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Method for Experimental Determination of Flutter Speed by Parameter Identification

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CONTENTS

SUMMARY

A method for flight flutter testing is proposed which enables one to determine the flutter dynamic pressure from flights flown far below the flutter dynamic pressure. The method is based on the identification of the coefficients of the equations of motion at low dynamic pressures, followed by the solution of these equations to compute the flutter dynamic pressure. The initial results of simulated data reported in the present work indicate that the method can accurately predict the flutter dynamic pressure, as described. If no insurmountable difficulties arise in the implementation of this method, it may significantly improve the procedures for flight flutter testing.

INTRODUCTION

The current procedures involving flight flutter testing are essentially based on the experimental determination of the modal damping coefficients and on the study of the variation of these coefficients with airspeed (Ruhlin and others, 1982; Russo and others, 1983; Roy and Walker, 1985). The flutter phenomenon involves an aeroelastic structural instability which may be violent in nature (explosive flutter), and, therefore, it may exhibit a rapid deterioration in modal dampings with speed increase. Therefore, a careful opening of the flight envelope is required to avoid a possible loss of the vehicle during these tests. As a result, a large number of flight tests are performed, involving a careful increase in flight speeds. These flight tests are both time consuming and costly; and they require flying at speeds that are close to the flutter speeds. Ruhlin and others (1982) say that for a reliable determination of flutter speed, flight flutter tests must be performed at dynamic pressures with values around 7 to 10 percent below the flutter dynamic pressure. Therefore, it is important to formulate a method that will permit a rapid and reliable determination of the flutter speed at speeds well below the flutter speed. Attempts to formulate such a method were made in the past by several investigators. Zimmerman and Weissenburger (1964) describe a method whereby the complex eigenvalues of the vehicle are determined at three different airspeeds. These eigenvalues are then used to derive the vehicle's characteristic equation at these three airspeeds. The coefficients of the characteristic equations are then used to evaluate the Routh-Hurwitz stability discriminants. Zimmerman and Weissenburger (1964) show that for binary systems, the Routh-Hurwitz stability discriminant varies in a quadratic fashion with the dynamic pressure. Hence, the complete parabolic shape can be determined from measurements at three different airspeeds, which can all be far below the critical flutter speed. Unfortunately, this method does not seem to work with systems that have more than 2 degrees of freedom.

A variation on the aforementioned method is suggested in Houbolt (1975) in an attempt to extend the method of Zimmerman and Weissenburger (1964) to systems with more than 2 degrees of freedom. However, the suggested variation appears to have difficulties similar to those existing in Zimmerman and Weissenburger (1964), and there is no indication regarding its use, or even of computed analytical results since it had been proposed in 1975.

Gaukroger and others (1973; 1980) make use of a different approach, much in accordance with the basic approach adopted in this work, that is, the coefficients of the equations of motion of the vehicle are identified, and the flight flutter speed is computed following the identification stage. However, the identification procedure in Gaukroger and others (1973; 1980) is different than the one presented in this work, and the results produced in Gaukroger and others (1973; 1980) relate to binary systems only, much the same as in Zimmerman and Weissenburger (1964). Even for the simplified binary systems presented in Gaukroger and others (1973; 1980), the identification of the coefficients of the equations of motion yielded substantial errors in the coefficients. However, the final values regarding the flutter speed were in reasonably good agreement with the theoretical flutter speeds.

In the following work, attempts will be made to identify the coefficients of the equations of motion of dynamic systems. Following this identification stage, applications to simulated flight flutter testing will be made. The examples presented include the identification of the equations of motion for a 5-degrees-of-freedom mass-string system; the identification of seven modes of a continuous simply supported uniform beam; the identification of a 9-degrees-of-freedom DAST (NASA's drone for aerodynamic and structural testing) aircraft model and computation of its flutter dynamic pressure; and finally, the identification of a 12-degrees-of-freedom YF-17 aircraft model and the eventual computation of its flutter dynamic pressure.

NOMENCLATURE

Variables

b	reference semichord length
g	structural damping coefficient
k	reduced frequency (= $\omega b/V$)
m	number of excitation vectors
n	number of modes
n_f	number of excitation frequencies
n_P	number of measurement points
Q_D	flight dynamic pressure
Q_F	flutter dynamic pressure
V	flight speed
β_i	ith aerodynamic lag term
μ	real part of eigenvalue λ
ζ	viscous damping coefficient
ρ	air density
λ	complex eigenvalue
ω	frequency of oscillation
ω_n	natural frequency of oscillation
ω_d	damped frequency of oscillation
ω_F	flutter frequency

Matrices

$[A]$	aerodynamic matrix
$[A_i]$	aerodynamic matrices defined in equation (16)
$[B]$	defined in equation (10)
$[B_R], [B_I]$	defined in equation (12)
$[C]$	defined in equation (5)
$[\bar{C}]$	damping matrix
$[C_T]$	total damping matrix defined in equation (21)
$[F]$	defined in equation (5)
$[\bar{F}]$	forcing matrix
$[K]$	defined in equation (5)
$[\bar{K}]$	stiffness matrix
$[K_T]$	total stiffness matrix defined in equation (21)
$[\bar{M}]$	mass matrix
$[q]$	response amplitudes
$[q]_i$	response amplitudes associated with the ith frequency of excitation
$[q_0]$	response matrix
$[T]$	defined in equation (9)
$[T_R], [T_I]$	defined in equation (12)
$[x]$	eigenvector
$[z]$	matrix of displacements (in physical coordinates)
$[\phi]$	mode shape matrix

ANALYTICAL APPROACH

Identification of the Equations of Motion

Let the equations of motion be given by

$$[\bar{M}][\ddot{q}_0] + [\bar{C}][\dot{q}_0] + [\bar{K}][q_0] = [\bar{F}]e^{i\omega t} \quad (1)$$

where all the coefficient matrices are real. It is desired to identify the coefficients of equation (1) by exciting the system over a range of frequencies using one or more forcing vectors, the magnitude of which need not be known.

Equation (1) can also be written as

$$(-[\bar{M}]\omega^2 + [\bar{C}]i\omega + [\bar{K}])[q] = [\bar{F}] \qquad (2)$$

where

$$[q_0] = [q]e^{i\omega t} \qquad (3)$$

Note that matrices $[\bar{M}]$, $[\bar{C}]$, and $[\bar{K}]$ are of order $n \times n$, where n is the number of degrees of freedom of the system, and $[q]$ and $[\bar{F}]$ are matrices of order $n \times m$, where m represents the number of fixed amplitude forcing columns used during the excitation.

If one assumes that $[\bar{F}]$ is known, and $[q]$ is measured, then it is possible to generate enough equations during a frequency sweep to determine the matrix coefficients $[\bar{M}]$, $[\bar{C}]$, and $[\bar{K}]$.

If $[\bar{F}]$ is assumed to be unknown, then it may be more convenient to premultiply equation (2) by $[\bar{M}]^{-1}$ to obtain

$$(-[I]\omega^2 + [C]i\omega + [K])[q] = [F] \qquad (4)$$

where

$$\begin{aligned} [C] &= [\bar{M}]^{-1}[\bar{C}] \\ [K] &= [\bar{M}]^{-1}[\bar{K}] \\ [F] &= [\bar{M}]^{-1}[\bar{F}] \end{aligned} \qquad (5)$$

and where $[I]$ is the unit matrix. Assume the system is excited with frequencies $\omega_1, \omega_2, \ldots \omega_{nf}$, with responses $[q]_1, [q]_2, \ldots [q]_{nf}$. Equation (4) can then be written in the form

$$\begin{aligned} &[C]i\omega_1[q]_1 + [K][q]_1 - [F] = \omega_1^2[q]_1 \\ &[C]i\omega_2[q]_2 + [K][q]_2 - [F] = \omega_2^2[q]_2 \\ &\vdots \\ &[C]i\omega_{nf}[q]_{nf} + [K][q]_{nf} - [F] = \omega_{nf}^2[q]_{nf} \end{aligned} \qquad (6)$$

Equation (6) can be written in a compact form, after transposing it, that is,

$$\begin{bmatrix} [q]_1^T & i\omega_1[q]_1^T & -[I] \\ [q]_2^T & i\omega_2[q]_2^T & -[I] \\ \vdots & & \\ [q]_{nf}^T & i\omega_{nf}[q]_{nf}^T & -[I] \end{bmatrix} \begin{bmatrix} [K]^T \\ [C]^T \\ [F]^T \end{bmatrix} = \begin{bmatrix} \omega_1^2[q]_1^T \\ \omega_2^2[q]_2^T \\ \vdots \\ \omega_{nf}^2[q]_{nf}^T \end{bmatrix} \qquad (7)$$

Equation (7) can be written as

$$[T]\begin{bmatrix} K^T \\ C^T \\ F^T \end{bmatrix} = [B] \qquad (8)$$

where

$$[T] = \begin{bmatrix} [q]_1^T & i\omega_1[q]_1^T & -[I] \\ [q]_2^T & i\omega_2[q]_2^T & -[I] \\ \vdots & \vdots & \vdots \\ [q]_{nf}^T & i\omega_{nf}[q]_{nf}^T & -[I] \end{bmatrix} \qquad (9)$$

$$[B] = \begin{bmatrix} \omega_1^2[q]_1^T \\ \omega_2^2[q]_2^T \\ \vdots \\ \omega_{nf}^2[q]_{nf}^T \end{bmatrix} \qquad (10)$$

Note that matrices $[T]$ and $[B]$ are complex matrices. If normal complex least square analysis is applied to equation (8), or if a generalized inverse of matrix $[T]$ is computed, the computed values for $[K]$, $[C]$, and $[F]$ are exact only if exact responses are considered when constructing matrix $[T]$. If errors are allowed to exist in the measured responses $[q]_i$, the resulting identified coefficients show large errors in the computed values. This extreme sensitivity to errors makes this formulation worthless for practical applications. This sensitivity to errors may be attributed to the fact that if matrix $[T]$ is complex, the resulting solution for the matrices $[K]$, $[C]$, and $[F]$ assumes complex form. This means that the number of unknowns in equation (8) is effectively doubled when allowing for the real and imaginary parts of each of the solution matrices. To cut down the number of unknowns, equation (8) will be reformulated so as to constrain the solution for matrices $[K]$, $[C]$, and $[F]$ to assume real values only. Later on, these constraints will impose limitations on the aerodynamic representation of the equations of motion, but this must be accepted if one wishes to avoid the aforementioned extreme sensitivity of the solution to measurement errors.

Equation (8) is split into its real and imaginary parts, while constraining the solution matrices to assume real values. Hence, equation (8) can be written as

$$\left.\begin{aligned} [T_R]\begin{bmatrix} K^{\mathrm{T}} \\ C^{\mathrm{T}} \\ F^{\mathrm{T}} \end{bmatrix} &= [B_R] \\ [T_I]\begin{bmatrix} K^{\mathrm{T}} \\ C^{\mathrm{T}} \\ F^{\mathrm{T}} \end{bmatrix} &= [B_I] \end{aligned}\right\} \qquad (11)$$

where

$$\left.\begin{aligned} [T] &= [T_R] + i[T_I] \\ [B] &= [B_R] + i[B_I] \end{aligned}\right\} \qquad (12)$$

Equation (11) can be written more compactly as

$$\begin{bmatrix} T_R \\ T_I \end{bmatrix}\begin{bmatrix} K^{\mathrm{T}} \\ C^{\mathrm{T}} \\ F^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} B_R \\ B_I \end{bmatrix} \qquad (13)$$

Equation (13) is solved herein using a generalized inverse algorithm for real matrices based on singular value decomposition (IMSL subroutine LSVDF), to yield

$$\begin{bmatrix} K^{\mathrm{T}} \\ C^{\mathrm{T}} \\ F^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} T_R \\ T_I \end{bmatrix}^{+}\begin{bmatrix} B_R \\ B_I \end{bmatrix} \qquad (14)$$

where the symbol $^{+}$ denotes the generalized inverse of the matrix. Note that matrices $[T_R]$ and $[T_I]$ are of order $(nf \times m) \times (2n + m)$ and $[B_R]$ and $[B_I]$ are of the order $(nf \times m) \times n$. Hence

$$\begin{bmatrix} T_R \\ T_I \end{bmatrix}$$

is of the order $(2nf \times m) \times (2n + m)$ and

$$\begin{bmatrix} B_R \\ B_I \end{bmatrix}$$

is of the order $(2nf \times m) \times n$. Clearly, the solution matrix

$$\begin{bmatrix} K^{\mathrm{T}} \\ C^{\mathrm{T}} \\ F^{\mathrm{T}} \end{bmatrix}$$

is of the order $(2n + m) \times n$. It should also be noted that if equation (1) relates to generalized coordinates, whereas measurements are made of physical coordinates z, then equation (14) can be used after transforming the measured z responses into the q coordinates using the transformation matrix $[\phi]$, that is,

$$[z] = [\phi][q] \qquad (15)$$

where the matrix $[\phi]$ is often chosen as the orthogonal mode-shape matrix. The matrix $[z]$ is of the order $n_p \times m$, where n_p is the number of measurement points of the physical coordinates; $[\phi]$ is of the order $n_p \times n$ and $[q]$ is of the order $n \times m$.

Formulation of the Flutter Equations

As already stated, the formulation of the flutter equations is constrained to equations with real coefficient matrices, following the aforementioned sensitivity to errors of the measured responses. The structural equations of motion can easily be brought to the form of equation (1). There remains to treat the aerodynamic coefficient matrix. This matrix is a function of the flight Mach number, the reduced frequency k, and the flight dynamic pressure Q_{D}. For any specific Mach number, the aerodynamic matrix $[A]$ can be approximated by the following Padé relation:

$$[A] = Q_{\mathrm{D}}\left([A_0] + [\bar{A}_1]ik + [\bar{A}_2](ik)^2 + \sum_{j=i}^{\ell}\frac{[\bar{A}_{2+j}]ik}{ik + \beta_j}\right) \qquad (16)$$

If one ignores all of the lag terms, equation (16) assumes the form

$$[A] = \frac{1}{2}\rho V^2[A_0] + \frac{1}{2}\rho V[A_1]i\omega + \frac{1}{2}\rho[A_2](i\omega)^2 \qquad (17)$$

where

$$\left.\begin{aligned} [A_1] &= b[\bar{A}_1] \\ [A_2] &= b^2[\bar{A}_2] \end{aligned}\right\} \qquad (18)$$

and where b represents a reference semichord length used to compute k.

The matrix $[A_2]$ represents aerodynamic inertia terms. These are normally small compared to the structural inertia terms and can therefore be ignored. Hence, the aerodynamic matrix can be written as

$$[A] = \frac{1}{2}\rho V^2[A_0] + \frac{1}{2}\rho V[A_1]i\omega \qquad (19)$$

Equation (19) is much in accordance with the British method of representation of the equations of motion for flutter analysis.

Introducing equation (19) into equation (1), one gets the following modified form of equation (4):

$$(-[I]\omega^2 + [C_T]i\omega + [K_T])[q] = [F] \qquad (20)$$

where

$$\left.\begin{aligned}[C_T] &= [C] + \tfrac{1}{2}\rho V[A_1] \\ [K_T] &= [K] + \tfrac{1}{2}\rho V^2[A_0]\end{aligned}\right\} \qquad (21)$$

If $[C_T]$ and $[K_T]$ are determined for two values Q_1 and Q_2 of dynamic pressures, then equation (21) yields

$$[A_0] = \frac{[K_T]_{Q=Q_2} - [K_T]_{Q=Q_1}}{Q_2 - Q_1} \qquad (22)$$

$$[A_1] = \frac{[C_T]_{Q=Q_2} - [C_T]_{Q=Q_1}}{\frac{Q_2}{V_2} - \frac{Q_1}{V_1}} \qquad (23)$$

$$[K] = [K_T]_{Q=Q_2} - Q_2[A_0] \qquad (24)$$

$$[C] = [C_T]_{Q=Q_2} - \frac{Q_2}{V_2}[A_1] \qquad (25)$$

These equations will form the basis for the flutter prediction to be presented in the following sections of this work.

SUGGESTED PROCEDURE FOR FLIGHT FLUTTER TESTING

Assuming that the coefficients of the equations of motion can be reasonably identified, the following procedure is suggested for flight flutter testing.

1. Choose a flight Mach number and keep it constant throughout this procedure.

2. Choose a flight altitude, and thus determine the value of the flight speed V_1 and the value of the dynamic pressure Q_1 for the specific conditions of this flight.

3. Make an excitation frequency sweep using forcing vectors $[F]_{Q_1}$ and recording the resulting responses $[q_1]_{Q_1}, [q_2]_{Q_1} \dots [q_{nf}]_{Q_1}$.

4. Change the flight altitude keeping the Mach number constant, and thus determine a new value of the flight speed V_2 and a new value for the dynamic pressure Q_2, with $Q_2 > Q_1$.

5. With these new values of V_2 and Q_2, repeat step 3 above with $[F]_{Q_2}$ to obtain $[q_1]_{Q_2}$, $[q_2]_{Q_2}, \dots [q_{nf}]_{Q_2}$.

6. With the values obtained in step 3, solve for $[K_T]_{Q=Q_1}$, $[C_T]_{Q=Q_1}$, and $[F]_{Q_1}$, using equation (20).

7. With the values obtained in step 5, solve for $[K_T]_{Q=Q_2}$, $[C_T]_{Q=Q_2}$, and $[F]_{Q=Q_2}$.

8. With the values of $[K_T]_{Q=Q_1}$, $[C_T]_{Q=Q_1}$, $[K_T]_{Q=Q_2}$, and $[C_T]_{Q=Q_2}$ obtained in steps 6 and 7, determine the matrices $[C]$, $[K]$, $[A_0]$, and $[A_1]$ using equations (22) to (25).

9. Solve, numerically, equation (20) for a range of values of V and Q to compute the dynamic pressure $Q_{F_{1-2}}$.

10. If one wishes to, one could choose $Q_3 > Q_2$ and repeat steps 2 and 3 to produce $[K_T]_{Q=Q_3}$, $[C_T]_{Q=Q_3}$, and $[F]_{Q=Q_3}$.

11. With the results associated with Q_2 and Q_3, repeat steps 8 and 9 to determine the flutter dynamic pressure $Q_{F_{2-3}}$ and check for agreement with $Q_{F_{1-2}}$.

12. Steps 10 and 11 can be repeated until reasonable convergence of computed flutter dynamic pressures is obtained.

PRESENTATION AND DISCUSSION OF RESULTS

Numerical Examples

Four numerical examples are presented in this work. Two of the examples relate to the identification of simple dynamic systems without aerodynamic contributions, and two examples relate to the mathematical models of the drone for aerodynamic and structural testing (DAST) and the YF-17 aircraft. The identification in the last two examples are followed by the computation of the predicted flutter dynamic pressures.

Example 1 — Mass-String System

Example 1 was chosen in an attempt to test the effectiveness and sensitivities of the proposed method in identifying the various matrix coefficients, including the forcing columns. The system is shown in figure 1. It comprises five lumped masses attached to a string. The viscous dampers were chosen so that the resulting damping matrix will not yield proportional damping. The resulting coefficient matrices with values of $T/m\ell = 300$ and $c/m = 1.8$ are shown in table 1. Both the damped and the undamped eigenvalues for this example are presented in table 2. The eigenvectors are presented in table 3.

The system is excited analytically with forces having constant amplitudes. The generated responses are then used to identify the system. These responses are also contaminated with errors generated by rounding the responses to two digits (thus introducing a nonzero mean error of up to 5 percent).

The results obtained using a single forcing column are summarized in tables 4 through 6. Table 4 shows the values obtained for the identified system, using exact responses with excitation frequency spanning over all the frequencies of the system. As can be seen, the identified system essentially coincides with the exact system. Table 5 is identical to table 4, except for the reduced frequency range used for excitation, spanning over three natural frequencies of the system, and leaving two natural frequencies outside of the excitation range. It can be seen that even for this case, the identified results are essentially equal to the exact system. This suggests that it may be advantageous to select a conservative range of frequencies in the analysis, and then increase the range should frequencies be found outside the excitation range. Table 6 shows the results obtained using responses contaminated by errors introduced while rounding these responses to two digits. It can be seen that while the identified stiffness matrix $[K]$ is correct to within 5 percent errors, the damping matrix shows larger errors of up to 40 percent. The identified forcing column values are essentially the same as the exact values. At this stage, it can be concluded that the identified system is very close to the original system, provided the response errors are small. Furthermore, the identified parameters which are most sensitive to errors are the damping matrix coefficients.

Results obtained using two different forcing vectors are presented in tables 7 and 8. The program assumes that each of these two forcing columns excites the structure at identical frequencies. This is not essential, and it may even be advantageous if, say, the second forcing vector excites the system at intermediate frequencies (which fall between the frequencies of excitation of the first forcing vector). Table 7 uses exact responses in obtaining the values of the identified system, whereas table 8 uses responses rounded to two digits. It can be seen that the exact responses yield, once again, exact coefficients for the identified system. Table 8 shows that the two forcing vectors had very large improvements in the coefficients when contaminated responses are used (by rounding them to two digits). It can be seen that the identified values of the stiffness matrix $[K]$ and the forcing vectors $[F]$ are essentially exact, whereas the identified values of the damping matrix are within 6 percent error, which is around the values of the errors introduced into the responses. Attempts to smooth the responses prior to their substitution in the $[T]$ matrix did not improve the results, and, in many cases, the attempts led to degraded results.

Finally, tables 9 and 10 present results for the same cases described in tables 7 and 8 but with five forcing columns. It can be seen that in this case, the identified results are essentially exact, even when the responses are contaminated by rounding them to two digits.

The results described indicate the advantage of using more than one forcing vector. It seems that for reasonable results, *at least two linearly independent forcing vectors need to be used.*

Tables 11 to 13 represent the results obtained when using responses contaminated with 5 percent random errors (instead of rounding the responses to the nearest two-digit values). Table 11 relates to the case where one forcing vector is used. It can be seen that large errors result, somewhat higher than those obtained when the responses were rounded to two digits. Peak errors in the stiffness matrix terms reach values around 40 percent, and peak errors in the damping matrix terms may reach values around 60 percent. However, even with these relatively large errors, the eigenvalues of the identified system are close to those of the exact system, with frequency errors reaching maximum values of around 3 percent and peak eigenvalue damping errors of around 28 percent (this error relates to the highest frequency mode, with much smaller values of errors in the lower modes). Even the eigenvectors obtained appear to be close to those of the exact system, with a possible exception of the 29.68 rad/sec mode.

Table 12 shows the results obtained when using responses contaminated by 5 percent random errors and two forcing vectors. Dramatic reduction in errors can be seen with peak errors in stiffness terms of around 8 percent and peak errors in damping terms of around 20 percent, with most other terms showing errors less than 10 percent. Note that the frequencies obtained in this case, for the identified system, are essentially exact. The eigenvalue damping errors are within 2.5 percent, and the eigenvectors are essentially identical to those of the exact system. Table 13 shows similar results when using five forcing vectors. Hence we can see once again the importance of using more than one excitation vector. Furthermore, we can see that the peak errors in the eigenvalues and eigenvectors of the identified system are appreciably smaller than the peak errors in the individual matrix terms.

Example 2 — Simply Supported Continuous Uniform Beam

Example 2 was chosen in an attempt to test the effects of the ignored higher modes on the identification of the desired lower modes. The analytical model for this beam is very simple, with pure sinusoidal mode shapes. The beam is allowed 3-percent damping to avoid infinite responses, and the objective set for this example was to identify the seven lowest modes. The frequencies were normalized with respect to the seventh natural frequency, rendering the following values for the first seven natural frequencies:

$$\left.\begin{array}{ll} \omega_1 = 1/49 = 0.020408 & ; \omega_2 = 4/49 = 0.081633 \\ \omega_3 = 9/49 = 0.18367 & ; \omega_4 = 16/49 = 0.32653 \\ \omega_5 = 25/49 = 0.51020 & ; \omega_6 = 36/49 = 0.73469 \\ \omega_7 = 1 & \end{array}\right\} \quad (26)$$

Since the damping coefficient ζ is set to 0.03, the real parts of the eigenvalues ($\omega_i\zeta$) therefore will assume the following values:

$$\left.\begin{array}{ll} \omega_1\zeta = 0.00061225 & ; \omega_2\zeta = 0.0024490 \\ \omega_3\zeta = 0.0055102 & ; \omega_4\zeta = 0.0097959 \\ \omega_5\zeta = 0.015306 & ; \omega_6\zeta = 0.022041 \\ \omega_7\zeta = 0.03 & \end{array}\right\} \quad (27)$$

The beam was analytically excited over a normalized frequency range from 0 to 1.05. Since damping is light, 600 values of equally spaced excitation frequencies were initially used. The analytical responses were computed at seven equally spaced locations along the beam using the contributions of the lowest 36 modes. In one set of cases, only seven modes were used to calculate the analytical responses, in order to evaluate the effects of truncation of the higher modes.

The results obtained using two forcing vectors and the exact computed analytical responses of the first seven modes only are summarized in table 14. Since physical coordinates are being used, the stiffness and damping matrices have little meaning beyond enabling the computation of the eigenvalues and eigenvectors of the identified system. As can be seen, the eigenvalues (both real and imaginary parts) obtained from the identified system are essentially exact (see eqs. (26) and (27)). Table 15 presents results similar to those appearing in table 14, except that in this case, the exact responses were computed using the contributions of the lowest 36 modes. As can be seen, the identified frequencies are essentially exact, with some errors in the damping values, especially for the highest frequency. All the results to be presented from here on relate to analytical responses computed using 36 modes of the continuous beam.

When the system is identified using responses contaminated with errors, as for example 5 percent random errors, the resulting eigenvalues and eigenvectors show extremely large errors. It appears as if the higher frequencies "fold down" and mingle among the exact values of the lower frequencies. The reason for these large errors was eventually traced to the small responses associated with the higher modes. Since a least squares technique is effectively used in solving equation (13), it appears that the small values for the responses associated with the higher modes carry little weight in the least squares expression when compared with the relatively large responses associated with the lower modes. To overcome this problem, some weighting needs to be given to the high-frequency equations. Hence a weighting has been introduced.

Table 16 shows the results obtained while using a weighting proportional to the frequency of excitation, with responses contaminated with 5 percent random errors. As can be seen, excellent results are obtained for all frequencies, except for the damping of the first mode. At this stage, it is important to note that in this beam example, the ratio between the highest and the lowest frequencies has a value of 49. This is a large ratio that does not exist in any practical flutter example. Nevertheless, the results appearing in table 16 can be improved by using a weighting which is constant (= FREQC) up to a frequency equal to FREQC and from thereon using a weighting equal to the frequency of excitation. Table 17 shows such a result when using FREQC = 0.14. As can be seen, excellent results are obtained except for the highest mode, where 6 percent error in frequency is obtained and around 25 percent error in damping. The errors associated with all other modes are much smaller. No further efforts were made to improve these results since it was felt that the frequency ratio between the highest and lowest modes was too high to affect any practical flutter problem. Table 18 is similar to table 16 except that the excitation frequency range, spanning from 0 to 1.12, is subdivided into 14 subranges totaling 218 excitation frequencies instead of the 600 frequencies previously used. More frequencies were allowed around the resonance frequencies and less frequencies when far from resonance. The subranges were weighted to simulate integration of the square of the errors along the frequency axis. As can be seen from table 18, the results obtained are essentially identical to those shown in table 16.

Finally, table 19 shows the results obtained using 14 physical responses contaminated by 5 percent random errors and transformed to the seven generalized coordinates using the known mode shapes. Here again, 14 excitation subranges were used with FREQC = 0.04. As can be seen, excellent results are obtained except for the 10 percent error in the damping of the first mode.

Example 3 — Flutter of the DAST Aircraft

The aforementioned two examples were very useful in helping to formulate the identification problem and the excitation forms so as to yield results which can accept contamination errors. Example 3, together with example 4 which relates to the flutter of the YF-17 aircraft, are intended to test whether the aerodynamic simplifications made in equation (18) are valid, and also whether the identification procedure can yield reasonable flutter predictions from low dynamic pressure simulated flights.

The three views of the drone for aerodynamic and structural testing (DAST) are shown in figure 2. The mathematical model of the DAST consists of two rigid body modes (plunge and pitch) and seven elastic modes ranging from around 10 to around 128 Hz. The responses of the DAST aircraft were obtained for Mach number M = 0.9 using an 'exact' aerodynamic modeling of the aircraft with four lag terms in the Padé representation. The range of excitation between 0.5 to 900 rad/sec was divided into eight subranges with a total of 256 excitation frequencies. The exciting generalized forces were chosen so that all the ratios between the active generalized forces and the generalized masses are of the same order of magnitude for the elastic modes. For the rigid body modes, the active generalized forces were chosen to be between two to three orders of magnitude smaller.

The root locus plot for the 'exact' mathematical model is presented in figure 3. It can be seen that the dynamic pressure at flutter is given by $Q_F = 547$ lb/ft^2 and the flutter frequency is given by $\omega_F = 114$ rad/sec. In all of the DAST root locus plots presented in this work, the dynamic pressure increments have the value of 25 lb/ft^2, and the dynamic pressure Q is varied from 0 to 750 lb/ft^2. The root locus plots are truncated so as to show only those roots whose real parts lie within the range of -50 to $+20$.

As already stated earlier, the flutter prediction is based on the identification of the equations of motion, and the identification stage in this example ran into two difficulties. The first difficulty arises since the 'exact' analytical scheme for solving the flutter equations assumes undamped structural system, with damping associated with aerodynamic terms only. Therefore, this system could not be excited at zero dynamic pressure (that is, with zero damping) since the responses at resonance will be infinite. To overcome this difficulty, 3 percent structural damping was assumed while calculating the responses of the 'exact' system. The second difficulty, of a more severe nature, occurred at the identification stage since the matrix, for which the generalized inverse was sought, showed a strong singularity thus indicating it was rank deficient. The source of this difficulty was eventually traced to the two rigid body modes. These modes are unaffected by the structural damping added to the system, and therefore, their damping remains zero. In addition, at zero dynamic pressure, their stiffnesses are also zero so that the frequency responses to a single excitation vector of both the pitch and the plunge *generalized* coordinates vary identically as $1/\omega^2$ thus yielding a $[T]$ matrix (see eq. (9)) with two columns identical to two other columns. At this stage, one could have proceeded with the generalized inverse by equating to zero the two smallest singular values of matrix $[T]$, but this would have left the $[T]$ matrix ill conditioned at small values of Q, thus undermining the purpose of this work, that is, to predict flutter by identifying the system at relatively low Q values. However, once the source of the difficulty was traced, the fix was relatively simple, and it involved using two different vectors of excitation, where particular emphasis was placed on different excitation of the rigid body modes. Once this was done, excellent identification results were obtained, much in accordance with the results described earlier for the beam and for the mass-string system. This observation is important since around flutter speed there is a tendency of two modes to coalesce, thus yielding two modes with identical frequencies (but with different dampings) and thus possibly giving rise to the difficulty described earlier. Hence, it is concluded that the *excitation by more than one forcing vector is essential* for the identification to be successful.

Based on the aforementioned conclusions, the system identification stage of the flutter example treated herein is performed *using two forcing vectors.* It is expected that the use of a large number of forcing vectors may improve the numerical results, but by the same token, it may turn the method to be practically unappealing owing to the difficulties involved when a large number of exciters is needed. Hence all the results to be presented for the DAST were obtained from a two-vector excitation system (that is, from at least two shakers), spanning a frequency range between 0.5 to 900 rad/sec.

The root locus plot obtained by solving equation (20) for different values of Q, after identifying the system at $Q = 0$ and at $Q = 150$ lb/ft^2, using exact responses, is shown in figure 4. Figure 5 shows similar results, except that this time the system was identified at $Q = 150$ lb/ft^2 and at $Q = 250$ lb/ft^2. It can be seen from figure 4 that for the identification performed at the lower values of Q, the predicted dynamic pressure is $Q_F \approx 560$ lb/ft^2 with flutter frequency $\omega_F \approx 113$ rad/sec. These values are in excellent agreement with those of the 'exact' system (fig. 3), especially if allowance is made for the discrepancy involving the 3 percent structural damping introduced into the identified system. It should be noted that the effect of the 3 percent structural damping manifests itself in the form of displaced values for $Q = 0$. These displaced values should lie along a line with a slope of $g/2$, that is of 0.015, to the vertical axis and passing through the origin. All the root locus plots presented in this work, including those relating to example 4 which deals with the YF-17 aircraft simulation, clearly show this effect. Figure 5 is identical to figure 4, except that in this case the system was identified at $Q = 150$ lb/ft^2 and $Q = 250$ lb/ft^2. The flutter dynamic pressure computed in this case yields $Q_F = 553$ lb/ft^2 and $\omega_F = 113$ rad/sec. This is once again in excellent agreement with the values obtained using the 'exact' mathematical model. At this stage, it may be concluded that the aerodynamic simplifications introduced in equation (19) are adequate and have a negligible effect on the flutter dynamic pressure and on the flutter frequency.

Figures 6 and 7 are similar to figures 4 and 5, respectively, with the only exception that the responses used during the identification stage were contaminated with 5 percent random errors. It can be seen that the effects of these errors on the flutter dynamic pressure and the flutter frequency are not noticeable, yielding essentially the same values as for the case where no errors were introduced into the responses.

At this stage, it should be stated that the identification stage for this example needed no weighting for the high modes since the ratio between the highest and the lowest elastic modes was much smaller than in the beam example (example 2). Furthermore, it should be mentioned that the condition number of the matrix

$$\begin{bmatrix} T_R \\ T_I \end{bmatrix}$$

defined as the ratio between its highest and lowest singular values, was greatly improved by scaling its columns so as to yield equal maximum values in all columns. This scaling of columns led to the reduction of the condition number by at least *two orders of magnitude*, and thus leading to a solution less sensitive to contamination errors.

Example 4 — Flutter of the YF-17 Aircraft

The aerodynamic damping values for the DAST example were large as manifested by the scales of the abscissas of the root locus plots (after truncation) spanning from -50 to $+20$. It was felt that the identification procedure might falter in cases where a much lower aerodynamic damping is involved. Therefore, it was decided to test the identification method on the simulated data of the YF-17 aircraft which exhibits relatively low aerodynamic damping values. It will be shown that the (truncated) abscissas for this case span from -3 to $+4$ instead of the -50 to $+20$ for the DAST.

The plan view of the YF-17 aircraft is shown in figure 8. The flutter mathematical model of the aircraft includes 10 elastic modes, with natural frequencies varying from around 4.6 to around 62 Hz, and 2 rigid body modes (plunge and pitch), thus yielding a total of 12 modes. The excitation range for generating responses varies from 0.5 to 450 rad/sec. Two forcing vectors were used, with relative values between the various generalized forces in accordance with the procedure described for the DAST example.

Figure 9 shows the root locus plot using 'exact' aerodynamics with four lag terms and the 'exact' matrix coefficients. Here, and in all other YF-17 root locus plots presented in this work, the increments in the dynamic pressure are 5 lb/ft^2, with the dynamic pressure varying from $Q = 0$ to $Q = 110$ lb/ft^2. It can be seen that the flutter dynamic pressure is given by $Q_F = 84$ lb/ft^2, and the flutter frequency assumes the value $\omega_F = 37.4$ rad/sec.

Figures 10 and 11 present results for the case where no errors are introduced into the responses of the aircraft. Figure 10 shows the results obtained for the case where the system is identified at $Q = 0$ and at $Q = 20$ lb/ft^2. As can be seen, the results obtained are essentially exact, yielding $Q_F \approx 86.5$ lb/ft^2 and $\omega_F \approx 37.4$ rad/sec. In this example, a 1 percent structural damping was assumed. This was done considering the light aerodynamic damping in this example. Figure 11 shows results similar to those shown in figure 10, but for the case where the system is identified at $Q = 20$ and $Q = 35$ lb/ft^2. Here again, essentially exact values are obtained, with $Q_F \approx 87$ lb/ft^2 and $\omega_F \approx 36.8$ rad/sec. Similar to the DAST example, the origins of the root locus branches (at $Q = 0$) all lie on a straight line passing through the origin, with a slope of $g/2$ $(= 0.005)$ to the ordinate.

Figures 12 and 13 show the results obtained when the responses are contaminated with 5 percent random errors. It can be seen that the results are essentially exact if one allows for the effects of the 1 percent structural damping, yielding $Q_F = 86.5$ lb/ft^2 and $\omega_F = 37.4$ rad/sec for the system identified at $Q = 0$ and $Q = 20$ lb/ft^2, and $Q_F = 86.1$ lb/ft^2 and $\omega_F = 37.1$ rad/sec for the system identified at $Q = 20$ and $Q = 35$ lb/ft^2.

CONCLUDING REMARKS

The results presented in this work have shown that it is possible to identify the coefficients of the aeroelastic equations of motion of a dynamic system. The method is robust only if the coefficients of the equations of motion are real and the solution procedure uses this fact. It is also found that for robustness, more than one excitation vector needs to be used. This is true in all the cases treated herein, but is particularly essential when few modes have either the same value of frequency, or have frequencies with close values. In these latter cases, particular attention is needed in providing different excitation vectors, with particular emphasis on the generalized forces associated with the identical or close frequencies. This is particularly true for the rigid body modes of the aircraft where not only the frequencies are the same, or almost the same, but also the dampings assume identical or close values. These rigid body modes cannot be readily ignored during the identification procedure, since they have very strong aerodynamic coupling terms with the elastic modes, thus leading to some unexplained elastic distortions at low frequencies. These distortions greatly affect the identified results (when rigid body modes are ignored). The results obtained indicate that in all cases, two excitation vectors, with constant amplitudes throughout the frequency sweep, yield very good results. In the case of the beam where the elastic modes were spread over a large frequency range, some weighting of the equations was necessary. However, none of the other examples required any weighting, and it is not believed that practical flutter examples will ever need it. It was also found that scaling of the matrix columns, to equal maximum values before performing the generalized inverse procedure, improved robustness by reducing the ratio between the largest and the smallest singular values by at least two orders of magnitude.

Indeed, it was surprising to find out how good the introduced aerodynamic approximations were. Although these approximations appear to yield aerodynamic coefficients that are identical to the British representation, nevertheless they are different in that the British aerodynamic coefficients relate to a constant reduced frequency, whereas in the present method, the values of the identified coefficients include the effects of the reduced frequency on the different modes. Finally, it is gratifying to find that in both flutter examples treated herein, the flutter dynamic pressures and flutter frequencies are accurately predicted from simulated flight data gathered at dynamic pressures far below Q_F. However, it is still too early to state whether this method can be turned into a practical flight testing method. To do this, one needs to test the method in ground vibrations to gauge the errors introduced by the experimental setup and their effects on the identified system. In parallel, physical shaker locations should be analytically evaluated in some flutter examples, and the method need be reformulated for possible aerodynamic excitations using control surfaces with or without excitation vanes. Wind-tunnel and flight validation tests need to be performed, and, if successful, the method can be adapted to cope also with aircraft having active flutter suppression systems.

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TABLE 1. THE COEFFICIENT MATRICES FOR THE 5-DEGREES-OF-FREEDOM MASS-STRING SYSTEM

$$[C] = \begin{bmatrix} 5.4 & -1.8 & 0 & 0 & -1.8 \\ 1.8 & 5.4 & -1.8 & -1.8 & 0 \\ 0 & -1.8 & 3.6 & 1.8 & 0 \\ 0 & -1.8 & 1.8 & 5.4 & -1.8 \\ -1.8 & 0 & 0 & -1.8 & 5.4 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 600 & -300 & 0 & 0 & 0 \\ -300 & 600 & -300 & 0 & 0 \\ 0 & -300 & 600 & -300 & 0 \\ 0 & 0 & -300 & 600 & -300 \\ 0 & 0 & 0 & -300 & 600 \end{bmatrix}$$

$$[M] = \lceil I \rfloor$$

TABLE 2. EIGENVALUES OF EXAMPLE WITH 5 DEGREES OF FREEDOM

Mode no.	ω_n	λ: μ	λ: ω_d	ζ
1	8.96575	−0.241154	8.96251	0.0268972
2	17.3205	−2.70000	17.1088	0.15885
3	24.4949	−1.80000	24.4287	0.0734847
4	30.0000	−4.50000	29.6606	0.15
5	33.4607	−3.35885	33.2916	0.100382

TABLE 3. EIGENVECTORS OF EXAMPLE WITH 5 DEGREES OF FREEDOM

Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
0.2887	0.5000	0.5773	−0.5000	−0.2887
0.5000	0.5000	0	0.5000	0.5000
0.5773	0	−0.5773	0	−0.5773
0.5000	−0.5000	0	−0.5000	0.5000
0.2887	−0.5000	0.5773	0.5000	−0.2887

TABLE 4. IDENTIFICATION OF MASS-STRING SYSTEM USING EXACT RESPONSES

(n = 5, nfreq = 100, wfbegin = 0, wfend = 40.0, nfrce = 1)

SINGULAR VALUES SING(I)

$0.805129E+02$	$0.610284E+02$	$0.389048E+02$	$0.304252E+02$	$0.939689E+01$
$0.906324E+01$	$0.698794E+01$	$0.188909E+01$	$0.149979E+01$	$0.374279E+00$
$0.146845E+00$				

INPUT MATRIX K

$0.600000E+03$	$-0.300000E+03$	$0.000000E+00$	$0.000000E+00$	$0.000000E+00$
$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$	$0.000000E+00$	$0.000000E+00$
$0.000000E+00$	$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$	$0.000000E+00$
$0.000000E+00$	$0.000000E+00$	$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$
$0.000000E+00$	$0.000000E+00$	$0.000000E+00$	$-0.300000E+03$	$0.600000E+03$

IDENTIFIED MATRIX K

$0.600000E+03$	$-0.300000E+03$	$-0.223821E-12$	$0.913047E-12$	$-0.436984E-12$
$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$	$-0.318323E-11$	$0.224532E-11$
$-0.216360E-11$	$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$	$-0.909495E-12$
$0.448352E-11$	$-0.909495E-12$	$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$
$-0.440892E-11$	$0.116529E-11$	$0.682121E-12$	$-0.300000E+03$	$0.600000E+03$

INPUT MATRIX C

$0.540000E+01$	$-0.180000E+01$	$0.000000E+00$	$0.000000E+00$	$-0.180000E+01$
$-0.180000E+01$	$0.540000E+01$	$-0.180000E+01$	$-0.180000E+01$	$0.000000E+00$
$0.000000E+00$	$-0.180000E+01$	$0.360000E+01$	$-0.180000E+01$	$0.000000E+00$
$0.000000E+00$	$-0.180000E+01$	$-0.180000E+01$	$0.540000E+01$	$-0.180000E+01$
$-0.180000E+01$	$0.000000E+00$	$0.000000E+00$	$-0.180000E+01$	$0.540000E+01$

IDENTIFIED MATRIX C

$0.540000E+01$	$-0.180000E+01$	$0.204212E-13$	$0.807132E-13$	$-0.180000E+01$
$-0.180000E+01$	$0.540000E+01$	$-0.180000E+01$	$-0.180000E+01$	$-0.972555E-13$
$-0.959233E-13$	$-0.180000E+01$	$0.360000E+01$	$-0.180000E+01$	$0.101252E-12$
$0.245137E-12$	$-0.180000E+01$	$-0.180000E+01$	$0.540000E+01$	$-0.180000E+01$
$-0.180000E+01$	$0.429878E-12$	$-0.313194E-12$	$-0.180000E+01$	$0.540000E+01$

INPUT MATRIX F**(T)

$0.100000E+03$	$0.000000E+00$	$0.000000E+00$	$0.000000E+00$	$0.000000E+00$

IDENTIFIED MATRIX F**(T)

$0.100000E+03$	$0.125233E-12$	$-0.229150E-12$	$0.452971E-12$	$-0.367706E-12$

TABLE 5. IDENTIFICATION OF MASS-STRING SYSTEM USING EXACT RESPONSES WITH A REDUCED FREQUENCY EXCITATION RANGE (SMALLER THAN THE SYSTEM'S FREQUENCY RANGE)

(n = 5, nfreq = 100, wfbegin = 0, wfend = 27.0, nfrce = 1, ntruncate = 0)

SINGULAR VALUES SING(I)

$0.999001E+02$	$0.644175E+02$	$0.467998E+02$	$0.157411E+02$	$0.112027E+02$
$0.744586E+01$	$0.204753E+01$	$0.144638E+01$	$0.917581E+00$	$0.394669E-01$
$0.141374E-01$				

INPUT MATRIX K

$0.600000E+03$	$-0.300000E+03$	$0.000000E+00$	$0.000000E+00$	$0.000000E+00$
$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$	$0.000000E+00$	$0.000000E+00$
$0.000000E+00$	$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$	$0.000000E+00$
$0.000000E+00$	$0.000000E+00$	$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$
$0.000000E+00$	$0.000000E+00$	$0.000000E+00$	$-0.300000E+03$	$0.600000E+03$

IDENTIFIED MATRIX K

$0.600000E+03$	$-0.300000E+03$	$-0.380709E-10$	$0.414815E-10$	$-0.275548E-10$
$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$	$-0.355840E-10$	$0.235332E-10$
$-0.214584E-11$	$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$	$0.187583E-10$
$0.480327E-11$	$-0.710543E-11$	$-0.300000E+03$	$0.600000E+03$	$-0.300000E+03$
$-0.122498E-10$	$0.221121E-10$	$-0.285922E-10$	$-0.300000E+03$	$0.600000E+03$

INPUT MATRIX C

$0.540000E+01$	$-0.180000E+01$	$0.000000E+00$	$0.000000E+00$	$-0.180000E+01$
$-0.180000E+01$	$0.540000E+01$	$-0.180000E+01$	$-0.180000E+01$	$0.000000E+00$
$0.000000E+00$	$-0.180000E+01$	$0.360000E+01$	$-0.180000E+01$	$0.000000E+00$
$0.000000E+00$	$-0.180000E+01$	$-0.180000E+01$	$0.540000E+01$	$-0.180000E+01$
$-0.180000E+01$	$0.000000E+00$	$0.000000E+00$	$-0.180000E+01$	$0.540000E+01$

IDENTIFIED MATRIX C

$0.540000E+01$	$-0.180000E+01$	$-0.814460E-12$	$0.106604E-11$	$-0.180000E+01$
$-0.180000E+01$	$0.540000E+01$	$-0.180000E+01$	$-0.180000E+01$	$0.864198E-12$
$-0.300759E-12$	$-0.180000E+01$	$0.360000E+01$	$-0.180000E+01$	$-0.533795E-12$
$0.940359E-13$	$-0.180000E+01$	$-0.180000E+01$	$0.540000E+01$	$-0.180000E+01$
$-0.180000E+01$	$0.791159E-12$	$-0.135580E-11$	$-0.180000E+01$	$0.540000E+01$

INPUT MATRIX F**(T)

$0.100000E+03$	$0.000000E+00$	$0.000000E+00$	$0.000000E+00$	$0.000000E+00$

IDENTIFIED MATRIX F**(T)

$0.100000E+03$	$0.959233E-12$	$-0.814904E-12$	$0.536904E-12$	$-0.112710E-11$

TABLE 6. IDENTIFICATION OF MASS-STRING SYSTEM USING EXACT RESPONSES ROUNDED TO TWO DIGITS

(n = 5, nfreq = 100, wfbegin = 0, wfend = 40.0, nfrce = 1, ntruncate = 1, nsing = 0)

SINGULAR VALUES SING(I)

0.807032E+02	0.610709E+02	0.388331E+02	0.303359E+02	0.939338E+01
0.910814E+01	0.698627E+01	0.189392E+01	0.150738E+01	0.392295E+00
0.156853E+00				

INPUT MATRIX K

0.600000E+03	−0.300000E+03	0.000000E+00	0.000000E+00	0.000000E+00
−0.300000E+03	0.600000E+03	−0.300000E+03	0.000000E+00	0.000000E+00
0.000000E+00	−0.300000E+03	0.600000E+03	−0.300000E+03	0.000000E+00
0.000000E+00	0.000000E+00	−0.300000E+03	0.600000E+03	−0.300000E+03
0.000000E+00	0.000000E+00	0.000000E+00	−0.300000E+03	0.600000E+03

IDENTIFIED MATRIX K

0.586000E+03	−0.287116E+03	0.974557E+01	−0.316641E+02	0.263701E+02
−0.286081E+03	0.572231E+03	−0.285391E+03	0.615751E+01	−0.784484E+01
−0.206888E+01	−0.273644E+03	0.563153E+03	−0.265827E+03	−0.223147E+02
−0.682018E+01	−0.812837E+01	−0.268178E+03	0.550117E+03	−0.264529E+03
0.286527E+01	−0.600799E−01	−0.807834E+01	−0.282137E+03	0.588391E+03

INPUT MATRIX C

0.540000E+01	−0.180000E+01	0.000000E+00	0.000000E+00	−0.180000E+01
−0.180000E+01	0.540000E+01	−0.180000E+01	−0.180000E+01	0.000000E+00
0.000000E+00	−0.180000E+01	0.360000E+01	−0.180000E+01	0.000000E+00
0.000000E+00	−0.180000E+01	−0.180000E+01	0.540000E+01	−0.180000E+01
−0.180000E+01	0.000000E+00	0.000000E+00	−0.180000E+01	0.540000E+01

IDENTIFIED MATRIX C

0.561371E+01	−0.102137E+01	−0.710652E+00	0.703446E+00	−0.224857E+01
−0.235185E+01	0.509210E+01	−0.754696E+00	−0.304576E+01	0.112021E+01
0.689283E+00	−0.237106E+01	0.296940E+01	−0.638892E+00	−0.124476E+01
−0.366219E+00	−0.104917E+01	−0.211044E+01	0.516889E+01	−0.141857E+01
−0.170601E+01	−0.210497E+00	0.267120E+00	−0.184889E+00	0.541996E+01

INPUT MATRIX F**(T)

0.100000E+03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

IDENTIFIED MATRIX F**(T)

0.997144E+02	−0.781761E+00	0.186927E+01	−0.144992E+01	0.462906E+00

TABLE 7. IDENTIFICATION OF MASS-STRING SYSTEM USING EXACT RESPONSES AND TWO FORCING VECTORS
(n = 5, nfreq = 100, wfbegin = 0, wfend = 40.0, nfrce = 2, nderv = 0, ntruncate = 0, ndigits = 3, nsing = 0)

SINGULAR VALUES SING(I)

0.113830E + 03	0.820036E + 02	0.577449E + 02	0.407324E + 02	0.275308E + 02
0.127581E + 02	0.826819E + 01	0.774559E + 01	0.289272E + 01	0.280360E + 01
0.118339E + 01	0.714605E + 00			

INPUT MATRIX K

0.600000E + 03	−0.300000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00
−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.000000E + 00	0.000000E + 00
0.000000E + 00	−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.000000E + 00
0.000000E + 00	0.000000E + 00	−0.300000E + 03	0.600000E + 03	−0.300000E + 03
0.000000E + 00	0.000000E + 00	0.000000E + 00	−0.300000E + 03	0.600000E + 03

IDENTIFIED MATRIX K

0.600000E + 03	−0.300000E + 03	−0.710543E − 12	0.227374E − 12	−0.383693E − 12
−0.300000E + 03	0.600000E + 03	−0.300000E + 03	−0.568434E − 13	0.369482E − 12
0.207478E − 11	−0.300000E + 03	0.600000E + 03	−0.300000E + 03	−0.312639E − 12
−0.483169E − 12	−0.795808E − 12	−0.300000E + 03	0.600000E + 03	−0.300000E + 03
−0.179057E − 11	0.113687E − 11	−0.397904E − 12	−0.300000E + 03	0.600000E + 03

INPUT MATRIX C

0.540000E + 01	−0.180000E + 01	0.000000E + 00	0.000000E + 00	−0.180000E + 01
−0.180000E + 01	0.540000E + 01	−0.180000E + 01	−0.180000E + 01	0.000000E + 00
0.000000E + 00	−0.180000E + 01	0.360000E + 01	−0.180000E + 01	0.000000E + 00
0.000000E + 00	−0.180000E + 01	−0.180000E + 01	0.540000E + 01	−0.180000E + 01
−0.180000E + 01	0.000000E + 00	0.000000E + 00	−0.180000E + 01	0.540000E + 01

IDENTIFIED MATRIX C

0.540000E + 01	−0.180000E + 01	−0.737188E − 13	−0.762723E − 13	−0.180000E + 01
−0.180000E + 01	0.540000E + 01	−0.180000E + 01	−0.180000E + 01	0.370814E − 13
−0.852651E − 13	−0.180000E + 01	0.360000E + 01	−0.180000E + 01	0.657252E − 13
0.699441E − 13	−0.180000E + 01	−0.180000E + 01	0.540000E + 01	−0.180000E + 01
−0.180000E + 01	0.534017E − 13	−0.981437E − 13	−0.180000E + 01	0.540000E + 01

INPUT MATRIX F**(T)

0.100000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00	0.000000E + 00
0.000000E + 00	0.000000E + 00	0.000000E + 00	0.000000E + 00	0.100000E + 03

IDENTIFIED MATRIX F**(T)

0.100000E + 03	0.186517E − 12	−0.110134E − 12	0.834888E − 13	−0.162537E − 12
0.524025E − 13	−0.186517E − 12	0.214939E − 12	−0.188294E − 12	0.100000E + 03

TABLE 8. IDENTIFICATION OF MASS-STRING SYSTEM WITH RESPONSES ROUNDED TO TWO DIGITS, USING TWO FORCING VECTORS

(n = 5, nfreq = 100, wfbegin = 0, wfend = 40.0, nfrce = 2, nderv = 0, ntruncate = 1, ndigits = 2, nsing = 0)

SINGULAR VALUES SING(I)

0.114106E + 03	0.820496E + 02	0.576723E + 02	0.406613E + 02	0.274326E + 02
0.127880E + 02	0.825774E + 01	0.773024E + 01	0.289335E + 01	0.281242E + 01
0.119176E + 01	0.715271E + 00			

INPUT MATRIX K

0.600000E + 03	−0.300000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00
−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.000000E + 00	0.000000E + 00
0.000000E + 00	−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.000000E + 00
0.000000E + 00	0.000000E + 00	−0.300000E + 03	0.600000E + 03	−0.300000E + 03
0.000000E + 00	0.000000E + 00	0.000000E + 00	−0.300000E + 03	0.600000E + 03

IDENTIFIED MATRIX K

0.596843E + 03	−0.294724E + 03	−0.176846E + 01	−0.863264E + 00	0.114821E + 01
−0.296257E + 03	0.593089E + 03	−0.297839E + 03	0.470965E + 00	−0.196209E + 01
0.798027E + 00	−0.298236E + 03	0.599565E + 03	−0.298236E + 03	0.798027E + 00
−0.196209E + 01	0.470965E + 00	−0.297839E + 03	0.593089E + 03	−0.296257E + 03
0.114821E + 01	−0.863264E + 00	−0.176846E + 01	−0.294724E + 03	0.596843E + 03

INPUT MATRIX C

0.540000E + 01	−0.180000E + 01	0.000000E + 00	0.000000E + 00	−0.180000E + 01
−0.180000E + 01	0.540000E + 01	−0.180000E + 01	−0.180000E + 01	0.000000E + 00
0.000000E + 00	−0.180000E + 01	0.360000E + 01	−0.180000E + 01	0.000000E + 00
0.000000E + 00	−0.180000E + 01	−0.180000E + 01	0.540000E + 01	−0.180000E + 01
−0.180000E + 01	0.000000E + 00	0.000000E + 00	−0.180000E + 01	0.540000E + 01

IDENTIFIED MATRIX C

0.557895E + 01	−0.170116E + 01	0.978228E − 01	−0.850484E − 02	−0.180889E + 01
−0.193914E + 01	0.539453E + 01	−0.177369E + 01	−0.169785E + 01	0.865564E − 01
−0.199781E − 01	−0.196868E + 01	0.349926E + 01	−0.196868E + 01	−0.199781E − 01
0.865564E − 01	−0.169785E + 01	−0.177369E + 01	0.539453E + 01	−0.193914E + 01
−0.180889E + 01	−0.850484E − 02	0.978228E − 01	−0.170116E + 01	0.557895E + 01

INPUT MATRIX F**(T)

0.100000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00	0.000000E + 00
0.000000E + 00	0.000000E + 00	0.000000E + 00	0.000000E + 00	1.000000E + 03

IDENTIFIED MATRIX F**(T)

0.100219E + 03	−0.259031E + 00	0.140905E + 00	0.458221E − 01	0.918566E − 01
0.918566E − 01	0.458221E − 01	0.140905E + 00	−0.259031E + 00	−0.100219E + 03

TABLE 9. IDENTIFICATION OF MASS-STRING SYSTEM USING EXACT RESPONSES AND FIVE FORCING COLUMNS
(n = 5, nfreq = 100, wfbegin = 0, wfend = 40.0, nfrce = 5, nderv = 0, ntruncate = 0, ndigits = 3, nsing = 0)

SINGULAR VALUES SING(I)

0.278734E + 03	0.100354E + 03	0.811368E + 02	0.671963E + 02	0.579641E + 02
0.312493E + 02	0.995577E + 01	0.941263E + 01	0.907758E + 01	0.901629E + 01
0.861631E + 01	0.483233E + 01	0.415057E + 01	0.195707E + 01	0.192773E + 01

INPUT MATRIX K

0.600000E + 03	−0.300000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00
−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.000000E + 00	0.000000E + 00
0.000000E + 00	−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.000000E + 00
0.000000E + 00	0.000000E + 00	−0.300000E + 03	0.600000E + 03	−0.300000E + 03
0.000000E + 00	0.000000E + 00	0.000000E + 00	−0.300000E + 03	0.600000E + 03

IDENTIFIED MATRIX K

0.600000E + 03	−0.300000E + 03	0.214954E − 12	0.170530E − 12	0.568434E − 13
−0.300000E + 03	0.600000E + 03	−0.300000E + 03	−0.255795E − 12	0.397904E − 12
0.203662E − 11	−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.265534E − 12
−0.312639E − 12	0.454747E − 12	−0.300000E + 03	0.600000E + 03	−0.300000E + 03
0.142109E − 11	−0.156319E − 11	0.103560E − 11	−0.300000E + 03	0.600000E + 03

INPUT MATRIX C

0.540000E + 01	−0.180000E + 01	0.000000E + 00	0.000000E + 00	−0.180000E + 01
−0.180000E + 01	0.540000E + 01	−0.180000E + 01	−0.180000E + 01	0.000000E + 00
0.000000E + 00	−0.180000E + 01	0.360000E + 01	−0.180000E + 01	0.000000E + 00
0.000000E + 00	−0.180000E + 01	−0.180000E + 01	0.540000E + 01	−0.180000E + 01
−0.180000E + 01	0.000000E + 00	0.000000E + 00	−0.180000E + 01	0.540000E + 01

IDENTIFIED MATRIX C

0.540000E + 01	−0.180000E + 01	−0.651621E − 13	−0.275335E − 13	−0.180000E + 01
−0.180000E + 01	0.540000E + 01	−0.180000E + 01	−0.180000E + 01	0.293099E − 13
0.866313E − 13	−0.180000E + 01	0.360000E + 01	−0.180000E + 01	0.351885E − 14
0.648370E − 13	−0.180000E + 01	−0.180000E + 01	0.540000E + 01	−0.180000E + 01
−0.180000E + 01	−0.117240E − 12	0.129333E − 12	−0.180000E + 01	0.540000E + 01

INPUT MATRIX F**(T)

0.100000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00	0.000000E + 00
0.000000E + 00	0.000000E + 00	0.000000E + 00	0.000000E + 00	0.100000E + 03
0.000000E + 00	0.100000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00
0.000000E + 00	0.000000E + 00	1.000000E + 03	0.000000E + 00	0.000000E + 00
0.000000E + 00	0.000000E + 00	0.000000E + 00	1.000000E + 03	0.000000E + 00

IDENTIFIED MATRIX F**(T)

0.100000E + 03	−0.123279E − 11	−0.239980E − 12	0.586198E − 12	0.706990E − 12
−0.532907E − 13	0.355271E − 14	0.343008E − 12	−0.301981E − 12	0.100000E + 03
−0.319744E − 12	0.100000E + 03	−0.111739E − 12	−0.195399E − 12	−0.159872E − 12
−0.185281E − 12	0.327390E − 12	0.100000E + 03	0.233939E − 12	−0.251703E − 12
−0.188294E − 12	0.181188E − 12	0.182793E − 12	0.100000E + 03	−0.159872E − 12

TABLE 10. IDENTIFICATION OF MASS-STRING SYSTEM USING RESPONSES ROUNDED TO TWO DIGITS AND FIVE FORCING COLUMNS

(n = 5, nfreq = 100, wfbegin = 0, wfend = 40.0, nfrce = 5, ntruncate = 1, nsing = 0)

SINGULAR VALUES SING(I)

0.279289 E + 03	0.100350 E + 03	0.810687 E + 02	0.673052 E + 02	0.578963 E + 02
0.313014 E + 02	0.995633 E + 01	0.941196 E + 01	0.907725 E + 01	0.901826 E + 01
0.860725 E + 01	0.482696 E + 01	0.415145 E + 01	0.196900 E + 01	0.193199 E + 01

INPUT MATRIX K

0.600000 E + 03	−0.300000 E + 03	0.000000 E + 00	0.000000 E + 00	0.000000 E + 00
−0.300000 E + 03	0.600000 E + 03	−0.300000 E + 03	0.000000 E + 00	0.000000 E + 00
0.000000 E + 00	−0.300000 E + 03	0.600000 E + 03	−0.300000 E + 03	0.000000 E + 00
0.000000 E + 00	0.000000 E + 00	−0.300000 E + 03	0.600000 E + 03	−0.300000 E + 03
0.000000 E + 00	0.000000 E + 00	0.000000 E + 00	−0.300000 E + 03	0.600000 E + 03

IDENTIFIED MATRIX K

0.597600 E + 03	−0.296943 E + 03	−0.165657 E + 01	−0.843780 E − 01	0.107052 E + 00
−0.297177 E + 03	0.596373 E + 03	−0.297361 E + 03	−0.820422 E + 00	0.425542 E + 00
−0.164678 E + 01	−0.297408 E + 03	0.596766 E + 03	−0.297408 E + 03	−0.164678 E + 01
0.425542 E + 00	−0.820422 E + 00	−0.297361 E + 03	0.596373 E + 03	−0.297177 E + 03
0.107052 E + 00	−0.843780 E − 01	−0.165657 E + 01	−0.296943 E + 03	0.597600 E + 03

INPUT MATRIX C

0.540000 E + 01	−0.180000 E + 01	0.000000 E + 00	0.000000 E + 00	−0.180000 E + 01
−0.180000 E + 01	0.540000 E + 01	−0.180000 E + 01	−0.180000 E + 01	0.000000 E + 00
0.000000 E + 00	−0.180000 E + 01	0.360000 E + 01	−0.180000 E + 01	0.000000 E + 00
0.000000 E + 00	−0.180000 E + 01	−0.180000 E + 01	0.540000 E + 01	−0.180000 E + 01
−0.180000 E + 01	0.000000 E + 00	0.000000 E + 00	−0.180000 E + 01	0.540000 E + 01

IDENTIFIED MATRIX C

0.538372 E + 01	−0.179809 E + 01	−0.252365 E − 01	−0.163945 E − 01	−0.181218 E + 01
−0.178253 E + 01	0.539514 E + 01	−0.173583 E + 01	−0.180406 E + 01	0.375789 E − 01
−0.115444 E − 01	−0.179368 E + 01	0.350283 E + 01	−0.179368 E + 01	−0.115444 E − 01
0.375789 E − 01	−0.180406 E + 01	−0.173583 E + 01	0.539514 E + 01	−0.178253 E + 01
−0.181218 E + 01	−0.163945 E − 01	−0.252365 E − 01	−0.179809 E + 01	0.538372 E + 01

INPUT MATRIX F**(T)

0.100000 E + 03	0.000000 E + 00	0.000000 E + 00	0.000000 E + 00	0.000000 E + 00
0.000000 E + 00	0.000000 E + 00	0.000000 E + 00	0.000000 E + 00	0.100000 E + 03
0.000000 E + 00	0.100000 E + 03	0.000000 E + 00	0.000000 E + 00	0.000000 E + 00
0.000000 E + 00	0.000000 E + 00	1.000000 E + 03	0.000000 E + 00	0.000000 E + 00
0.000000 E + 00	0.000000 E + 00	0.000000 E + 00	0.100000 E + 03	0.000000 E + 00

IDENTIFIED MATRIX F**(T)

0.996082 E + 02	0.190199 E + 00	0.134360 E − 01	0.558700 E − 01	−0.148873 E − 01
−0.148873 E − 01	0.558700 E − 01	0.134360 E − 01	0.190199 E + 00	0.996082 E + 02
0.218553 E + 00	0.996508 E + 02	0.320199 E + 00	−0.179082 E + 00	0.815127 E − 01
0.133903 E + 00	0.655929 E + 00	0.990216 E + 02	0.655929 E + 00	0.133903 E + 00
0.815127 E − 01	−0.179082 E + 00	0.320199 E + 00	0.996508 E + 02	0.218553 E + 00

TABLE 11. IDENTIFICATION OF MASS-STRING SYSTEM WITH RESPONSES CONTAMINATED BY 5 PERCENT RANDOM ERRORS USING ONE FORCING VECTOR

(n = 5, nfreq = 100, wfbegin = 0, wfend = 40.0, nfrce = 1, nderv = 0, ntruncate = 0, ndigits = 0, nsing = 0, nrand = 1, ranpcent = 5.0)

SINGULAR VALUES SING(I)

0.810376E+02	0.611324E+02	0.389486E+02	0.305393E+02	0.947908E+01
0.916512E+01	0.697298E+01	0.188564E+01	0.150699E+01	0.412525E+00
0.221762E+00				

INPUT MATRIX K

0.600000E+03	-0.300000E+03	0.000000E+00	0.000000E+00	0.000000E+00
-0.300000E+03	0.600000E+03	-0.300000E+03	0.000000E+00	0.000000E+00
0.000000E+00	-0.300000E+03	0.600000E+03	-0.300000E+03	0.000000E+00
0.000000E+00	0.000000E+00	-0.300000E+03	0.600000E+03	-0.300000E+03
0.000000E+00	0.000000E+00	0.000000E+00	-0.300000E+03	0.600000E+03

IDENTIFIED MATRIX K

0.595943E+03	-0.254982E+03	-0.628979E+02	0.593688E+02	-0.418472E+02
-0.289290E+03	0.508327E+03	-0.156406E+03	-0.142342E+03	0.990645E+02
-0.645780E+00	-0.193836E+03	0.393238E+03	-0.699174E+02	-0.160582E+03
-0.106536E+02	-0.743161E+02	-0.120407E+03	0.374253E+03	-0.138458E+03
0.459025E+01	0.314581E+02	-0.760430E+02	-0.199994E+03	0.528483E+03

INPUT MATRIX C

0.540000E+01	-0.180000E+01	0.000000E+00	0.000000E+00	-0.180000E+01
-0.180000E+01	0.540000E+01	-0.180000E+01	-0.180000E+01	0.000000E+00
0.000000E+00	-0.180000E+01	0.360000E+01	-0.180000E+01	0.000000E+00
0.000000E+00	-0.180000E+01	-0.180000E+01	0.540000E+01	-0.180000E+01
-0.180000E+01	0.000000E+00	0.000000E+00	-0.180000E+01	0.540000E+01

IDENTIFIED MATRIX C

0.638825E+01	-0.256800E+01	-0.859907E+00	0.164740E+01	-0.350468E+01
-0.380275E+01	0.690763E+01	-0.147975E+00	-0.602997E+01	0.369787E+01
0.277855E+01	-0.466157E+01	0.245420E+01	0.320580E+01	-0.455018E+01
-0.226159E+01	0.116000E+01	-0.179366E+01	0.223212E+01	0.149027E+01
-0.705297E+00	-0.124906E+01	0.200627E+00	-0.734732E+00	0.413788E+01

INPUT MATRIX F**(T)

0.100000E+03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

IDENTIFIED MATRIX F**(T)

0.102411E+03	-0.331347E+01	0.637162E+01	-0.654855E+01	0.326556E+01

TABLE 11. Concluded.

	Identified system		Input system	
	Real	Imaginary	Real	Imaginary
λ	$-0.2392E+01$	$0.3438E+02$	$-0.3359E+01$	$0.3329E+02$
	$0.3221E+00$	$-0.4156E-01$	$0.2887E+00$	$0.1713E-15$
	$-0.5404E+00$	$0.7587E-01$	$-0.5000E+00$	$-0.2113E-14$
$\{x\}$	$0.5758E+00$	$0.0000E+00$	$0.5774E+00$	$0.0000E+00$
	$-0.4401E+00$	$-0.1071E+00$	$-0.5000E+00$	$-0.5711E-15$
	$0.2246E+00$	$0.9828E-01$	$0.2887E+00$	$-0.2284E-15$
λ	$-0.3898E+01$	$0.2891E+02$	$-0.4500E+01$	$0.2966E+02$
	$-0.5172E+00$	$-0.5276E-01$	$-0.5000E+00$	$0.4121E-15$
	$0.6099E+00$	$0.0000E+00$	$0.5000E+00$	$0.0000E+00$
$\{x\}$	$0.3381E+00$	$0.1481E+00$	$-0.1439E-14$	$0.3641E-15$
	$0.4282E-01$	$-0.3311E+00$	$-0.5000E+00$	$-0.6868E-15$
	$0.6161E-01$	$0.3260E+00$	$0.5000E+00$	$0.2060E-15$
λ	$-0.1870E+01$	$0.2439E+02$	$-0.1800E+01$	$0.2443E+02$
	$-0.5632E+00$	$-0.8426E-02$	$-0.5774E+00$	$0.7274E-15$
	$-0.1945E-01$	$0.4464E-02$	$0.6929E-15$	$0.6871E-15$
$\{x\}$	$0.5884E+00$	$0.0000E+00$	$0.5774E+00$	$0.0000E+00$
	$-0.2112E-02$	$0.2526E-01$	$-0.6103E-15$	$-0.3938E-15$
	$-0.5768E+00$	$-0.5298E-01$	$-0.5774E+00$	$-0.3810E-15$
λ	$-0.2661E+01$	$0.1711E+02$	$-0.2700E+01$	$0.1711E+02$
	$0.5018E+00$	$0.0000E+00$	$-0.5000E+00$	$-0.9099E-15$
	$0.5015E+00$	$0.1011E-01$	$-0.5000E+00$	$-0.4079E-15$
$\{x\}$	$-0.3410E-02$	$0.1263E-01$	$0.1697E-14$	$-0.9205E-15$
	$-0.4983E+00$	$-0.1199E-01$	$0.5000E+00$	$0.0000E+00$
	$-0.4978E+00$	$-0.1043E-01$	$0.5000E+00$	$0.3137E-16$
λ	$-0.2402E+00$	$0.8962E+01$	$-0.2412E+00$	$0.8963E+01$
	$0.2894E+00$	$0.5452E-02$	$0.2887E+00$	$0.1758E-15$
	$0.5006E+00$	$0.1672E-02$	$0.5000E+00$	$-0.1055E-15$
$\{x\}$	$0.5773E+00$	$0.0000E+00$	$0.5774E+00$	$0.0000E+00$
	$0.4993E+00$	$-0.4662E-02$	$0.5000E+00$	$-0.1407E-15$
	$0.2881E+00$	$-0.2236E-02$	$0.2887E+00$	$0.1934E-15$

TABLE 12. IDENTIFICATION OF THE MASS-STRING SYSTEM WITH RESPONSES CONTAMINATED BY 5 PERCENT ERRORS, USING TWO FORCING VECTORS

(n = 5, nfreq = 100, wfbegin = 0, wfend = 40, nfrce = 2, nderv = 0, ntruncate = 0, ndigits = 0, nsing = 0, nrand = 1, ranpcent = 5)

SINGULAR VALUES SING(1)

0.113636E + 03	0.821057E + 02	0.577159E + 02	0.407132E + 02	0.276802E + 02
0.127338E + 02	0.826123E + 01	0.776452E + 01	0.289924E + 01	0.281445E + 01
0.118886E + 01	0.736966E + 00			

INPUT MATRIX K

0.600000E + 03	−0.300000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00
−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.000000E + 00	0.000000E + 00
0.000000E + 00	−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.000000E + 00
0.000000E + 03	0.000000E + 00	−0.300000E + 03	0.600000E + 00	−0.300000E + 03
0.000000E + 00	0.000000E + 00	0.000000E + 00	−0.300000E + 03	0.600000E + 00

IDENTIFIED MATRIX K

0.594185E + 03	−0.291472E + 03	−0.656767E + 01	0.353434E + 01	−0.867322E + 00
−0.288592E + 03	0.576535E + 03	−0.277703E + 03	−0.142293E + 02	0.624185E + 01
−0.110985E + 02	−0.275517E + 03	0.573438E + 03	−0.278207E + 03	−0.817331E + 01
0.507562E + 01	−0.114907E + 02	−0.284552E + 03	0.584563E + 03	−0.293429E + 03
−0.104684E + 01	−0.125127E + 01	−0.202359E + 01	−0.297760E + 03	0.598571E + 03

INPUT MATRIX C

0.540000E + 01	−0.180000E + 01	0.000000E + 00	0.000000E + 00	−0.180000E + 01
−0.180000E + 01	0.540000E + 01	−0.180000E + 01	−0.180000E + 01	0.000000E + 00
0.000000E + 00	−0.180000E + 01	0.360000E + 01	−0.180000E + 01	0.000000E + 00
0.000000E + 00	−0.180000E + 01	−0.180000E + 01	0.540000E + 01	−0.180000E + 01
−0.180000E + 01	0.000000E + 00	0.000000E + 00	−0.180000E + 01	0.540000E + 01

IDENTIFIED MATRIX C

0.523275E + 01	−0.174659E + 01	−0.100059E + 00	−0.974456E − 01	−0.167062E + 01
−0.198011E + 01	0.541224E + 01	−0.166087E + 01	−0.166055E + 01	−0.109726E + 00
0.187856E + 00	−0.204428E + 01	0.334890E + 01	−0.217474E + 01	0.674523E − 01
−0.176856E − 01	−0.137403E + 01	−0.141109E + 01	0.568810E + 01	−0.160570E + 01
−0.191548E + 01	−0.108815E + 00	−0.275413E + 00	−0.195241E + 01	0.525610E + 01

INPUT MATRIX F**(T)

0.100000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00	0.000000E + 00
0.000000E + 00	0.000000E + 00	0.000000E + 00	0.000000E + 00	0.100000E + 03

IDENTIFIED MATRIX F**(T)

0.983363E+02	0.307570E+00	0.859843E+00	-0.122294E+01	0.499973E+00
0.106519E+01	-0.110180E+01	0.964407E+00	0.683939E-01	0.998547E+02

TABLE 12. Concluded.

	Identified system		Input system	
	Real	Imaginary	Real	Imaginary
λ	$-0.3446E+01$	$0.3329E+02$	$-0.3359E+01$	$0.3329E+02$
	$0.2869E+00$	$-0.9929E-02$	$0.2887E+00$	$0.1713E-15$
	$-0.4993E+00$	$0.6365E-02$	$-0.5000E+00$	$-0.2113E-14$
$\{x\}$	$0.5783E+00$	$0.0000E+00$	$0.5774E+00$	$0.0000E+00$
	$-0.5004E+00$	$0.8803E-02$	$-0.5000E+00$	$-0.5711E-15$
	$0.2882E+00$	$-0.1479E-01$	$0.2887E+00$	$-0.2284E-15$
λ	$-0.4395E+01$	$0.2968E+02$	$-0.4500E+01$	$0.2966E+02$
	$-0.4969E+00$	$0.1628E-01$	$-0.5000E+00$	$0.4121E-15$
	$0.5004E+00$	$-0.5428E-02$	$0.5000E+00$	$0.0000E+00$
$\{x\}$	$-0.2208E-02$	$-0.3406E-02$	$-0.1439E-14$	$0.3641E-15$
	$-0.4995E+00$	$0.1025E-01$	$-0.5000E+00$	$-0.6868E-15$
	$0.5028E+00$	$0.0000E+00$	$0.5000E+00$	$0.2060E-15$
λ	$-0.1729E+01$	$0.2443E+02$	$-0.1800E+01$	$0.2443E+02$
	$-0.5760E+00$	$0.1962E-01$	$-0.5774E+00$	$0.7274E-15$
	$-0.2400E-02$	$-0.9299E-02$	$0.6929E-15$	$0.6871E-15$
$\{x\}$	$0.5783E+00$	$0.0000E+00$	$0.5774E+00$	$0.0000E+00$
	$0.5488E-03$	$-0.7725E-02$	$-0.6103E-15$	$-0.3938E-15$
	$-0.5773E+00$	$0.1035E-01$	$-0.5774E+00$	$-0.3810E-15$
λ	$-0.2657E+01$	$0.1712E+02$	$-0.2700E+01$	$0.1711E+02$
	$-0.4993E+00$	$-0.4080E-02$	$-0.5000E+00$	$-0.9099E-15$
	$-0.4998E+00$	$-0.2474E-02$	$-0.5000E+00$	$-0.4079E-15$
$\{x\}$	$-0.9424E-03$	$0.4799E-02$	$0.1697E-14$	$-0.9205E-15$
	$0.5002E+00$	$0.2247E-03$	$0.5000E+00$	$0.0000E+00$
	$0.5007E+00$	$0.0000E+00$	$0.5000E+00$	$0.3137E-16$
λ	$-0.2418E+00$	$0.8962E+01$	$-0.2412E+00$	$0.8963E+01$
	$0.2891E+00$	$0.1210E-02$	$0.2887E+00$	$0.1758E-15$
	$0.5005E+00$	$-0.5869E-03$	$0.5000E+00$	$-0.1055E-15$
$\{x\}$	$0.5774E+00$	$0.0000E+00$	$0.5774E+00$	$0.0000E+00$
	$0.4992E+00$	$-0.1080E-01$	$0.5000E+00$	$-0.1407E-15$
	$0.2884E+00$	$0.7788E-04$	$0.2887E+00$	$0.1934E-15$

TABLE 13. IDENTIFICATION OF THE MASS-STRING SYSTEM WITH RESPONSES CONTAMINATED BY 5 PERCENT RANDOM ERRORS, USING FIVE FORCING VECTORS

(n = 5, nfreq =100, wfbegin = 0, wfend = 40, nfrce = 5, nderv = 0 ntruncate = 0, ndigits = 0, nsing = 0, nrand = 1, ranpcent = 5)

SINGULAR VALUES SING(I)

0.279744E + 03	0.100288E + 03	0.812816E + 02	0.674947E + 02	0.580173E + 02
0.313713E + 02	0.995525E + 01	0.941646E + 01	0.908247E + 01	0.901177E + 01
0.861687E + 01	0.484428E + 01	0.415906E + 01	0.199922E + 01	0.196229E + 01

INPUT MATRIX K

0.600000E + 03	−0.300000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00
−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.000000E + 00	0.000000E + 00
0.000000E + 00	−0.300000E + 03	0.600000E + 03	−0.300000E + 03	0.000000E + 00
0.000000E + 00	0.000000E + 00	−0.300000E + 03	0.600000E + 03	−0.300000E + 03
0.000000E + 00	0.000000E + 00	0.000000E + 00	−0.300000E + 03	0.600000E + 03

IDENTIFIED MATRIX K

0.591900E + 03	−0.290599E + 03	−0.464199E + 01	0.851605E + 00	0.151260E + 01
−0.288661E + 03	0.583348E + 03	−0.291235E + 03	−0.700458E + 00	−0.540150E + 01
−0.662487E + 01	−0.287400E + 03	0.586711E + 03	−0.288167E + 03	−0.282718E + 01
−0.237708E + 01	−0.943632E + 00	−0.287669E + 03	0.580414E + 03	−0.287120E + 03
0.379455E + 01	−0.407109E + 01	−0.381536E + 01	−0.288546E + 03	0.588515E + 03

INPUT MATRIX C

0.540000E + 01	−0.180000E + 01	0.000000E + 00	0.000000E + 00	−0.180000E + 01
−0.180000E + 01	0.540000E + 01	−0.180000E + 01	−0.180000E + 01	0.000000E + 00
0.000000E + 00	−0.180000E + 01	0.360000E + 01	−0.180000E + 01	0.000000E + 00
0.000000E + 00	−0.180000E + 01	−0.180000E + 01	0.540000E + 01	−0.180000E + 01
−0.180000E + 01	0.000000E + 00	0.000000E + 00	−0.180000E + 01	0.540000E + 01

IDENTIFIED MATRIX C

0.538023E + 01	−0.163016E + 01	0.680588E − 01	−0.598664E − 03	−0.168049E + 01
−0.169194E + 01	0.506214E + 01	−0.181617E + 01	−0.162600E + 01	−0.154139E + 00
−0.159521E + 00	−0.160840E + 01	0.334577E + 01	−0.188682E + 01	−0.260642E − 01
0.127531E − 01	−0.167744E + 01	−0.153805E + 01	0.522405E + 01	−0.162362E + 01
−0.172936E + 01	−0.112703E + 00	−0.518368E − 01	−0.160310E + 01	0.530303E + 01

INPUT MATRIX F**(T)

0.100000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00	0.000000E + 00
0.000000E + 00	0.000000E + 00	0.000000E + 00	0.000000E + 00	0.100000E + 03
0.000000E + 00	0.100000E + 03	0.000000E + 00	0.000000E + 00	0.000000E + 00
0.000000E + 00	0.000000E + 00	0.100000E + 03	0.000000E + 00	0.000000E + 00
0.000000E + 00	0.000000E + 00	0.000000E + 00	0.100000E + 03	0.000000E + 00

IDENTIFIED MATRIX F**(T)

0.999927E + 02	0.108633E + 01	−0.116052E + 01	−0.129725E + 00	0.193322E + 00
0.115166E + 01	−0.177106E + 01	−0.930904E − 02	0.143839E + 01	0.984132E + 02
0.663155E + 00	0.969599E + 02	0.247974E + 01	0.759462E + 00	−0.151197E + 01
−0.813813E + 00	0.103218E + 01	0.971246E + 02	0.341816E + 01	−0.129490E + 01
−0.958776E + 00	0.211859E + 01	0.605688E − 01	0.969814E + 02	0.231097E + 01

TABLE 13. Concluded.

	Identified system		Input system	
	Real	Imaginary	Real	Imaginary
λ	$-0.3200E+01$	$0.3331E+02$	$-0.3359E+01$	$0.3329E+02$
	$0.2856E+00$	$0.1153E-01$	$0.2887E+00$	$0.1713E-15$
	$-0.4967E+00$	$-0.1260E-01$	$-0.5000E+00$	$-0.2113E-14$
$\{x\}$	$0.5770E+00$	$0.0000E+00$	$0.5774E+00$	$0.0000E+00$
	$-0.5032E+00$	$0.5074E-02$	$-0.5000E+00$	$-0.5711E-15$
	$0.2920E+00$	$-0.1572E-02$	$0.2887E+00$	$-0.2284E-15$
λ	$-0.4246E+01$	$0.2970E+02$	$-0.4500E+01$	$0.2966E+02$
	$0.4998E+00$	$0.3023E-01$	$-0.5000E+00$	$0.4121E-15$
	$-0.4952E+00$	$-0.2047E-01$	$0.5000E+00$	$0.0000E+00$
$\{x\}$	$-0.7987E-02$	$0.2386E-02$	$-0.1439E-14$	$0.3641E-15$
	$0.5041E+00$	$0.0000E+00$	$-0.5000E+00$	$-0.6868E-15$
	$-0.4995E+00$	$-0.1633E-02$	$0.5000E+00$	$0.2060E-15$
λ	$-0.2398E+00$	$0.8963E+01$	$-0.2412E+00$	$0.8963E+01$
	$0.2888E+00$	$-0.2463E-02$	$0.2887E+00$	$0.1758E-15$
	$0.5003E+00$	$0.3370E-03$	$0.5000E+00$	$-0.1055E-15$
$\{x\}$	$0.5774E+00$	$0.0000E+00$	$0.5774E+00$	$0.0000E+00$
	$0.4997E+00$	$-0.4708E-02$	$0.5000E+00$	$-0.1407E-15$
	$0.2885E+00$	$-0.2799E-02$	$0.2887E+00$	$0.1934E-15$
λ	$-0.2674E+01$	$0.1711E+02$	$-0.2700E+01$	$0.1711E+02$
	$-0.4995E+00$	$-0.3862E-02$	$-0.5000E+00$	$-0.9099E-15$
	$-0.4997E+00$	$-0.6449E-02$	$-0.5000E+00$	$-0.4079E-15$
$\{x\}$	$-0.7201E-03$	$0.3104E-02$	$0.1697E-14$	$-0.9205E-15$
	$0.5000E+00$	$0.5524E-02$	$0.5000E+00$	$0.0000E+00$
	$0.5007E+00$	$0.0000E+00$	$0.5000E+00$	$0.3137E-16$
λ	$-0.1798E+01$	$0.2443E+02$	$-0.1800E+01$	$0.2443E+02$
	$-0.5773E+00$	$0.1467E-02$	$-0.5774E+00$	$0.7274E-15$
	$0.1570E-03$	$-0.1683E-02$	$0.6929E-15$	$0.6871E-15$
$\{x\}$	$0.5774E+00$	$0.0000E+00$	$0.5774E+00$	$0.0000E+00$
	$-0.3479E-03$	$-0.1367E-02$	$-0.6103E-15$	$-0.3938E-15$
	$-0.5773E+00$	$0.3366E-02$	$-0.5774E+00$	$-0.3810E-15$

TABLE 14. IDENTIFICATION RESULTS OF A SIMPLY SUPPORTED BEAM, USING TWO EXCITATION VECTORS AND EXACT RESPONSES OF THE FIRST SEVEN MODES ONLY

(n = 7, nfreq = 600, wfbegin = 0, wfend = 1.05, nfrcev = 2, nderv = 0, ntruncate = 0, ndigits = 0, nsing = 0, ngeneq = 7, ζ = 0.03, nrand = 0, ranpcent = 0)

INPUT MATRIX [F] TRANSPOSED

−0.100000 E + 02	0.000000 E + 00	0.000000 E + 00	0.400000 E + 02	0.000000 E + 00
0.000000 E + 00	−0.100000 E + 02			
0.000000 E + 00	0.100000 E + 02	−0.600000 E + 02	0.000000 E + 00	0.600000 E + 02
−0.100000 E + 02	0.000000 E + 00			

SINGULAR VALUES SING(I)

0.115942 E + 04	0.183762 E + 03	0.676501 E + 02	0.510157 E + 02	0.257955 E + 02
0.249105 E + 02	0.235481 E + 02	0.206399 E + 02	0.162942 E + 02	0.150041 E + 02
0.140976 E + 02	0.122655 E + 02	0.620207 E + 01	0.553820 E + 01	0.489011 E + 01
0.301421 E + 01				

IDENTIFIED MATRIX K

0.194333 E + 00	−0.198867 E + 00	0.104084 E + 00	−0.432996 E − 01	0.207858 E − 01
−0.103836 E − 01	0.441206 E − 02	−0.198867 E + 00	0.298417 E + 00	−0.242166 E + 00
0.124870 E + 00	−0.536832 E − 01	0.251978 E − 01	−0.103836 E − 01	0.104084 E + 00
−0.242166 E + 00	0.319203 E + 00	−0.252550 E + 00	0.129282 E + 00	−0.536832 E − 01
0.207858 E − 01	−0.432996 E − 01	0.124870 E + 00	−0.252550 E + 00	0.323615 E + 00
−0.252550 E + 00	0.124870 E + 00	−0.432996 E − 01	0.207858 E − 01	−0.536832 E − 01
0.129282 E + 00	−0.252550 E + 00	0.319203 E + 00	−0.242166 E + 00	0.104084 E + 00
−0.103836 E − 01	0.251978 E − 01	−0.536832 E − 01	0.124870 E + 00	−0.242166 E + 00
0.298417 E + 00	−0.198867 E + 00	0.441205 E − 02	−0.103835 E − 01	0.207857 E − 01
−0.432996 E − 01	0.104084 E + 00	−0.198867 E + 00	0.194333 E + 00	

IDENTIFIED MATRIX C

0.221459 E − 01	−0.141026 E − 01	0.295618 E − 02	−0.109798 E − 02	0.507200 E − 03
−0.249120 E − 03	0.105045 E − 03	−0.141026 E − 01	0.251020 E − 01	−0.152006 E − 01
0.346338 E − 02	−0.134710 E − 02	0.612245 E − 03	−0.249120 E − 03	0.295618 E − 02
−0.152006 E − 01	0.256092 E − 01	−0.154497 E − 01	0.356842 E − 02	−0.134710 E − 02
0.507200 E − 03	−0.109798 E − 02	0.346338 E − 02	−0.154497 E − 01	0.257143 E − 01
−0.154497 E − 01	0.346338 E − 02	−0.109798 E − 02	0.507200 E − 03	−0.134710 E − 02
0.356842 E − 02	−0.154497 E − 01	0.256092 E − 01	−0.152006 E − 01	0.295618 E − 02
−0.249120 E − 03	0.612245 E − 03	−0.134710 E − 02	0.346338 E − 02	−0.152006 E − 01
0.251020 E − 01	−0.141026 E − 01	0.105044 E − 03	−0.249120 E − 03	0.507199 E − 03
−0.109798 E − 02	0.295618 E − 02	−0.141026 E − 01	0.221459 E − 01	

IDENTIFIED MATRIX [F] TRANSPOSED

−0.154553 E − 01	0.624034 E − 02	−0.457675 E − 02	0.708310 E − 01	−0.457675 E − 02
0.624034 E − 02	−0.154553 E − 01			
−0.122724 E − 01	0.373314 E − 01	−0.900742 E − 01	0.550067 E − 08	0.900742 E − 01
−0.373314 E − 01	0.122724 E − 01			

TABLE 14. Concluded.

EIGENVALUES AND EIGENVECTORS OF IDENTIFIED SYSTEM

	Real	Imaginary	Real	Imaginary
λ	$-0.300000E-01$	$0.999550E+00$	$-0.153061E-01$	$0.509974E+00$
	$-0.127617E+00$	$0.138783E+00$	$-0.322855E+00$	$0.521317E+00$
	$0.235806E+00$	$-0.256438E+00$	$0.247103E+00$	$-0.398999E+00$
	$-0.308095E+00$	$0.335052E+00$	$0.133731E+00$	$-0.215937E+00$
$\{x\}$	$0.333479E+00$	$-0.362658E+00$	$-0.349456E+00$	$0.564270E+00$
	$-0.308095E+00$	$0.335052E+00$	$0.133731E+00$	$-0.215937E+00$
	$0.235805E+00$	$-0.256438E+00$	$0.247103E+00$	$-0.398999E+00$
	$-0.127617E+00$	$0.138783E+00$	$-0.322855E+00$	$0.521317E+00$
λ	$-0.220408E-01$	$0.734363E+00$	$-0.979592E-02$	$0.326384E+00$
	$0.344239E+00$	$-0.264681E+00$	$-0.228487E+00$	$0.650636E+00$
	$-0.486827E+00$	$0.374315E+00$	$-0.122446E-07$	$0.348675E-07$
	$0.344239E+00$	$-0.264681E+00$	$0.228487E+00$	$-0.650636E+00$
$\{x\}$	$0.782669E-07$	$-0.601784E-07$	$0.244892E-07$	$-0.697349E-07$
	$-0.344239E+00$	$0.264681E+00$	$-0.228487E+00$	$0.650636E+00$
	$0.486827E+00$	$-0.374315E+00$	$-0.367337E-07$	$0.104602E-06$
	$-0.344238E+00$	$0.264681E+00$	$0.228487E+00$	$-0.650636E+00$
λ	$-0.551020E-02$	$0.183591E+00$	$-0.612245E-03$	$0.203990E-01$
	$-0.170202E-01$	$0.469741E+00$	$-0.280115E-01$	$0.529895E+00$
	$-0.130267E-01$	$0.359524E+00$	$-0.517586E-01$	$0.979119E+00$
	$0.705001E-02$	$-0.194573E+00$	$-0.676258E-01$	$0.127928E+01$
$\{x\}$	$0.184226E-01$	$-0.508444E+00$	$-0.731977E-01$	$0.138468E+01$
	$0.705002E-02$	$-0.194573E+00$	$-0.676258E-01$	$0.127928E+01$
	$-0.130267E-01$	$0.359524E+00$	$-0.517586E-01$	$0.979119E+00$
	$-0.170202E-01$	$0.469741E+00$	$-0.280115E-01$	$0.529895E+00$
λ	$-0.244898E-02$	$0.815959E-01$		
	$-0.127306E+01$	$0.198449E+01$		
	$-0.180037E+01$	$0.280649E+01$		
	$-0.127306E+01$	$0.198449E+01$		
$\{x\}$	$-0.964816E-07$	$0.150399E-06$		
	$0.127306E+01$	$-0.198449E+01$		
	$0.180037E+01$	$-0.280649E+01$		
	$0.127306E+01$	$-0.198449E+01$		

TABLE 15. IDENTIFICATION OF A SIMPLY SUPPORTED BEAM USING TWO EXCITATION VECTORS AND EXACT RESPONSES COMPUTED FROM THE FIRST 36 MODES

(n = 7, nfreq = 600, wfbegin = 0, wfend = 1.05, nfrcev = 2, nderv = 0, ntruncate = 0, ndigits = 0, nsing = 0, ngeneq = 36, ζ = 0.03, nrand = 0, ranpcent = 0)

INPUT MATRIX [F] TRANSPOSED

−0.100000E + 02	0.000000E + 00	0.000000E + 00	0.400000E + 02	0.000000E + 00
0.000000E + 00	−0.100000E + 02			

0.000000E + 00	0.100000E + 02	−0.600000E + 02	0.000000E + 00	0.600000E + 02
−0.100000E + 02	0.000000E + 00			

SINGULAR VALUES SING(I)

0.115942E + 04	0.183763E + 03	0.676513E + 02	0.510163E + 02	0.256999E + 02
0.249300E + 02	0.235497E + 02	0.207162E + 02	0.162879E + 02	0.150063E + 02
0.141307E + 02	0.122596E + 02	0.620508E + 01	0.557900E + 01	0.488090E + 01
0.299861E + 01				

IDENTIFIED MATRIX K

0.194154E + 00	−0.198739E + 00	0.104114E + 00	−0.432543E − 01	0.204214E − 01
−0.981493E − 02	0.399402E − 02	−0.198931E + 00	0.298646E + 00	−0.242462E + 00
0.124891E + 00	−0.532921E − 01	0.246617E − 01	−0.100359E − 01	0.104582E + 00
−0.242851E + 00	0.319701E + 00	−0.252778E + 00	0.129492E + 00	−0.540456E − 01
0.211090E − 01	−0.449151E − 01	0.126791E + 00	−0.253560E + 00	0.324001E + 00
−0.253560E + 00	0.126791E + 00	−0.449151E − 01	0.211090E − 01	−0.540456E − 01
0.129492E + 00	−0.252778E + 00	0.319701E + 00	−0.242851E + 00	0.104582E + 00
−0.100359E − 01	0.246617E − 01	−0.532921E − 01	0.124891E + 00	−0.242462E + 00
0.298646E + 00	−0.198931E + 00	0.399401E − 02	−0.981492E − 02	0.204213E − 01
−0.432542E − 01	0.104114E + 00	−0.198739E + 00	0.194154E + 00	

IDENTIFIED MATRIX C

0.217318E − 01	−0.133241E − 01	0.234334E − 02	−0.491067E − 03	−0.207902E − 03
0.923199E − 04	−0.204040E − 04	−0.137570E − 01	0.245029E − 01	−0.150367E − 01
0.324092E − 02	−0.992960E − 03	0.773777E − 03	−0.403958E − 03	0.330504E − 02
−0.159270E − 01	0.267096E − 01	−0.164758E − 01	0.459770E − 02	−0.239643E − 02
0.106889E − 02	−0.272362E − 02	0.673824E − 02	−0.193230E − 01	0.293090E − 01
−0.193230E − 01	0.673824E − 02	−0.272362E − 02	0.106889E − 02	−0.239643E − 02
0.459770E − 02	−0.164758E − 01	0.267096E − 01	−0.159270E − 01	0.330504E − 02
−0.403959E − 03	0.773779E − 03	−0.992962E − 03	0.324092E − 02	−0.150367E − 01
0.245029E − 01	−0.137570E − 01	−0.204039E − 04	0.923197E − 04	−0.207901E − 03
−0.491068E − 03	0.234333E − 02	−0.133241E − 01	0.217318E − 01	

IDENTIFIED MATRIX [F] TRANSPOSED

−0.164802E − 01	0.826640E − 02	−0.748574E − 02	0.723171E − 01	−0.748574E − 02
0.826640E − 02	−0.164802E − 01			

−0.115302E − 01	0.368466E − 01	−0.890205E − 01	0.155712E − 08	0.890205E − 01
−0.368466E − 01	0.115302E − 01			

TABLE 15. Concluded.

EIGENVALUES AND EIGENVECTORS OF IDENTIFIED SYSTEM

	Real	Imaginary	Real	Imaginary
λ	$-0.326715E-01$	$0.100052E+01$	$-0.151021E-01$	$0.508125E+00$
	$-0.137688E+00$	$0.126354E+00$	$-0.868560E+00$	$0.201118E+00$
	$0.257350E+00$	$-0.232012E+00$	$0.662351E+00$	$-0.153594E+00$
	$-0.340528E+00$	$0.301044E+00$	$0.364247E+00$	$-0.839748E-01$
$\{x\}$	$0.374594E+00$	$-0.322730E+00$	$-0.934986E+00$	$0.216767E+00$
	$-0.340528E+00$	$0.301044E+00$	$0.364247E+00$	$-0.839747E-01$
	$0.257350E+00$	$-0.232012E+00$	$0.662351E+00$	$-0.153594E+00$
	$-0.137688E+00$	$0.126354E+00$	$-0.868560E+00$	$0.201118E+00$
λ	$-0.216203E-01$	$0.735270E+00$	$-0.976925E-02$	$0.326353E+00$
	$0.240588E+00$	$-0.319456E+00$	$-0.152368E+01$	$-0.126739E+00$
	$-0.340289E+00$	$0.452064E+00$	$0.146985E-03$	$0.167436E-03$
	$0.240523E+00$	$-0.319117E+00$	$0.152176E+01$	$0.124938E+00$
$\{x\}$	$0.554076E-07$	$-0.744552E-07$	$0.162007E-06$	$0.120932E-07$
	$-0.240523E+00$	$0.319117E+00$	$-0.152176E+01$	$-0.124938E+00$
	$0.340289E+00$	$-0.452064E+00$	$-0.147309E-03$	$-0.167461E-03$
	$-0.240588E+00$	$0.319456E+00$	$0.152368E+01$	$0.126739E+00$
λ	$-0.543022E-02$	$0.183542E+00$	$-0.598416E-03$	$0.204009E-01$
	$0.132048E+00$	$0.659236E+00$	$-0.153027E+00$	$0.119589E+01$
	$0.100591E+00$	$0.505031E+00$	$-0.282772E+00$	$0.220972E+01$
	$-0.552808E-01$	$-0.272466E+00$	$-0.369534E+00$	$0.288712E+01$
$\{x\}$	$-0.143211E+00$	$-0.713236E+00$	$-0.400043E+00$	$0.312498E+01$
	$-0.552808E-01$	$-0.272466E+00$	$-0.369534E+00$	$0.288712E+01$
	$0.100591E+00$	$0.505031E+00$	$-0.282772E+00$	$0.220972E+01$
	$0.132048E+00$	$0.659236E+00$	$-0.153027E+00$	$0.119589E+01$
λ	$-0.240710E-02$	$0.815859E-01$		
	$-0.274884E+01$	$0.209235E+01$		
	$-0.388817E+01$	$0.295844E+01$		
	$-0.274910E+01$	$0.209214E+01$		
$\{x\}$	$-0.210089E-06$	$0.157266E-06$		
	$0.274910E+01$	$-0.209214E+01$		
	$0.388817E+01$	$-0.295844E+01$		
	$0.274884E+01$	$-0.209235E+01$		

TABLE 16. IDENTIFICATION RESULTS OF A SIMPLY SUPPORTED BEAM, USING TWO EXCITATION VECTORS WITH RESPONSES CONTAMINATED WITH 5 PERCENT RANDOM ERRORS AND A WEIGHTING EQUAL TO THE VALUE OF THE EXCITATION FREQUENCY

(n = 7, nfreq = 600, wfbegin = 0, wfend = 1.05, nfrcev = 2, nderv = 1, ntruncate = 0, ndigits = 0, nsing = 0, ngeneq = 36, ζ = 0.03, nrand = 1, ranpcent = 5.0)

INPUT MATRIX [F] TRANSPOSED

$-0.100000E+02$	$0.000000E+00$	$0.000000E+00$	$0.400000E+02$	$0.000000E+00$
$0.000000E+00$	$-0.100000E+02$			
$0.000000E+00$	$0.100000E+02$	$-0.600000E+02$	$0.000000E+00$	$0.600000E+02$
$-0.100000E+02$	$0.000000E+00$			

SINGULAR VALUES SING(I)

$0.237224E+02$	$0.174868E+02$	$0.157876E+02$	$0.153762E+02$	$0.152525E+02$
$0.139268E+02$	$0.120465E+02$	$0.102533E+02$	$0.558572E+01$	$0.548147E+01$
$0.451168E+01$	$0.314783E+01$	$0.240969E+01$	$0.156592E+01$	$0.145036E+01$
$0.775893E+00$				

IDENTIFIED MATRIX K

$0.194037E+00$	$-0.197202E+00$	$0.104531E+00$	$-0.447668E-01$	$0.192443E-01$
$-0.966129E-02$	$0.258762E-02$	$-0.199102E+00$	$0.296341E+00$	$-0.242142E+00$
$0.125673E+00$	$-0.515716E-01$	$0.254169E-01$	$-0.950152E-02$	$0.104795E+00$
$-0.240188E+00$	$0.319467E+00$	$-0.252661E+00$	$0.127390E+00$	$-0.536652E-01$
$0.205805E-01$	$-0.444199E-01$	$0.123388E+00$	$-0.251767E+00$	$0.320933E+00$
$-0.250127E+00$	$0.124360E+00$	$-0.447381E-01$	$0.211586E-01$	$-0.525785E-01$
$0.128766E+00$	$-0.250131E+00$	$0.319142E+00$	$-0.241455E+00$	$0.105488E+00$
$-0.104754E-01$	$0.244911E-01$	$-0.531836E-01$	$0.122582E+00$	$-0.242522E+00$
$0.297521E+00$	$-0.199418E+00$	$0.396093E-02$	$-0.948409E-02$	$0.199482E-01$
$-0.415137E-01$	$0.103638E+00$	$-0.197591E+00$	$0.194141E+00$	

IDENTIFIED MATRIX C

$0.262822E-01$	$-0.481538E-02$	$0.114940E-01$	$0.840471E-02$	$0.105123E-01$
$0.104392E-01$	$0.613716E-02$	$-0.214737E-01$	$0.132307E-01$	$-0.268256E-01$
$-0.721552E-02$	$-0.114161E-01$	$-0.108850E-01$	$-0.645165E-02$	$0.104369E-01$
$-0.672375E-02$	$0.326974E-01$	$-0.109544E-01$	$0.820027E-02$	$0.252650E-02$
$0.574916E-02$	$-0.600837E-02$	$0.532516E-02$	$-0.128883E-01$	$0.314542E-01$
$-0.929245E-02$	$0.119797E-01$	$-0.241120E-02$	$0.247068E-02$	$-0.447280E-02$
$-0.219335E-02$	$-0.267557E-01$	$0.134074E-01$	$-0.286496E-01$	$-0.227531E-02$
$-0.223546E-04$	$0.292216E-02$	$0.520518E-02$	$0.134540E-01$	$-0.165027E-02$
$0.391381E-01$	$-0.558495E-02$	$-0.594804E-03$	$-0.124381E-02$	$-0.352173E-02$
$-0.646521E-02$	$-0.474596E-02$	$-0.220491E-01$	$0.166587E-01$	

IDENTIFIED MATRIX [F] TRANSPOSED

$-0.166417E-01$	$0.807763E-02$	$-0.626105E-02$	$0.685902E-01$	$-0.669438E-02$
$0.841053E-02$	$-0.168291E-01$			
$-0.127982E-01$	$0.389780E-01$	$-0.897487E-01$	$0.557997E-04$	$0.900097E-01$
$-0.387366E-01$	$0.129581E-01$			

TABLE 16. Concluded.

EIGENVALUES AND EIGENVECTORS OF IDENTIFIED SYSTEM

	Real	Imaginary	Real	Imaginary
λ	$-0.313972E-01$	$0.997180E+00$	$-0.132651E-01$	$0.507559E+00$
	$0.146912E+00$	$-0.118723E+00$	$0.790101E+00$	$-0.328308E+00$
	$-0.273202E+00$	$0.217969E+00$	$-0.614467E+00$	$0.258013E+00$
	$0.359497E+00$	$-0.282088E+00$	$-0.327083E+00$	$0.133653E+00$
$\{x\}$	$-0.389716E+00$	$0.301566E+00$	$0.851886E+00$	$-0.359346E+00$
	$0.360092E+00$	$-0.279645E+00$	$-0.340308E+00$	$0.134216E+00$
	$-0.273526E+00$	$0.215638E+00$	$-0.595234E+00$	$0.253976E+00$
	$0.146199E+00$	$-0.116340E+00$	$0.785755E+00$	$-0.328943E+00$
λ	$-0.222915E-01$	$0.734884E+00$	$-0.972426E-02$	$0.326416E+00$
	$-0.189441E+00$	$0.335951E+00$	$0.116336E+01$	$-0.670824E+00$
	$0.268932E+00$	$-0.471846E+00$	$-0.771226E-03$	$-0.302198E-04$
	$-0.189100E+00$	$0.334469E+00$	$-0.115978E+01$	$0.669000E+00$
$\{x\}$	$0.231661E-03$	$-0.198012E-03$	$0.937335E-03$	$-0.977512E-03$
	$0.188239E+00$	$-0.335391E+00$	$0.117194E+01$	$-0.675492E+00$
	$-0.266216E+00$	$0.472775E+00$	$0.358804E-03$	$0.161411E-02$
	$0.188359E+00$	$-0.333609E+00$	$-0.117755E+01$	$0.676923E+00$
λ	$-0.556408E-02$	$0.184213E+00$	$-0.175400E-02$	$0.217996E-01$
	$0.205240E+01$	$-0.968370E+00$	$-0.516602E+01$	$0.443004E+01$
	$0.158480E+01$	$-0.732183E+00$	$-0.943054E+01$	$0.792588E+01$
	$-0.863179E+00$	$0.408463E+00$	$-0.123650E+02$	$0.104736E+02$
$\{x\}$	$-0.221694E+01$	$0.105813E+01$	$-0.134179E+02$	$0.113346E+02$
	$-0.850616E+00$	$0.406636E+00$	$-0.123457E+02$	$0.103220E+02$
	$0.157362E+01$	$-0.739520E+00$	$-0.961499E+01$	$0.807067E+01$
	$0.206999E+01$	$-0.965541E+00$	$-0.519187E+01$	$0.431724E+01$
λ	$-0.243812E-02$	$0.826824E-01$		
	$0.378141E+01$	$-0.124705E+01$		
	$0.543130E+01$	$-0.182984E+01$		
	$0.377352E+01$	$-0.130637E+01$		
$\{x\}$	$-0.829433E-02$	$-0.287140E-01$		
	$-0.383286E+01$	$0.128532E+01$		
	$-0.546253E+01$	$0.190522E+01$		
	$-0.386050E+01$	$0.133358E+01$		

TABLE 17. IDENTIFICATION RESULTS OF A SIMPLY SUPPORTED BEAM, USING TWO EXCITATION VECTORS WITH RESPONSES CONTAMINATED WITH 5 PERCENT RANDOM ERRORS AND A WEIGHTING EQUAL TO THE EXCITATION FREQUENCY (FREQC = 0.143)

(n = 7, nfreq = 600, wfbegin = 0, wfend = 1.05, nfrcev = 2, slope = 1.0, freqc = 0.143, ntruncate = 0, ndigits = 0, nsing = 0, ngeneq = 36, ζ = 0.03, nrand = 1, ranpcent = 5.0)

INPUT MATRIX [F] TRANSPOSED

−0.100000E + 02	0.000000E + 00	0.000000E + 00	0.400000E + 02	0.000000E + 00
0.000000E + 00	−0.100000E + 02			
0.000000E + 00	0.100000E + 02	−0.600000E + 02	0.000000E + 00	0.600000E + 02
−0.100000E + 02	0.000000E + 00			

SINGULAR VALUES SING(I)

0.165538E + 03	0.265154E + 02	0.175181E + 02	0.157818E + 02	0.154157E + 02
0.143656E + 02	0.123272E + 02	0.102826E + 02	0.572261E + 01	0.558805E + 01
0.460589E + 01	0.341661E + 01	0.339965E + 01	0.241690E + 01	0.225009E + 01
0.156487E + 01				

IDENTIFIED MATRIX K

0.191268E + 00	−0.189166E + 00	0.949225E − 01	−0.366809E − 01	0.123112E − 01
−0.423379E − 02	−0.807628E − 03	−0.204011E + 00	0.286695E + 00	−0.214866E + 00
0.939855E − 01	−0.301565E − 01	0.209433E − 01	−0.124693E − 01	0.123832E + 00
−0.237463E + 00	0.276081E + 00	−0.193518E + 00	0.911082E − 01	−0.561219E − 01
0.369615E − 01	−0.682306E − 01	0.126117E + 00	−0.208132E + 00	0.256750E + 00
−0.210622E + 00	0.129428E + 00	−0.664270E − 01	0.361135E − 01	−0.527017E − 01
0.976965E − 01	−0.205038E + 00	0.287898E + 00	−0.240579E + 00	0.119472E + 00
−0.143292E − 01	0.202502E − 01	−0.363314E − 01	0.100892E + 00	−0.223970E + 00
0.291086E + 00	−0.203114E + 00	0.353561E − 02	−0.555050E − 02	0.129156E − 01
−0.339695E − 01	0.957134E − 01	−0.192589E + 00	0.193633E + 00	

IDENTIFIED MATRIX C

0.325611E − 01	0.890313E − 02	0.218270E − 01	0.187690E − 01	0.187896E − 01
0.228144E − 01	0.121872E − 01	−0.318580E − 01	−0.998016E − 02	−0.388969E − 01
−0.176628E − 01	−0.196196E − 01	−0.317158E − 01	−0.166098E − 01	0.208421E − 01
0.173162E − 01	0.378228E − 01	−0.100135E − 01	0.945393E − 02	0.244870E − 01
0.162525E − 01	−0.123922E − 01	−0.114572E − 01	−0.985126E − 02	0.388534E − 01
−0.512958E − 02	−0.530941E − 02	−0.100132E − 01	−0.270362E − 03	−0.463345E − 02
−0.177505E − 01	−0.441811E − 01	−0.364018E − 03	−0.258721E − 01	−0.299473E − 02
0.676093E − 02	0.112436E − 01	0.227179E − 01	0.307692E − 01	0.133590E − 01
0.441799E − 01	−0.727912E − 03	−0.404830E − 02	−0.532077E − 02	−0.116405E − 01
−0.142738E − 01	−0.119017E − 01	−0.248995E − 01	0.138725E − 01	

IDENTIFIED MATRIX [F] TRANSPOSED

−0.141370E − 01	0.276009E − 02	0.497559E − 03	0.622341E − 01	−0.126673E − 02
0.438555E − 02	−0.147322E − 01			
−0.133482E − 01	0.401532E − 01	−0.911538E − 01	0.865668E − 03	0.900501E − 01
−0.390701E − 01	0.131305E − 01			

TABLE 17. Concluded.

EIGENVALUES AND EIGENVECTORS OF IDENTIFIED SYSTEM

	Real	Imaginary	Real	Imaginary
λ	−0.226244 E − 01	0.941635 E + 00	−0.145619 E − 01	0.463997 E + 00
	0.104314 E + 00	−0.161006 E + 00	0.360523 E + 00	−0.457260 E + 00
	−0.189204 E + 00	0.288699 E + 00	−0.270052 E + 00	0.343298 E + 00
	0.250850 E + 00	−0.372212 E + 00	−0.207899 E + 00	0.275907 E + 00
$\{x\}$	−0.284159 E + 00	0.407824 E + 00	0.514694 E + 00	−0.689617 E + 00
	0.276029 E + 00	−0.390375 E + 00	−0.273547 E + 00	0.365992 E + 00
	−0.217993 E + 00	0.308782 E + 00	−0.281785 E + 00	0.366737 E + 00
	0.119389 E + 00	−0.169492 E + 00	0.444747 E + 00	−0.587045 E + 00
λ	−0.223600 E − 01	0.730970 E + 00	−0.995764 E − 02	0.324418 E + 00
	−0.296017 E − 01	0.357516 E + 00	−0.107296 E + 01	0.728541 E + 00
	0.439583 E − 01	−0.501931 E + 00	−0.954964 E − 02	0.990619 E − 02
	−0.298898 E − 01	0.355523 E + 00	0.108986 E + 01	−0.746506 E + 00
$\{x\}$	0.208935 E − 03	−0.436421 E − 02	−0.147426 E − 01	0.139928 E − 01
	0.286879 E − 01	−0.347741 E + 00	−0.108239 E + 01	0.736158 E + 00
	−0.412769 E − 01	0.494417 E + 00	0.987291 E − 02	−0.108948 E − 01
	0.296058 E − 01	−0.350852 E + 00	0.107716 E + 01	−0.728473 E + 00
λ	−0.596619 E − 02	0.182808 E + 00	−0.633177 E − 03	0.204200 E − 01
	−0.136115 E + 01	0.124983 E + 01	−0.399930 E + 01	0.471547 E + 01
	−0.106456 E + 01	0.975268 E + 00	−0.743550 E + 01	0.840955 E + 01
	0.612368 E + 00	−0.500339 E + 00	−0.971781 E + 01	0.111727 E + 02
$\{x\}$	0.144026 E + 01	−0.141106 E + 01	−0.105472 E + 02	0.121141 E + 02
	0.567736 E + 00	−0.539516 E + 00	−0.967205 E + 01	0.111150 E + 02
	−0.105771 E + 01	0.997902 E + 00	−0.737276 E + 01	0.878663 E + 01
	−0.136202 E + 01	0.126901 E + 01	−0.397045 E + 01	0.472814 E + 01
λ	−0.236946 E − 02	0.817272 E − 01		
	−0.810124 E + 00	−0.171025 E + 01		
	−0.118387 E + 01	−0.246641 E + 01		
	−0.829845 E + 00	−0.168920 E + 01		
$\{x\}$	−0.426308 E − 02	0.238807 E − 02		
	0.839828 E + 00	0.171779 E + 01		
	0.122304 E + 01	0.246024 E + 01		
	0.855094 E + 00	0.173676 E + 01		

TABLE 18. IDENTIFICATION RESULTS OF A SIMPLY SUPPORTED BEAM, USING TWO EXCITATION VECTORS WITH RESPONSES CONTAMINATED WITH 5 PERCENT RANDOM ERRORS, A WEIGHTING EQUAL TO THE EXCITATION FREQUENCY, AND 14 SUBRANGES OF EXCITATION FREQUENCIES

(n = 7, nranges = 14, nfreq = 6, 25, 6, 25, 6, 25, 6, 25, 6, 25, 6, 25, 6, 25, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, wfbegin = 0, $1.76E-002, 2.32E-002, 7.22E-002, 9.12E-002$, 0.161, 0.205, 0.286, 0.364, 0.44, 0.56, 0.644, 0.82, 0.88, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, wfend = $1.76E-002, 2.32E-002, 7.22E-002, 9.12E-002$, 0.161, 0.205, 0.286, 0.364, 0.44, 0.56, 0.644, 0.82, 0.88, 1.12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, nfrcev = 2, slope = 1.0, freqc = 0, ntruncate = 0, ndigits = 0, nsing = 0, ngeneq = 36, ζ = 0.03, nrand = 1, ranpcent = 5.0)

INPUT MATRIX [F] TRANSPOSED

$0.000000E+00$	$0.000000E+00$	$0.000000E+00$	$0.500000E+02$	$0.000000E+00$
$0.000000E+00$	$0.000000E+00$			
$0.000000E+00$	$0.100000E+02$	$-0.600000E+02$	$0.000000E+00$	$0.600000E+02$
$-0.100000E+02$	$0.000000E+00$			

SINGULAR VALUES SING(I)

$0.154728E+01$	$0.763904E+00$	$0.697949E+00$	$0.672401E+00$	$0.641849E+00$
$0.596769E+00$	$0.490812E+00$	$0.444835E+00$	$0.291199E+00$	$0.235478E+00$
$0.207062E+00$	$0.177566E+00$	$0.146209E+00$	$0.992819E-01$	$0.614743E-01$
$0.476817E-01$				

IDENTIFIED MATRIX K

$0.193354E+00$	$-0.197943E+00$	$0.101628E+00$	$-0.406954E-01$	$0.177047E-01$
$-0.945415E-02$	$0.357336E-02$	$-0.197432E+00$	$0.298574E+00$	$-0.240295E+00$
$0.122998E+00$	$-0.500582E-01$	$0.248456E-01$	$-0.917192E-02$	$0.102093E+00$
$-0.240734E+00$	$0.314237E+00$	$-0.247315E+00$	$0.123093E+00$	$-0.532834E-01$
$0.196581E-01$	$-0.412701E-01$	$0.122582E+00$	$-0.247054E+00$	$0.317501E+00$
$-0.245141E+00$	$0.122862E+00$	$-0.418250E-01$	$0.191931E-01$	$-0.516991E-01$
$0.125272E+00$	$-0.248485E+00$	$0.314779E+00$	$-0.241140E+00$	$0.103661E+00$
$-0.981144E-02$	$0.246518E-01$	$-0.522248E-01$	$0.124163E+00$	$-0.240821E+00$
$0.298263E+00$	$-0.199496E+00$	$0.372971E-02$	$-0.916989E-02$	$0.183821E-01$
$-0.412183E-01$	$0.101132E+00$	$-0.196322E+00$	$0.193769E+00$	

IDENTIFIED MATRIX C

$0.180176E-01$	$-0.206961E-01$	$-0.170855E-02$	$-0.432949E-02$	$-0.144930E-02$
$0.214452E-02$	$0.294256E-02$	$-0.683449E-02$	$0.338379E-01$	$-0.987023E-02$
$0.102743E-01$	$0.187719E-02$	$0.976027E-03$	$-0.433979E-02$	$-0.342985E-02$
$-0.258424E-01$	$0.180825E-01$	$-0.210031E-01$	$-0.262011E-02$	$-0.313762E-02$
$0.188661E-02$	$0.495965E-02$	$0.181619E-01$	$-0.879909E-02$	$0.366078E-01$
$-0.680676E-02$	$0.144186E-01$	$0.140785E-04$	$-0.507772E-02$	$-0.142892E-01$
$-0.155315E-02$	$-0.254373E-01$	$0.172355E-01$	$-0.260363E-01$	$-0.115776E-02$
$0.803895E-02$	$0.976601E-02$	$0.637976E-02$	$0.114627E-01$	$-0.593177E-02$
$0.369223E-01$	$-0.850457E-02$	$-0.534445E-02$	$-0.759989E-02$	$-0.477437E-02$
$-0.509375E-02$	$-0.122459E-02$	$-0.196320E-01$	$0.184871E-01$	

IDENTIFIED MATRIX [F] TRANSPOSED

$-0.149776E-02$	$0.312827E-02$	$-0.231955E-02$	$0.803777E-01$	$-0.323477E-02$
$0.330069E-02$	$-0.161662E-02$			
$-0.133848E-01$	$0.393812E-01$	$-0.900116E-01$	$-0.439619E-03$	$0.908613E-01$
$-0.398884E-01$	$0.137242E-01$			

TABLE 18. Concluded.

EIGENVALUES AND EIGENVECTORS OF IDENTIFIED SYSTEM

	Real	Imaginary	Real	Imaginary
λ	$-0.333129E-01$	$0.990801E+00$	$-0.143679E-01$	$0.508946E+00$
	$0.137753E+00$	$-0.126715E+00$	$0.820647E-01$	$0.475874E+00$
	$-0.260102E+00$	$0.235549E+00$	$-0.648627E-01$	$-0.358249E+00$
	$0.336037E+00$	$-0.306315E+00$	$-0.333049E-01$	$-0.198755E+00$
$\{x\}$	$-0.368147E+00$	$0.325396E+00$	$0.906997E-01$	$0.518059E+00$
	$0.343169E+00$	$-0.303283E+00$	$-0.330466E-01$	$-0.199993E+00$
	$-0.263621E+00$	$0.237781E+00$	$-0.646619E-01$	$-0.363117E+00$
	$0.139747E+00$	$-0.126105E+00$	$0.825250E-01$	$0.476766E+00$
λ	$-0.224623E-01$	$0.734756E+00$	$-0.956934E-02$	$0.326546E+00$
	$0.681439E-01$	$0.354674E+00$	$-0.152734E+01$	$-0.106448E+00$
	$-0.995215E-01$	$-0.501762E+00$	$0.333349E-03$	$-0.445495E-03$
	$0.678031E-01$	$0.353226E+00$	$0.151766E+01$	$0.851435E-01$
$\{x\}$	$0.229698E-03$	$-0.374665E-03$	$0.787200E-03$	$0.681113E-03$
	$-0.697351E-01$	$-0.355207E+00$	$-0.153013E+01$	$-0.977051E-01$
	$0.924596E-01$	$0.502190E+00$	$0.177169E-02$	$-0.242099E-02$
	$-0.660361E-01$	$-0.352054E+00$	$0.152831E+01$	$0.883642E-01$
λ	$-0.526201E-02$	$0.184412E+00$	$-0.203771E-02$	$0.212589E-01$
	$0.211342E+01$	$0.916286E+00$	$-0.133305E+01$	$0.315479E+01$
	$0.160245E+01$	$0.705553E+00$	$-0.243086E+01$	$0.581827E+01$
	$-0.879670E+00$	$-0.374084E+00$	$-0.321803E+01$	$0.774930E+01$
$\{x\}$	$-0.230402E+01$	$-0.100166E+01$	$-0.340816E+01$	$0.829124E+01$
	$-0.880857E+00$	$-0.377411E+00$	$-0.316847E+01$	$0.768464E+01$
	$0.162417E+01$	$0.701211E+00$	$-0.241293E+01$	$0.589532E+01$
	$0.209181E+01$	$0.929200E+00$	$-0.132634E+01$	$0.320070E+01$
λ	$-0.258316E-02$	$0.823099E-01$		
	$0.418531E+01$	$0.321875E+00$		
	$0.592605E+01$	$0.384253E+00$		
	$0.419057E+01$	$0.223368E+00$		
$\{x\}$	$0.409464E-01$	$-0.127519E+00$		
	$-0.416480E+01$	$-0.435140E+00$		
	$-0.590389E+01$	$-0.506359E+00$		
	$-0.414171E+01$	$-0.324332E+00$		

TABLE 19. IDENTIFICATION RESULTS OF A SIMPLY SUPPORTED BEAM, USING TWO EXCITATION VECTORS AND RESPONSES CONTAMINATED WITH 5 PERCENT RANDOM ERRORS, WITH A WEIGHTING PROPORTIONAL TO THE EXCITATION FREQUENCY (FREQC = 0.04), 14 FREQUENCY SUBRANGES, AND 14 DISPLACEMENT POINTS

(n = 7, nranges = 14, nfreq = 6, 25, 6, 25, 6, 25, 6, 25, 6, 25, 6, 25, 6, 25, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, wfbegin = 0, $1.76E-002$, $2.32E-002$, $7.22E-002$, $9.12E-002$, 0.161, 0.205, 0.286, 0.364, 0.44, 0.56, 0.644, 0.82, 0.88, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, wfend = $1.76E-002$, $2.32E-002$, $7.22E-002$, $9.12E-002$, 0.161, 0.205, 0.286, 0.364, 0.44, 0.56, 0.644, 0.82, 0.88, 1.12, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, nfrcev = 2, slope = 1.0, freqc = $4.0E-002$, ntruncate = 0, ndigits = 0, nsing = 0, ngeneq = 36, ζ = 0.03, nrand = 1, ranpcent = 5.0)

INPUT MATRIX [F] TRANSPOSED

$0.000000E+00$	$0.000000E+00$	$0.000000E+00$	$0.500000E+02$	$0.000000E+00$
$0.000000E+00$	$0.000000E+00$			
$0.000000E+00$	$0.100000E+02$	$-0.600000E+02$	$0.000000E+00$	$0.600000E+02$
$-0.100000E+02$	$0.000000E+00$			

SINGULAR VALUES SING(I)

$0.150828E+01$	$0.693129E+00$	$0.687091E+00$	$0.343049E+00$	$0.321284E+00$
$0.312184E+00$	$0.246224E+00$	$0.227471E+00$	$0.146511E+00$	$0.117884E+00$
$0.103657E+00$	$0.905946E-01$	$0.742751E-01$	$0.502146E-01$	$0.346260E-01$
$0.307532E-01$				

IDENTIFIED MATRIX K

$0.417696E-03$	$-0.376368E-04$	$0.577179E-04$	$-0.116407E-03$	$0.309257E-03$
$-0.948580E-03$	$-0.726364E-03$	$-0.293144E-05$	$0.673807E-02$	$0.165086E-04$
$-0.246370E-04$	$0.118328E-03$	$0.836068E-03$	$0.281029E-02$	$-0.576808E-04$
$0.327139E-04$	$0.336914E-01$	$-0.994578E-04$	$-0.210542E-04$	$0.209427E-02$
$-0.975619E-04$	$0.198869E-03$	$0.193379E-04$	$-0.109194E-03$	$0.106530E+00$
$0.185181E-03$	$0.718313E-03$	$-0.118919E-03$	$-0.661482E-04$	$-0.384768E-03$
$0.267418E-04$	$-0.319997E-03$	$0.258991E+00$	$0.197241E-03$	$0.145409E-02$
$-0.392488E-03$	$0.637571E-03$	$0.452296E-03$	$-0.245112E-03$	$-0.642817E-03$
$0.538941E+00$	$0.270588E-03$	$0.443842E-03$	$0.745831E-03$	$0.131058E-02$
$0.130944E-02$	$0.243329E-02$	$-0.348219E-03$	$0.986410E+00$	

IDENTIFIED MATRIX C

$0.102570E-02$	$-0.932000E-03$	$0.235836E-03$	$-0.341272E-03$	$-0.200306E-03$
$0.216651E-03$	$0.104659E-02$	$-0.469890E-04$	$0.491991E-02$	$0.627058E-03$
$0.185366E-03$	$-0.980276E-04$	$0.156793E-04$	$0.274113E-02$	$0.547004E-03$
$-0.334275E-04$	$0.108109E-01$	$0.736009E-05$	$-0.149580E-03$	$-0.283810E-03$
$-0.688270E-03$	$0.356010E-02$	$0.926320E-03$	$0.981349E-04$	$0.195570E-01$
$-0.661696E-03$	$-0.698401E-03$	$0.140366E-02$	$-0.740172E-02$	$-0.246667E-02$
$-0.212517E-02$	$-0.347155E-05$	$0.311698E-01$	$-0.192195E-03$	$0.636600E-03$
$0.585366E-02$	$0.746605E-02$	$0.430854E-03$	$0.767516E-03$	$0.252518E-03$
$0.442020E-01$	$0.299515E-03$	$0.596656E-01$	$0.207848E-02$	$0.126733E-01$
$0.437421E-03$	$-0.441657E-02$	$0.974302E-03$	$0.584156E-01$	

IDENTIFIED MATRIX [F] TRANSPOSED

$0.210166E-01$	$-0.182372E-03$	$-0.210425E-01$	$-0.235332E-03$	$0.209816E-01$
$0.103170E-03$	$-0.205939E-01$			
$0.728854E-04$	$-0.172108E-01$	$0.151139E-03$	$0.389858E-01$	$0.313262E-04$
$-0.549493E-01$	$-0.268878E-03$			

TABLE 19. Concluded.

EIGENVALUES AND EIGENVECTORS OF IDENTIFIED SYSTEM

	Real	Imaginary	Real	Imaginary
λ	$-0.291867E-01$	$0.992726E+00$	$-0.155838E-01$	$0.508665E+00$
	$-0.130905E-02$	$0.181201E-03$	$-0.196555E-02$	$0.143550E-02$
	$-0.817368E-04$	$0.400412E-02$	$-0.684683E-03$	$0.578240E-03$
	$0.473325E-03$	$-0.561057E-03$	$0.288623E-03$	$0.590042E-03$
$\{x\}$	$-0.130569E-02$	$0.962781E-03$	$-0.118525E-02$	$0.478200E-02$
	$0.728860E-03$	$0.208503E-02$	$-0.186203E+01$	$0.463903E+00$
	$-0.167267E-03$	$0.877872E-03$	$-0.402491E-02$	$0.206364E-02$
	$0.662414E+00$	$0.772220E+00$	$0.490241E-02$	$-0.736849E-02$
λ	$-0.221033E-01$	$0.733795E+00$	$-0.977345E-02$	$0.326246E+00$
	$0.164676E-02$	$-0.156350E-02$	$0.369938E-02$	$0.260896E-02$
	$-0.167904E-02$	$0.108943E-02$	$0.616446E-03$	$-0.189395E-02$
	$-0.410293E-02$	$0.333366E-02$	$0.404650E-02$	$-0.673022E-03$
$\{x\}$	$-0.978995E-03$	$0.243450E-02$	$-0.302620E+01$	$0.343042E+00$
	$-0.429218E-03$	$0.103778E-02$	$-0.640637E-02$	$0.925718E-03$
	$-0.108910E+01$	$0.644654E+00$	$-0.154629E-02$	$0.196912E-02$
	$0.115579E-03$	$0.217372E-02$	$0.460804E-02$	$-0.194712E-03$
λ	$-0.545432E-03$	$0.204317E-01$	$-0.245120E-02$	$0.820300E-01$
	$-0.363861E+00$	$-0.124249E+01$	$0.905285E-02$	$0.560156E-02$
	$0.693616E-03$	$-0.917895E-03$	$-0.698376E+00$	$0.428296E+00$
$\{x\}$	$-0.103411E-02$	$-0.197433E-02$	$0.724993E-03$	$-0.628659E-03$
	$-0.167317E-03$	$0.256213E-02$	$0.443937E-03$	$0.412003E-03$
	$0.647801E-03$	$-0.515616E-03$	$-0.140696E-02$	$0.120021E-03$
	$-0.543437E-03$	$-0.828621E-03$	$0.131706E-02$	$0.294413E-03$
λ	$-0.540642E-02$	$0.183466E+00$	$0.628536E-03$	$-0.253410E-03$
	$-0.134446E-02$	$-0.155821E-04$		
	$-0.158621E-02$	$-0.181347E-02$		
	$-0.500162E+00$	$0.332308E+00$		
$\{x\}$	$-0.654223E-03$	$0.655906E-03$		
	$-0.515363E-03$	$-0.932810E-03$		
	$0.489200E-03$	$-0.215634E-03$		
	$0.147443E-02$	$0.792550E-03$		

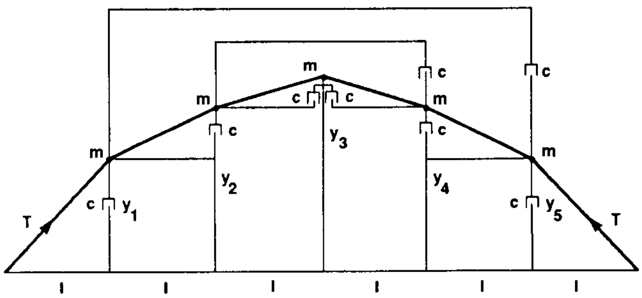

Figure 1. Description of the dynamic system used as an example.

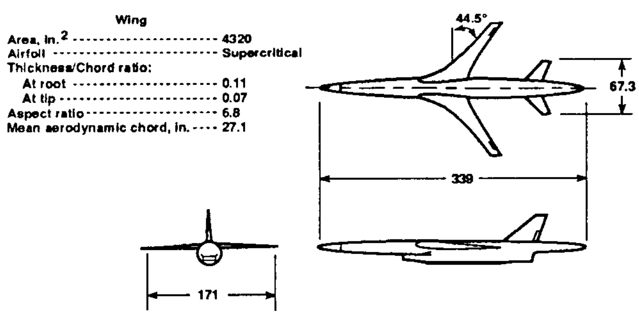

Figure 2. Three-view drawing of drone research vehicle. All linear dimensions are in inches.

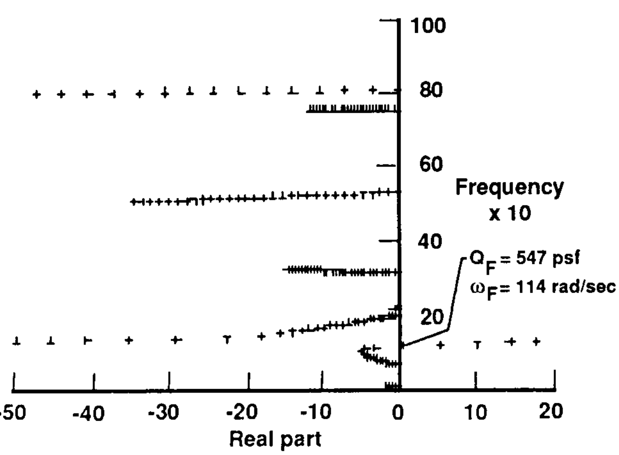

Figure 3. Root locus plot of "exact" system.

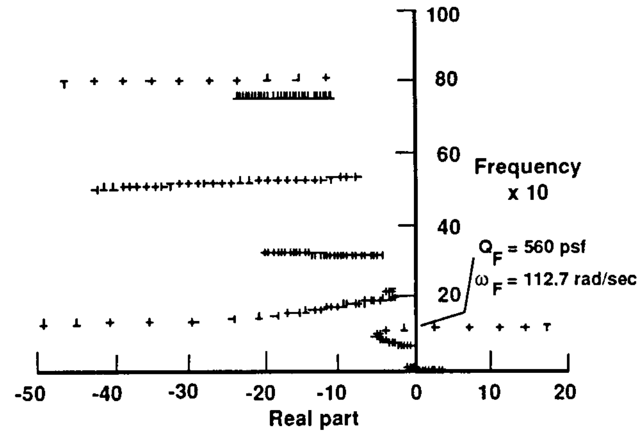

Figure 4. Drone for aerodynamic and structural testing (DAST) — results obtained using "exact" responses for $Q = 0$ and $Q = 150$ lb/ft^2.

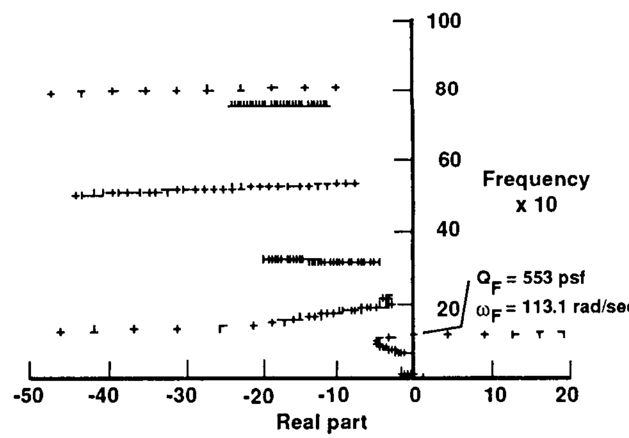

Figure 5. DAST—results obtained using "exact" responses for $Q = 150$ and $Q = 250$ lb/ft^2.

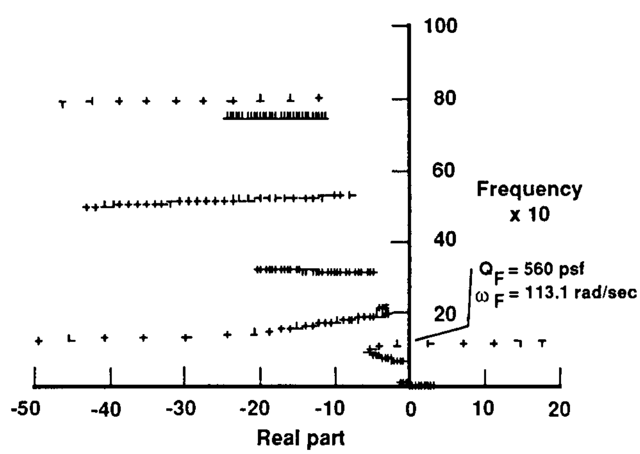

Figure 6. DAST—results obtained using responses with 5 percent random errors, for $Q = 0$ and $Q = 150$ lb/ft^2.

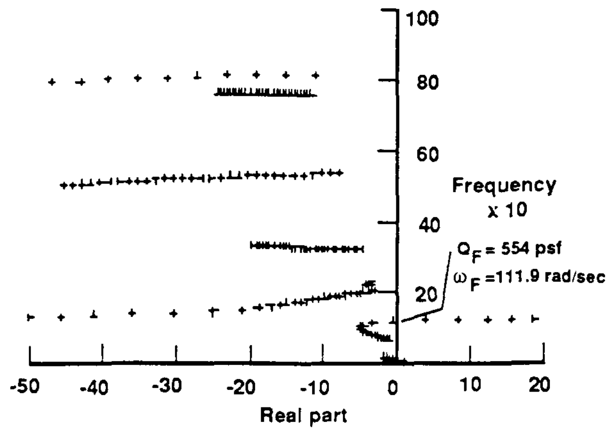

Figure 7. DAST—results obtained using responses with 5 percent random errors, for Q = 150 and Q = 250 lb/ft^2.

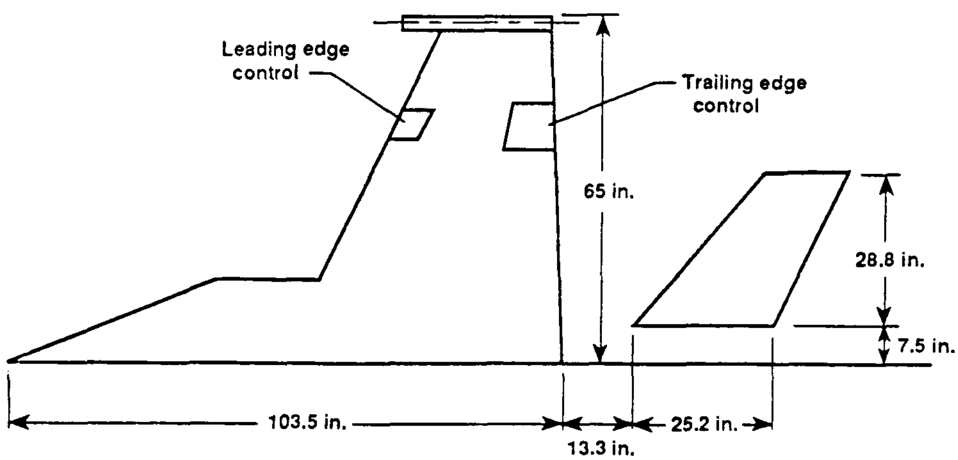

Figure 8. Plan view of YF-17 flutter model.

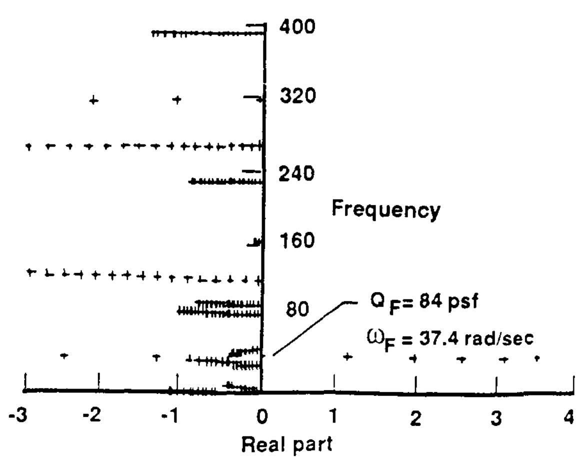

Figure 9. YF-17 aircraft—"exact" results.

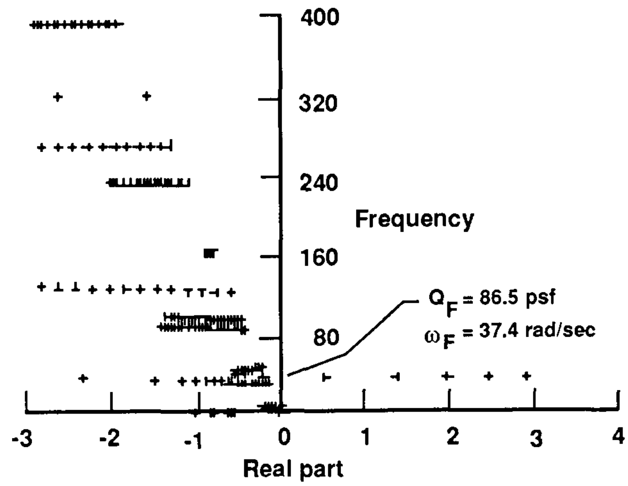

Figure 10. YF-17 aircraft—results obtained using "exact" responses for $Q = 0$ and $Q = 20$ lb/ft^2.

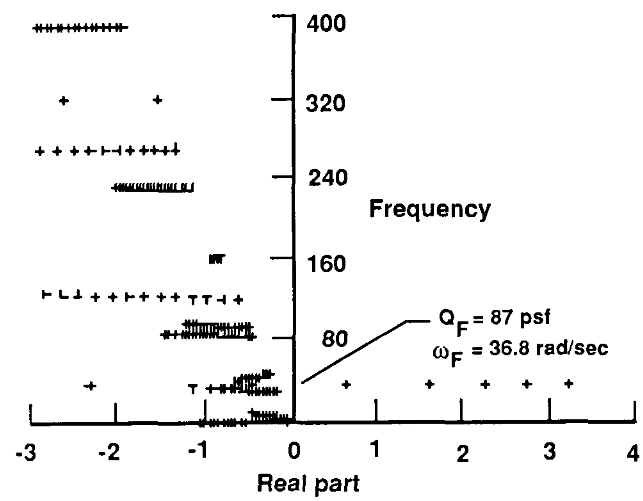

Figure 11. YF-17 aircraft—results obtained using "exact" responses for $Q = 20$ and $Q = 35$ lb/ft^2.

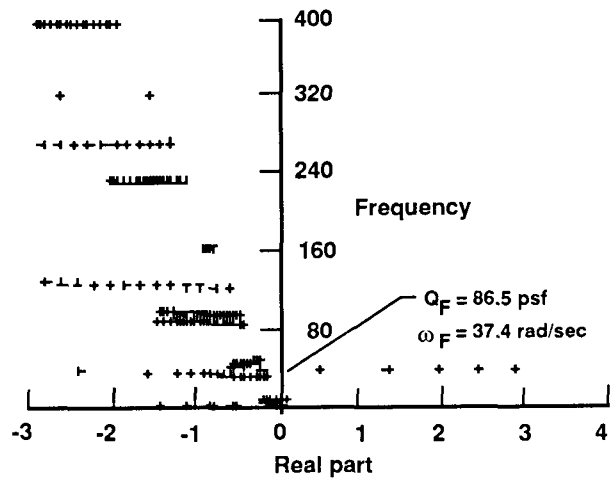

Figure 12. YF-17 aircraft—results obtained using responses contaminated with 5 percent random errors at $Q = 0$ and $Q = 20$ lb/ft^2.

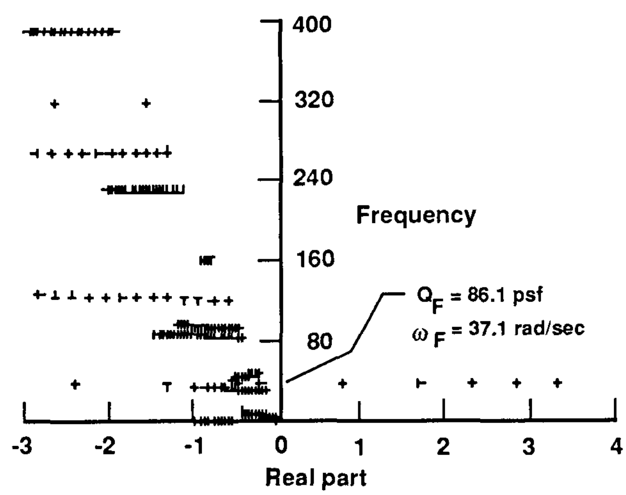

Figure 13. YF-17 aircraft—results obtained using responses contaminated with 5 percent random errors at $Q = 20$ and $Q = 35$ lb/ft^2.

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16. Abstract

A method for flight flutter testing is proposed which enables one to determine the flutter dynamic pressure from flights flown far below the flutter dynamic pressure. The method is based on the identification of the coefficients of the equations of motion at low dynamic pressures, followed by the solution of these equations to compute the flutter dynamic pressure. The initial results of simulated data reported in the present work indicate that the method can accurately predict the flutter dynamic pressure, as described. If no insurmountable difficulties arise in the implementation of this method, it may significantly improve the procedures for flight flutter testing.

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