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FINAL REPORT

ON

NONLINEAR AND ADAPTIVE CONTROL

NASA GRANT NAG 2-297
MIT OSP NO. 95178

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JULY 1989

LIDS-FR-1891

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NASA HEADQUARTERS (1 copy)

SUMMARY

This final report overviews the research on Nonlinear and Adaptive Control carried out at the MIT Laboratory for Information and Decision Systems under NASA grant NAG 2-297 for the time period 1 June 1984 to 31 January 1989. Participating faculty were Professors Gunter Stein, Lena Valavani, and Michael Athans (principal investigator). The grant monitors are Dr. George Meyer (NASA Ames Research Center) and Mr. Jarrell R. Elliott (NASA Langley Research Center).

The primary thrust of the research was to conduct fundamental research in the theories and methodologies for designing complex high-performance multivariable feedback control systems; and to conduct feasibility studies in application areas of interest to our NASA sponsors that point out advantages and shortcomings of available control system design methodologies.

1. ROBUST ADAPTIVE CONTROL

Our research support under this NASA grant started shortly following the completion of the Ph.D. thesis of C.E. Rohrs under the supervision of Professors Valavani, Athans, and Stein. This research had uncovered major shortcomings with available adaptive control algorithms, which were proven to be globally stable under certain mathematical assumptions. We showed, by a combination of analysis and simulations, that existing adaptive control algorithms could become unstable in the presence of unmodeled dynamics and unmeasurable disturbances. Our research was originally received with some hostility in the Decision and Control Conferences and the American Control Conferences and resulted in many heated discussions.

Eventually, by 1985, the adaptive control community became convinced that existing adaptive control algorithms could break into instability. The so-called *Rohrs et al counterexample* became the test benchmark by which modifications of adaptive algorithms were tested on. Soon a new field of international research on the *Robust Adaptive Control Problem* was born. Research on this topic has and still is vigorously pursued by many distinguished researchers at present; nobody as yet has arrived on a simple modification to the original adaptive algorithms that preserves global stability and robustness to unmodeled dynamics.

Intermittent Adaptation and Variable Dead-Zones.

The results of Rohrs *et al* pointed out that a potential villain in the destruction of adaptive control stability was that the combination of certain types of reference inputs, disturbances, and unmodeled dynamics provided spurious, and unwanted, information to the (explicit or implicit) adaptive identification scheme. These errors, unless accounted for, could interact with the feedback mechanism and result in instability. Hence, we decided to initiate a research effort that would desensitize the adaptive system from such spurious information. Similar philosophy was followed by other researchers, e.g. Peterson and Narendra, by the used of a fixed dead-zone whose width was adjusted *a priori* based upon estimates of the size of the unknown disturbances. Only output error signals that exceeded the dead-zone were used to update the parameters of the adaptive compensators. The problem was that this dead-zone could be very conservative; also, previous researchers did not account for the impact of high-frequency modeling errors. These unmodeled dynamics could interact with both reference inputs and disturbances and introduce additional spurious signals that would confuse the identification algorithm.

The doctoral thesis of D. Orlicki and subsequent publications, see Refs. [1] and [3], addressed this class of problems. We focused upon the philosophy of *Intermittent Adaptation* realized by passing the output error through a variable dead-zone; the size of the dead-zone was varied in real time by carrying certain computations, over and above those necessary to implement the classical adaptive algorithms. In this research, we were able to develop new algorithms, of the MRAC type, which have guaranteed local stability properties in the presence of unmodeled dynamics and unmeasurable disturbances. The instability of the classical MRAC schemes was prevented by the intermittent adaptation; as discussed above, this technique prevents the updating of uncertain plant parameters whenever the identification information is of dubious quality due to the simultaneous presence of unmodeled dynamics and disturbances which cannot be measured. Thus, we only adapt whenever we are sure that the real- time signals contain relevant information.

It is a highly nontrivial manner to decide, in real-time, when to adapt and when to (temporarily) stop the adaptation. The new algorithms of Orlicki *et al* involve the real-time monitoring of easily measurable signals, and require the capability of computing discrete Fast Fourier transforms (DFFT's) for those signals. Intermittent adaptation is implemented by blending the real-time spectral information generated by the DFFT's with variants of the model reference algorithms. The algorithms can be implemented through the use of a dead-zone nonlinearity whose width changes in real time based upon the DFFT calculations. To the best of our knowledge, this was the first time that an adaptive control algorithm had been developed that requires extensive real-time spectral calculations so as to guarantee stability-robustness. Due to the very significant real-time computational requirements only limited simulation results were obtained; these results were encouraging but could not be used with confidence to pinpoint the advantages and shortcomings of this class of algorithms in a practical setting.

One can question the practical utility of adaptive algorithms that require so many spectral calculations to control a relatively simple process. Nonetheless, one should not lose sight of the experience of adaptive signal-processing in which spectral calculations to improve performance are used routinely. The adaptive control problem is much harder than the adaptive signal processing problem, because in addition to improved performance one has to worry also about the stability of the adaptive feedback control problem.

The most important by-product of that research was a detailed appreciation of the immense complexity of the adaptive control problem. In point of fact, we became convinced that new and different approaches to the robust adaptive control problem must be developed. There are simply too many hard questions, only tangentially related to adaptive control, that must be posed first, and of course answered, before we can proceed with confidence to using adaptive control to regulate physical systems, and especially multivariable ones. These questions motivated our subsequent research.

Robust Adaptive Identification in the Time and Frequency Domains.

Classical adaptive control algorithms use a postulated dynamic system order, i.e. a transfer function with fixed numbers of poles and zeros, and then use (explicit or implicit) identification to improve the prior estimate of the model uncertain parameters. In robust adaptive control this is necessary, but by no means sufficient. What is required is the development of a new class of adaptive identification algorithms which, with a finite amount of data, produce not only a better nominal model, but in addition generate a bound in the frequency domain that captures the presence of possible high-frequency model errors. Such bounding of model errors in the frequency domain is required by all nonadaptive design methods so as to ensure stability-robustness by limiting the bandwidth of the closed-loop system. Such identification algorithms do not exist in the classical identification literature; such questions were not even posed. Thus, we believe that is essential to develop such algorithms and then to incorporate them in the adaptive control problem.

A major milestone along these lines has been completed with the publication of Richard LaMaire's doctoral thesis and related articles, see Refs. [23], [24], and [40]. In this research, we viewed the robust adaptive control problem as a combination of a robust identifier (estimator) and a robust control-law redesign algorithm. Current robust control design methodologies, such as the LQG/LTR methodology,[7], [8], [10], require: 1) a nominal model, and 2) a frequency-domain bounding function on the modelling error associated with the nominal model. A new robust estimation technique, which we call a 'guaranteed' estimator, has been developed to provide these two pieces of information for a plant with unstructured uncertainty and an additive output disturbance. This guaranteed estimator uses parametric time-domain estimation techniques to identify a nominal model, and non-parametric frequency-domain estimation techniques to identify a frequency-domain bounding function on the modelling error. This bounding function is generated using discrete Fourier transforms (DFT's) of finite-length input/output data.

Several assumptions are required by the guaranteed estimator. In addition to a priori assumptions of the structure of the nominal model along with coarse, worst-case values of the parameters, we assume that the unmeasurable disturbance is bounded and that a magnitude bounding function on the Fourier transform of the disturbance is known. Further, we assume prior knowledge of a bounding function on the unstructured uncertainty of the plant relative to our choice of nominal model structure. These assumptions allow our time-domain estimator to be made robust to the effects of unstructured uncertainty and bounded disturbances. That is, our time-domain estimator updates the parameters of our nominal model only when there is good (uncorrupted) information. Similarly, the frequency-domain estimator, which has been developed, only updates the model and current bounding function on the modelling error when there is good information. In summary, the guaranteed estimator provides a nominal model plus a guaranteed bounding function, in the frequency-domain, as to how good the model is. Accuracy guarantees in the identifier part of the adaptive controller can be used by the control-law redesign part of the adaptive controller to ensure closed-loop stability, assuming the control-law is updated sufficiently slowly.

All equations for both the time-domain and the frequency-domain plant identifier have been developed. Also, in the frequency-domain, all equations that are used to compute the bound on the modeling error have been derived. The maximum possible effect, in the plant output, due to the unstructured uncertainty and the disturbance is computed using real-time DFTs of the input and the a priori assumed bound on the disturbance. The identification algorithm only updates the parameter estimates when the output error between the actual and predicted plant output is greater than the maximum possible error signal due to the unstructured uncertainty and the disturbance.

Additional issues concerning the guaranteed estimator relate to the fact that we are estimating a continuous-time plant with a discrete-time identifier. For example, the choice of sampling period for the estimator limits both the bandwidth of the adaptive control system as well as the accuracy of the estimator at high frequencies.

Because of the extensive real-time spectral calculations, we decided to use the CYBER supercomputer at Princeton which is available for use by the MIT community at no cost for CPU time. Numerical examples which are simple enough to demonstrate the ideas yet rich enough to capture the potential pitfalls have been designed and simulated. The simulation results indicate that for the systems tested the time-domain identification algorithm did not work very well. On the other hand, the frequency-domain algorithms worked much better.

In closed-loop identification simulations the richness of the command signal was often not sufficient to excite the plant dynamics so that the identification algorithms could work properly. For this reason, we developed an "intelligent" scheme which would monitor the progress of the identification algorithm and inject probing signals at the appropriate frequencies at the plant input so as to enhance identification. Of course, this would deteriorate (temporarily) performance since a disturbance was injected intentionally in the feedback loop. Better identification, accompanied by higher loop-gains and bandwidths, would improve overall command-following and disturbance-rejection performance after the probing signals were terminated.

The algorithms require extensive real time computations. For sluggish plants the computational requirements are not severe. However, in order to identify and control plants with very lightly damped dynamics truly extensive CPU requirements exist. For example, in our simulation studies involving a second order plant with lightly damped poles the Cyber 205 supercomputer was too slow, for real time control, by a factor of two so as to achieve a closed-loop bandwidth of 5 rad/sec.

These findings cast a tone of pessimism, with respect to CPU requirements, in using real-time identification and high-performance adaptive control for typical aerospace plants that are characterized by lightly damped dynamics and dominant high-frequency modeling errors. On the other hand, parallel computer architectures can be exploited in this class of algorithms. Thus, more research along these lines is required.

Robust Compensator Design

Our research pinpointed the need for a good initial guess for an adaptive compensator, whose parameters are then updated, in real-time, by the adaptive algorithm. We developed techniques that design the best (from the viewpoint of good command-following and disturbance-rejection) nonadaptive compensator for the given prior plant uncertainty information. It is yet unknown how to design such nonadaptive compensators that exhibit this property of "best" performance-robustness.

Such a robust design technique will prove useful in a number of ways. First, it will yield a systematic procedure for designing feedback systems for uncertain plants with performance guarantees. Thus, the feedback loop will be guaranteed to be stable and, in addition, will meet minimum performance specifications for all possible plant perturbations. Second, the solution of this robust design problem will also enable us to quantitatively address one of the most fundamental questions in adaptive control: *what are the performance benefits of adaptive control?* While much attention has been paid to the development of many specific adaptive algorithms, very little consideration has been given to this issue at the heart of the adaptive control problem. Practical adaptive systems rely upon external persistently exciting signals (to ensure good identification), slow sampling (which helps stability-robustness to unmodeled high frequency dynamics) in addition to extensive real-time computation (to provide safety nets and turn-off the adaptive algorithm when it exhibits instability). All these "gimmicks" degrade command-following and disturbance-rejection performance and tend to neutralize the hoped-for benefits of an adaptive compensator. In light of these circumstances it is imperative that the decision to use adaptive control, for a real engineering application, must be based upon a quantitative assessment of costs and benefits. One of the main goals of this research project is to quantitatively evaluate the performance benefits of an adaptive control system vis-a-vis the best fixed-parameter nonadaptive compensator for a linear plant. Note that for a nonlinear system the parameters of such compensators can be fine-tuned using gain scheduling.

In his doctoral thesis of Mr. David Milich and subsequent publications, see Refs. [21], [29], and[35], has examined design techniques which will yield the "best" fixed-parameter nonadaptive compensator for a plant characterized by significant structured, as well as unstructured, uncertainty. The "best" compensator is defined as the one that meets the posed performance (i.e. command-following, disturbance-rejection, insensitivity to sensor noise) specifications and stability-robustness over the entire range of possible plants.

Some of the key issues, and severe difficulties, in the design process have been identified. Conditions for stability-robustness and performance-robustness in the presence of significant structured and unstructured uncertainty have been developed. An a-priori magnitude bound, as a function of frequency, on the unstructured uncertainty is assumed known. In order to reduce the conservatism of the stability and performance conditions with respect to the structured uncertainty, directional information (in the complex plane) associated with the plant-parameter variations is exploited. Unfortunately, this directional information turns out to be closely associated with the so-called *Real-μ problem*, i.e. the problem of calculating structured singular values for real -- rather than complex-valued -- plant modeling errors; this problem has been studied by Doyle and is generically very difficult. Its solution appears to be beyond the state of the art, at least in the near future.

The only reasonable alternative appears to be to translate the prior knowledge of structured uncertainty into an equivalent unstructured uncertainty. It is still a very hard problem to design a compensator with guaranteed performance characteristics in the presence of these modeling errors. We have transformed the problem into what Doyle calls the *μ-synthesis problem,* which unfortunately is also very hard to solve. From a technical point of view, the *μ-synthesis problem* involves a blend a co-prime factorizations, structured singular value theory, and H^{∞}-optimization. Doyle has developed a method, called the D,K iteration, which converges to local minima. Milich's results provide an alternative to the D,K iteration method.

While the analysis aspect of LTI feedback design is well-established, the μ-synthesis problem remains open. The purpose of this research has been to develop a methodology (based on μ) for the synthesis of robust feedback systems. That is, the design process will ensure the resulting feedback system is stable and performs satisfactorily in the event the actual physical plant differs from the design model (as it surely will). The motivation for an alternative to D,K iteration is due to the nonconvex nature of the μ-synthesis problem. Nonconvexity may lead to local minima, therefore it is essential that several independent methods be available to examine the problem.

Our research has produced a new approach to the design of LTI feedback systems. We call it the "Causality Recovery Methodology (CRM)". For a given plant, the Youla parameterization describes all stabilizing compensators in terms of a stable, causal operator Q. LTI feedback design may be viewed as simply a procedure for choosing the appropriate Q to meet certain performance specifications. Thus, the design process imposes two constraints on the free parameter Q: (1) stability and causality (i.e. Q must be an H_∞ function); (2) Q must produce a closed-loop system that satisfies some performance specification. The design objective of interest here is performance robustness, which can be stated in terms of a frequency domain inequality using the structured singular value.

The CRM initially lifts the restriction of compensator causality and the synthesis problem with uncertainty is examined at each frequency. A feasible set of Q's in the space of complex matrices satisfying the performance specification is constructed. Causality is then recovered via an optimization problem which minimizes the Hankel norm (i.e. the measure of noncausality) of Q over the feasible set. If the problem is well posed (i.e. the performance specifications are not too stringent given the amount of modeling uncertainty), the resulting compensator nominally stabilizes the feedback system and guarantees robust stability and performance.

The theoretical foundation for the methodology have been established. Next, a research algorithm was written so that we can obtain numerical results. It was applied to two design examples to demonstrate its effectiveness. Excellent robust performance was obtained. However, the current generation of our CRM algorithms require very extensive *off-line* computational resources, because of the several optimization problems that must be solved to design the robust compensator.

2. NONLINEAR CONTROL SYSTEMS.

A significant portion of the grant resources was devoted to the development of methodologies, theories, and design techniques that will advance the state of the art in multivariable control system design. During this reporting period we have made some significant progress in this area.

Direct Nonlinear Control Synthesis Using the NMBC/NLOR Method.

Our goal in this project was to develop an integrated approach to nonlinear feedback control synthesis. The integration methodologies involve the blending of concepts and theories from (a) state-space representations, dynamic optimal control theory, and Lyapunov stability theory, and (b) from input-output operator-theoretic representations and conic-sector stability results.

The traditional method for designing a nonlinear feedback control system involves the linearization of the nonlinear dynamics at several operating conditions, the design of linear compensators at each operating condition, and finally the use of gain-scheduling to transform the family of linear compensators into a nonlinear one. What we are looking for are methods that bypass the linearization steps, and can yield directly a nonlinear dynamic compensator that meets the posed performance and stability-robustness specifications.

Our research philosophy in the area of nonlinear feedback control exploits the valuable lessons that we have learned during the past five years from the integration of time-domain and frequency-domain methods for linear feedback systems:

(a) Performance and stability-robustness specifications are most naturally expressed in an input-output context.

(b) The design of the dynamic compensator is most easily accomplished via a time-domain optimization-based algorithm, which should have guaranteed nominal-stability, and stability-robustness properties. However, the resultant control system need not be optimal in a well defined mathematical context.

(c) Any succesful design must lead to a compensator that creates an *approximate inverse* to the plant dynamics for the class of command-reference and disturbance inputs that dictate control system performance.

Mr. D. B. Grunberg, in his doctoral research and other publications, see Refs. [4], [15], [20], [27], has developed such a direct design methodology for nonlinear systems. The structure of the nonlinear compensator involves a nonlinear model of the plant, together with nonlinear feedback loops inside the compensator. Thus we deal with a Nonlinear Model Based Compensator (NMBC). We have exploited the structural and the mathematical properties of the NMBC and have shown that, under suitable mathematical assumptions, the NMBC dynamics can be modified using a nonlinear loop-operator recovery (NLOR) process. We refer to this methodology as NMBC/NLOR.

We have shown that under some, not very restrictive, assumptions the *Extended Kalman Filter* (EKF) is guaranteed to be a good estimator in the nonlinear control context, because - just as the linear Kalman Filter -- the EKF has certain guaranteed stability and, more important, robustness properties. Thus the EKF can be used to design a *Filter Operator Loop* (FOL) which can serve as the "target" designs in the NMBC/NLOR context.

We have also shown that if the nonlinear plant is in, or can be transformed to, the so-called *controller and observer form,* we can easily carry out the NLOR process in which asymptotically the loop operator of the nonlinear feedback control system approaches the FOL. In fact, the NMBC/NLOR process when applied at the plant input, where we are trying to recover the desirable characteristics of a full-state feedback design based only on limited output measurements, works even if the nonlinear plant is *only* in the so-called controller form .

The theoretical results have been illustrated using a simple nonlinear pendulum to carry out numerical simulations and evaluations.

Systems with Multiple Saturation Nonlinearities.

The goal of this project is to develop new theory and methodologies for the analysis and synthesis of linear multivariable control systems that contain several saturation nonlinearities. We seek to develop modifications to the purely linear design methodologies, such as LQR, LQG, LQG/LTR, and H-∞ optimization, to explicitly take into account the problems associated with multiple saturation (magnitude and/or rate) nonlinearities in the control actuation channels.

There are several problems that can arise when a control system that has many saturation nonlinearities is designed by purely linear means. The most serious problem is that of stability; it is possible for a control system, which is stable when the actuators are not saturated, to become unstable when one or more controls become saturated. Such instability can happen if large command signals are applied or disturbances of large magnitude are present. The second class of problems are associated with performance. If the saturation limits are ignored in the purely linear design phase, it may happen that large crossover frequencies are specified by the designer. The actuators may not be able to provide the gain necessary to attain the required bandwidths; also, rate-limiting may not allow the physical controls to change as rapidly as a purely linear design demands. Hence, redesign must take place. However, in multivariable designs, by far the most serious degradation to the control system performance occurs because the saturation nonlinearities distort the direction of the commanded control. Changes in the direction of the control vector cause oscillatory responses which may be unacceptable from a performance viewpoint. Also, transient performance suffers when saturation nonlinearities interact with integrators in the control loop; the so-called reset windup phenomenon. Reset windup keeps the nonlinearities saturated longer than necessary, and as a consequence transient responses are characterized by large overshoots.

Our research tried to examine these stability and performance problems associated with multiple saturations in a unified manner. Most of the existing theory is either too complex or incomplete. It is possible to deal with saturation nonlinearities using optimal control theory, and derive necessary conditions using Pontryagin's maximum principle; unfortunately, this only provides us with open-loop solutions through the solution of complex two point boundary value problems for high-order plants. Most other approaches are based upon Lyapunov theory, which does not capture in a straightforward way the input-output behavior necessary for design.

In our research, we have focused attention to the changes in the direction of the control signals that are induced by the saturating elements. The fact that we cannot deliver the "correct" magnitude should not produce any unpleasant effects except that the settling times should increase. What we want is to avoid is the highly oscillatory transients and unstable behavior. This appears to be more related to the changes in the directions of the control vectors.

The new results are described in the Ph.D. thesis of Petros Kapasouris and related publications; see Refs. [5], [28], [37], and [38]. We were able to come up with simple, yet elegant, ways of attacking the problem. The algorithms are different depending on whether or not the compensator is stable or unstable.

For closed-loop designs that use stable compensators to control stable plants, the concept is to have the command-following response of the MIMO system mimick, to the extent possible by the presence of the saturation nonlinearities, the transient response of the linear system. The idea is to monitor and adjust in real-time the tracking error vector, which acts as the input to the dynamic compensator so that the compensator never generates signals that will drive the system into saturation. In this manner, we are able to maintain the necessary "directional" properties of the design which are required to carry-out the *approximate plant inversion* and substitution of the "desired" dynamics in the forward loop associated with modern multivariable design methodologies. Note that if we allow arbitrary saturation of the nonlinearities, the directional properties of the linear design become distorted; as a consequence, we destroy the approximate plant inversion property of our compensator. The method under study controls the signal levels so that the system always works in the linear region. *This key idea appears to solve all at once the undesirable stability, performance, and reset-windup issues.* Of course, as to be expected, the speed of response (rise time, settling time etc) to commands of large magnitude is reduced compared to the design without saturation nonlinearities.

In order to implement this scheme one has to execute some off-line and some on-line computations. The off-line computations require the computation of the boundary of a convex compact set, with several nondifferentiable points. This set is defined over a Euclidean space whose dimension is that of the dynamic compensator. The on-line computations calculate a (pseudo)gradient vector to the boundary of the set, and adjust a scalar which reduces the instanteneous size of the tracking error vector. This causes the dynamic compensator to generate a control signal that never saturates.

We have used some linearized dynamics of the F-8 aircraft, to which we added a fictitious flaperon, to test these ideas. In this setting we command changes in both the flight path and pitch angles; these are to be controlled using the elevator and the flaperon. In this set of transient simulations the results show excellent nonlinear responses.

For feedback designs that contain open-loop unstable plants, or unstable compensators, it is important to limit the set of initial states, disturbances and commands so that the system can be stabilized. Assuming that the system is at rest and that the disturbance environment is such that the system can be stabilized, then the problem is to limit in an intelligent manner the size of the command (reference) vector. This is accomplished by a method that modulates the size of the command vector, and the rate at which it is applied, so that the controls do not saturate; eventually, the full command vector is applied. The nature of the computations is similar as in the open-loop stable case. However, the dimension of the underlying sets is now much larger.

We have used a model of the AFTI F-16 aircraft, which is open-loop unstable, to test the algorithm. As before, we are using the aircraft elevon and flaperon to control the pitch and flight path angles. Once more, the transient responses are excellent.

Similar ideas can be used to handle rate saturation, and simultaneous magnitude and rate saturation. Also, the same concepts can be used to ensure that certain state and/ or output variables do not exceed prespecified limits (often introduced on the basis of safety considerations).

Gain Scheduled Control Systems

Gain scheduling is a common engineering method used to design controllers for systems with nonlinear and/or parameter varying dynamics. In the nonlinear case, the dynamics are linearized at several operating points, and a linear compensator is designed for each linearized plant. The parameters of the compensator are then interpolated, or scheduled, in between operating points, thus resulting in a global compensator. The procedure for linear parameter varying dynamics is identical to that above, except that the linearization is omitted.

Despite the lack of a sound theoretical analysis, gain scheduling is a design methodology which is known to work in many engineering applications (e.g. jet engines, submarines, and aircraft). In the absence of such an analysis, a complete and systematic design methodology has yet to emerge. In its place, a collection of intuitive ideas has develop into heuristics for gain scheduled designs. Two common examples are: "the scheduling variable should vary slowly" and "the scheduling variable should capture the plant's nonlinearities." Thus, a sound analysis of various gain scheduling scenarios would prove very useful in better understanding these designs. Hopefully, this analysis would formalize the popular notions regarding the design of gain scheduled control systems. The analysis would then be used towards the ultimate goal to develop a complete and systematic gain scheduling design framework.

Research results have been obtained by Mr. Jeff Shamma in his Ph.D. thesis and subsequent publications; see Refs. [26], [34], [36], and [39]. We have identified and analyzed three different gain scheduling scenarios: 1) Linear plants scheduling on an exogenous parameter, 2) Nonlinear plants scheduling on a reference input trajectory, and 3) Nonlinear plants scheduling on the plant output.

The first case of linear parameter varying plants can be described as follows. Using the gain scheduling procedure outlined above, the resulting closed-loop global design can be modeled as a linear parameter varying system. This feedback system has the property that for each *frozen* value of the parameter, the closed-loop dynamics have excellent feedback properties (by design), such as robust stability, robust performance, disturbance rejection etc. However, these properties need not carry over to the time varying case. In fact, even nominal stability can be lost in the presence of parameter time variations. Thus, we have developed sufficient conditions for stability and stability-robustness for linear parameter varying systems. More precisely, we have shown that stability and stability-robustness is maintained for sufficiently slow time variations. This is not surprising since the original local designs were based on line *time-invariant* approximations to the time varying plant. Research is ongoing regarding the possible conservatism of these stability tests. However, these tests have been used to guarantee stability of a gain scheduled design for the F-8 aircraft reported in [Stein *et.al*, "Adaptive Control Laws for F-8 Flight Test", IEEE Trans. on Auto. Control, Vol. AC-22, No. 5, October 1977].

Various additional insights have been obtained regarding the design for such parameter varying systems. Recall that in the case of nominal stability, it was shown that stability is maintained for sufficiently slow parameter variations. However, a quantitative statement of this condition reveals that the restrictions on the parameter variations critically depend on an overshoot-like property of the closed loop design. This overshoot-like property is very sensitive to the scaling of the compensator state-variable. In fact, it is possible that a rescaling of compensator state-variables can significantly alter the stability properties of the resulting closed loop design. This distinction is important since only input/output aspects of the compensator (such as its frozen parameter frequency response) have been the focus of gain scheduling designs.

New insights have also been obtained in the analysis of the stability-robustness of the parameter varying system. The sufficient conditions for stability-robustness are very similar to their time-invariant counterparts in that they take the form of frequency-domain inequalities. However, these inequalities must be evaluated along a line parallel to the $j\omega$-axis in the left half s-plane. This implies that different information must be available regarding the nature of the unmodeled dynamics. In the absence of such information, it is shown that one can still use the time-invariant stability-robustness tests. However these tests must be satisfied with a greater degree of relative stability.

Guaranteed global properties for the cases of a nonlinear plant scheduling on either a reference trajectory or the plant output have also been analyzed. For such systems one has that, at each moment in time, the linearized closed loop system has excellent feedback properties. As in the parameter varying case, it is reasonable to ask under what conditions do these properties carry over to the global nonlinear case. In the case of scheduling on a reference trajectory, it was shown that these properties are maintained if 1) The reference command trajectory is sufficiently slow & 2) The reference command trajectory and corresponding reference control trajectory do not excite the unmodeled dynamics. In the case of scheduling on the plant output, it was shown that the various feedback properties are maintained if 1) The plant output is a naturally slow variable & 2) The plant output captures the bulk of plants nonlinearities.

The main idea behind all of these results may be summarized as follows.

Gain-scheduled designs are based on *linear time-invariant approximations* of the true plant. If one wishes the feedback properties of the local designs to carry over to the global design, the true plant should not differ greatly from the approximate design plants. It turns out that in the case of scheduling on an exogenous parameter, this amounts to requiring the parameter to vary sufficiently slowly. In the case of a nonlinear plant scheduling on a plant output, this amounts to requiring the plant output to vary sufficiently slowly and capture the plant nonlinearities. Note that these are precisely the intuitive ideas which have guided existing gain-scheduled designs. However, this analysis has formalized these notions and transformed them into quantitative statements.

Sliding Mode Controllers for Multivariable Systems.

Sliding mode control is a technique within the variable structure methodologies which has been used to design SISO nonlinear systems, in controllable canonical form, and for a limited class of multivariable systems. Mr. Benito Fernandez, in his Ph.D. thesis [31], has developed a new methodology for designing nonlinear multivariable controllers using the sliding mode concepts, including guarantees of closed loop nominal stability, stability-robustness to unmodeled dynamics, and performance.

A major feature of the methodology is the relationship between the input-output linearization of invertible nonlinear systems and the sliding mode approach, when the error dynamics on the sliding mode surfaces are chosen to be linear and time-invariant.

3. FEASIBILITY STUDIES

A small portion of the grant resources has been devoted to the study of specific design oriented studies, which are of interest to our NASA sponsors. These numerical feasibility studies help to point out the strengths and weaknesses of different design methodologies. References [6], [9], [11], [12], [13], [14], [30], [33] and [41] summarize our research on designing controls for different aerospace systems. We discuss a subset of these in more detail below.

Forward-Swept Wing Aircraft Studies.

A multivariable control synthesis feasibility study, which has been completed and documented in W. Quinn's SM thesis [12], relates to the multivariable control of forward-swept wing aircraft, similar to the X-29. There are certain generic problems associated with the control of the longitudinal dynamics of such aircraft which arise from the highly unstable open-loop aircraft characteristics and their dynamic interactions with the flexure and torsional wing bending modes. We wanted to understand the interplay between the multivariable flight control system that must stabilize the inherently unstable airframe and the degree of modeling necessary associated with the wing bending modes.

Although we had obtained the X-29 rigid dynamics from Dreyden Research Center, we did not have any information on the flexible dynamics. For this reason, we decided to use a model of a forward-swept wing aircraft developed at purdue University. The Perdue model is similar, but not identical, to the X-29 and it did include the first wing bending mode (at about 68 radians per second) and the first wing torsional mode (at about 270 radians per second). In the longitudinal axis one could control independently the canard and the flaperon control surfaces.

Studies by Honeywell Inc. on similar aircraft had posed the control of the longitudinal dynamics as a SISO problem, slaving the motions of the canard and the flaperon surfaces. We wanted to see what benefits, if any, could be obtained through independent dynamic coordinated control of the canard and flaperons. The physical flaperon characteristics are such that one cannot expect a large normal acceleration from their use, but it may be possible to use them in conjuction with the canard to independently control two longitudinal variables, the pitch attitude and the angle of attack, provided the corresponding commands were restricted to be small in magnitude.

We employed the LQG/LTR methodology [7], [10], throughout. We found that in order to have any reasonable performance for the flight condition examined the closed-loop bandwidth must be about 10 rad/sec. As a consequence we had to explicitly model the wing bending mode, but we could ignore the wing torsional mode without experiencing instability problems.

In order to assess the potential benefits of independent flaperon control we designed three different control systems, two SISO ones and a two-input two-output (TITO) one using the same performance and stability-robustness specifications. One SISO design used only the canard to follow pitch commands. In the second SISO design we again used only the canard to follow angle-of-attack commands. In the TITO design we used both the canard and the flaperons to follow independent commands (of small magnitude) in both pitch and angle-of-attack simultaneously.

We found that the flaperons can be quite effective in preventing the uncontrolled output in the SISO designs to drift off, while maintaining effectively the same performance for the control of the main variable. In the TITO design there was a very high degree of dynamic coordination between the canard and the flaperon surfaces. In other words, one does not lose anything by using the flaperons independently from the canard; there are benefits in a small signal environment from controlling both pitch and angle of attack independently.

Twin-Lift Helicopter Systems.

A multivariable control synthesis feasibility study, [13], [14], relates to to the development of an automatic flight control system (AFCS) for two helicopters jointly lifting a heavy payload. We became interested in studying the so-called Twin Lift Helicopter System (TLHS) because of its importance to NASA and industry, and because it represents an extraordinarily complex control problem.

For simplicity our study focussed only on the longitudinal rigid body dynamics of the TLHS near hover. A seven degree of freedom (three per helicopter; one for load) linear model was used throughout the research endeavour. The helicopters modeled were Sikorsky UH-60A Blackhawks.

Since our study focussed on the planar dynamics, only four controls (two per helicopter) were relevant. These controls, of course, were the helicopter cyclic pitch controls and collective pitch controls. Because we had four independent controls, we could independently control at most four outputs. Throughout the study, we focussed our attention upon following commands in horizontal and vertical velocity, while explicitly regulating the motion of the payload and the horizontal separation between the helicopters.

We examined two versions of the problem. The simplest case assumes that the tether lengths are equal. Under this assumption, it is possible to find a coordinate system in which the 4-input 4-output multivariable control design problem splits into three separate design problems: two SISO designs and one 2-input 2-output design. The more complex case assumes that the tether lengths are not equal, so that at equilibrium the helicopters are at different altitude. In this case, we have one SISO design problem and one 3-input 3-output design problem.

The results of the research indicate that the TLHS is very hard to control, even under full automatic control. The difficulty arises from several sources. One source of difficulty is attributable to three unstable poles associated with the longitudinal configuration. Because of these unstable poles any AFCS would require a minimum bandwidth just to stabilize the system. This minimum bandwidth would be accompanied by minimum pitch rates and control rates (for fixed reference commands). These rates, of course, depend on the amplitude and spectral content of the applied reference commands and of disturbances. Based on these facts alone one might conjecture that pre-filtering of references, so as to command smooth transitions, would be necessary in order to ensure "passenger friendly" pitch rates and realistic control rates. In order to keep these rates reasonable we kept the bandwidth of the closed loop system near the minimum.

Another source of difficulty arises from the presence of three lightly damped pole-pairs which lie within an octave of the minimum bandwidth required to stabilize the system. Because of the large phase lag which these poles contribute near crossover, it follows that to have nice robustness properties we require a great deal of lead. Although this lead helps with stability robustness, it requires quite large pitch and control rates, even when "reasonable" sudden step-commands are suddenly applied for all four outputs simultaneously. In fact, it was found that as our robustness properties improved, the control effort required increased rapidly. This implies that we must trade-off robustness for "reasonable" controls. It is this trade-off which makes the Twin Lift control problem so difficult. This was a trade-off that we were not willing to accept. To reduce the trade-offs involved we found that pre-filtering of references was imperative.

In summary, the research has confirmed our physical intuition about TLHS dynamics. In order to rapidly attenuate any load motions, the helicopters must undergo significant pitching motions and rapid changes in their vertical separations. Because of this we feel that for applications of the TLHS in which precise control of the load position is necessary in the presence of significant wind disturbances, we believe that additional research be carried out to more fully understand how to best control such TLHS along all axes, including studies that are directed toward the cost/benefit tradeoffs associated with active tether control to control "easier" the motion of the suspended mass without adverse motion of the helicopters as in the present design.

Adaptive Redesign Strategies Following Failures

It is important to develop both high level (symbolic) and low level(quantitative) strategies for coping with control surface failures in aircraft. To compensate for a control surface failure, sufficient redundancy in the control authority must be provided by other control surfaces, thrust and moment producing mechanisms. To understand these issues, presently configured aircraft provide an opportunity for the development of such strategies.

Control failures in aircraft are not uncommon. Military aircraft can expect frequent damage to their control surfaces from enemy fire. However, even civil aircraft undergo such failures. A brief survey yielded almost 30 cases in which there were failures of controls other than engines. In all but five of these incidents, such malfunctions resulted in crashes, and loss of life to passengers and crew. In about half of these cases, the flight could have ended safely if the pilot had acted in a correct and timely manner; unfortunately, present procedures and training are inadequate to prevent many such accidents because corrective action must be taken extremely fast. What is needed is an automated means of helping the pilot to utilize the implicit multivariable redundancy of his many surfaces and thrust producing mechanisms so as to recover positive control of the aircraft.

The Ph.D. thesis of E. Wagner [30], has made important strides toward the development of an on-board automated aid advisory for a C-130 aircraft. A rule-based expert system was developed to handle elevator-jam failures for the C-130 aircraft and its value illustrated using extensive simulations. This expert system produces an intelligent guide to pre-simulations of alternative controls (elevator tab, collective ailerons, symmetric flaps and engine thrust) using a high fidelity model of the aircraft. Pre-simulation of a recovery strategy was crucial because (a) often even a few degrees of available deflections could make all the difference, and (b) side-effects of doing the wrong thing could be devastating. The rule-based system was programmed using the OPS5 program.

Multivariable Designs for the F-18/HARV Aircraft.

Mr. Voulgaris in his SM thesis [33], [41], has used the H_2 and H_∞ control system design methodologies to design multivariable control systems using the dynamics of the F-18/HARV aircraft provided to us by the NASA Langley Research Center and to compare the similarities and differences. It was found that both design methodologies gave essentially the same performance, when the specifications were the same.

PUBLICATIONS

The following publications have been supported in full or in part by NASA grant NAG 2-297 since its inception 1 June 1984. Copies of these publications have been transmitted to the grant monitors, NASA headquarters, and publications office as required.

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