coordinate frame relative to which the dynamics equations will be written. We will also assume that the object has been partitioned into n parts. Each of these parts will then be considered as a part of the last link of each arm. Figure 2.2 illustrates one of these parts together with the last link of arm i.

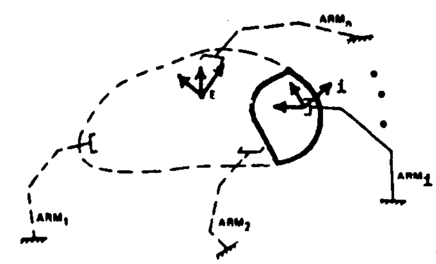


Figure 2.2. Schematic drawing of the partitioned load and its integration with the last link of arm i.

If ${}^iJ_i$ is the pseudo-inertia matrix of the last link represented in the i coordinate frame, its representation in the E frame is obtained from:

$${}^EJ_i = \int_{\text{last link}} {}^E\underline{r}_i \, {}^E\underline{r}_i^T \, dm_i \qquad (2.2)$$

Since the arms are considered to be rigidly attached to the load, there is a constant transformation relating the i and E frames, ${}^E_iT$. It then follows that

$${}^EJ_i = \int_{\text{last link}} {}^E_iT \, {}^i\underline{r}_i(dm_i) \, ({}^E_iT \, {}^i\underline{r}_i)^T = {}^E_iT \, {}^iJ_i \, {}^i_ET \qquad (2.3)$$

where ${}^EJ_i$ is the pseudo-inertia matrix of the last link of the ith arm expressed in the E frame. Let us now assume that ${}^EJ_{Li}$ is the pseudo-inertia matrix of the ith partition of the load. This means that

$$ {}^{E}J_{Ei} = \int_{\text{partition } i} {}^{E}x_E \, {}^{E}x_E^T \, dm_E \qquad (2.4) $$

where partition i is as shown schematically in Figure 2.2. Utilizing equations (2.3) and (2.4), we can now define a new last link for the ith arm which is essentially the combination of the original last link plus a portion of the load. Also note that the coordinate frame of the last link is now E instead of the frame i.

Although in principle one can use joint space dynamics equations for the system of multiple manipulators, the derivation is considerably simplified when the so-called operational space formulation (or Cartesian state space dynamics) is used. The relationship between the joint space and the Cartesian space formulation can be derived simply by utilizing the manipulator Jacobian as [5,10]:

$$ \dot{x} = J(q)\, \dot{q} \qquad (2.5) $$

where J is the manipulator Jacobian and x is the position/orientation of the last link coordinate frame. The Jacobian used here is a general one, meaning that one can consider it to be defined relative to the last frame, the world frame, or in fact any other desired frame. Taking the time derivative of (2.5) results in

$$ \ddot{x} = \dot{J}(q)\, \dot{q} + J(q)\, \ddot{q} \qquad (2.6) $$

Substituting $\ddot{q}$ from (2.6) into (2.1), we obtain

$$ M(q) \;\; J^{-1}(q)\; [\ddot{x} - \dot{J}\dot{q}] \;\; + V(q,\dot{q}) + G(q) = \tau $$

or

$$ M_x(q)\, \ddot{x} + V_x(q,\dot{q}) + G_x(q) = F \qquad (2.7) $$

where

$$ M_x(q) = J^{-T}(q)\, M(q)\, J^{-1}(q) \qquad (2.8) $$

$$ V_x(q,q) = J^{-1}[\; V(q,\dot{q}) - M(q)\; J^{-1}(q)\; \dot{J}(q)\; \dot{q}] \qquad (2.9) $$

$$ G_x(q) = J^{-T}(q)\; G\;(q) \qquad (2.10) $$

In equation (2.7), F represents an equivalent generalized force vector applied at the end-point (E-frame). Let us denote by f the interaction generalized force vector on one of the robot arms at the E-frame. The equations of motion are therefore

$$ M_{xi}(q_i)\, \ddot{x} + V_{xi}(q_i,\dot{q}_i) + G_{xi}(q_i) = F_i + f_i \quad , \; i = 1,\ldots,n \qquad (2.11) $$

Since f_i's are the internal forces created at point E, their sum is zero.

$$ \sum_{i=1}^{n} f_i = 0 $$

The equation of motion, therefore, may be obtained by simply adding the equations (2.11)

$$ [\sum_{i=1}^{n} M_{xi}(q_i)]\, \ddot{x} + \sum_{i=1}^{n} V_{xi}(q_i,\dot{q}_i) + \sum_{i=1}^{n} G_{xi}(q_i) = \sum_{i=1}^{n} F_i \qquad (2.12) $$

In order to simplify the notation, let us define

$$ M_x(Q) = \sum_{i=1}^{n} M_{xi}(q_i) \qquad (2.13) $$

$$ V_x(Q,Q) = \sum_{i=1}^{n} V_{xi}(q_i,\dot{q}_i) \qquad (2.14) $$

$$ G_x(Q) = \sum_{i=1}^{n} G_{xi}(q_i) \qquad (2.15) $$

$$ F = \sum_{i=1}^{n} F_i $$

where

$$ Q = (q_1^T,\, q_2^T, \ldots, q_n^T) $$

Now, equation (2.12) can be written in the simpler form

$$ M_x(Q)\, \ddot{x} + V_x(Q,\dot{Q}) + G_x(Q) = F \qquad (2.16) $$

Note that M_x is still a positive definite matrix. The interaction generalized forces, i.e. f_i's, can be computed by solving for $\ddot{X}$ in equation (2.16) and substituting the results in (2.11)

$$f_i = M_{xi}(q_i)\, M_x^{-1}(Q)\, [F - V_x(Q,\dot{Q}) - G_x(Q)]$$
$$+ V_{xi}(q_i,\dot{q}_i) + G_{xi}(q_i) - F_i \quad , \; i = 1,\ldots,n \qquad (2.17)$$

In summary, if the individual joint torques are given, one can compute the equivalent F_i's by utilizing the Jacobian matrices. Equation (2.16) can then be used to solve the forward dynamics of the multiple arm system. Finally, the interaction forces can be computed from equation (2.17). Implicit in the above derivation are the constraint equations

$$x = K_1(q_1) = K_2(q_2) \ldots = K_n(q)$$

where K_i is the forward kinematics relation for arm i.

3. DYNAMICS OF MULTIPLE COOPERATING ROBOT MANIPULATORS WITH EXTERNAL CONSTRAINTS

In this section we will derive the equations of motion when the object that is being held by n robot manipulators is in contact with a frictionless rigid environment. Let C be a coordinate frame on the object to represent the generalized constraint forces. This coordinate frame, in general, will be distinct from the E-frame. With a slight modification, equation (2.16) can be transformed so that the generalized force vector F can be represented in the C frame. If ${}^E_C T$ is a homogeneous transformation that relates the C frame to the E frame, then ${}^E_C J$ can be obtained to relate the generalized force vectors in these two frames. The Jacobian matrix ${}^E_C J$ is given from

$${}^E_C J = \left[\begin{array}{c|c} {}^E_C R & {}^E P_{cORG} \times {}^E_C R \\ \hline 0 & {}^E_C R \end{array}\right] \qquad (3.1)$$

where

$${}^E_C T = \left[\begin{array}{c|c} {}^E_C R & {}^E P_{cORG} \\ \hline 0 & 1 \end{array}\right] \qquad (3.2)$$

In general, since the C-frame can be time varying, ${}^E_C T$ and consequently ${}^E_C J$ are also time varying matrices. Let x_c denote the position/orientation of the C-frame. Then

$$\dot{x}_c = {}^C_E J\, \dot{x} \qquad (3.3)$$

$$\ddot{x}_c = {}^C_E \dot{J}\, \dot{x} + {}^C_E J\, \ddot{x} \qquad (3.4)$$

and hence equation (2.16) is

$$M_{xc}(Q)\, \ddot{X}_c + V_{xc}(Q,\dot{Q}) + G_{xc}(Q) = F_c \qquad (3.5)$$

where M_{xc}, V_{xc}, G_{xc} are defined by a set of matrix equations (2.8) - (2.10) with J replaced by ${}^C_E J$ and

$$F_c = ({}^E_C J)^T\, F.$$

Let us assume that $\phi(x_c) \in R^m$, $m \leq 6$, represents the constraint function such that

$$\phi(x_c) = 0 \;\forall\; t \geq 0 \qquad (3.6)$$

If we denote by f_c the generalized constraint force vector, then the equation of motion is

$$M_{xc}(Q)\, \ddot{X}_c + V_{xc}(Q,\dot{Q}) + G_{xc}(Q) = F_c + f_c \qquad (3.7)$$

The constraint force f_c can be obtained by noting that the virtual work performed by f_c is equal to zero and by using the Lagrange multiplier method as discussed in [6] and [7].

The statement of virtual work is

$$f_c^T\, \delta x_c = 0 \qquad (3.8)$$

From the constraint equation (3.6) we can write

$$\frac{\partial\phi(X_c)}{\partial X_c}\,\delta X_c = 0 \tag{3.9}$$

We can now use the Lagrange multiplier $\lambda \in R^m$ to relate (3.8) and (3.9) as

$$\left[f_c^T - \lambda^T \frac{\partial\phi(X_c)}{\partial X_c}\right]\delta X_c = 0$$

Since δX_c is an arbitrary infinitesimal displacement vector, it is not equal to zero. Therefore

$$f_c = D^T(X_c)\,\lambda \tag{3.10}$$

where

$$D(X_c) = \partial\phi(X_c)\,/\,\partial X_c \tag{3.11}$$

To find λ, we differentiate (3.6) with respect to time twice

$$\dot\phi(X_c) = D(X_c)\,\dot X_c = 0 \tag{3.12}$$

$$\ddot\phi(X_c) = \dot D(X_c)\,\dot X_c + D(X_c)\,\ddot X_c = 0 \tag{3.13}$$

and substitute for $\ddot X_c$ from (3.7)

$$\dot D(X_c)\,\dot X_c + D(X_c)\,M_{xc}^{-1}(Q)\,[\,F_c + D^T(X_c)\,\lambda - V_{xc}(Q,\dot Q) - G_{xc}(Q)\,] = 0$$

which gives λ as

$$\lambda = [\,D(X_c)\,M_{xc}^{-1}(Q)\,D^T(X_c)\,]^{-1}\,\{\,D(X_c)\,M_{xc}^{-1}(Q)\,[V_{xc}(Q,\dot Q) + G_{xc}(Q) - F_c] - \dot D(X_c)\,\dot X_c\,\} \tag{3.14}$$

Equation (3.7) together with equation (3.14) completely characterize the equation of motion of the multiple cooperating robot with external motion constraints on the object.

4. CONTROL OF MULTIPLE COOPERATING ROBOTS

In this section we will introduce a control technique which is based on the operational space formulation to control the position of the object and interaction forces of the object with the external environment. The approach presented here is based on distributing the actuation forces among the arms such that each arm contributes to the motion of the object and to its own inertial forces in a predetermined manner. Let us begin by introducing a decoupling generalized force such that

$$F_c = -[f_{cd} + K_{fd}(\dot f_{cd} - \dot f_c) + K_{fp}(f_{cd} - f_c)] + M_{xc}(Q)\,[\ddot X_{cd} + K_{pd}(\dot X_{cd} - \dot X_c) + K_{pp}(X_{cd} - X_c)]$$
$$+\; V_{xc}(Q,\dot Q) + G_{xc}(Q) \tag{4.1}$$

where f_{cd} is the desired interaction force vector between the object and the external environment.

We must now show that the above choice for F_c does result in the multiple robotic system following the specified trajectory and applying the desired forces on the constraint surface. In order to simplify the derivation and without loss of generality, let us assume that the position and force subspaces are reordered in the equations such that the first 6-m elements of the position vector correspond to the position controlled subspace and the last m elements belong to the force controlled subspace. This means that, for example,

$$\ddot{x}_{cd} = \begin{bmatrix} \ddot{x}'_{cd} \\ \hline 0 \end{bmatrix} \begin{array}{l} \} \ 6-m \\ \} \ m \end{array} \tag{4.2}$$

$$\dot{x}_{cd} = \begin{bmatrix} \dot{x}'_{cd} \\ \hline 0 \end{bmatrix} \begin{array}{l} \} \ 6-m \\ \} \ m \end{array} \tag{4.3}$$

and

$$f_{cd} = \begin{bmatrix} 0 \\ \hline f'_{cd} \end{bmatrix} \begin{array}{l} \} \ 6-m \\ \} \ m \end{array} \tag{4.4}$$

Let us partition the mass matrix as

$$M_{xc} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{4.5}$$

and its inverse as

$$M_{xc}^{-1} = \begin{bmatrix} M'_{11} & M'_{12} \\ M'_{21} & M'_{22} \end{bmatrix} \tag{4.6}$$

where $M_{11} \in R^{6-m \times 6-m}$. With the above decomposition of the force-position subspaces, the D matrix (see equation 3.11) can be written as

$$D(x_c) = [\ 0 \mid I_{mxm}\ ] \tag{4.7}$$

and

$$\dot{D}(x_c) = 0 \tag{4.8}$$

With the above choice for $\phi(x_c)$, we can now compute the closed loop dynamics of the multiple manipulator system with F_c as the generalized control force vector given by equation (4.1). Let us partition the actual force, acceleration, and velocity vectors as

$$f_c = \begin{bmatrix} f_{c1} \\ f_{c2} \end{bmatrix}, \quad \ddot{x}_c = \begin{bmatrix} \ddot{x}_{c1} \\ \ddot{x}_{c2} \end{bmatrix}, \quad \dot{x}_c = \begin{bmatrix} \dot{x}_{c1} \\ \dot{x}_{c2} \end{bmatrix} \tag{4.9}$$

and also select K_{fp}, K_{fd}, K_{pd}, K_{pp} as

$$K_{fd} = \begin{bmatrix} 0 & 0 \\ 0 & k_{fp} \end{bmatrix}, \quad K_{fp} = \begin{bmatrix} 0 & 0 \\ 0 & k_{fd} \end{bmatrix}, \quad K_{pp} = \begin{bmatrix} k_{pp} & 0 \\ 0 & 0 \end{bmatrix}, \quad K_{pd} = \begin{bmatrix} k_{pd} & 0 \\ 0 & 0 \end{bmatrix} \tag{4.10}$$

where k_{ij}'s are diagonal matrices. From equation (4.1) F_c is given by

$$F_c = \begin{bmatrix} 0 \\ f'_{cd} \end{bmatrix} - \begin{bmatrix} 0 \\ k_{fd}\, \dot{e}_f \end{bmatrix} - \begin{bmatrix} 0 \\ k_{fp}\, e_f \end{bmatrix} + M_{xc} \left\{ \begin{bmatrix} \ddot{x}'_{cd} \\ 0 \end{bmatrix} + \begin{bmatrix} k_{pd}\, \dot{e}_p \\ 0 \end{bmatrix} + \begin{bmatrix} k_{pp}\, e_p \\ 0 \end{bmatrix} \right\} + V_{xc} + G_{xc} \tag{4.11}$$

where $e_f = f'_{cd} - f_{c2}$ and $\dot{e}_p = \dot{x}'_{cd} - \dot{x}_{c1}$

The reaction force f_c can be computed by substituting from equation (4.11) for F_c in equation (3.10) and (3.14).

After some manipulation of the equations, one obtains

$$
\underline{f}_e = \begin{bmatrix} 0 \\ f'_{ed} + k_{fd}\dot{e}_f + k_{fp}\, e_f \end{bmatrix} \tag{4.12}
$$

Substituting from (4.9) for $\underline{f}_e$, we have

$$
\underline{f}_{e2} = f'_{ed} + k_{fd}\,\dot{e}_f + k_{fp}\, e_f
$$

or

$$
k_{fd}\,\dot{e}_f + (k_{fp} + I)\, e_f = 0
$$

This result indicates that the generalized reaction force will be equal to the desired force. The control time constant can be designed by selecting the proper k_{fd} and k_{fp} diagonal gain matrices.

Similarly, by substituting from equation (4.1) into equation (3.6) and using equation (4.12), we obtain

$$
M_{xc}(Q) \begin{bmatrix} \ddot{e}_p + k_{pd}\,\dot{e}_p + k_{pp}\, e_p \\ 0 \end{bmatrix} = 0 \tag{4.13}
$$

This means that in the position subspace, the multiple manipulator system can be controlled by the proper choice of the k_{pd} and k_{pp} matrices.

To summarize, a generalized force vector as given by equation (4.1) was found such that both the position of the object and its interaction forces with an environment can be controlled. Note that this equation does not specify how the generalized force vector will be realized by a system of redundant actuators of multiple cooperating manipulators. One choice is to let each arm supply enough torques at its joints to provide the necessary actuation for its own links plus a portion of the object's. Similarly, one can determine a priori how much each arm must contribute to the realization of the interaction forces between the object and the environment (for more details, please see Ref. 8). The internal forces at the origin of the E-frame can also be controlled in the position subspace by adding force vectors to $\underline{F}_i$ (see equation 2.17) such that one has

$$
F = \sum_{i=1}^{n} (F_i + \underline{F}'_i)
$$

or

$$
\sum_{i=1}^{n} \underline{F}'_i = 0
$$

This means that the total effect of adding the $\underline{F}'_i$ does not affect the motion of the multiple manipulator system while realizing the desired internal forces.

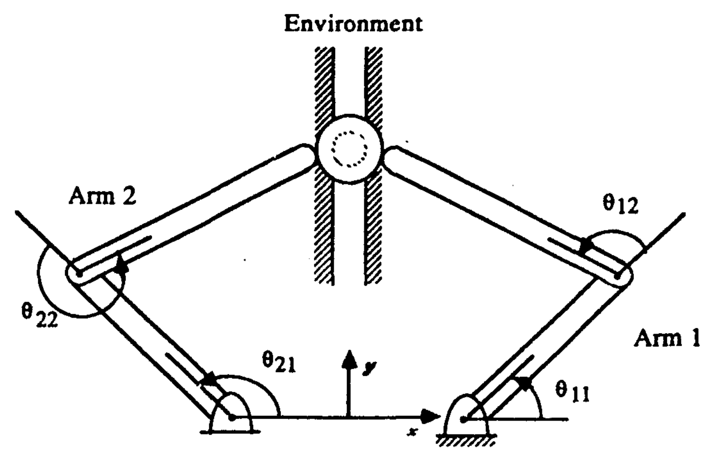


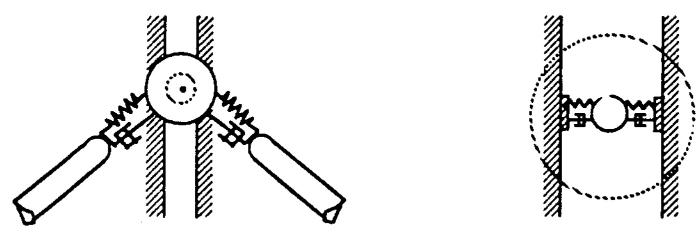
Fig. 5.1 Schematic Drawing of a Pair of Two-Link Cooperating Manipulators

5. EXAMPLE

In order to validate the proposed theory, a simulation study was undertaken. The model consisted of simple planar robotic system composed of a pair of two-link revolute manipulators. The object was assumed to a point mass attached to the upper link of each of the manipulators. ace each manipulator has only t degrees of freedom, the object was assumed to be in contact with the upper links only through interaction forc rather than through forces and torques.

Two cases were considered. In the first case, it was assumed that there were no environmental constrain This case represents a pure transport problem where two manipulators cooperate in moving an object. In t second case, an environment was assumed which restricted the motion of the object in the x direction. In bo cases, the desired motion in the y direction was obtained from a constant acceleration trajectory. The desir motion in the x direction was set equal to the initial x value (i.e., x = 0) in the first case, and the desir interaction force between the object and the environment was set equal to zero in the second case.

The simulation of closed kinematic chains can either be performed by computing the interaction forces bas on the dynamics equations (such as those developed in this paper) or by assuming the existance of stiff sprin (or spring-dashpot) at the contact points. Since in actual experiments, one uses force/torque sensors to obta these interaction forces (torques), the second approach is used in this study. Figure 5.2 illustrates t modeling of the contact mechanism between the upper links and the object and between the object and t environment. Figure 5.3 shows the overall simulation block diagram.



(a) (b)

Figure 5.2 Detailed modeling of the connections between (a) the arms and the object, and (b) the object and the external constraint

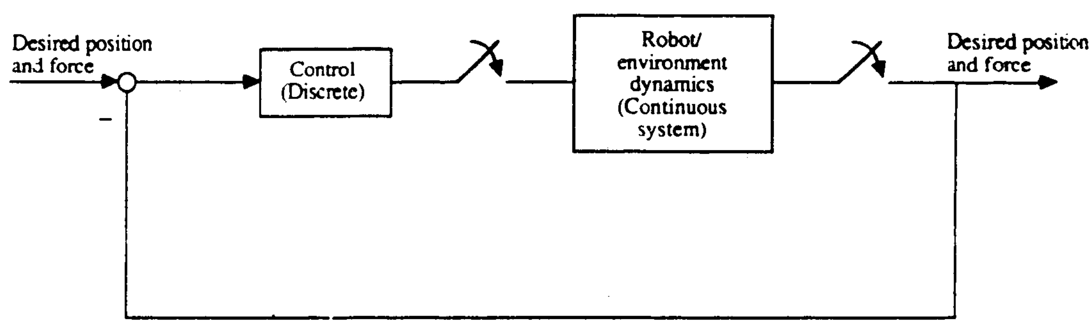


Figure 5.3 Simplified block diagram of the simulation study

Although this paper does not attempt to address the digital control aspects of the problem, the simulation stu was made more realistic by separating the continuous and discrete parts as shown in Figure 5.3.

Figures 5.4 through 5.6 show the responses of the system for the following set of parameters and control gains.

link lengths	= .04 [m]	, equal for all lengths
link masses	= 4.0 [Kg]	, for lower links
	= 2.0 [Kg]	, for upper links
object mass	= 2.0 [Kg]	
Initial joint angles:	$\theta_{11} = 30^\circ$, $\theta_{12} = 120^\circ$	
	$\theta_{21} = 150^\circ$, $\theta_{22} = -120^\circ$	
Position loop gains	$k_{pp} = 4900.0$, $k_{pd} = 98.0$	
Force loop gains	$k_{fp} = 1.0$, $k_{fd} = 0$	
Spring constants	500,000 [N/m] for all of the springs	
Damping constants	140.00 [N/(m/sec)]	
Sampling period	1 msec	

Figure 5.4 shows the tracking capability of the object moved by the arms. Since the difference between the position of the object and the tip of each upper link is insignificant, the plots show the desired vs. actual position of the upper links. Figure 5.5 shows the tracking of the object in the second case (motion constrained in the x direction). Figure 5.6 shows the interaction forces between the object and the environment. The simulation study indicates that the control algorithm developed in this paper yields excellent results. It must be understood that several important practical problems such as friction, flexibility of the links and joints, link parameter errors, etc., were not included in this simulation study. Further research and study is necessary to include such effects.

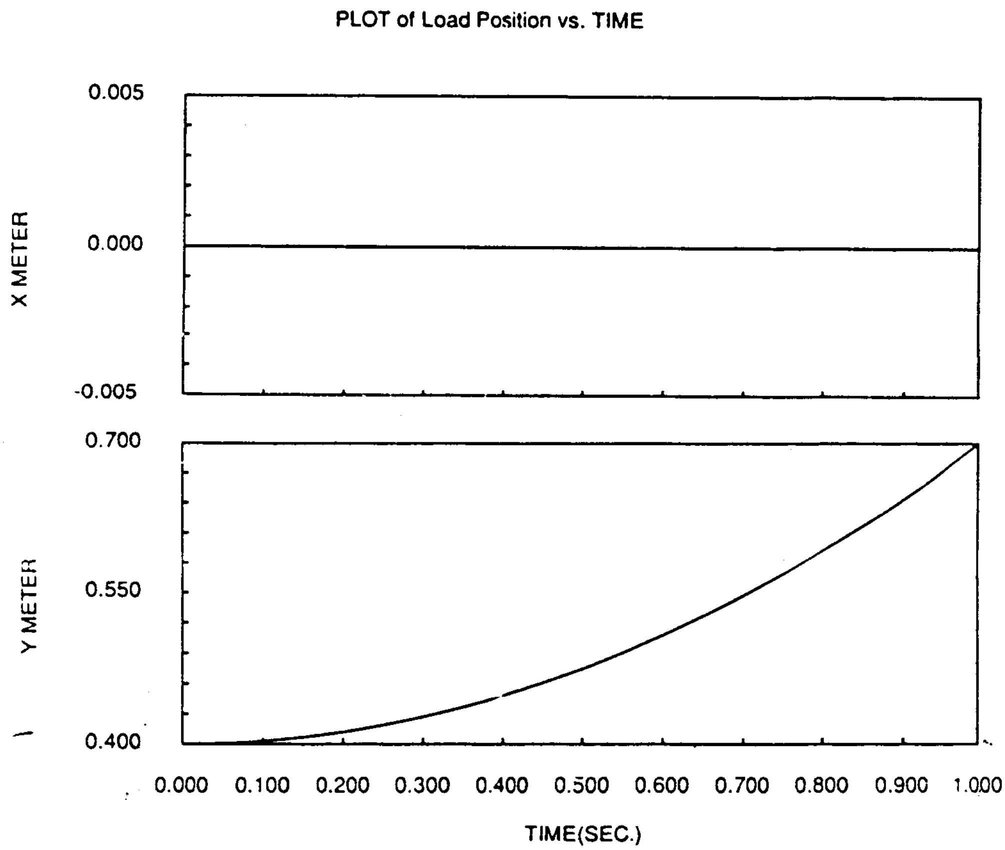


Figure 5.4 Pure Position Control with Cooperating Arms

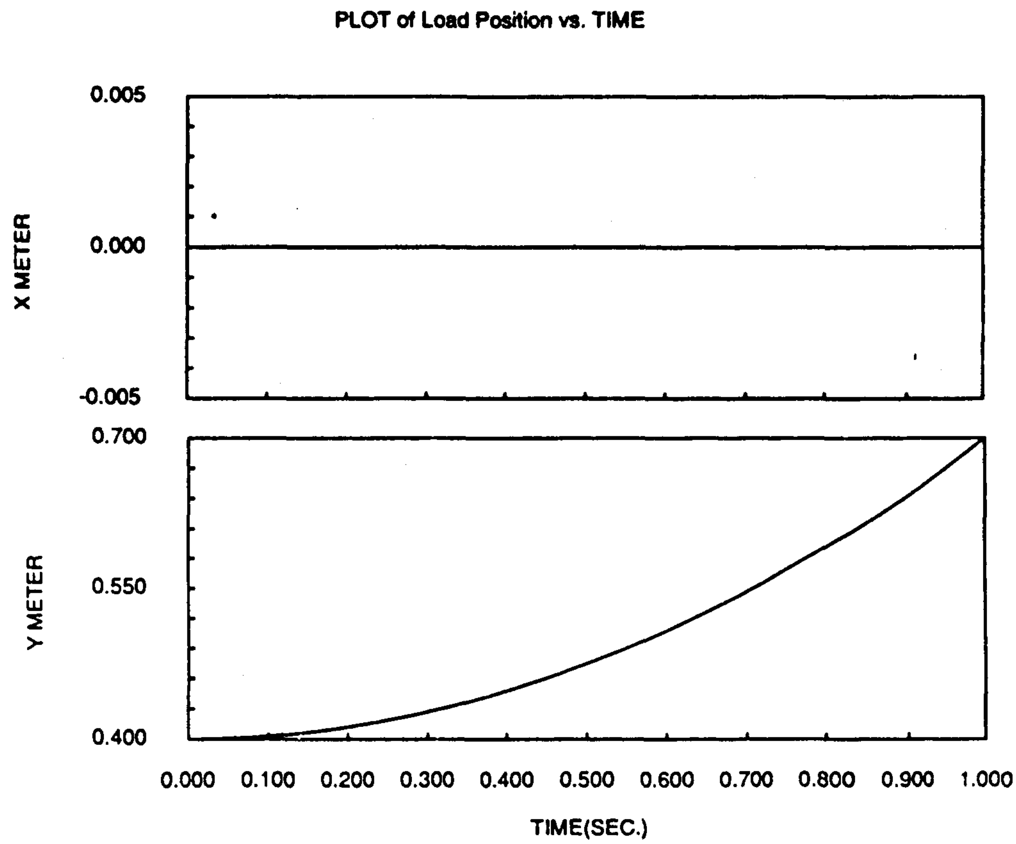


Figure 5.5 Position Tracking in Position/Force Control with Cooperating Arms

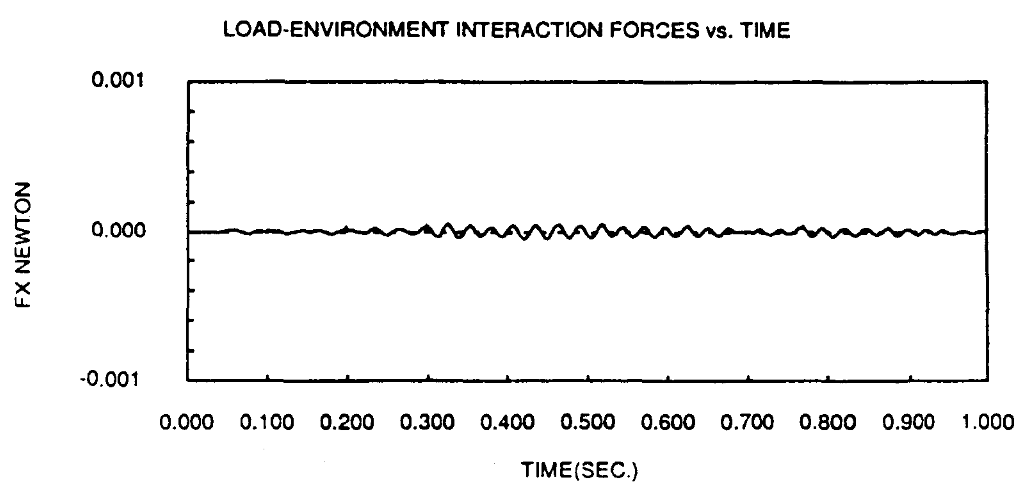


Figure 5.6 Force Tracking in Position/Force Control with Cooperating Arms

6. CONCLUSIONS

This paper presented a theory for the position and force control of multiple manipulators holding an object which is in contact with an environment. The derivation is for n manipulators each having six degrees of freedom. The control is based on the Cartesian formulation of the arm dynamics and extends the single-arm hybrid position/force control concept to the case of multiple arms. Simple but realistic simulation studies confirmed that the developed control concept results in excellent position tracking and force control.

7. ACKNOWLEDGEMENT

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8. REFERENCES

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