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# Simple Robust Control Laws for Robot Manipulators, Part I: Non-adaptive Case

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Abstract

A new class of exponentially stabilizing control laws for joint level control of robot arms is introduced. It has been recently recognized that the nonlinear dynamics associated with robotic manipulators have certain inherent passivity properties. More specifically, the derivation of the robotic dynamic equations from the Hamilton's principle gives rise to natural Lyapunov functions for control design based on total energy considerations. Through a slight modification of the energy Lyapunov function and the use of a convenient lemma to handle third order terms in the Lyapunov function derivatives, closed loop exponential stability for both the set point and tracking control problem is demonstrated. The exponential convergence property also leads to robustness with respect to frictions, bounded modeling errors and instrument noise. In one new design, the nonlinear terms are decoupled from real-time measurements which completely removes the requirement for on-line computation of nonlinear terms in the controller implementation. In general, the new class of control laws offers alternatives to the more conventional computed torque method, providing trade offs between robustness, computation and convergence properties. Furthermore, these control laws have the unique feature that they can be adapted in a very simple fashion to achieve asymptotically stable adaptive control.

## 1. Introduction

The problem of joint level control for the multi-link rigid articulated robot arm is addressed in this paper. Accurate measurements of the joint variables, either angular or displacement, and the joint velocities are assumed available. Traditionally, this problem has been treated by the PID algorithm. Since the justification of using PID control is based on either linearization or some local stability argument [1], its application is limited to small angle maneuvers. Large excursions usually require partitioning a desired trajectory into intermediate points and PID control is used to drive the arm between adjacent points. This approach is less than satisfactory since global stability and adequate performance are not guaranteed. This then motivates the computed torque method [2] which compensates for nonlinear terms in the robot dynamics. Assuming that the robot dynamics are known exactly, the compensated system appears like a decoupled system of double integrators and the closed loop dynamics can be shaped into desirable forms.

A different approach has been advanced in the past few years. It is based on the recognition that robot arms belong to the class of natural systems, which means time invariant, unconstrained and lying in a conservative force field [3]. It is natural to investigate if the structure specific to this class of systems can be exploited in controller design. It has long been known [3,4 and earlier] that negative proportional (generalized position) and derivative (generalized velocity), or equivalently PD, feedback globally asymptotically stabilized natural systems. The stability analysis is based on a Lypaunov function motivated by total energy considerations. Application of this result to robot arms has been relatively recent. In particular, global asymptotic stability under joint level PD feedback with gravity compensation has been shown [5-8]. Application to the tracking problem is more difficult due to the time varying nature of the problem. More specifically, the stability analysis requires a generalization of the invariance principle to time-varying systems; this issue has been partially addressed in [9-10].

In this paper, we will introduce a new class of exponentially stabilizing control laws for both the set point and tracking control problems. The stability proof is achieved by making use of a particular class of energy-like Lyapunov functions in conjunction with a useful lemma for addressing the higher order terms in the Lyapunov function derivatives. In the set point control case, Lyapunov functions based on various artificial potential fields are used to derive control laws possessing desired properties. These include set point controllers having simple PD or PD+bias structures and the ability to handle joint stop constraints. In the tracking control case, this new class of Lyapunov functions avoids the need for a generalized invariance principle, which, as mentioned above, has been the major source of difficulty in existing approaches. This leads to a new class of exponentially stabilizing tracking control laws. In one design among this new class, the nonlinear terms are decoupled from real-time measurements which completely removes the requirement for on-line computation on nonlinear terms in the controller implementation. This result is believed to have no counterpart in the present day literature. In general, the new class of control laws offers alternatives to the more conventional computed torque method, providing tradeoffs between robustness, computation and convergence properties. Furthermore, these control laws have the unique property that they can be adapted in a very simple fashion to achieve asymptotically stable adaptive control. This last property will be elaborated on in the companion paper [13]. The closed loop exponential stability also allows the robustness property to be established with respect to viscous and Coulomb friction, bounded modeling error and instrument noise.

This paper is organized as follows: Some background derivations, identities, notations, lemmas and releva results in the literature are covered in Sec. 2. Several useful set point controllers based on different artificial potential energies are presented in Sec. 3. A new Lyapunov function is also introduced to demonstrate exponential convergence. In Sec. 4, a new family of exponentially stabilizing tracking control laws are derive We will discuss the trade off between the ease of implementation and the strength of assumptions for these controllers. Their robustness properties are also analyzed in this section. Finally, conclusions are drawn in Sec. 5 together with a table summarizing all of the controllers presented in this paper and the conditions for stability. Due to the size limitation of this paper, the detail derivations are given in [2].

## 2. Background

### 2.1 Robot Dynamic Equation

In this section, the dynamic equation of robot manipulator is derived. At the first glance, it appears as a complex, tightly coupled set of nonlinear equations. However, based on derivation from Hamiltonian principle, the nonlinearity actually contains a great deal of structure. As a result, some important identities are developed in the next section on which the rest of the paper is based.

An n-link rigid robot arm belongs to the class of so-called natural systems with the kinetic and potential energies given by

$$T = \frac{1}{2} q_2^T M(q_1) q_2$$
$$U = -q_2^T u + g(q_1) \tag{2.1}$$

where

$T$ = kinetic energy , $U$ = potential energy , $q_1$ = joint angle or position vector $\epsilon R^n$ ,

$q_2$ = joint velocity vector $\epsilon R^n$ , $M(q_1)$ = mass inertia matrix $\epsilon R^{nxn}$ , $g(q_1)$ = gravitational potential energy ,

$u$ = joint torque force vector $\epsilon R^n$

Note that since all the analysis is done at joint level, the arm can be redundant (more than 6 joints). To derive the differential form of the robot dynamics, first set up the Lagrangian

$$L = T - U$$

Then apply the Lagrange's Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_2}\right) - \frac{\partial L}{\partial q_1} = 0$$

This gives the dynamic equation of robot motion:

$$\dot{q}_1 = q_2$$
$$M(q_1)\dot{q}_2 = -C(q_1,q_2)q_2 - k(q_1) + u \tag{2.2}$$

where

$$C(q_1,q_2) = \sum_{i=1}^{n} [(e_i \, q_2^T M_i(q_1))^T - \frac{1}{2}(e_i \, q_2^T M_i(q_1))] \tag{2.3}$$

$e_i$ = ith unit vector

$$M_i(q_1) = \frac{\partial M(q_1)}{\partial q_{1i}}$$

$$k(q_1) = \frac{\partial g(q_1)}{\partial q_{1i}}$$

$q_{1i}$ = ith component of $q_1$

The term $C(q_1,q_2)q_2$ represents the coriolis and centrifugal forces and $k(q_1)$ represents the gravity load. Note that $C(q_1, q_2)$ is determined entirely from the mass-inertia matrix. Many desirable properties, for example, inherent passivity, well-posedness of solution (no finite escape under any bounded control), existence of solution to optimal control problem [14] etc. are the consequence of this additional structure. Other important properties of (2.2) include that $M(q_1)$ and $M_i(q_1)$ are symmetric and $M(q_1)$ is positive definite, for all $q_1 \epsilon R^n$. For later use, the matrices $C(q_1,q_2)$ and $M_i(q_1)$ are interpreted as $R^{nxn}$ valued function of two n-vectors ($q_1$ and $q_2$) and one n-vector ($q_1$), respectively.

### 2.2 Some Useful Identities

Some key identities that will be used throughout this paper and the companion paper [13] are derived in this section. First define some notations:

$$M_D(q_1,Z) = \sum_{i=1}^{n} M_i(q_1) z\, e_i^T \tag{2.4}$$

$$\dot{M}(q_1,q_2) = \frac{d}{dt} M(q_1) \tag{2.5}$$

$$J(q_1,z) = \sum_{i=1}^{n} [(e_i z^T M_i(q_1)) - (e_i z^T M_i(q_1))^T] \tag{2.6}$$

$$r(q_1,q_2,q_{2d}) = (q_2-q_{2d})^T [\tfrac{1}{2} \dot{M}(q_1,q_2)(q_2-q_{2d}) - C(q_1,q_2)q_2] \tag{2.7}$$

$q_{1d}$, $q_{2d}$ = desired joint position and joint velocities

$$\Delta q_1 = q_1 - q_{1d}$$
$$\Delta q_2 = q_2 - q_{2d} \tag{2.8}$$

Again, $M_D$ and J are regarded as $R^{nxn}$ valued function of two n-vector arguments. Note that J is a skew symmetric matrix, i.e., $J+J^T = 0$.

Identity 1

$$\dot{M}(q_1,q_2)z = M_D(q_1,z)q_2 \tag{2.9}$$

Identity 2

$$C(q_1,z)z = \frac{1}{2}(M_D(q_1,z) - J(q_1,z))z \tag{2.10}$$

Identity 3

$$J(q_1,z) = M_D^T(q_1,z) - M_D(q_1,z) \tag{2.11}$$

Identity 4

$$M_D^T(q_1,x)y = M_D^T(q_1,y)x \tag{2.12}$$

Identity 5

$$r(q_1,q_2,q_{2d}) = \frac{1}{2}\ \Delta q_2^T (J(q_1,q_2)q_{2d} - M_D(q_1,q_{2d})q_2) \tag{2.13}$$

$$= \frac{1}{2}\ \Delta q_2^T (J(q_1,q_{2d})q_2 - M_D(q_1,q_2)q_{2d}) \tag{2.14}$$

$$= \frac{1}{2}\ \Delta q_2^T (J(q_1,q_{2d})q_{2d} - M_D(q_1,q_2)q_{2d}) \tag{2.15}$$

$$= \frac{1}{2}\ \Delta q_2^T (J(q_1,q_2)q_2 - M_D(q_1,q_{2d})q_2) \tag{2.16}$$

Identity 6

$$M_D(q_1,\Delta q_2)q_2 - C(q_1,q_2)q_2 - \frac{1}{2}(J(q_1,q_2)q_{2d} - M_D(q_1,q_{2d})q_2)$$

$$= M_D(q_1,\Delta q_2)q_2 - C(q_1,q_2)q_2 - \frac{1}{2}(J(q_1,q_{2d})q_2 - M_D(q_1,q_2)q_{2d})$$

$$= \frac{1}{2}(M_D^T(q_1,\Delta q_2)\Delta q_2 + M_D^T(q_1,q_{2d})\Delta q_2 - M_D(q_1,q_{2d})\Delta q_2 + M_D(q_1,\Delta q_2)q_{2d}) \tag{2.17}$$

$$M_D(q_1,\Delta q_2)q_2 - C(q_1,q_2)q_2 - \frac{1}{2}(J(q_1,q_{2d})q_{2d} - M_D(q_1,q_2)q_{2d})$$

$$= (M_D^T(q_1,q_{2d}) - M_D(q_1,q_{2d}))\Delta q_2 + \frac{1}{2} M_D^T(q_1,\Delta q_2)\Delta q_2 + \frac{1}{2} M_D(q_1,\Delta q_2)q_{2d} \tag{2.18}$$

$$M_D(q_1,\Delta q_2)q_2 - C(q_1,q_2)q_2 - \frac{1}{2}(J(q_1,q_2)q_2 - M_D(q_1,q_{2d})q_2)$$

$$= \frac{1}{2}(M_D(q_1,\Delta q_2)\Delta q_2 + M_D(q_1,\Delta q_2)q_{2d}) \tag{2.19}$$

## 2.3 Important Lemmas

In this section, two important stability lemmas are presented that will play pivotal roles in later sections. The first lemma is essentially a local stability theorem that establishes a region of convergence. The second lemma generalizes the first result to the Lagrange stability case. The proofs of these lemmas are given in [23].

Lemma 2-1

Given a dynamical system

$$\dot{x}_i = f_i(x_1,\ldots,x_N, t) \quad , \quad x_i \in R^{n_i} \quad ; \quad t \geq 0$$

Let $f_i$'s be locally Lipschitz with respect to $x_1,\ldots,x_N$ uniformly in t on bounded intervals and continuous in t for $t \geq 0$.

Suppose a function $V: R^{n_1 \times \ldots \times n_N} \times R_+ \to R_+$ is given such that

$$V(x_1,\ldots,x_N, t) = \sum_{i,j=1}^{N} x_i^T P_{ij}(x_1,\ldots,x_j, t)x_j \ ,$$

V is positive definite in $x_1,\ldots,x_N$

$$\dot{V}(x_1,\ldots,x_N, t) \leq -\sum_{i\epsilon I_1} (\alpha_i - \sum_{j\epsilon I_{2_i}} \gamma_{ij}||x_j(t)||^{k_{ij}})||x_i(t)||^2 \tag{2.20}$$

where $\alpha_i, \gamma_{ij}, k_{ij} > 0, \ I_{2_i} \subset I_1 \subset \{1,\ldots,N\}$

Let $\xi_i > 0$ be such

$$\xi_i||x_i||^2 \leq V(x_1,\ldots,x_N, t) \tag{2.21}$$

Let $V_o \triangleq V(x_1(0),\ldots,x_N(0), 0)$

If $\forall \ i \ \epsilon \ I_1$,

$$\alpha_i > \sum_{j\epsilon I_{2_i}} \gamma_{ij} \left(\frac{V_o}{\xi_j}\right)^{\frac{k_{ij}}{2}} \tag{2.22}$$

then $\forall \ \lambda_i \ \epsilon \ (0, \alpha_i - \sum_{j\epsilon I_{2_i}} \gamma_{ij} (\frac{V_o}{\xi_j})^{\frac{k_{ij}}{2}})$,

$$\dot{V}(x_1,\ldots,x_N, t) \leq -\sum_{i\epsilon I_1} \lambda_i \ ||x_i||^2 \qquad \forall \ t \geq 0 \tag{2.23}$$

In the above lemma, we choose to bound over $\sum_{j\epsilon I_{2_i}} \gamma_{ij} \ ||x_j(t)||^{k_{ij}}$

(Condition (2.20) reflects that choice). This choice is arbitrary; in fact, we can extract any quadratic cross term from

$$\sum_{j\epsilon I_{2_i}} \gamma_{ij} \ ||x_j(t)||^{k_{ij}} \ ||x_i(t)||^2$$

and overbound the rest. After completing square stability condition similar to (2.22) can be stated. We do not pursue this generalization here.

Lemma 2-2

Suppose in Lemma 2-1,

$$\dot{V}(x_1,\ldots,x_N,t) \leq -\sum_{i\epsilon I_1} (\alpha_i - \sum_{j\epsilon I_{2_i}} \gamma_{ij} \ ||x_j||^{k_{ij}}) \ ||x_i||^2 + \rho \tag{2.24}$$

$$I_1 = \{1,\ldots,N\}$$

Let $p = \sup_{x_i \epsilon R^{n_i}, t\epsilon R+} ||P(x_1,\ldots,x_N,t)|| < \infty \ , \ [P]_{ij} = P_{ij}$

If $\forall \ i \ \epsilon \ I_1$

$$\alpha_i > \frac{\rho p}{V_o} + \sum_{j\epsilon I_{2_i}} \gamma_{ij} \left(\frac{V_o}{\xi_j}\right)^{\frac{k_{ij}}{2}} \tag{2.25}$$

then

$$\limsup_{t \to \infty} V(x_1(t),\ldots,x_N(t),t) \leq \frac{\rho}{\lambda} \tag{2.26}$$

$$\text{where} \quad \lambda = \frac{1}{p} \min_{i\epsilon I_1} (\alpha_i - \sum_{j\epsilon I_{2_i}} \gamma_{ij} \left(\frac{V_o}{\xi_j}\right)^{\frac{k_{ij}}{2}}) \tag{2.27}$$

Furthermore, the convergence to the set $\{(x_1,\ldots,x_N): ||x_k||^2 \leq \frac{\rho}{\xi_k \lambda} \ , \ k\epsilon I_1\}$ is exponential with rate $\lambda$.

The utility of this result is mainly in robustness analysis. Basically, given a bounded set of possible initial conditions, excluding a neighborhood of the origin, $\alpha_i$'s must be large enough in the sense of (2.25) for the trajectories to be uniformly bounded. What if $V_o = 0$? We can shift $t = 0$ to some finite time later when $V_t \neq 0$. In practice, a neighborhood around origin can usually be excluded since some robust locally stable control algorithm such as PID takes over.

## 2.4 Recent Results

Some of the recent results related to Lyapunov analysis of robot systems are reviewed in this section. For the set point control problem, [5-10] has applied the result that linear negative feedback of generalized position and velocities globally asymptotically stabilizes a natural system to robot manipulators. We will restate this result, mention work for the tracking problem in [9,10,18] and state some open issues that will be addressed in the remainder of this paper and in [13].

**Theorem 2-1**

Consider (2.2) with the control law

$$u = -K_p \Delta q_1 - K_v \dot{q}_2 + k(q_1), \quad K_p > 0,\ K_v > 0 \tag{2.28}$$

The null state of $(\Delta q_1, q_2)$ - system is a globally asymptotically stable equilibrium.

The main idea of this approach is to shape the potential field in such a way that it is globally convex and attains a global minimum at $\Delta q_1 = 0$. Complete damping (in the terminology of [3]) is added through the derivative feedback to drive the system to the minimum potential energy state which by design is the desired state. To be specific, suppose the desired potential field is $U^*(\Delta q_1)$. The total energy under this potential is

$$V = T + U^* \tag{2.29}$$

Rewrite V as

$$V = T + U^o + U^* - U^o = V^o + U^* - U^o$$

where $U^o$ is the original potential energy without external force fields, and $V^o$ is the corresponding total energy. Let $p = M(q_1)q_2$ be the generalized momentum. From Hamilton's equation,

$$\dot{V} = \left(\frac{\partial V^o}{\partial p}\right)^T u + q_2^T\left(\frac{\partial U^*}{\partial \Delta q_1} - \frac{\partial U^o}{\partial \Delta q_1}\right)$$

$$= q_2^T\left(u + \frac{\partial U^*}{\partial \Delta q_1} - \frac{\partial U^o}{\partial \Delta q_1}\right) \tag{2.30}$$

Hence, to drive the desired total energy to its minimum state, we can select ([5])

$$u = -K_v q_2 - \frac{\partial U^*}{\partial \Delta q_1} + \frac{\partial U^o}{\partial \Delta q_1} \tag{2.31}$$

Then $\dot{V} = -q_2^T K_v q_2$. From the fact that $-2\dot{q}_2^T K_v q_2$ is uniformly bounded ($\dot{V} \leq 0$), $q_2(t) \to 0$ as $t \to \infty$ [19].

$$\dot{p} = -\frac{\partial V^o}{\partial q_1} + u$$

$$= -\frac{\partial T}{\partial q_1} - \frac{\partial U^o}{\partial \Delta q_1} + u$$

$$= -\frac{\partial T}{\partial q_1} - K_v q_2 - \frac{\partial U^*}{\partial \Delta q_1} \tag{2.32}$$

Since $\frac{\partial T}{\partial q_1} \to 0$, $\frac{\partial U^*}{\partial \Delta q_1} \to 0$, also. Hence, if $U^*(\Delta q_1)$ is globally convex with minimum at $\Delta q_1 = 0$, $\Delta q_1(t) \to 0$ as $t \to \infty$. If $U^*(\Delta q_1) = \frac{1}{2}\Delta q_1^T k_p \Delta q_1$, Theorem 2-1 is immediately obtained.

Obviously, any other potential field convex in $\Delta q_1$ that has global minimum at $\Delta q_1 = 0$ (and no local minima) can be used. We will use this idea in the next section to address the joint stop issue.

This control law is very appealing in its simplicity and obvious robustness with respect to modeling error in mass matrix, and centrifugal and coriolis terms. Generalization to the tracking problem is partially addressed in [9, 10]. A control algorithm is given in [9], but it lacks stability analysis. In [10], Metrosov's Theorem [11] is used. A question remains on the necessity and applicability of the Metrosov's Theorem to the tracking problem. One version of the tracking control law in Section 4 is the same as in [10], but the stability issue is resolved more completely. Nonadaptive version of the tracking control laws in [10, 18] do yield global asymptotic stability. However, the simple structure of (2.28) is lost; even for set point control, full model information is needed.

Based on the above very brief review of the currently available pertinent results, it is evident that the following issues remain open:

1. Can we get away with no gravity information, thus achieving a "universal" (arm independent) set point control law?

2. Computed torque achieves exponential stability. Are schemes based on energy Lyapunov functions inherently inferior (e.g., only asymptotic stability is possible) or have we not been clever enough in choosing the Lyapunov functions?

3. The tracking problem produces a time-varying system. Can the Invariance Principle still be applied?

4. How far can we reduce the on-line computation requirement (thus allow increasing performance) for both set point and tracking problems? What is the price to be paid?

5. Can we ensure any robustness properties with respect to friction, instrument noise, modeling errors?

6. How does one incorporate joint stop constraints?

7. Would these schemes (set point and tracking controls) still work if unknown parameters are adapted?

The rest of this paper will be devoted to answering issues 1-6. The last item is addressed in [13] and [24].

2.5 Computed Torque from Lyapunov Perspective

In Section 2.4, we introduced the total energy Lyapunov function (2.29) to derive a simple set point control law. The computed torque method can also be motivated in the same manner with a different Lyapunov function. For generality, we will consider the tracking case. Let

$$V(\Delta q_1,\Delta q_2) = \frac{1}{2}||\Delta q_2||^2 + \frac{1}{2}\Delta q_1^T K_p \Delta q_1 \qquad (2.33)$$

Calculate derivative along solution,

$$\dot V = \Delta q_2^T(M^{-1}(q_1)(-C(q_1,q_2)q_2 - k(q_1) + u) - \dot q_{2d} + K_p\Delta q_1)$$

If the computed torque control is used

$$u = k(q_1) + C(q_1,q_2)q_2 + M(q_1)(\dot q_{2d} - K_p\Delta q_1 - K_v\Delta q_2) \qquad (2.34)$$

then

$$\dot V \leq -\Delta q_2^T K_v\Delta q_2$$

From the same line of reasoning as before, the closed loop system is globally asymptotically stable. However, we know that the closed loop system is linear, therefore it is exponentially stable. This means that we should look for a better Lyapunov function. An obvious choice is to add in a cross term in (2.33). Then

$$V(\Delta q_1,\Delta q_2) = \frac{1}{2}||\Delta q_2||^2 + \frac{1}{2}\Delta q_1^T(K_p + cK_v)\Delta q_1 + c\Delta q_1^T\Delta q_2 \qquad (2.35)$$

where c is small constant so that V is positive definite. Take derivative and apply (2.34),

$$\dot V = -\Delta q_2^T K_v\Delta q_2 + c\Delta q_2^T K_v\Delta q_1 + c||\Delta q_2||^2 - c\Delta q_1^T K_p\Delta q_1 - c\Delta q_1^T K_v\Delta q_2$$
$$= -(\sigma_{min}(K_v) - c)\,||\Delta q_2||^2 - c\Delta q_1^T K_p\Delta q_1$$

which shows closed loop exponential stability.

Note that in (2.34), in contrast to (2.28), even for set point case, full model nonlinearity cancellation is needed. The approach in this paper is to use the energy Lyapunov function instead of (2.33) to generate control laws. We will see in later sections that this affords a much larger class of controls which contains much simpler structure in certain cases (especially for set point control).

## 3. New Results on PD Set Point Control

### 3.1 Simple PD Controls

In this section, we will explore using different U* in the controller design. The following has been suggested in [5]:

$$U^*(\Delta q_1) = \frac{1}{2}\Delta q_1^T K_p\Delta q_1 + g(\Delta q_1+q_{1d}) - g(q_{1d}) - \Delta q_1^T k(q_{1d}) \qquad (3.1)$$

If $K_p$ is sufficiently large, $\Delta q_1 = 0$ is the global minimum of $U(\Delta q_1)$. Hence, a simpler control law can be used:

$$u = - K_p \Delta q_1 - K_v q_2 + k(q_{1d}) \quad (3.2)$$

Suppose each joint is constrained between joint stops:

$$q_{1i}^{(\ell)} \le q_{1i} \le q_{1i}^{(h)} \quad (3.3)$$

and the set point is in the interior of the joint inputs:

$$q_{1i}^{(\ell)} < \underline{q}_{1i} \le q_{1id} \le \bar{q}_{1i} < q_{1i}^{(h)} \quad (3.4)$$

Let the desired potential function be

$$U^*(\Delta q_1) = \frac{1}{2} \Delta q_1^T K_p \Delta q_1 + \sum_{i=1}^{n} (H_i(\Delta q_{1i}+q_{1id}) + L_i(\Delta q_{1i}+q_{1id})) \quad (3.5)$$

where $H_i$ and $L_i$ are appropriate upper and lower barrier potential functions for joint i [23].

Then, $\Delta q_1 = 0$ is a global minimum of $U^*(\Delta q_1)$ [23]. From (2.31)

$$u = - K_v q_2 - K_p \Delta q_1 - \dot{H}(\Delta q_1) - \dot{L}(\Delta q_1) + k(q_1) \quad (3.6)$$

Similarly, if $K_p$ is sufficiently large $(K_p + \frac{\partial k}{\partial \Delta q_1} (\Delta q_1 + q_{1d}) > 0)$, the following control law also achieves global asymptotic stability:

$$u = - K_v q_2 - K_p \Delta q_1 - \dot{H}(\Delta q_1) - \dot{L}(\Delta q_1) + k(q_{1d}) \quad (3.7)$$

Control laws (3.2), (3.6) and (3.7) still require information on the gravity load. It is interesting to ask if this last piece of model information can be removed. This case corresponds to the desired potential energy

$$U^*(\Delta q_1) = \frac{1}{2} \Delta q_1^T K_p \Delta q_1 + g(\Delta q_1+q_{1d}) \quad (3.8)$$

The corresponding control law is

$$u = - K_p \Delta q_1 - K_v \Delta q_2 \quad (3.9)$$

From section 2.4,

$$\frac{\partial U^*}{\partial \Delta q_1} (\Delta q_1) = K_p \Delta q_1 + k(\Delta q_1+q_{1d}) \to 0$$

This implies

$$\limsup_{t\to\infty} ||\Delta q_1(t)|| \le \sigma_{min}(K_p) \sup_{q_1 \in R^n} || k(q_1)||$$

If $K_p + \frac{\partial k(q_1)}{\partial q_1} > 0 \ \forall q_1 \in R^n$, $- K_p k(\Delta q_1+q_{1d})$ is a contraction map in $\Delta q_1$. Then $\exists !\ q_1^*$ such that

$$K_p(q_1^* - q_{1d}) + k(q_1^*) = 0 \quad (3.10)$$

$$\lim_{t\to\infty} q_1(t) = q_1^*$$

This result suggests a very simple, robust and practical control scheme. The feedback gain $K_p$ can be chosen large enough to justify the use of PID control [1, ch. 6 of 19] which is locally stable. Typically, $k(q_1)$ and $\frac{\partial k(q_1)}{\partial q_1}$ are composed of trigonometric functions, therefore, they are uniformly bounded.

3.2 PD Control with Exponential Convergence Rate

Use of the Invariance Principle in Section 2.4 only shows asymptotic stability. Some guaranteed rate of convergence is highly desirable not just for performance reasons but for robustness analysis and adaptive control also. In Section 2.5, a Lyapunov function with cross term has been used to show exponential stability. This suggests a similar modification here. The result is summarized below.

**Theorem 3-1**

Given the control law (2.31)

$$u = -K_v q_2 - \frac{\partial U^*}{\partial q_1} + \frac{\partial U^o}{\partial q_1} \tag{2.31}$$

Suppose $\exists\, \nu > 0$ such that

$$\Delta q_1^T \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) > \nu \, ||\Delta q_1||^2 \tag{3.11}$$

and $U^*(\Delta q_1)$ has a global minimum at $\Delta q_1 = 0$, then the closed loop $(\Delta q_1, q_2)$ system is exponentially stable.

**Proof:**

Modify the total energy Lyapunov function (2.29) to

$$V = T + U^* + c\Delta q_1^T p + \frac{1}{2} c\Delta q_1^T K_v \Delta q_1$$

where c is a small constant so that V is positive definite in p and $q_1$. Without loss of generality, $U^*$ can be considered positive definite in $q_1$ (by adding an appropriate constant). Then from (2.32),

$$\dot V = q_2^T\left(u + \frac{\partial U^*}{\partial q_1} - \frac{\partial U^o}{\partial q_1}\right) + c\, q_2^T p + c\Delta q_1^T\left(-\frac{\partial T}{\partial q_1} - \frac{\partial U^o}{\partial q_1} + u\right) + c\, q_2^T K_v \Delta q_1$$

$$= -q_2^T K_v q_2 + c\, q_2^T M(q_1) q_2 - c\,\Delta q_1^T \frac{\partial T}{\partial q_1} - c\,\Delta q_1^T \frac{\partial U^*}{\partial q_1}$$

Let

$$\mu \triangleq \sup_{q_1 \in R^n} ||M(q_1)|| \tag{3.12}$$

$$\xi_1 = \inf_{||\Delta q_1||=1} [U^*(\Delta q_1) + \frac{1}{2} c\Delta q_1^T K_v \Delta q_1 - \frac{1}{2} c\mu \ell^2] > 0 \quad \text{for some constant } \ell. \tag{3.13}$$

From Lemma 2-1, $\forall\, \lambda_2 \in (0, \alpha_2 - \gamma_{21} (\frac{V_o}{\xi_1})^{\frac{1}{2}})$,

$$\dot V \le -\alpha_1 ||\Delta q_1||^2 - \lambda_2 ||\Delta q_2||^2$$

Note

$$\frac{\partial T}{\partial q_1} = \frac{1}{2} \sum_{i=1}^{n} e_i\, q_2^T M_i(q_1)\, q_2 = \frac{1}{2} M_D^T(q_1, q_2) q_2 \tag{3.14}$$

Then

$$\dot V \le -(\sigma_{min}(K_v) - c\mu)||q_2||^2 - c\,\nu\, ||\Delta q_1||^2 - \frac{c}{2} \Delta q_1^T M_D^T(q_1, q_2) q_2$$

Define

$$\eta_1 = \sup_{q_1} \sum_{i=1}^{n} ||M_i(q_1)|| \tag{3.15}$$

Then

$$\dot V \le -\alpha_1 ||\Delta q_1||^2 - \alpha_2 ||\Delta q_2||^2 + \gamma_{21} ||\Delta q_1||\, ||\Delta q_2||^2$$

where

$$\alpha_1 = c\nu$$

$$\alpha_2 = \sigma_{min}(K_v) - c\mu$$

$$\gamma_{21} = \frac{1}{2} c\, \eta_1$$

Choose

$$c < \sigma_{min}(K_v)\left(\mu + \frac{1}{2} \eta_1 \left(\frac{V_o}{\xi_1}\right)^{\frac{1}{2}}\right)^{-1} \tag{3.16}$$

where $V_o = V\big|_{t=0}$ and

$\leq -\lambda V$

for some $\lambda > 0$. Hence, the closed loop system is exponentially stable. ∎

Given any $U^*$ according to (3.11), $K_v > 0$ and initial condition, there always exists c that satisfies (3.16). Even though c is not needed in the implementation, its maximum allowable size affects the convergence rate. The artificial potentials $U^* = 1/2\ \Delta q_1^T K_p \Delta q_1$, (3.1) and (3.5) all satisfy the assumptions of Theorem 3-1. Therefore, the corresponding closed loop systems are exponentially stable. For the potential given by (3.8) and $K_p$ large enough $(K_p + \frac{\partial k}{\partial q_1} > 0)$, we can add a constant to $U^*$ so that $U^*$ is positive definite in $q_1 - q_1^*$ and (3.11) is satisfied for $\Delta q_1 = q_1 - q_1^*$, where $q_1$ solves (3.10). Then Theorem 3-1 implies exponential convergence of $q_1$ to $q_1^*$ which is within $\sigma_{min}(K_p) \sup_{q_1} ||k(q_1)||$ from the true $q_{1d}$.

## 4. New Results in Tracking Control

### 4.1 Exponentially Stable Algorithms

Frequently a robot is required to follow a prespecified path for continuous action at the end effector (e.g., arc welding), tracking of a moving target (e.g. pick and place operation from conveyer belt) or other high level objectives (e.g., time optimality, collision avoidance, arm singularity avoidance, etc.). This can be posed as the problem of tracking the desired positions and velocities $(q_{1d}, q_{2d})$ by $(q_1, q_2)$. In this section, we will extend the basic ideas put forth in Section 3 to the tracking problem. The error equation is now in the form

$$\Delta\dot{q}_1 = \Delta q_2$$

$$M(q_1)\Delta\dot{q}_2 = -C(q_1,q_2)q_2 - k(q_1) + u - M(q_1)\dot{q}_{2d} \tag{4.1}$$

We will first state several direct generalizations of Theorem 3-1. An energy type Lyapunov function similar to (2.33) used in Section 2.5 to motivate computed torque is used here.

<u>Theorem 4-1</u>

Consider (4.1) with the control law

$$u = -K_v\Delta q_2 + k(q_1) - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + M(q_1)\dot{q}_{2d} - D(q_1,q_2,q_{2d}) \tag{4.2}$$

where D is given by any one of the following expressions

$$D(q_1,q_2,q_{2d}) = \frac{1}{2}(J(q_1,q_2)q_{2d} - M_D(q_1,q_{2d})q_2) \tag{4.2a}$$

$$D(q_1,q_2,q_{2d}) = \frac{1}{2}(J(q_1,q_{2d})q_2 - M_D(q_1,q_2)q_{2d}) \tag{4.2b}$$

$$D(q_1,q_2,q_{2d}) = \frac{1}{2}(J(q_1,q_{2d})q_{2d} - M_D(q_1,q_2)q_{2d}) \tag{4.2c}$$

$$D(q_1,q_2,q_{2d}) = \frac{1}{2}(J(q_1,q_2)q_2 - M_D(q_1,q_{2d})q_2) \tag{4.2d}$$

Assume $\exists \nu > 0$ such that

$$\Delta q_1^T \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) > \nu\ ||q_1||^2 \tag{4.3}$$

and $U^*(\Delta q_1)$ is positive definite in $\Delta q_1$. Then the null state of the $(\Delta q_1, \Delta q_2)$ system is a globally exponentially stable equilibrium.

<u>Proof:</u> Use the following Lyapunov function

$$V(\Delta q_1,\Delta q_2) = \frac{1}{2}\Delta q_2^T M(q_1)\Delta q_2 + U^*(\Delta q_1) + c\ \Delta q_1^T M(q_1)\Delta q_2 + \frac{1}{2} c\ \Delta q_1^T K_v \Delta q_1 \tag{4.4}$$

where c is a small constant, such that V is positive definite in $\Delta q_1$ and $\Delta q_2$. Take derivative along solution:

$$\dot{V}(\Delta q_1,\Delta q_2) = \Delta q_2^T(M(q_1)\Delta\dot{q}_2 + \frac{1}{2}M_D(q_1,\Delta q_2)q_2 + \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + c\ M(q_1)\Delta q_2$$

$$+ c\ K_v\Delta q_1) + c\ \Delta q_1^T(M(q_1)\Delta\dot{q}_2 + M_D(q_1,\Delta q_2)q_2)$$

Substitute (4.1) and (4.2) and use (2.7)

$$\dot V(\Delta q_1,\Delta q_2) = \Delta q_2^T(-C(q_1,q_2)q_2 - k(q_1) + u - M(q_1)\dot q_{2d} + \frac{1}{2} M_D(q_1,\Delta q_2)q_2$$

$$+ \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + c\, M(q_1)\Delta q_2 + c\, K_v\Delta q_1)$$

$$+ c\, \Delta q_1^T(-C(q_1,q_2)q_2 - k(q_1) + u - M(q_1)\dot q_{2d} + M_D(q_1,\Delta q_2)q_2)$$

$$= r(q_1,q_2,q_{2d}) - \Delta q_2^T(K_v - cM(q_1))\Delta q_2 - \Delta q_2^T D(q_1,q_2,q_{2d})$$

$$+ c\Delta q_1^T(M_D(q_1,\Delta q_2)q_2 - C(q_1,q_2)q_2)$$

$$- c\Delta q_1^T \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) - c\Delta q_1^T D(q_1,q_2,q_{2d})$$

Apply Identity 5, $r - \Delta q_2^T D = 0$. Define $\eta_1$, $\eta_2$ as follows

$$\eta_1 = \sup_{q_1 \in R^n} \sum_{i=1}^{n} ||M_i(q_1)||$$

$$\eta_2 = \sup_{q_{2d}} ||q_{2d}||\eta_1$$

From Identity 6,

$$|c\Delta q_1^T(M_D(q_1,\Delta q_2)q_2 - C(q_1,q_2)q_2 - D(q_1,q_2,q_{2d})|$$

$$\leq c||\Delta q_1||(a\eta_2||\Delta q_2|| + \frac{\eta_1}{2}||\Delta q_2||^2)$$

where $a=\frac{3}{2}$ for (4.2a) , $a=\frac{3}{2}$ for (4.2b) , $a=\frac{5}{2}$ for (4.2c) , $a=\frac{1}{2}$ for (4.2d)

Hence,

$$\dot V(\Delta q_1,\Delta q_2) \leq -(\sigma_{min}(K_v) - c\mu)\,||\Delta q_2||^2 - c\,\nu\,||\Delta q_1||^2$$

$$+ c\,||\Delta q_1||(a\eta_2||\Delta q_2|| + \frac{\eta_1}{2}||\Delta q_2||^2)$$

Completing square for the cross term,

$$\dot V(\Delta q_1,\Delta q_2) \leq -\alpha_1||\Delta q_1||^2 - \alpha_2||\Delta q_2||^2 + \gamma_{21}||\Delta q_1||\,||\Delta q_2||^2 \qquad (4.5)$$

where

$$\alpha_1 = c(\nu - \frac{1}{2}a\,\eta_2\rho^2)$$

$$\alpha_2 = \sigma_{min}(K_v) - c(\mu + \frac{1}{2}\frac{a\eta_2}{\rho^2})$$

$$\gamma_{21} = \frac{1}{2}c\,\eta_1$$

Given $\nu$, choose $\rho^2 < \frac{2\alpha_1}{a\eta_2}$. By Lemma 2-1, for

$$c < \sigma_{min}(K_v)\,(\mu + \frac{1}{2}\frac{a\eta_2}{\rho^2} + \frac{1}{2}\eta_1(\frac{V_o}{\xi_1})^{\frac{1}{2}})^{-1}$$

($V_o$, $\xi_1$ are as defined from the proof of Theorem 3-1) and $\forall\ \lambda_2 \in (0, \alpha_1 - \gamma_{21}(\frac{V_o}{\xi_1})^{\frac{1}{2}})$

$$\dot V \leq -\alpha_1||\Delta q_1||^2 - \lambda_2||\Delta q_2||^2$$

$$\leq -\lambda V$$

for some $\lambda > 0$. Hence, the closed loop system is exponentially stable.

A common Lyapunov function used for tracking problem has been [9, 10]

$$V(\Delta q_1,\Delta q_2) = \frac{1}{2}\Delta q_2^T M(q_1)\Delta q_2 + U^*(\Delta q_1)$$

In this case, a generalization of Invariance Principle to time-varying case is required. There are two possibilities. The result in [Theorem A.7.6, 21] appears promising, but we must verify that (4.1) is positive pre-compact (in the sence defined in [21]). A more direct route is to use [Lemma 1, 22] which states that if $\Delta\ddot{q}_2$ and $\Delta\dot{q}_2$ are both bounded uniformly in t (it follows from $\dot{V} \leq 0$), then $\Delta q_2(t) \to 0$ implies $\Delta\dot{q}_2(t) \to 0$.

Note that $U^*(\Delta q_1)$ does not depend on time explicitly. This restriction eliminates some of the candidates used in set point case. How to generalize to the case of $U^*(\Delta q_1,t)$ and $\frac{\partial U^*}{\partial t}(\Delta q_1,t)$ not necessarily negative semidefinite is currently under investigation.

Control laws (4.2a-d) all have same stability property nominally. When $q_2$ is a very noise measurement, as is typically the case, (4.2c) which only uses $q_2$ once may have better robustness.

Note that all the controllers have structures very similar to computer torque; in fact, if all occurrences of $q_{2d}$ are replaced by $q_2$, then the nonlinear compensation is exactly the same as the case of computer torque. However, in their present forms, (4.2a-d) cannot take advantage of well known recursive algorithms for inverse dynamics computation [11, 12]. Therefore, we next present slightly modified versions that can be implemented with these algorithms.

Corollary 4-1

Consider (4.1) with the control law

$$u = -K_v\Delta q_2 + k(q_1) - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + M(q_1)\dot{q}_{2d} + C(q_1,q_{2d})q_{2d} \tag{4.6}$$

where $U^*(\Delta q_1)$ satisfies the same assumptions as in Theorem 4-1.

If

$$\sigma_{min}(K_v) > \frac{\eta_2}{2}$$

then the null state of the $(\Delta q_1,\Delta q_2)$ system is a globally exponentially stable equilibrium.

Corollary 4-2

Consider (4.1) with the control law

$$u = -K_v\Delta q_2 + k(q_1) - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + M(q_1)\dot{q}_{2d} + C(q_1,q_2)q_2 \tag{4.7}$$

where $U^*(\Delta q_1)$ satisfies the assumptions as in Theorem 4-1. Given a set of possible initial conditions, if $K_v$ is sufficiently large, then the closed loop system is exponentially stable with respect to that set.

If $U^*(\Delta q_1) = \frac{1}{2}\Delta q_1^T K_p \Delta q_1$, (4.7) is actually a modification of the computed torque method with $K_p$, $K_v$ replacing $M(q_1)K_p$, $M(q_1)K_v$.

So far we have generated many control laws that are similar to computed torque. However, the ones just requiring $K_v > 0$, $\nu > 0$ are not easily implementable, and the easily implementable ones need stronger conditions ($K_v$ sufficiently large). The next control law that we shall present is of very appealing structure: the real time update computations are linear and the off-line computation can take advantage of efficient algorithms (e.g., Newton-Euler type). The trade-off is that $K_v$ and $\nu$ must both be large enough for a given set of initial conditions.

Theorem 4-2

Consider (4.1) with the control law

$$u = -K_v\Delta q_1 + k(q_{1d}) - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + M(q_{1d})\dot{q}_{2d} + C(q_{1d},q_{2d})q_{2d} \tag{4.8}$$

where $U^*(\Delta q_1)$ satisfies the assumptions as in Theorem 4.1. Given a set of possible initial conditions, if $K_v$ and $\nu$, are sufficiently large, then the closed loop system is exponentially stable with respect to that set.

Typically, $q_2(0) = q_{2d}(0) = 0$ and $\Delta q_1(0)$ is always within $2\pi$. Hence $V_o$ is bounded above, and the result is essentially a global one. This scheme requires both $\nu$ and $K_v$ large enough. This requirement is made easier by shifting the computational burden to offline thus allowing very high sampling rates which in turn means high gains can be tolerated.

## 4.2 Robustness

Lyapunov analysis provides a useful approach to study the robustness issue. Robustness is a much abused word in the literature. Here, we use insensitive design to mean preservation of stability (in the sense of Lagrange) under sufficiently small perturbations. Furthermore, the size of the ultimate bound should vary continuously with the size of perturbations. By robust design we mean a controller design that preserves stability under prescribed size of perturbations. In this section, we will examine frictions, both viscous and Coulomb type, bounded modeling error and bounded actuator and sensor noises.

Friction forces can be approximately modeled by Coulomb friction, or dry friction, and viscous friction due to oil lubrication. For joint i, the frictional force is given by

$$f_{fric}^{(i)} = - F_{1i} \operatorname{sgn}(q_{2i}) - F_{2i} q_{2i} \quad , \quad F_{1i}, F_{2i} > 0 \qquad (4.9)$$

Equation (4.1) is then

$$M(q_1)\Delta\dot{q}_2 = - C(q_1,q_2)q_2 - k(q_1) + u - M(q_1)\dot{q}_{2d} - F_1 \operatorname{sgn}(q_2) - F_2 q_2$$

where $F_1$ and $F_2$ are diagonal matrices with elements $F_{1i}$, $F_{2i}$, respectively, $\operatorname{sgn}(q_2)$ represents a vector with elements $\operatorname{sgn}(q_{2i})$.

From [23], the set point controller is both insensitive and robust. The tracking controllers are also insensitive with respect to frictions. For robust design for a given level of friction, $\sigma_{min}(K_v)$ and $\nu$ should be chosen large enough.

Next we consider modeling error in controller implementation. Model parameters k, M, $M_D$ can all be written linearly in constant parameters. Assume bounded errors are incurred in these parameters. Control laws (4.2) are in the form

$$u = - K_v \Delta q_2 - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + k(q_1) + M(q_1)\dot{q}_{2d} - D(q_1,q_2,q_{2d}) + \Delta_1 + \Delta_2$$

The additional terms in $\dot{V}$ are

$$- (C\Delta q_1 + \Delta q_2)^T (\Delta_1 + \Delta_2)$$

which, after overbounding [23], becomes

$$(c||\Delta q_1|| + ||\Delta q_2||)\ (\delta_1 + \delta_2 + \delta_3\ ||\Delta q_2|| + \delta_4\ ||\Delta q_2||^2)$$

After completing squares, the overbound over the extra terms in $\dot{V}$ become

$$\frac{c}{2}\ (\delta_3 s_3^{\ 2} + \frac{(\delta_1+\delta_2)}{s_1^{\ 2}})\ ||\Delta q_1||^2 + (\delta_3 + \frac{c\delta_3}{2s_3^{\ 2}} + \frac{\delta_1+\delta_2}{2\ s_2^{\ 2}})\ ||\Delta q_2||^2$$

$$+ c\delta_4\ ||\Delta q_1||\ ||\Delta q_2||^2 + \delta_4\ ||\Delta q_2||^3 + \frac{1}{2}\ (\delta_1+\delta_2)\ (cs_1^{\ 2} + s_2^{\ 2})$$

For $\delta_1$, $\delta_5$ sufficiently small, $\exists\ \alpha_1, \alpha_2, \rho > 0$ such that

$$\dot{V} \le - \alpha_1 ||\Delta q_1||^2 - \alpha_2\ ||\Delta q_2||^2 + c(\frac{\eta_1}{2} + \delta_4)\ ||\Delta q_1||\ ||\Delta q_2||^2 + \delta_4\ ||\Delta q_2||^3 + \rho$$

By Lemma 2-2, if $V_o > 0$ and $\delta_1, \delta_5$ are small, the system remains Lagrange stable and the ultimate bound vanishes as $\delta_1$, $\delta_5 \to 0$. Hence, the design is insensitive with respect to modeling errors. For robust design for a given level of modeling uncertainties, $\sigma_{min}(K_v)$ and $\nu$ should be large enough.

Finally, suppose bounded errors $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$ are incurred in $q_1$, $q_2$ and u, respectively. Control laws (4.2) are now in the form

$$u = - K_v\ \Delta q_2 - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + k(q_1) + M(q_1)\dot{q}_{2d} - D(q_1,q_2,q_{2d}) + \Delta_1 + \Delta_2$$

Follow the same steps as before, we overbound the extra terms in $\dot{V}$ by sums of $\|\Delta q_1\|^2$, $\|\Delta q_2\|^2$, $\|\Delta q_1\|\ \|\Delta q_2\|^2$ and constant terms. Again use Lemma 2-2 to conclude that the controller is insensitive to bounded instrument noises and robust for a given level of noise if $\sigma_{min}(K_v)$ and $\nu$ are sufficiently large.

As an aside, it should be noted that similar results as derived here hold for computed torque techniques also. For the case of set point control under friction, the insensitivity and robustness properties of the design here do not follow directly from the analysis.

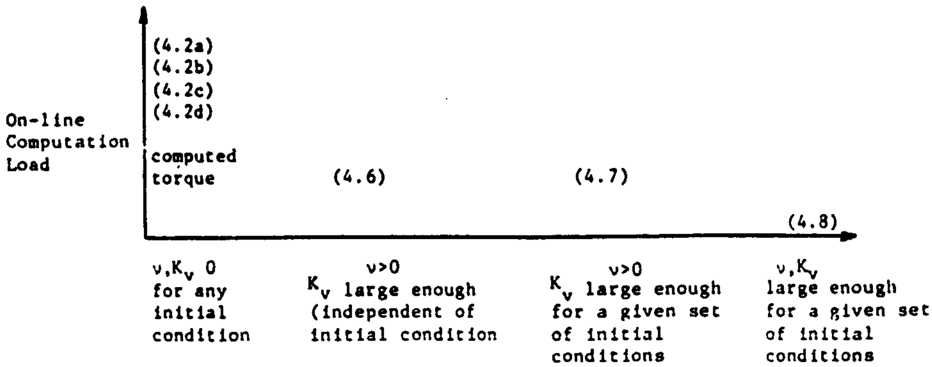
There are obviously many more practical implementation issues not addressed here: sampling effect, actuator saturation, joint and arm flexibilities and instrument dynamics. They are currently under investigation in the same framework.

## 5. Summary

We have introduced a new class of exponentially stabilizing control laws for the joint level control of robot manipulators (summarized in Table I). The stability result is achieved by making use of a particular class of energy-like Lyapunov functions (of the form (4.4)) in conjunction with a useful lemma (Lemma 2-1) for addressing higher order terms in Lyapunov function derivatives. This approach avoids the need for a generalization of the invariance principle to time-varying systems, which has been the major source of difficulty in the past [9,10].

In the set point control case, by incorporating artificial potential fields in the Lyapunov function, we have derived a class of exponentially stabilizing, PD + potential shaping type of control laws. Several useful potential fields have been examined resulting in simple structures: PD and PD+bias, and the ability to handle joint stop constraints with PD + joint-stop-barrier controller.

In the tracking control case, the modified Lyapunov function leads to a new class of exponentially stable control laws. This class of control laws offers an alternative to the conventional computed torque method and provides trade-offs between on-line computation (which directly relates to performance through maximum sampling rate) and condition for stability. In one new design, (4.8), the nonlinear structure is decoupled from the real-time measurements which completely removes the requirement for on-line nonlinear computation. The chart below illustrates the trade-offs in the various tracking control laws.




The framework of Lyapunov stability analysis also allows robustness issues to be directly addressed. Specifically, insensitivity property (preservation of stability under small perturbation) and condition for robust design (preservation of stability under a specified amount of perturbation) for this new class of controllers have been derived with respect to viscous and Coulomb friction, modeling error and bounded instrument noise.

The new stability anslysis and controller design teqhniques presented in this paper open up many promising avenues for future research. In particular, our current research directions include: ways to incorporate time-varying artificial potential fields in the tracking problem and the generalization of the exponentially stabilizing joint-level control laws to the task space.

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| Equation Number | Control Laws | Condition for Exponential Stability | Comments |
| --- | --- | --- | --- |
| (2.31) | $u = -K_v q_2 - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + \frac{\partial U^o}{\partial \Delta q_1}(\Delta q_1)$ | $U^*(\Delta q_1)$ has global minimum at $\Delta q_1 = 0$ <br> $\nu > 0$ <br> $\Delta q_1^T \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) > \nu \lVert \Delta q_1 \rVert^2$ $K_v > 0$ | General Set Point Control |
| (2.28) | $u = -K_v q_2 - K_p \Delta q_1 + k(q_1)$ | $K_p > 0$ , $K_v > 0$ | $U^*(\Delta q_1) = \frac{1}{2} \Delta q_1^T K_p \Delta q_1$ |
| (3.2) | $u = -K_v q_2 - K_p \Delta q_1 + k(q_{1d})$ | $K_p > \sup_{q_1} \frac{\partial k(q_1)}{\partial q_1}$ , $K_v > 0$ | $U^*(\Delta q_1) = \frac{1}{2} \Delta q_1^T K_p \Delta q_1 + g(\Delta q_1 + q_{1d}) - g(q_{1d})$ <br> $- \Delta q_1^T k(q_{1d})$ |
| (3.6) | $u = -k_v q_2 - K_p \Delta q_1 - \dot{H}(\Delta q_1) - \dot{L}(\Delta q_1) + k(q_1)$ | $K_p > 0$ , $K_v > 0$ , $\underline{q}_{1i} \le q_{1id} \le \bar{q}_{1i}$ | $U^*(q_1) = \frac{1}{2} \Delta q_1^T K_p \Delta q_1 + \sum_{i=1}^{n} (H_i(\Delta q_{1i} + q_{1id}) + L_i(\Delta q_{1i} + q_{1id}))$ |
| (3.7) | $u = -K_v q_2 - K_p \Delta q_1 - \dot{H}(\Delta q_1) - \dot{L}(\Delta q_1) + k(q_{1d})$ | $K_p > \sup_{q_1} \frac{\partial k(q_1)}{\partial \Delta q_1}$ , $K_v > 0$, $\underline{q}_{1i} \le q_{1id} \le \bar{q}_{1i}$ | $U^*(\Delta q_1) = \frac{1}{2} \Delta q_1^T K_p \Delta q_1 + g(\Delta q_1 + q_{1d}) - g(q_{1d})$ <br> $- \Delta q_1^T k(q_{1d}) + \sum_{i=1}^{n} (H_i(\Delta q_{1i} + q_{1id}) + L_i(\Delta q_{1i} + q_{1i}))$ |
| (3.9) | $u = -K_v q_2 - K_p \Delta q_1$ | $\limsup_{t \to \infty} \lVert \Delta q_1(t) \rVert \le \sigma_{min}(K_p) \sup_{q_1} \lVert k(q_1) \rVert$ , $K_v > 0$ <br> If $K_p > \sup_{q_1} \frac{\partial k(q_1)}{\partial q_1}$ , $\lim_{t \to \infty} q_1(t) = q_1^*$ , $K_p(q_1^* - q_{1d}) + k(q_1^*) = 0$ | $U^*(\Delta q_1) = \frac{1}{2} \Delta q_1^T K_p \Delta q_1 + g(\Delta q_1 + q_{1d})$ <br> Global Lagrange Stability |
| (2.34) | $u = M(q_1)(-K_p \Delta q_1 - K_v \Delta q_2 - \dot{q}_{2d}) + k(q_1) + C(q_1, q_2) q_2$ | $K_p > 0$ , $K_v > 0$ | Computed Torque. Newton-Euler Algorithm can be used to update control law. |
| (4.2a) | $u = -K_v \Delta q_2 - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + M(q_1)\dot{q}_{2d} - \frac{1}{2}(J(q_1, q_2) q_{2d} - M_D(q_1, q_{2d}) q_2)$ | $K_v > 0$ $U^*$ has global minimum at $\Delta q_1 = 0$ <br> $U^*$ time - invariant <br> $\nu > 0$ $\Delta q_1^T \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) > \nu \lVert \Delta q_1 \rVert^2$ | Convergence rate depends on initial condition. Newton-Euler algorithm not applicable |
| (4.2b) | $u = -K_v \Delta q_2 - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + M(q_1)\dot{q}_{2d} - \frac{1}{2}(J(q_1, q_{2d}) q_2 - M_D(q_1, q_2) q_{2d})$ | Same as (4.2a) | Same as (4.2a) |
| (4.2c) | $u = -K_v \Delta q_2 - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + M(q_1)\dot{q}_{2d} - \frac{1}{2}(J(q_1, q_{2d}) q_{2d} - M_D(q_1, q_2) q_{2d})$ | Same as (4.2a) | Same as (4.2a) |
| (4.2d) | $u = -K_v q_2 - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + M(q_1)\dot{q}_{2d} - \frac{1}{2}(J(q_1, q_2) q_2 - M_D(q_1, q_{2d}) q_2)$ | Same as (4.2a) | Same as (4.2a) |
| (4.6) | $u = -K_v \Delta q_2 - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + k(q_1) + M(q_1) q_{2d} + C(q_1, q_{2d}) q_{2d}$ | Same condition on $U^*$ as in (4.2a) ; $K_v > \frac{n_2}{2}$ | Newton-Euler algorithm can be applied to $(q_1, q_{2d}, \dot{q}_{2d})$. Convergence rate depends on initial condition. |
| (4.7) | $u = -K_v \Delta q_2 - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + k(q_1) + M(q_1) q_{2d} + C(q_1, q_2) q_2$ | $\nu > 0$ , $K_v$ sufficiently large with respect to a given set of initial conditions | Modified Computed Torque. Newton-Euler algorithm can be applied to $(q_1, q_2, \dot{q}_{2d})$. Convergence rate depends on initial condition. |
| (4.8) | $u = -K_v \Delta q_2 - \frac{\partial U^*}{\partial \Delta q_1}(\Delta q_1) + k(q_{1d}) + M(q_{1d}) q_{2d} + C(q_{1d}, q_{2d}) q_{2d}$ | $\nu$ , $K_v$ sufficiently large with respect to a given set of initial conditions | Nonlinear compensation can be computed off-line by applying Newton-Euler algorithm to $(q_{1d}, q_{2d}, \dot{q}_{2d})$. Convergence rate depends on initial condition. |

TABLE I - SUMMARY OF CONTROL LAWS -