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## **NEUTRON STARS AND WHITE DWARFS IN GALACTIC HALOS?**

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# ABSTRACT

We consider the possibility that galactic halos are composed of stellar remnants such as neutron stars and white dwarfs. On the basis of a simple model for the evolution of galactic halos, we follow the history of halo matter, luminosity, and metal and helium abundances. We assume conventional yields for helium and the heavier elements. By comparing with the observational constraints, which may be considered as fairly conservative, we find that, for an exponentially decreasing star formation rate (SFR) with e-folding time  $\tau$ , only values between  $6 \times 10^8 \lesssim \tau \lesssim 2 \times 10^9$  years are allowed together with a very limited range of masses for the initial mass function (IMF). Star formation is allowed for  $2M_{\odot} \lesssim m \lesssim 8M_{\odot}$  if  $\tau = 2 \times 10^9$  years, and for  $4M_{\odot} \lesssim m \lesssim 6M_{\odot}$  if  $\tau = 10^9$  years. For  $\tau = 6 \times 10^8$  years, the lower and upper mass limits merge to  $\sim 5M_{\odot}$ . We conclude that, even though the possibility of neutron stars as halo matter may be ruled out, that of white dwarfs may still be a viable hypothesis, though with very stringent constraints on allowed parameters, that merits further consideration.

## I. INTRODUCTION

There is compelling theoretical and observational evidence for believing in the existence of baryonic dark matter, in addition to the usual non-baryonic dark matter, in the universe. According to the theory of big-bang nucleosynthesis (Yang et al. 1984; Kurki-Suonio et al. 1989), the baryon density of the universe in units of the critical density,  $\Omega_B$ , is limited to the range  $0.01 \lesssim \Omega_B \lesssim 0.1$ . On the other hand, luminous matter such as stars can only account for  $\Omega_L \lesssim 0.01$ . Possible sites for the baryonic dark matter include the intergalactic medium (IGM) between galaxies and galactic halos surrounding galaxies.

The most direct evidence for the existence of baryons in the IGM comes from the Lyman  $\alpha$  absorption lines in high red-shift quasar spectra. These systems are generally interpreted as being due to intervening gas clouds of almost pure H and He, distributed almost randomly in space and possibly confined by the pressure of a relatively tenuous surrounding gas (Sargent et al. 1980). The baryonic density contained in the Lyman  $\alpha$  clouds is model dependent and may be as much as  $\Omega_c h^2 \sim 0.02$ . Here  $h$  is the present value of the Hubble constant in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

It is known that there are massive dark halos around spiral galaxies, even though the nature of the halo mass is unknown. Strong evidence for the existence of dark, massive halos comes from the rotation curves of spiral galaxies which remain relatively flat out to 2-3 Holmberg radii (see, for example, Krumm and Salpeter 1979). The inferred halo mass is several times larger than either the luminous mass consistent with standard population synthesis models based on a solar neighborhood initial mass function or the mass required to account for the dynamical mass within a Holmberg radius.

The possibility that galactic halos are composed of some form of baryonic matter has been previously considered. For example, Hegyi and Olive (1983 and 1986) discussed the halos of spiral galaxies composed of snowball-like objects, gas, and low-mass stars or Jupiter-like objects. According to them, snowball-like objects should evaporate within the collapse time scale of the halos which is much shorter than the age of the galaxies. Halos composed of gas should heat up quickly, again, in a collapse time scale and emit an excessive x-ray background. If halos are made up of low-mass stars or Jupiter-like objects, they should produce too much infrared radiation to be compatible with observations, unless the slope of the IMF  $x \gtrsim 1.7$  in units where  $x = 1.35$  for the Salpeter mass function. Observations tend to indicate that the IMF turns over ( $x < 0$ ) for masses less than  $\sim 0.2 M_\odot$  (Scalo 1986). From the above arguments, they concluded that such forms of baryonic dark matter in the halo are not very plausible. Similar arguments were applied to the halo of elliptical galaxies by Hegyi and Olive (1989).

Here, we will consider the possibility that galactic halos are comprised of stellar remnants such as neutron stars and white dwarfs. Although Hegyi and Olive (1983, 1986, and 1989) also considered this possibility, they concluded that in the conversion of the halo mass into stellar remnants, heavy elements would be produced in sufficient abundances so as to over-enrich the disk. In reaching this conclusion, it was assumed that star formation proceeded with a constant SFR and the range of stellar masses was taken to be  $m > 2M_{\odot}$  (up to  $\sim 100M_{\odot}$ ). It was also assumed that for spiral galaxies, a fraction  $\epsilon$  of the halo material is mixed into the disk. It was concluded that to avoid over-contamination of the disk,  $\epsilon < 3 \times 10^{-4}$ . For elliptical galaxies,  $\epsilon$  was taken to be 1 and a limit was placed on the slope of the IMF. In this case, over-contamination occurred if  $\alpha < 10$ . On this basis it was concluded that stellar remnants are an unlikely candidate for the dark matter of galactic halos.

In this paper, we will make a more detailed analysis of the evolution of the halo. For a constant SFR and a standard IMF, we agree qualitatively with the conclusions of Hegyi and Olive (1983, 1986, and 1989). Instead of setting limits to  $\epsilon$ , we will vary the parameters in the SFR and IMF in an attempt to find a consistent set of parameters such that the disk abundances of metals and helium are below observational bounds. We also find that a major constraint comes from the lower limit on the mass-to light ratio, i.e., we cannot permit large numbers of main-sequence stars to be still burning in the halo. In §II, we establish the basic equations governing the history of halo matter, metal and helium abundances in halos and disks, and halo luminosity. In §III, we describe several observational constraints which determine the plausibility of halos made up of neutron stars and white dwarfs. In §IV, we present our results derived from the basic equations by imposing these observational constraints, and in §V, we discuss various implications.

## II. BASIC EQUATIONS

### a) *Evolution of the Galactic Halo*

We begin by defining our equations and notation which follow those of Tinsley (1980, 1981). The creation function, or the number density of stars born over the mass and time intervals  $(m, m + dm)$ ,  $(t, t + dt)$ , in the galactic halo is assumed to be separable, i.e.,

$$C(m, t) = \phi(m)\psi(t), \quad (2.1)$$

where  $\phi$  is the IMF and  $\psi$  is the SFR. The IMF is normalized so that

$$\int_{m_L}^{m_U} m\phi(m)dm = 1, \quad (2.2)$$

where  $m_L$  and  $m_U$  are the lower and upper mass limits for star formation. In terms of  $\phi$  and  $\psi$ , the total mass densities of stars, remnants, and gas as a function of time are expressed as

$$M_S(t) = \int_0^t [1 - \phi_u(t - t')]\psi(t')dt', \quad (2.3)$$

$$M_R(t) = \int_0^t [\phi_u(t - t') - R(t - t')]\psi(t')dt', \quad (2.4)$$

$$M_G(t) = M_T - \int_0^t [1 - R(t - t')]\psi(t')dt', \quad (2.5)$$

where  $M_T$  is the total mass density of the halo. In what follows, we take  $10^{12}M_\odot$  as the halo mass and  $R_H \sim 100kpc$  as the halo radius, yielding an average mass density  $M_T = \rho_H \sim 1.6 \times 10^{-26} gcm^{-3}$ . Here, the mass fraction of stars ever born which have already left the main-sequence is given by

$$\phi_u(t) = \int_{m_1(t)}^{m_U} m\phi(m)dm, \quad (2.6)$$

where  $m_1(t)$  is the main-sequence turnoff mass as a function of age, and the mass fraction of stars returned as gas is given by

$$R(t) = \int_{m_1(t)}^{m_{SN}} (m - m_R)\phi(m)dm, \quad (2.7)$$

where  $m_R$  is the mass of the remnants and  $m_{SN}$  is the upper mass limit for the supernova explosion which may differ from  $m_U$ . If  $m_{SN} < m < m_U$ , we assume that black holes are formed.

We assume that the IMF of the halo stars is not very different from that of the disk stars and have used the IMF of the disk field stars given in Scalo (1986). From

his Table 7, by assuming the age of the galaxy,  $T_0$ , to be 15 Gyr and a constant SFR in the disk, the IMF has been fitted to a power-law relation

$$\phi(m) \propto \begin{cases} 88m^{1.3}, & \text{for } 0 \leq m/M_\odot \leq 0.18, \\ 0.80m^{-1.4}, & \text{for } 0.18 \leq m/M_\odot \leq 1.2, \\ m^{-2.8}, & \text{for } 1.2 \leq m/M_\odot \leq 63. \end{cases} \quad (2.8)$$

The slope of the IMF  $x$  is defined by  $\phi \propto m^{-(1+x)}$ . Notice the turnover ( $x < 0$ ) for very low-mass stars. For the main-sequence turnoff mass,  $m_1(t)$ , a formula for the stellar lifetimes from Bahcall and Piran (1983) has been used

$$T(m) = \begin{cases} 10 - 3.6 \log m + 1.0(\log m)^2, & \text{for } 1 \lesssim m/M_\odot \lesssim 10^2, \\ 6.3, & \text{for } m/M_\odot \gtrsim 10^2, \end{cases} \quad (2.9)$$

which agrees very closely with the values given in Scalo (1986). The mass of remnants (especially neutron stars and white dwarfs),  $m_R$ , has been calculated with the formula

$$m_R(m) = \begin{cases} 0.15m + 0.38, & \text{for } 1.0 \leq m/M_\odot \leq 6.8, \\ 1.4, & \text{for } m/M_\odot \geq 6.8, \end{cases} \quad (2.10)$$

taken from Iben and Renzini (1983) (for their parameter  $\eta = 1$ ). We consider  $m_L$ ,  $m_U$ ,  $m_{SN}$ , and the SFR as free parameters. In particular, we use an exponentially decreasing SFR

$$\psi(t) = \psi(0) \exp(-t/\tau), \quad (2.11)$$

and allow  $\tau$  and  $\psi(0)$  to vary.

### b) Metal and Helium Production

If the mass fractions of metals and helium of a dying star with mass  $m$ , which were once processed through stellar nucleosynthesis and are returned back to the halo, are  $m_z/m$  and  $m_y/m$ , the total mass fraction returned to the halo in metals is

$$P_z(t) = \int_{m_1(t)}^{m_{SN}} m_z \phi(m) dm, \quad (2.12)$$

and that in helium is

$$P_y(t) = \int_{m_1(t)}^{m_{SN}} m_y \phi(m) dm. \quad (2.13)$$

Then, one can show that the metallicity in the halo, which is defined by

$$z_H(t) \equiv \frac{M_z(t)}{M_G(t)}, \quad (2.14)$$

where  $M_z(t)$  is the mass density of metals, is expressed approximately by

$$z_H(t) = \int_0^t \frac{1}{M_G(t')} \frac{dM_{z,eject}(t')}{dt'} dt', \quad (2.15)$$

where

$$M_{z,eject}(t) = \int_0^t P_z(t-t')\psi(t')dt' \quad (2.16)$$

is the total mass density of metals ever ejected from dying stars. In deriving the above relation, we have assumed that  $z$  changes little between birth and death of the stars and that  $Z \ll y$ . We use the approximation that the metallicity of the star at the time of its birth is equal to the metallicity of the halo gas at that time and that these elements are returned to the halo at the death of the star. We have also made the assumption that  $R(t) \gg P_z(t)$ , which is generally true. The above approximation is a weaker one than the commonly-used instantaneous recycling approximation, and we do not expect it to produce any serious errors for most of the cases of interest. In the same way, one can show that the helium abundance in the halo, defined by

$$y_H(t) \equiv \frac{M_y(t)}{M_G(t)}, \quad (2.17)$$

where  $M_y(t)$  is the mass density of helium, is expressed approximately by

$$y_H(t) = y_{H,i} + (1 - y_{H,i}) \int_0^t \frac{1}{M_G(t')} \frac{dM_{y,eject}(t')}{dt'} dt', \quad (2.18)$$

where  $y_{H,i}$  is the initial, primordial helium abundance in the halo and

$$M_{y,eject}(t) = \int_0^t P_y(t-t')\psi(t')dt' \quad (2.19)$$

is the total mass density of helium ever ejected from dying stars. We have assumed that the increase in  $y_H$  is small compared with  $y_{H,i}$ . Again, we have used similar approximations and assumptions as in the case of the metals.

The yield of metals,  $m_z(m)$ , has been calculated from the estimated yields of Arnett (1978). Although  $\sim 1.7$  times smaller at  $m = 12M_\odot$  and  $\sim 1.5$  times larger at  $m = 16M_\odot$ , the yields of Arnett agree reasonably well with those of Woosley and Weaver (1986) for massive stars.  $m_z$  has been interpolated from Table 4 of Arnett (1978) for  $m \gtrsim 8M_\odot$ , and assumed to be negligible for  $m \lesssim 8M_\odot$ . Similarly, the yield of helium,  $m_y(m)$ , has been interpolated from Table 4 of Arnett (1978) for  $m \gtrsim 10M_\odot$  and from Figure 2 of Audouze and Tinsley (1976) for  $4M_\odot \lesssim m \lesssim 10M_\odot$ , and assumed to be negligible for  $m \lesssim 4M_\odot$ . The initial primordial helium abundance  $y_{H,i}$ , has been set to be 0.23.

In order to calculate the contamination of the disk by metals and helium from halo stars, we assume the following: i) there is no star formation in the disk; ii) the halo gas moves in orbits perpendicular to the disk with typical velocity  $v_H$ ; iii) a small fraction  $\epsilon$  of metals and helium is mixed into the disk when ejecta pass through the disk. Then the metallicity in the disk is calculated approximately by

$$\begin{aligned} z_D(t) &= \int_0^t \frac{\zeta \epsilon \rho_{z,H}(t') v_H S_D}{(\Omega_D / \Omega_H) M_T} dt' \\ &= \frac{3 \Omega_H}{4 \Omega_D} \frac{\zeta \epsilon v_H}{R_H} \left( \frac{R_D}{R_H} \right)^2 \int_0^t z_H(t') dt', \end{aligned} \quad (2.20)$$

where  $\rho_{z,H}$  is the mass density of metals in the halo,  $S_D$  is the area of the disk,  $\Omega_D/\Omega_H$  is the ratio of mass in the disk to that of the halo,  $R_H$  is the radius of the halo, and  $R_D$  is the radius of the disk. Here,  $\zeta$  is a factor which accounts for the effect of radial orbits, in addition to tangential orbits, equal to one for tangential orbits and much greater than one for radial orbits. If  $R_D \sim R_H$ , the above approximation with only tangential orbits is expected to be good and  $\zeta \sim 1$ . However, if  $R_D \ll R_H$ , mixing through radial orbits becomes dominant and  $(R_D/R_H)^2 \zeta \lesssim 1$ . In the same way, the helium abundance in the disk is calculated approximately by

$$y_D(t) = y_{D,i} + \frac{3}{4} \frac{\Omega_H}{\Omega_D} \frac{\zeta \epsilon v_H}{R_H} \left( \frac{R_D}{R_H} \right)^2 \int_0^t (y_H(t') - y_{H,i}) dt', \quad (2.21)$$

where  $y_{D,i}$  is the initial, primordial helium abundance in the disk. In the calculations, we use  $\Omega_D/\Omega_H \sim 0.1$  and  $v_H \sim 200 \text{ km/s}$  and set  $y_{D,i}$  equal to  $y_{H,i}$ . On the other hand, the disk mass  $\sim 10^{11} M_\odot$  yields  $R_D \sim 20 \text{ kpc}$ , if the column density of the disk is  $\sigma_D \sim 65 M_\odot \text{ pc}^{-2}$ , corresponding to the value near the Sun (Bahcall 1984). Hence, we assume  $0.1 \lesssim \zeta (R_D/R_H)^2 \lesssim 1$ .

An independent effect is due to the mixing efficiency of halo gas with disk gas. The value of the mixing efficiency  $\epsilon$  is estimated as follows: Assume that the halo gas is mixed into the disk by turbulent motion. Then, the mixing length is approximately

$$l \sim \sqrt{D \tau_d}, \quad (2.22)$$

where  $D$  is the diffusion coefficient given by

$$D \sim \frac{1}{3} v_t \lambda_t, \quad (2.23)$$

$\tau_d$  is the diffusion time scale,  $v_t$  is the rms turbulent velocity, and  $\lambda_t$  is the characteristic scale of turbulence. Using  $\tau_d \sim 3 \times 10^9 \text{ years}$ ,  $v_t \sim 100 \text{ km/s}$ , and  $\lambda_t \sim 1 \text{ kpc}$ , the mixing length becomes  $l \sim 10 \text{ kpc}$  and the mixing efficiency becomes  $\epsilon \sim 0.1$ . However, the above estimate for  $\epsilon$  should be considered as a minimum value since there may be other mixing mechanisms such as two body interaction of particles. In what follows, we shall combine the preceding arguments with an empirical parameters,  $\zeta \epsilon (R_D/R_H)^2 \sim 0.03$ , but values between  $0.003 \lesssim \zeta \epsilon (R_D/R_H)^2 \lesssim 0.1$  will also be considered. Values of  $\zeta \epsilon (R_D/R_H)^2$  larger than 0.1 will be so restrictive that we do not consider them here.

### c) Luminosity

The evolution of the luminosity density of the halo can be expressed as

$$L_H(t) = \int_0^t Q(t-t') \psi(t') dt', \quad (2.24)$$

where

$$Q(t) = \int_{m_L}^{m_1(t)} L(m) \phi(m) dm, \quad (2.25)$$



and  $L(m)$  is the luminosity of a star with mass  $m$ . Using  $L(m)$  in the B-spectral bands interpolated from the tabulated mass-luminosity relation (Allen 1976),  $L_{H,B}/M_T$  is calculated and compared with the observed mass-to-light ratio of the halo in the B-spectral band. Red giants will make an additional contribution to the halo light which we have ignored. In our model calculations, all the above equations have been integrated numerically using Simpson's rule.

### III. CONSTRAINTS

We now describe several observational constraints which limit the possible values of free parameters,  $m_L$ ,  $m_U$ ,  $m_{SN}$ ,  $\tau$ , and  $\psi(0)$ . The first constraint comes from the metallicity and helium abundances in the disk. The Sun with an age of  $4.5 \times 10^9$  years has a metallicity  $z \sim 0.02$  and a helium abundance  $y \sim 0.265$ . According to the empirical age-metallicity relation of the disk obtained using F dwarf stars by Twarog (1980), the mean metallicity of the disk does not vary much between the present ( $1.5 \times 10^{10}$  years) and  $10^{10}$  years, but decreases by about a factor of 5 between  $10^{10}$  years and  $3 \times 10^9$  years. On the other hand, the extragalactic observations of metal and helium abundances indicate that the helium abundance could increase almost linearly with metallicity (Maeder 1983). For our purpose, we assume that the upper limits on the present metallicity and helium abundance of the disk, when only the contributions from the halo stars are accounted for, are

$$z_{D,p} \lesssim 0.02, \quad (3.1)$$

and

$$y_{D,p} \lesssim y_{D,i} + 0.05. \quad (3.2)$$

Since there is also some self-enrichment of metals and helium in the disk by the disk stars, which is expected to be substantial, the above upper limits on  $z_{D,p}$  and  $y_{D,p}$  should be considered as fairly conservative. Also, when appropriate, we will take  $z_D(3 \times 10^9 \text{ years}) \lesssim 0.004$ .

The next constraint comes from the gas mass density in the halo. If the halo predominantly consists of gas, it should be hot gas with a temperature  $T_H \sim 10^6 K$  (Spitzer 1956) and will emit x-rays. The observed, diffuse, soft x-ray background will therefore limit the allowed amount of halo gas density. Let  $C$  be the clumpiness parameter defined by

$$C \equiv \frac{\langle \rho_B^2 \rangle}{\langle \rho_B \rangle^2}, \quad (3.3)$$

where  $\rho_B$  is the baryonic mass density. From Silk (1973), the constraints based on the observed x-ray background in the energy range  $\lesssim 1 \text{ keV}$  limit

$$\delta C \Omega_B^2 \lesssim 3, \quad (3.4)$$

where  $\delta \sim h^3$  and  $T_H = 10^6 K$  can be used to set a limit on the net contribution of unresolved gaseous galactic halos. For  $\Omega_B \sim 0.1$  and  $C \Omega_B \sim 8\pi G \rho_G / 3H_0^2 \sim 900 h^{-2} M_G / M_T$ , we get an upper limit at present,

$$\frac{M_G}{M_T} \lesssim \frac{1}{30h}. \quad (3.5)$$

If the halo gas were to consist of smaller clouds, instead of a single massive cloud, we would get a more stringent limit on  $M_G$  (Tarter and Silk 1974; Hegyi and Olive

1986). Hence, again a fairly conservative limit would be that the present  $M_G$  should be less than 10% of the total mass density of the halo, or

$$\frac{M_G}{M_T} \lesssim 0.1. \quad (3.6)$$

The final constraint comes from the observed mass-to-light ratio of the halo. In our case, the most stringent constraint is at the shortest observable wavelengths, since we consider stars with  $m \gtrsim 1M_\odot$  which evolve into neutron stars and white dwarfs over a time scale less than the age of the universe. In particular, we have used the mass-to-light ratio in the B spectral band. From Faber and Gallagher (1979), on the scale of binary galaxies, the mass-to-light ratio of a spiral galaxy in the B spectral band is

$$\left(\frac{M}{L}\right)_B \sim (70 - 100)h \left(\frac{M_\odot}{L_\odot}\right)_B. \quad (3.7)$$

Since most of the light comes from the disk rather than the halo, a fairly conservative upper limit on the present mass-to-light ratio of the halo in the B spectral band would be  $\sim 100(M_\odot/L_\odot)_B$ , or

$$\left(\frac{M}{L}\right)_{H,B} \gtrsim 100 \left(\frac{M_\odot}{L_\odot}\right)_B. \quad (3.8)$$

In Hegyi and Olive (1986), the mass-to-light ratio of NGC4565 was computed in the I-band. To calculate  $M/L$ , the projected mass and luminosity density were determined and their ratio  $\sigma_M/\sigma_L = M/L$ . The projected halo mass density was evaluated in terms of the 21cm rotational velocity  $v = 253 \text{ km s}^{-1}$  (Krumm and Salpeter 1979). It was found that

$$\sigma_M \sim \frac{v^2}{4G} \frac{1}{r} = \frac{3.62 \times 10^9}{r} \frac{M_\odot}{\text{kpc}^2}. \quad (3.9)$$

Using the surface brightness data (Hegyi and Gerber 1977), the projected luminosity density was fit to  $\sigma_L = (a/r) + b$ . A  $2\sigma$  upper limit on  $a$  corresponds to a  $2\sigma$  lower limit on  $M/L$ . In the I-band, it was found that  $(M/L)_I > 76(M_\odot/L_\odot)$  (Hegyi and Olive 1986). For comparison the  $2\sigma$  lower bound on  $M/L$  in the K-band is  $(M/L)_K > 38(M_\odot/L_\odot)$  (Boughn, Saulson, and Seldner 1981). We have now also used data on NGC4565 in the R-band (Hegyi and Gerber 1977) and performed a similar fit to  $\sigma_L$ . We find a  $2\sigma$  lower limit

$$\left(\frac{M}{L}\right)_R > 113 \frac{M_\odot}{L_\odot}. \quad (3.10)$$

Because we expect the limit on  $M/L$  in the B-band to be larger still, we feel that the case of (3.8) is indeed conservative. (In the case of the elliptical galaxy M87, a similar investigation was made in the V-band (Hegyi and Olive 1989) where it was found that  $(M/L)_V > 565(M_\odot/L_\odot)$ .)

## IV. RESULTS

Among the free parameters,  $\psi(0)$  is set by fixing the present value of  $M_G/M_T$  for given  $m_L$ ,  $m_U$ ,  $m_{SN}$ , and  $\tau$ . However, as expected from the equations in §IIa,  $\psi(0)$  is determined mainly by  $\tau$ , and, for most of the cases that we have considered, is approximately given by

$$\frac{\psi(0)\tau}{M_T} \sim 3 - 5. \quad (4.1)$$

In what follows, we fix the value of  $\psi(0)$  by assuming  $M_G/M_T = 0.01$  at present, except when we limit the free parameters from the constraint  $M_G/M_T \leq 0.1$ . Also we assume that  $m_{SN} = m_U$ . (But we will return to the possibility that  $m_{SN} < m_U$  in §V.) Then, we are left only with three parameters,  $m_L$ ,  $m_U$ , and  $\tau$ , whose values will be limited by the three constraints described in §III, using the equations and relations in §II.

From the constraint  $M_G/M_T \leq 0.1$  at present and the condition that at all times  $M_G/M_T > 0$ , the values of  $m_L$  for given  $m_U$  and  $\tau$  are limited. Generally,  $\psi(0)$  can not be too large since  $M_G/M_T$  becomes less than 0 at an intermediate time (see equation (2.5)). Also,  $\psi(0)$  can not be too small since  $M_G/M_T$  becomes larger than 0.1 at present even though it is always larger than 0. However, if  $m_L$  is sufficiently large,  $\psi(0)$  can have a large enough value to suppress  $M_G/M_T$  below 0.1 at present, while allowing  $M_G/M_T$  to always be larger than 0, because the stars formed in this case die quickly due to their short lifetimes (see equation (2.9)). On the other hand, if  $m_L$  is small, a large fraction of the halo mass is trapped in low-mass stars and  $\psi(0)$  can not have a sufficiently large value without having  $M_G/M_T$  be smaller than 0. Therefore, the lower bound on  $m_L$  depends sensitively on  $\psi(0)$ , and hence on  $\tau$ . In Figure 1, the line which divides the  $\tau - m_L$  plane into the regions with  $M_G/M_T \leq 0.1$  and  $M_G/M_T > 0.1$  is plotted for  $m_U = 8M_\odot$ . For the larger values of  $m_U$ , the line shifts downwards since the halo mass locked into massive stars is recycled quickly. For the smaller  $m_U$ , it shifts upwards. However, the dependence on the value of  $m_U$  is weak, since the IMF,  $\phi(m)$ , falls off faster than  $m^{-1}$  for  $m \geq 1.17M_\odot$  (see equation (2.8)) and the bulk of the stellar mass is stored into low-mass stars rather than massive stars. In the figure, the smaller the value of  $\tau$ , the larger the lower bound on  $m_L$ .

The constraints that  $z_{D,p} \leq 0.02$  and  $y_{D,p} \leq y_{D,i} + 0.05$  apply mainly to the values of  $m_L$  and  $m_U$ . In our model, the metal and helium abundances increase because of ejection from dying massive stars ( $m \gtrsim 8M_\odot$  for metals and  $m \gtrsim 4M_\odot$  for helium). Since the stellar lifetimes of such massive stars are short compared to the age of the galaxy (see equation (2.9)), they can be approximated to die as soon as they are born. In that case, only the number of such massive stars is important in order to determine  $z_{D,p}$  and  $y_{D,p}$  and not the formation epoch. Hence,  $z_{D,p}$  and  $y_{D,p}$  depend only weakly on the value of  $\tau$  which determines the epoch of star formation. Figure 2 shows the contours which correspond to  $z_{D,p} = 0.02$  (long-dashed line),

$y_{D,p} = 0.28$  (long-dashed line), and  $m_L = m_U$  (solid line) on the  $m_L - m_U$  plane for  $\zeta\epsilon(R_D/R_H)^2 = 0.1$  (Figure 2a); 0.03 (Figure 2b); 0.01 (Figure 2c); and 0.003 (Figure 2d). It also shows the contours of  $z_{D,p} = 0.01$  (short-dashed line) and  $y_{D,p} = 0.255$  (short-dashed line) for comparison. Here, we have used  $\tau = 10^{10}$  years and  $y_{D,i} = 0.23$ . Even though their dependence is weak, the lines corresponding to  $z_{D,p} = 0.02$  and  $y_{D,p} = 0.28$  move upwards for smaller  $\tau$ , since the halo gas is consumed more rapidly in this case and has less time to be mixed into the disk. From these plots, we can exclude  $m_L \gtrsim 4.5M_\odot$  for  $\zeta\epsilon(R_D/R_H)^2 = 0.1$ ,  $m_L \gtrsim 5M_\odot$  for  $\zeta\epsilon(R_D/R_H)^2 = 0.03$ ,  $m_L \gtrsim 5.5M_\odot$  for  $\zeta\epsilon(R_D/R_H)^2 = 0.01$ , and  $m_L \gtrsim 6.5M_\odot$  for  $\zeta\epsilon(R_D/R_H)^2 = 0.003$ .

From Figures 1 and 2, we can see that, for instance,  $\tau$  should be larger than  $6 \times 10^8$  years if  $\zeta\epsilon(R_D/R_H)^2 = 0.03$ . However, if  $\tau$  is too large, the present value of  $L_{H,B}/M_T$  becomes too large to be compatible with the observed mass-to-light ratio. The present value of  $L_{H,B}/M_T$  depends on the number of stars that survived up to the present, which, on the other hand, depends on the epoch of their formation, their lifetimes, and their masses, since more massive stars are more luminous. Because the epoch of star formation is determined by  $\tau$  and only low-mass stars have lifetimes long enough to survive up to the present, the present value of  $L_{H,B}/M_T$  is basically decided by the values of  $\tau$  and  $m_L$  and depends weakly on the value of  $m_U$ . Figure 3 shows the present  $L_{H,B}/M_T$  as a function of  $\tau$  for  $m_L = 2M_\odot$  and  $m_U = 8M_\odot$  (solid line) and for  $m_L = 4M_\odot$  and  $m_U = 6M_\odot$  (dashed line). The curve moves downwards for larger  $m_L$ , since the number of the surviving stars is smaller. It moves upwards for smaller  $m_L$ . If  $\tau \gtrsim 2 \times 10^9$  years, the present value of  $L_{H,B}/M_T$  is always larger than  $1/100(L_\odot/M_\odot)_B$  for the allowed values for  $m_L$  and  $m_U$  and we can exclude those values of  $\tau$ . From Figure 3, it is also clear that the limit on  $\tau$  is not very sensitive to the constraint imposed on  $M/L$  when  $(M/L)_B \sim 100(M_\odot/L_\odot)_B$ . For example, there would be little difference had we chosen  $(M/L)_B > 565(M_\odot/L_\odot)_B$  based on the elliptical galaxy M87.

With the constraints in §III, which we consider to be fairly conservative, only those values between  $6 \times 10^8 \lesssim \tau \lesssim 2 \times 10^9$  years are allowed, if  $\zeta\epsilon(R_D/R_H)^2 = 0.03$ , with a very limited mass range for star formation. For instance, star formation is allowed only for  $2M_\odot \lesssim m \lesssim 8M_\odot$  if  $\tau = 2 \times 10^9$  years, and for  $4M_\odot \lesssim m \lesssim 6M_\odot$  if  $\tau = 10^9$  years. For  $\tau = 6 \times 10^8$  years, the lower and upper mass limits merge to  $\sim 5M_\odot$ . It is important to note that these requirements imply that only a very small portion of the IMF mass range is allowed. Though one may argue that this fine tuning of the IMF could conceivably arise because of the pristine initial conditions of the halo, after a short time the metallicity of the halo grows and star formation should not be all that different from that in the disk. In this case, it seems quite unnatural to consider only a slice of the IMF. On the other hand, values between  $8 \times 10^8 \lesssim \tau \lesssim 2 \times 10^9$  years for  $\zeta\epsilon(R_D/R_H)^2 = 0.1$ ,  $4 \times 10^8 \lesssim \tau \lesssim 2 \times 10^9$  years for  $\zeta\epsilon(R_D/R_H)^2 = 0.01$ , and  $2 \times 10^8 \lesssim \tau \lesssim 2 \times 10^9$  years for  $\zeta\epsilon(R_D/R_H)^2 = 0.003$  are allowed.

We have also run a model with  $m_L = 2M_\odot$  and  $m_U = 100M_\odot$  with a constant SFR to compare with the naive calculation by Hegyi and Olive (1983, 1986). We find that for  $z_D < 0.02$ ,  $\epsilon \lesssim 4 \times 10^{-3}$  which is to be compared with  $z_D < 10^{-5}$  and  $\epsilon \lesssim 3 \times 10^{-4}$ . In addition to being more stringent, a more severe constraint comes from the mass-to-light ratio. In this case  $(M/L)_B \sim (1/4)(M_\odot/L_\odot)_B$ . The halo is more luminous than the disk.

Figure 4 shows an example of the time evolution of  $M_S/M_T$ ,  $M_R/M_T$ , and  $M_G/M_T$  (solid lines),  $y_H$  and  $y_D$  (short-dashed lines), and  $L_H/M_T$  (long-dashed line) for  $m_L = 4M_\odot$ ,  $m_U = 6M_\odot$ ,  $\tau = 10^9$  years,  $y_{H,i} = y_{D,i} = 0.23$ , and  $\zeta\epsilon(R_D/R_H)^2 = 0.03$ . Gas is initially the dominant component of the halo mass, but soon stars and then stellar remnants take over. At present, about 99% of the total mass is locked inside stellar remnants, while stars contribute a negligible amount of mass. The gas accounts for 1% of the halo mass and is fixed at this value in all of our models. The helium abundances of the halo and disk increase gradually with time, and at present  $y_{H,p} \sim 0.95$  and  $y_{D,p} \sim 0.28$ . In this case,  $z_H = z_D = 0$  always since  $M_U < 8M_\odot$ . In Figure 5, the time evolution of  $z_D$ ,  $y_D$ , and  $dy_D/dz_D$  is shown for  $m_L = 3M_\odot$  and  $m_U = 9M_\odot$  (solid lines), allowed if  $\zeta\epsilon(R_D/R_H)^2 = 0.01$ , and for  $m_L = 3M_\odot$  and  $m_U = 28M_\odot$  (short-dashed lines), allowed if  $\zeta\epsilon(R_D/R_H)^2 = 0.003$ .  $dy_D/dz_D$  initially increases rapidly and later flattens. The luminosity in Figure 4 increases and decreases with the mass density of stars, as expected. In this case, the halo has a peak luminosity  $L_{H,B}/M_T \sim 40(L_\odot/M_\odot)_B$  and is brighter than  $L_{H,B}/M_T \sim 10(L_\odot/M_\odot)_B$  for  $\sim 0.1T_0$ . Even though the peak luminosity and the duration of the bright period change for different values of the free parameters, the integrated luminosity over the age of the galaxy does not change very much for most of the cases that we have calculated, and it is approximately given by

$$\int \frac{L_{H,B}}{M_T} dt = (3 - 3.5) \frac{L_{\odot B} T_0}{M_\odot}. \quad (4.2)$$

This corresponds to  $(6 - 7) \times 10^{63} \text{ erg}$  in the B-spectral band for a galaxy of nominal halo mass of  $10^{12} M_\odot$ , or an efficiency of conversion of rest mass to radiation of greater than or of order 0.3%.

## V. DISCUSSION

In this paper, we have considered the hypothesis that galactic halos are composed of stellar remnants such as neutron stars and white dwarfs. This possibility has been tested by calculating the evolution of halo matter, luminosity, and metallicity and helium abundance on the basis of a simple model, and by comparing them with the following three constraints: i) the present disk metallicity and helium abundance contributions from halo stars should be smaller than 0.02 and 0.05, respectively; ii) the present mass density of the halo gas should be less than 10% of the total mass density; iii) the present mass-to-light ratio of the halo in the B-spectral band should be larger than  $100(M_{\odot}/L_{\odot})_B$ .

In our evolutionary model of the halo, we have assumed that the disk of the galaxy forms no later than the halo. The reason for this is clear. If the disk forms after the period of rapid star formation in the halo, the chemical abundances of the disk will be far too high. For example, even in the case where metals are not formed in the halo as in the model shown in Figure 4, after only 1 billion years following the initiation of star formation in the halo, the helium abundance is  $\sim 40\%$ , and after 2 billion years it is up to  $\sim 60\%$ . Clearly the disk must form before the first generation of stars die. Despite the fact that the disk forms at about the same time as the halo, we must require star formation in the disk to be quite different. That is, we must choose a different SFR and IMF. Though we do not have an explanation for this difference, the process of halo and disk formation is understood poorly enough so that we can not rigorously exclude this possibility. An alternative hypothesis is that the disk forms out of a fresh supply of primordial material after halo star formation, and the associated supply of enriched gas is mostly exhausted to the level of a percent or less. In this case, the time delay between halo and disk formation could be several billion years.

Throughout this paper, we have taken  $m_{SN} = m_U$ . It is of course quite possible that  $m_{SN} < m_U$  in which case all stars with masses  $m_{SN} < m < m_U$  end up as black holes without the ejection of any processed material. Typically one might expect that  $m_{SN} \gtrsim 50M_{\odot}$ . However, for a first generation of stars, the absence of heavy elements may prevent mass loss and  $m_{SN}$  could be quite a bit smaller. If  $m_L > m_{SN}$ , it is not difficult to produce a halo of black holes (this is especially straightforward if  $m_L$  is taken to be very large). Thus as in previous investigations, we can not exclude black holes as a halo dark matter candidate. Although one can perhaps choose  $m_{SN}$  to be small initially, after some enrichment associated with one or two generations of stars, it seems unlikely that  $m_{SN} < 10M_{\odot}$ , and we could not plausibly argue for a halo exclusively containing low mass black holes.

Based on our constraints on the metallicity and helium abundance, we conclude that if  $\zeta\epsilon(R_D/R_H)^2 = 0.03$ , stars should have formed only with masses between  $2M_{\odot} \lesssim m \lesssim 8M_{\odot}$  and SFR duration between  $6 \times 10^8 \text{ years} \lesssim \tau \lesssim 2 \times 10^9 \text{ years}$  in order

to be compatible with the above constraints. This indicates that, even though the possibility of neutron stars as halo matter may be ruled out, there exists a small window for the possibility of white dwarfs constituting a major component of the galactic halo. If  $\zeta(\epsilon(R_D/R_H)^2)$  is smaller, it is possible that more massive stars which end up as neutron stars have also formed. However, considering the fact that the above constraints are fairly conservative, more severe limits could enable us to eventually exclude the possibility of white dwarfs as well as that of neutron stars as halo matter.

Our models also assume that the metallicity and increase in the helium abundance are entirely due to the halo. Because this is probably unrealistic, we have considered the case where  $z_D \lesssim 0.01$  and  $\Delta y_D \lesssim 0.025$ , i.e., we allow the disk to process half of its helium and heavy elements. These results are also shown in Figure 2. Only for the smaller value of  $\zeta(\epsilon(R_D/R_H)^2)$  is this difference important. For example, when  $\zeta(\epsilon(R_D/R_H)^2) = 0.003$ ,  $m_U$  is reduced from  $\sim 50M_\odot$  to  $\sim 25M_\odot$ .

In Figure 5, we also see that a further constraint is available from the calculation of  $dy/dz$ . Unless  $\zeta(\epsilon(R_D/R_H)^2)$  is very small and a sizable mass range is allowed in the IMF, we find that  $dy/dz$  would be very large,  $\gtrsim 100$ . When  $\zeta(\epsilon(R_D/R_H)^2) = 0.003$ , then  $dy/dz$  takes more typical values  $\sim 1 - 3$ . The reason for this is that when  $\zeta(\epsilon(R_D/R_H)^2) \gtrsim 0.01$ , the IMF contains very few stars producing metals and  $dy/dz$  becomes large. When a more standard IMF is used, metal producing massive stars exist and  $dy/dz$  falls. We note that in all of the models considered no deuterium is produced and thus we still require big-bang nucleosynthesis. Also, unless  $m_U \gtrsim 14M_\odot$  there is a marked lack of oxygen in the heavy elements produced. In most of the cases where  $m_U < 20M_\odot$ , we expect the chemical abundances to differ from those observed in extreme population II stars.

A direct observational test of the baryonic dark matter hypothesis would be to search for halo white dwarfs. Models by Tamanaha et al. (1989) suggest that proposed surveys should be capable of detecting nearby halo white dwarfs, provided that the halo formed no more than 6 billion years prior to the local (solar neighborhood) disk. Our model, however, requires the disk to have formed soon after the halo and any white dwarf component should therefore be detectable with a modest improvement in current limits.

It is of interest to note that the recently reported (Matsumoto et al. 1988) but not yet confirmed submillimeter excess in the cosmic background radiation may require substantial energy input of an early generation of stars. For example, Lacey and Field (1988) point out that the nuclear energy release associated with formation of the observed luminous component of galaxies falls short by an order of magnitude or more in providing an adequate energy source to heat up the intergalactic medium and distort via Comptonization the cosmic blackbody photons. Dust models for the submillimeter excess (Bond, Carr, and Hogan 1989; Adams et al. 1989) also require a very substantial stellar energy input that is more readily obtained if halo dark matter consists of stellar remnants. For example, with a luminous galaxy density



of  $\sim 0.01h^3Mpc^{-3}$ , equivalent to  $\Omega \sim 0.03h(M_T/10^{12}M_\odot)$ , baryonic dark halos forming at redshift  $z_*$  yield a diffuse radiation background, even if we only count the rest-frame B-band, of energy density  $\sim 1.3h^3(1+z_*)^{-1}eV/cm^3$ : for comparison, the submillimeter excess may amount to  $\sim 0.05eV/cm^3$ .

In summary, we have found that only if i) the disk forms no later than the halo; ii) the SFR in the halo is rapidly decreasing with an e-folding time of  $(0.6 - 2) \times 10^9$  years; iii) the IMF mass range in the halo contained only the segment  $2 - 8M_\odot$ ; and iv) halo material avoided substantial mixing with the disk, can dead remnants such as neutron stars and white dwarfs make up the dark matter in galactic halos. In other words, star formation in the disk is very different from that in the halo. These conclusions are based primarily on the constraints from the mass-to-light ratio in the halo  $(M/L)_B \gtrsim 100(M_\odot/L_\odot)_B$  and the abundances of helium ( $\Delta y \lesssim 0.05$ ) and metals ( $z_D \lesssim 0.02$ ). Our arguments presume a *reasonable* ( $0.1 - 100M_\odot$ ) range for the primordial IMF: if this assumption is relaxed, one opens the Pandora's box of Jupiter-like objects (see, however, constraints discussed in Hegyi and Olive 1986) or very massive black holes. In this mass range, only white dwarfs survive as a possible candidate for baryonic dark matter, provided that a *very* specific form is adopted for the IMF and SFR. If one relaxes the constraint imposed by our assumption about yields, black holes would be a more likely form for baryonic dark matter. We urge observers to take our constraints sufficiently seriously to mount a search for any stellar signature of relics, such as white dwarfs, that may constitute an observable component of a dark baryonic halo. Ultimately, the only constraints on baryonic dark matter will come from observations. Of course, non-baryonic dark matter (for a review see Primack, Seckel, and Sadoulet 1988), for which there are numerous particle candidates, all hitherto undetected, remains as a viable alternative.

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## FIGURE CAPTIONS

- Fig. 1.— The line which divides the  $\tau$ – $m_L$  plane into the regions with  $M_G/M_T \lesssim 0.1$  and  $M_G/M_T \gtrsim 0.1$  at the present. Here, we have used  $m_U = 8M_\odot$ . The values of  $\tau$  and  $m_L$  in the region with  $M_G/M_T \gtrsim 0.1$  at the present can be excluded.
- Fig. 2.— The contours which correspond to  $z_{D,p} = 0.02$  (long-dashed line),  $y_{D,p} = 0.28$  (long-dashed line),  $z_{D,p} = 0.01$  (short-dashed line),  $y_{D,p} = 0.255$  (short-dashed line), and  $m_L = m_U$  (solid line) on the  $m_L$ – $m_U$  plane. Here, we have used  $\tau = 10^{10}$  years,  $y_{D,i} = 0.23$ , and  $\zeta\epsilon(R_D/R_H)^2 = 0.1$  (2a); 0.03 (2b); 0.01 (2c); and 0.003 (2d). The region with  $z_{D,p} \gtrsim 0.02$  or  $y_{D,p} \gtrsim 0.28$  can be excluded.
- Fig. 3.— The present  $L_{H,B}/M_T$  as a function of  $\tau$  for  $m_L = 2M_\odot$  and  $m_U = 8M_\odot$  (solid line), and  $m_L = 4M_\odot$  and  $m_U = 6M_\odot$  (dashed line).  $L_{B,H}/M_T$  for  $m_L = 3M_\odot$  and  $m_U = 28M_\odot$  is also calculated and almost identical with that for  $m_L = 4M_\odot$  and  $m_U = 28M_\odot$ . The present value of  $L_{H,B}/M_T$  is always larger than  $1/100(L_\odot/M_\odot)_B$  for  $\tau \gtrsim 2 \times 10^9$  years, and we can exclude these values of  $\tau$ .
- Fig. 4.— The time evolution of  $M_S/M_T$ ,  $M_R/M_T$ , and  $M_G/M_T$  (solid lines),  $y_H$  and  $y_D$  (short-dashed lines), and  $L_{H,B}/M_T$  (long-dashed line) for  $m_L = 4M_\odot$ ,  $m_U = 6M_\odot$ ,  $\tau = 10^9$  years,  $y_{H,i} = y_{D,i} = 0.23$ , and  $\zeta\epsilon(R_D/R_H)^2 = 0.03$ . In this case,  $z_H = z_D = 0$  always since  $\dot{M}_U < 8M_\odot$ .
- Fig. 5.— The time evolution of  $z_D$ ,  $y_D$ , and  $dy_D/dz_D$  for  $m_L = 3M_\odot$ ,  $m_U = 9M_\odot$ , and  $\zeta\epsilon(R_D/R_H)^2 = 0.01$  (solid lines), and for  $m_L = 3M_\odot$ ,  $m_U = 28M_\odot$ , and  $\zeta\epsilon(R_D/R_H)^2 = 0.003$  (short-dashed lines).

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