NASA Technical Memorandum 101614

CWG - A Fortran Program for Mutual Coupling in a Planar Array of Circular Waveguide-fed Apertures

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June 1989

(NASA-TM-101614) CWG: A FORTRAN PROGRAM FOR MUTUAL COUPLING IN A PLANAR ARRAY OF CIRCULAR WAVEGUIDE-FED APERTURES (NASA. Langley Research Center) 46 p CSCL 17I

N89-27063

Unclas

G3/32 0224584

NASA

National Aeronautics and
Space Administration

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Introduction

Circular waveguide-fed apertures or conical horns are often used as elements of planar array antennas. The purpose of this present paper is to document a computer code which calculates the mutual coupling between circular apertures in a conductive plane. The program is quite general in that the apertures do not have to be the same sizes, nor do they have to be polarized in the same direction. In addition, several waveguide modes (TE and/or TM) may be specified in the apertures and the mutual coupling between all combinations of apertures and modes will be calculated. The program also allows multiple layers of homogeneous dielectrics to be placed over the aperture array. Outside the layered region, one can specify either a homogeneous half-space or a perfect reflecting surface. The program is a revision of an earlier one (ref.1) in an attempt to overcome some difficulties encountered with the earlier version and to expand its versatility.

Symbols

a_i	Radius of i_{th} aperture.
a_j	Radius of j_{th} aperture.
d_n	Thickness of n_{th} homogeneous dielectric layer.
E_i	i_{th} aperture electric field polarization vector.
E_j	j_{th} aperture electric field polarization vector.
$J_m(z)$	Bessel function of the first kind of order m and argument z.
$J'_m(z)$	Derivative of $J_m(z)$ with respect to z.
j	$\sqrt{-1}$
k_0	Wave propagation constant in free space, $2\pi/\lambda$.
k_n	Wave propagation constant in n_{th} layer.
k_{N+1}	Wave propagation constant in homogeneous half-space.
m_i	First index of waveguide mode in i_{th} aperture.
m_j	First index of waveguide mode in j_{th} aperture.
n_i	Second index of waveguide mode in i_{th} aperture.
n_j	Second index of waveguide mode in j_{th} aperture.
N	Number of homogeneous layers outside array plane.
R	Center-to-center spacing between apertures.
$R_{\parallel}(\beta)$	Reflection coefficient for plane wave with electric field polarized parallel to plane of incidence.
$R_{\perp}(\beta)$	Reflection coefficient for plane wave with electric field polarized perpendicular to plane of incidence.
$R^p_{\parallel}(\beta)$	Plane wave reflection coefficient for parallel polarization for exterior scattering problem.
$R^p_{\perp}(\beta)$	Plane wave reflection coefficient for perpendicular polarization for exterior scattering problem.
S_{ij}	Mutual coupling between i_{th} and j_{th} apertures for one mode in each aperture.
[S]	Complex square matrix whose elements are S_{ij}.
TE	Denotes transverse (to z) electric field.
TM	Denotes transverse (to z) magnetic field.
x,y,z	Cartesian coordinate variables.

x_i	x-coordinate of i_{th} aperture center.
x_j	x-coordinate of j_{th} aperture center.
y_i	y-coordinate of i_{th} aperture center.
y_j	y-coordinate of j_{th} aperture center.
Y_{ij}	Mutual admittance between i_{th} and j_{th} apertures for one mode in each aperture.
$[Y]$	Complex square matrix whose elements are Y_{ij}.
$[Yo]$	Complex diagonal matrix whose elements are the waveguide modal characteristic admittances.
β	Normalized radial propagation constant in cylindrical coordinate system.
δ	Distance from outer surface of N_{th} dielectric layer to exterior scattering boundary.
ε_0	Permittivity of free space.
ε_1	Permittivity of region just outside aperture plane.
ε_n	Permittivity of n_{th} homogeneous layer.
ε_{N+1}	Permittivity of homogeneous half-space outside of N_{th} layer.
ε_p	Equivalent permittivity just inside of exterior scattering boundary.
λ	Wavelength in free space.
μ_0	Permeability of free space.
μ_1	Permeability of region just outside of aperture.
μ_n	Permeability of n_{th} homogeneous layer.
μ_{N+1}	Permeability of homogeneous half-space outside of N_{th} layer.
μ_p	Equivalent permeability just inside of exterior scattering boundary.
ϕ_i	Polarization of i_{th} aperture with respect to y-axis.
ϕ_j	Polarization of j_{th} aperture with respect to y-axis.
ϕ_P	$\phi_j - \phi_i$.
χ_{mn}	n_{th} zero of $J_m(z)$.
χ'_{mn}	n_{th} zero of $J'_m(z)$.

Theory

The analytical formulation for the electromagnetic interaction between apertures in a planar array has been developed earlier (ref.1) and will not be repeated here.

The mutual interaction between the various modes among all the apertures in the array is obtained from the appropriate coefficients of the scattering matrix, [S], as follows:

$$[S] = \Big[[Y_o] - [Y] \Big] \Big[[Y_o] + [Y] \Big]^{-1} \tag{1}$$

Where $[Y_o]$ is a diagonal matrix whose elements are the characteristic admittances of the waveguide modes and $[Y]$ is the admittance matrix whose elements are the mutual admittances between each waveguide mode and all modes in each waveguide aperture. Therefore, for example, if the array consisted of 5 apertures and each aperture field was assumed to be the superposition of 7 waveguide modes (TE and/or TM), then (1) would consist of square complex matrices each of size 35 by 35. The sizes of the matrices can expand very rapidly if one does not use some "engineering judgment" in selecting the modes to approximate the aperture fields. The expressions for calculating the elements of the admittance matrix (i.e., the complex mutual admittance between pairs of waveguide modes) are derived in reference 1.

$$Y_{ij} = -\sqrt{\varepsilon_0/\mu_0}\ \sqrt{\gamma_{m_i}\gamma_{m_j}}\ \int_0^\infty \Big\{ W_1(\beta)\ \xi_i(\beta)\ \xi_j(\beta)\ U_{ij}(\beta) - W_2(\beta)\ \zeta_i(\beta)\ \zeta_j(\beta)\ V_{ij}(\beta) \Big\}\, \beta\, d\beta \tag{2}$$

where,

$$\gamma_{m_i} = 1 \quad \text{for } m_i = 0$$
$$= 2 \quad \text{otherwise.}$$

$W_1(\beta)$ and $W_2(\beta)$ are defined as

$$W_1(\beta) \quad = \quad \left(\frac{1 - R_{\parallel}(\beta)}{1 + R_{\parallel}(\beta)}\right) \Big/ \sqrt{1-\beta^2} \tag{3}$$

$$W_2(\beta) \quad = \quad \left(\frac{1 - R_{\perp}(\beta)}{1 + R_{\perp}(\beta)}\right) \sqrt{1-\beta^2} \tag{4}$$

where (for real angles) β can be interpreted as the sine of the angle of incidence for a plane wave with $R_{\parallel}(\beta)$ and $R_{\perp}(\beta)$ being the reflection coefficients in the outward direction evaluated at the aperture boundary for parallel and perpendicular polarization at an air-dielectric interface. Notice from equation 2 that the mutual admittance is a weighted sum over all angles of incidence, including real space ($0 \le \beta \le 1$) and invisible space ($\beta > 1$). An alternate form for $W_1(\beta)$ and $W_2(\beta)$ is

$$W_1(\beta) \quad = \quad \left\{\frac{-jk_0\varepsilon_1}{\varepsilon_0}\right\} \left(\frac{g_1(\beta)}{g_1'(\beta)}\right) \tag{5}$$

$$W_2(\beta) \quad = \quad \left\{\frac{\mu_0}{-jk_0\mu_1}\right\} \left(\frac{f_1'(\beta)}{f_1(\beta)}\right) \tag{6}$$

where $g_1(\beta)$ and $f_1(\beta)$ are the solutions to the wave equations in the external region evaluated at the aperture plane, and $g_1'(\beta)$ and $f_1'(\beta)$ are the corresponding derivatives in the direction normal to the aperture plane. ε_1 and μ_1 are the permittivity and permeability of the external medium evaluated just outside the aperture plane.

By applying the boundary conditions of continuity of tangential fields at the interface of each homogeneous dielectric layer, one can evaluate the mutual admittance between aperture modal fields with a multilayer dielectric medium outside (as shown in figure 1).

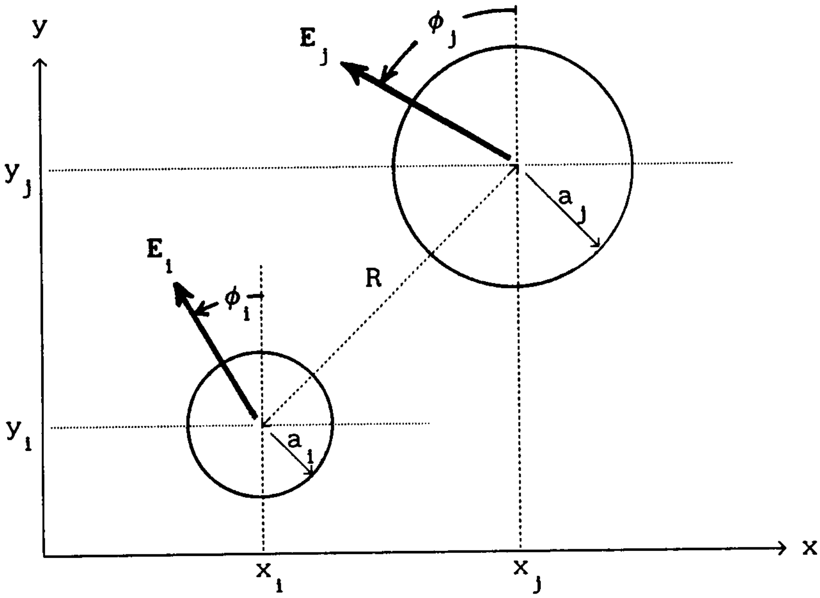


Figure 1a. Coordinate geometry for the i_{th} and j_{th} elements of a planar array of circular waveguide-fed apertures.

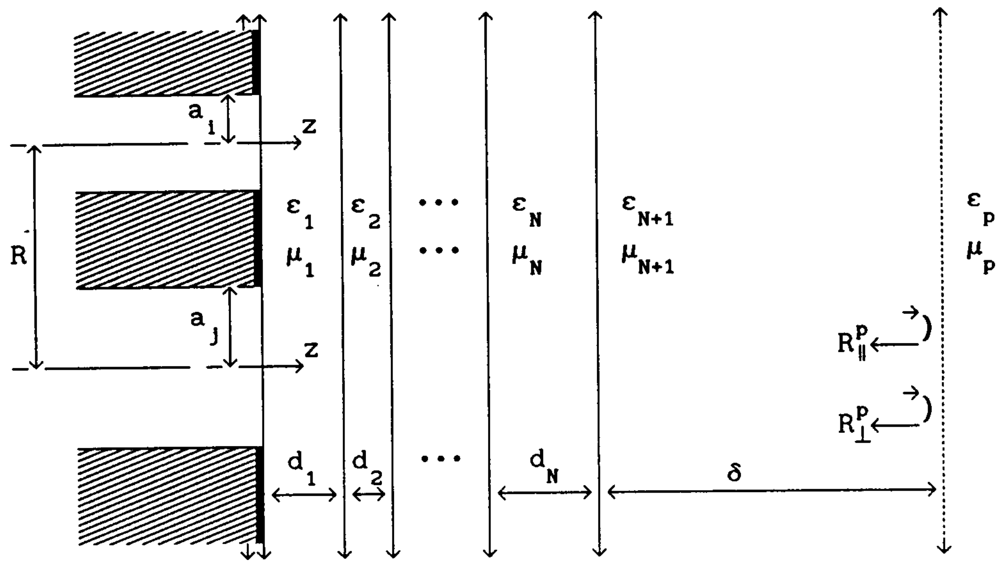


Figure 1b. Cross-section of i_{th} and j_{th} circular waveguide-fed apertures radiating into a multi-layer dielectric with an exterior arbitrary reflecting boundary.

Applying boundary conditions at layer interfaces, results in the following recursive equations,

$$\left(\frac{f_n'(\beta)}{f_n(\beta)}\right) = \left\{ \frac{(k_z)\left\{ \sin(k_z d_n) + \left(\frac{\mu_n}{k_z \mu_{n+1}}\right)\left(\frac{f_{n+1}'(\beta)}{f_{n+1}(\beta)}\right) \cos(k_z d_n) \right\}}{\left\{ \cos(k_z d_n) - \left(\frac{\mu_n}{k_z \mu_{n+1}}\right)\left(\frac{f_{n+1}'(\beta)}{f_{n+1}(\beta)}\right) \sin(k_z d_n) \right\}} \right\} \tag{7}$$

$$\left(\frac{g_n'(\beta)}{g_n(\beta)}\right) = \left\{ \frac{(k_z)\left\{ \sin(k_z d_n) + \left(\frac{\varepsilon_n}{k_z \varepsilon_{n+1}}\right)\left(\frac{g_{n+1}'(\beta)}{g_{n+1}(\beta)}\right) \cos(k_z d_n) \right\}}{\left\{ \cos(k_z d_n) - \left(\frac{\varepsilon_n}{k_z \varepsilon_{n+1}}\right)\left(\frac{g_{n+1}'(\beta)}{g_{n+1}(\beta)}\right) \sin(k_z d_n) \right\}} \right\} \tag{8}$$

for $n = N, (N-1), (N-2), \ldots, 3, 2, 1$. where k_z is defined as

$$k_z = \begin{cases} \sqrt{k_n^2 - k_0^2\beta^2} & ; \ \beta \le k_n/k_0 \\ -j\sqrt{k_0^2\beta^2 - k_n^2} & ; \ \beta > k_n/k_0 \end{cases}$$

Then starting with

$$\left(\frac{f_{N+1}'(\beta)}{f_{N+1}(\beta)}\right) = \left(\frac{g_{N+1}'(\beta)}{g_{N+1}(\beta)}\right) = \begin{cases} -j\sqrt{k_{N+1}^2 - k_0^2\beta^2} & ; \ k_0\beta \le k_{N+1} \\ -\sqrt{k_0^2\beta^2 - k_{N+1}^2} & ; \ k_0\beta > k_{N+1} \end{cases} \tag{9}$$

equations 7 and 8 are evaluated successively from the outermost homogeneous layer to the layer just outside of the aperture plane.

In the special case of a perfect electric conductor of infinite extent on the outer surface of the N_{th} layer, $f_{N+1}(\beta)=0$ and $g'_{N+1}(\beta)=0$; therefore, the starting conditions for equations 7 and 8 become

$$\left(\frac{f'_N(\beta)}{f_N(\beta)}\right) = - k_z \left\{\frac{\cos(k_z d_N)}{\sin(k_z d_N)}\right\} \quad (10a)$$

$$\left(\frac{g'_N(\beta)}{g_N(\beta)}\right) = k_z \left\{\frac{\sin(k_z d_N)}{\cos(k_z d_N)}\right\} \quad (10b)$$

If the medium outside of the N_{th} dielectric layer is not a homogeneous half-space, the starting conditions (equation 9) are replaced by

$$\left(\frac{f'_{N+1}(\beta)}{f_{N+1}(\beta)}\right) = -j\ K(\beta) \left(\frac{\mu_p}{\mu_{N+1}}\right)\left\{\frac{1 - R^p_\perp(\beta)\ \exp(-j2\delta\ K(\beta))}{1 + R^p_\perp(\beta)\ \exp(-j2\delta\ K(\beta))}\right\} \quad (11a)$$

$$\left(\frac{g'_{N+1}(\beta)}{g_{N+1}(\beta)}\right) = -j\ K(\beta) \left(\frac{\varepsilon_p}{\varepsilon_{N+1}}\right)\left\{\frac{1 + R^p_\parallel(\beta)\ \exp(-j2\delta\ K(\beta))}{1 - R^p_\parallel(\beta)\ \exp(-j2\delta\ K(\beta))}\right\} \quad (11b)$$

where,

$$K(\beta) = \begin{cases} \sqrt{k^2_{N+1} - k_0^2\beta^2} & ;\ k_0\beta \le k_{N+1} \\ -j\sqrt{k_0^2\beta^2 - k^2_{N+1}} & ;\ k_0\beta > k_{N+1} \end{cases}$$

$R^p_\perp(\beta)$ and $R^p_\parallel(\beta)$ are the plane wave reflection coefficients, evaluated at the external scattering boundary (shown dashed in figure 1b) for perpendicular and parallel polarization of the electric field vector with respect to the plane of incidence. ε_p and μ_p are the equivalent permittivity and equivalent permeability just inside of the external scattering boundary. The starting conditions in equation 11 allow for future modifications of the computer code to include problems for which the plane wave reflection is known from solutions to exterior scattering problems, such as objects in the vicinity of the array or an inhomogeneous ionized plasma.

The other quantities in equation 2 are related to the Fourier transforms of the aperture modal fields as derived in reference 1.

For the TE modes in the i_{th} aperture:

$$\zeta_i^{TE}(\beta) = \frac{\left(\chi'_{m_i n_i}\right)^2 (k_0 a_i)\, J'_{m_i}(k_0 a_i \beta)}{\left\{\left(\chi'_{m_i n_i}\right)^2 - \left(k_0 a_i \beta\right)^2\right\} \sqrt{\left(\chi'_{m_i n_i}\right)^2 - m_i^2}} \tag{12}$$

$$\xi_i^{TE}(\beta) = \frac{(k_0 a_i)\, m_i\, J_{m_i}(k_0 a_i \beta) \Big/ (k_0 a_i \beta)}{\sqrt{\left(\chi'_{m_i n_i}\right)^2 - m_i^2}} \tag{13}$$

For the TM modes in the i_{th} aperture:

$$\zeta_i^{TM}(\beta) = 0 \tag{14}$$

$$\xi_i^{TM}(\beta) = \frac{(k_0 a_i)(k_0 a_i \beta)\, J_{m_i}(k_0 a_i \beta)}{\left(\chi_{m_i n_i}\right)^2 - \left(k_0 a_i \beta\right)^2} \tag{15}$$

For the j_{th} aperture, the i subscripts are replaced with j in equations 12-15.

If R=0, then the i_{th} and j_{th} apertures are coincident and equation 2 is the mutual admittance between modal fields in the same aperture. When R=0, the orthogonality of the modal functions results in $U_{ij}(\beta)=V_{ij}(\beta)=0$ except when $m_i=m_j$. When R=0 and $m_i=m_j$; $U_{ij}(\beta)$ and $V_{ij}(\beta)$ are as given in Table I.

Table I
($R=0;\ m_i = m_j$)

Modes	$U_{ij}(\beta)$	$V_{ij}(\beta)$
$TE_i\ TE_j$	$-\left(\gamma_{m_i}-1\right)\ \cos\left(m_i\phi_p\right)$	$\left(2/\gamma_{m_i}\right)\ \cos\left(m_i\phi_p\right)$
$TM_i\ TM_j$	$-\left(2/\gamma_{m_i}\right)\ \cos\left(m_i\phi_p\right)$	0
$TE_i\ TM_j$	$\left(\gamma_{m_i}-1\right)\ \sin\left(m_i\phi_p\right)$	0
$TM_i\ TE_j$	$\left(\gamma_{m_i}-1\right)\ \sin\left(m_i\phi_p\right)$	0

Defining the following terms:

$$R = \sqrt{(x_j-x_i)^2 + (y_j-y_i)^2} \tag{16}$$

$$\phi = \arc\tan\left(\frac{y_j-y_i}{x_j-x_i}\right) - \phi_i \tag{17}$$

$$B_{\pm}(\beta) = J_{m_j \pm m_i}\left(k_0 \beta R\right) \tag{18}$$

$$C_{\pm} = \cos\ \left(\ (m_j \pm m_i)\phi\ -\ m_j\phi_p\ \right) \tag{19}$$

$$S_{\pm} = \sin\ \left(\ (m_j \pm m_i)\phi\ -\ m_j\phi_p\ \right) \tag{20}$$

$U_{ij}(\beta)$ and $V_{ij}(\beta)$ for $R>0$ are given in Table II.

Table II
(R > 0)

Modes	$U_{ij}(\beta)$	$V_{ij}(\beta)$
TE_i TE_j	$(-1)^{m_j} \left(C_+ B_+(\beta) - (-1)^{m_i} C_- B_-(\beta) \right)$	$(-1)^{m_j} \left(C_+ B_+(\beta) + (-1)^{m_i} C_- B_-(\beta) \right)$
TM_i TM_j	$-(-1)^{m_j} \left(C_+ B_+(\beta) + (-1)^{m_i} C_- B_-(\beta) \right)$	0
TE_i TM_j	$(-1)^{m_j} \left(S_+ B_+(\beta) - (-1)^{m_i} S_- B_-(\beta) \right)$	0
TM_i TE_j	$(-1)^{m_j} \left(S_+ B_+(\beta) - (-1)^{m_i} S_- B_-(\beta) \right)$	0

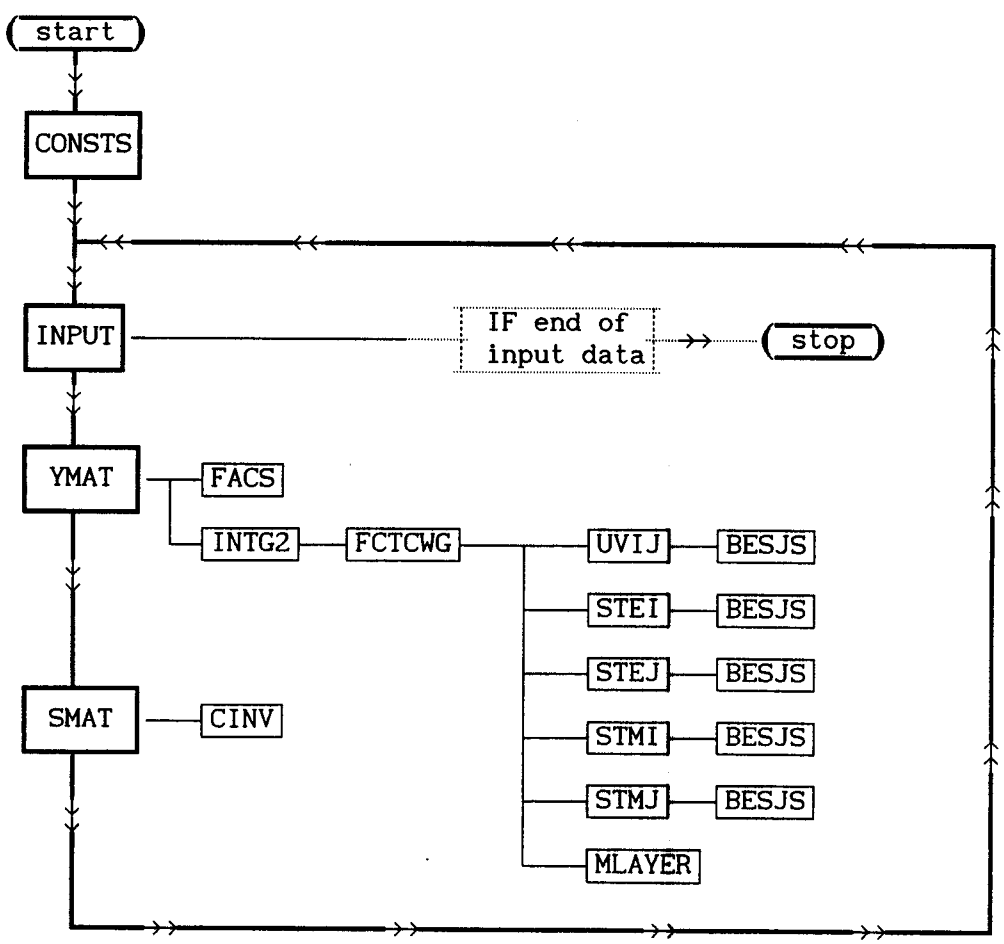
References

1. M. C. Bailey: "Analysis of Finite-size Phased Arrays of Circular Waveguide Elements", NASA Technical Report, R-408, April 1974.

Appendix
Computer Program

The computer code is written in FORTRAN IV and uses double precision variables throughout, except for integers. It was found that double precision was necessary in order to obtain sufficient accuracy on a VAX computer. The input data is read from logical unit 5. Results are output to logical unit 6. A flow chart indicating the subroutine calls is given and also a description of the input parameters with sample input and output data. The computer code has been verified against results in reference 1.

Subroutine Flowchart



Definition of Input Data

NHOLE	Number of apertures in array.
NMODE	Number of modes per aperture.
NUMTE	Number of TE-modes per aperture.
NUMTM	Number of TM-modes per aperture.
	(NOTE: same modes assumed for all apertures).
MIJ(I)	First index of I-th TE mode.
NIJ(I)	Second index of I-th TE mode.
MIJP(I)	First index of I-th TM mode.
NIJP(I)	Second index of I-th TM mode.
AIJ(I)	Radius of I-th aperture.
XIJ(I)	x-coordinate of center of I-th aperture.
YIJ(I)	y-coordinate of center of I-th aperture.
PHIJP(I)	Polarization of I-th aperture (degrees CCW).
F	Frequency (Hertz).
FCON	Factor for converting input to centimeters.
ER	Dielectric constant of waveguide interior.
CEP	Relative epsilon outside of layered region.
CUP	Relative mu outside of layered region.
NLAY	Number of homogeneous dielectric layers.
D(I)	Thickness of I-th homogeneous layer.
CE(I)	Relative epsilon of I-th layer.
CU(I)	Relative mu of I-th layer.

Input Data

(1) Read NHOLE, NMODE, NUMTE, NUMTM
Check for end-of-file on unit 5.

(2) If NUMTE=0, skip to (3).
Read MIJ(I),NIJ(I) ; I=1 To NUMTE

(3) If NUMTM=0, skip to (4).
Read MIJP(I),NIJP(I) ; I=1 To NUMTM

(4) Read AIJ(I),XIJ(I),YIJ(I),PHIJP(I) ; I=1 To NHOLE

(5) Read F, FCON, ER

(6) Read CEP, CUP

(7) Read NLAY

(8) If NLAY=0, skip to (9).
Read D(I),CE(I),CU(I) ; I=1 To NLAY

(9) Perform calculations.
Read next set of input data (1).

Sample Input Data

```
2 1 1 0
11
0.75 0.0 0.0 0.0
0.75 0.0 2.5 0.0
6.0D09 2.54 1.0
(1.0,0.0) (1.0,0.0)
1
0.18 (2.6,-0.0156) (1.0,0.0)
```

Sample Output Data

```
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
MUTUAL COUPLING OF CIRCULAR APERTURES
RADIATING INTO MULTI-LAYERS
NUMBER OF LAYERS =     1

LAYER   THICKNESS     COMPLEX EPSILON           COMPLEX MU
  1      0.1800    (   2.60000,  -0.01560)   (  1.00,   0.00)

OUTSIDE HALF-SPACE EPSILON =             1.00000            0.00000
OUTSIDE HALF-SPACE MU      =             1.00000            0.00000

****   INPUT DIMENSIONS IN INCHES   ****
****  FREQUENCY = .600000E+10 HERTZ   ****

++++++++ APERTURE ARRAY GEOMETRY +++++++++
HOLE   RADIUS      X        Y       POL.
  1    0.7500    0.000    0.000    0.000
  2    0.7500    0.000    2.500    0.000

MODE 1 = TE-11

CY( 1, 1)=( 0.3415E-02, 0.1691E-02)  BB= 17.00   IH= 1  IM= 1  JH= 1  JM= 1
CY( 1, 2)=( 0.3443E-04,-0.3158E-03)  BB= 15.00   IH= 1  IM= 1  JH= 2  JM= 1

YMN( 1)=( 0.1695E-02, 0.0000E+00)   IHOLE= 1   IMODE= 1
YMN( 2)=( 0.1695E-02, 0.0000E+00)   IHOLE= 2   IMODE= 1

         SCATTERING MATRIX
S( 1, 1)=(-0.4036E+00,-0.1964E+00)     -6.9570 DB   -154.0525 DEG
S( 1, 2)=( 0.1871E-01, 0.3199E-01)    -28.6224 DB     59.6795 DEG
S( 2, 1)=( 0.1871E-01, 0.3199E-01)    -28.6224 DB     59.6795 DEG
S( 2, 2)=(-0.4036E+00,-0.1964E+00)     -6.9570 DB   -154.0525 DEG
++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
C*****************  CWG ***********************************************
C     Mutual admittance and scattering matrix calculation for          *
C     circular waveguide array with TE-MN and TM-MN modes.             *
C     NOTE:  Field equations of Marcuvitz show                         *
C            TM modes polarized 90 degrees to TE modes.                *
C            program modified to rotate polarization of TM modes       *
C            by PI/2 such that TE-11 and TM-11 are polarized the same.*
C                                                                      *
C     Program can handle multiple homogeneous layers over apertures.   *
C     (INPUT on unit #5)  (OUTPUT on unit #6)                          *
C                                                                      *
C       PROGRAM:  M.C. Bailey                                          *
C                 Hampton, VA                                   (1989) *
C**********************************************************************
102   FORMAT(1X,79(1H+))
      WRITE(6,102)
      CALL  CONSTS
100   WRITE(6,102)
      CALL INPUT
      CALL YMAT
      CALL SMAT
      GO TO 100
      END
```

```
SUBROUTINE CONSTS
IMPLICIT REAL*8 (A,B,D-H,O-Z)
IMPLICIT COMPLEX*16 (C)
COMMON /ZEROS/ XMNP(9,6),XMN(9,6)
COMMON /CONST/ CJ,PI,TWOPI,DTOR,RTOD,BIG,SMALL,BMAX,YO,MAX,LAYM
DATA XMNP(1,1),XMNP(1,2),XMNP(1,3)/3.831706,7.0155867,10.173468/
DATA XMNP(2,1),XMNP(2,2),XMNP(2,3)/1.84118,5.33144,8.53632/
DATA XMNP(3,1),XMNP(3,2),XMNP(3,3)/3.05424,6.70613,9.96947/
DATA XMNP(4,1),XMNP(4,2),XMNP(4,3)/4.20119,8.01524,11.34592/
DATA XMNP(5,1),XMNP(5,2),XMNP(5,3)/5.31755,9.28240,12.68191/
DATA XMNP(6,1),XMNP(6,2),XMNP(6,3)/6.41562,10.51986,13.98719/
DATA XMNP(7,1),XMNP(7,2),XMNP(7,3)/7.50127,11.73494,15.26818/
DATA XMNP(8,1),XMNP(8,2),XMNP(8,3)/8.57784,12.93239,16.52937/
DATA XMNP(9,1),XMNP(9,2),XMNP(9,3)/9.64742,14.11552,17.77401/
DATA XMNP(1,4),XMNP(1,5),XMNP(1,6)/13.323692,16.47063,19.615859/
DATA XMNP(2,4),XMNP(2,5),XMNP(2,6)/11.70600,14.86359,18.01553/
DATA XMNP(3,4),XMNP(3,5),XMNP(3,6)/13.17037,16.34752,19.51291/
DATA XMNP(4,4),XMNP(4,5),XMNP(4,6)/14.58585,17.78875,20.97248/
DATA XMNP(5,4),XMNP(5,5),XMNP(5,6)/15.96711,19.19603,22.40103/
DATA XMNP(6,4),XMNP(6,5),XMNP(6,6)/17.31284,20.57551,23.80358/
DATA XMNP(7,4),XMNP(7,5),XMNP(7,6)/18.63744,21.93172,25.18393/
DATA XMNP(8,4),XMNP(8,5),XMNP(8,6)/19.94185,23.26805,26.54503/
DATA XMNP(9,4),XMNP(9,5),XMNP(9,6)/21.22906,24.58720,27.88927/
DATA XMN(1,1),XMN(1,2),XMN(1,3)/2.4048256,5.5200781,8.6537279/
DATA XMN(2,1),XMN(2,2),XMN(2,3)/3.8317060,7.0155867,10.1734681/
DATA XMN(3,1),XMN(3,2),XMN(3,3)/5.1356223,8.4172441,11.6198412/
DATA XMN(4,1),XMN(4,2),XMN(4,3)/6.3801619,9.7610231,13.0152007/
DATA XMN(5,1),XMN(5,2),XMN(5,3)/7.5883427,11.0647095,14.3725367/
DATA XMN(6,1),XMN(6,2),XMN(6,3)/8.7714838,12.3386042,15.7001741/
DATA XMN(7,1),XMN(7,2),XMN(7,3)/9.93611,13.58929,17.00382/
DATA XMN(8,1),XMN(8,2),XMN(8,3)/11.08637,14.82127,18.28758/
DATA XMN(9,1),XMN(9,2),XMN(9,3)/12.22509,16.03777,19.55454/
DATA XMN(1,4),XMN(1,5),XMN(1,6)/11.7915344,14.9309177,18.071064/
DATA XMN(2,4),XMN(2,5),XMN(2,6)/13.32369,16.47063,19.61586/
DATA XMN(3,4),XMN(3,5),XMN(3,6)/14.79595,17.95982,21.11700/
DATA XMN(4,4),XMN(4,5),XMN(4,6)/16.22347,19.40942,22.58273/
DATA XMN(5,4),XMN(5,5),XMN(5,6)/17.61597,20.82693,24.01902/
DATA XMN(6,4),XMN(6,5),XMN(6,6)/18.98013,22.21780,25.43034/
DATA XMN(7,4),XMN(7,5),XMN(7,6)/20.32079,23.58608,26.82015/
DATA XMN(8,4),XMN(8,5),XMN(8,6)/21.64154,24.93493,28.19119/
DATA XMN(9,4),XMN(9,5),XMN(9,6)/22.94517,26.26681,29.54566/
DATA CJ,BIG,SMALL/(0.0D0,1.0D0),1.0D+38,1.0D-38/
DATA BMAX,MAX,LAYM/200.0,40,600/
PI=2.0*DASIN(1.0D0)
YO=1.0/(120.0*PI)
DTOR=PI/180.0
RTOD=180.0/PI
TWOPI=2.0*PI
RETURN
END
```

```
      SUBROUTINE INPUT
c.......................................................................
c:  Reads input data                                                   :
c.......................................................................:
      IMPLICIT REAL*8 (A,B,D-H,O-Z)
      IMPLICIT COMPLEX*16 (C)
      LOGICAL LPOL,LSIZE,LSAME,LCON,LPLAZ
      COMMON DIA,MMM,NHOLE,NMODE,NUMTE,NUMTM,F,FCON,ER,LPOL,LSIZE,LSAME
      COMMON /LAYER/ CEP,CUP,NLAY,D(601),CE(601),CU(601),CEUP,LCON,LPLAZ
      COMMON /CONST/ CJ,PI,TWOPI,DTOR,RTOD,BIG,SMALL,BMAX,YO,MAX,LAYM
      COMMON /ARRYS/ MIJ(20),NIJ(20),MIJP(20),NIJP(20),
     A               CY(40,40),CA(40,40),CB(40,40),IPIV(40),INDX(40,2),
     B               AIJ(40),XIJ(40),YIJ(40),PHIJP(40)
101   FORMAT(1X)
102   FORMAT(1X,79(1H+))
C************************  INPUT DATA  *********************************
C**********************************************************************
C     NHOLE = Number of apertures in array.                            *
C     NMODE = Number of modes per aperture.                            *
C     NUMTE = Number of TE-modes per aperture.                         *
C     NUMTM = Number of TM-modes per aperture.                         *
C             NOTE:  (NUMTE+NUMTM)=NMODE                               *
C                    The same modes are assumed in all apertures.      *
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++*
      READ(5,*,END=9999)NHOLE,NMODE,NUMTE,NUMTM
      MMM=NHOLE*NMODE
      IF(MMM.LE.MAX) GO TO 120
      WRITE(6,110)
110   FORMAT(1X'(NHOLE*NMODE) EXCEEDS DIMENSION OF CY')
      STOP
120   WRITE(6,130)
130   FORMAT(1X'MUTUAL COUPLING OF CIRCULAR APERTURES')
      IF(NUMTE.GT.20) GO TO 500
      IF(NUMTM.GT.20) GO TO 500
      IF(NUMTE.GT.NMODE) GO TO 500
      IF(NUMTM.GT.NMODE) GO TO 500
      IF(NMODE.GT.40) GO TO 500
      IF((NUMTE+NUMTM).NE.NMODE) GO TO 500
      IF(NUMTE.EQ.0) GO TO 150
C**********************************************************************
C     MIJ(I),NIJ(I)    = M,N indices of I-th TE mode.                  *
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++*
      READ(5,140) ((MIJ(I),NIJ(I)),I=1,NUMTE)
140   FORMAT(20(2I1,1X))
150   IF(NUMTM.EQ.0) GO TO 170
C**********************************************************************
C     MIJP(I),NIJP(I) = M,N indices of I-th TM mode.                   *
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++*
      READ(5,160) ((MIJP(I),NIJP(I)),I=1,NUMTM)
160   FORMAT(20(2I1,1X))
```

```
170   CONTINUE
C*************************************************************************
C     AIJ(I)    = Radius of I-th aperture.                               *
C     XIJ(I)    = X-coordinate of center of I-th aperutre.               *
C     YIJ(I)    = Y-coordinate of center of I-th aperture.               *
C     PHIJP(I) = Angular rotation of XI-axis WRT X-axis (degrees CCW).  *
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++*
      READ(5,*) ((AIJ(I),XIJ(I),YIJ(I),PHIJP(I)),I=1,NHOLE)
C*************************************************************************
C     F    = Frequency (Hertz).                                          *
C     FCON = Factor for converting input dimensions to centimeters.      *
C            (2.54 for input in inches) (1.0 for input in centimeters).*
C     ER   = Relative dielectric constant of material filling waveguides
C........................................................................*
C              For input dimensions in wavelengths set:                  *
C                  F=3.0E10                                              *
C                  FCON=1.0                                              *
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++*
      READ(5,*)F,FCON,ER
C*************************************************************************
C     CEP = Complex (relative) Epsilon of half-space outside of layers.*
C     CUP = Complex (relative) Mu of half-space outside of layers.      *
C                                                                        *
C     Input format is:  (A,-B) (C,-D)                                    *
C     For free-space outside:  (1.0,0.0) (1.0,0.0)                       *
C     For a metal sheet approximation:   (-1.0E10,0.0) (1.0,0.0)         *
C                                  or CABS(CEP*CUP) greater than 1.0E09  *
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++*
      READ(5,*)CEP,CUP
C*************************************************************************
C     NLAY = Number of homogeneous layers over apertures.               *
C........................................................................*
C            NLAY=0    (half-space).                                     *
C++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++*
      READ(5,*)NLAY
      IF(NLAY.GT.LAYM)STOP 4444
      IF(NLAY.EQ.0) GO TO 210
C*************************************************************************
C     NOTE: If NLAY=0, do not inter the following data.                  *
C                                                                        *
C     D(I)  = Thickness of I-th layer.                                   *
C     CE(I) = Complex Epsilon of I-th layer (relative to free space).   *
C     CU(I) = Complex Mu of I-th layer (relative to free space).        *
C                                                                        *
C     NOTE:  Values of CE(I) must be input with format:  (A,-B)          *
C            Values of CU(I) must be input with format:  (C,-D)          *
C            where dielectric loss tangent = B/A                         *
C                                                                        *
```

```
C     CAUTION:                                                          *
C     An input value of 0.0 for B could produce erroneous results.      *
C     A value of 0.0001 for the dielectric loss tangent should yield    *
C     results which approximate a lossless dielectric in most cases.    *
C     (Suggest varying B to determine sensitivity to small values)      *
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++*
      DO 180 I=1,NLAY
      READ(5,*)D(I),CE(I),CU(I)
180   CONTINUE
      WRITE(6,190)NLAY
190   FORMAT(1X'RADIATING INTO MULTI-LAYERS'/1X'NUMBER OF LAYERS = 'I4/)
      WRITE(6,200)
200   FORMAT(1X'LAYER'3X'THICKNESS'5X'COMPLEX EPSILON'10X'COMPLEX MU')
210   IF(DABS(ER).LT.1.0D-04)ER=1.0
      IF(DABS(FCON).LT.1.0D-04)FCON=1.0
      XLAM=(30./FCON)/(F*1.0D-09)
      IF(DABS(XLAM-1.0).LT.0.0001)XLAM=1.0
      OMEGA=TWOPI*F
      IF(NLAY.EQ.0) GO TO 280
      DO 270 I=1,NLAY
      WRITE(6,220)I,D(I),CE(I),CU(I)
220   FORMAT(1XI4,2XF9.4,3X'('F10.5','F9.5')'3X'('F6.2','F6.2')')
      D(I)=D(I)/XLAM
      EPR=DREAL(CE(I))
      EPI=DIMAG(CE(I))
      IF(EPI.LE.-0.0001*EPR)GO TO 270
      IF(EPI.LT.0.0)GO TO 240
      EPI=-EPI
      WRITE(6,230)I
230   FORMAT(1X'NOTE:  The imaginary part of the dielectric constant'/
     A8X'for the layered region must be negative.'/
     B8X'This has been corrected for layer 'I2/)
240   IF(EPI.LE.-0.0001*EPR)GO TO 260
      TANL=DABS(EPI/EPR)
      WRITE(6,250)I,TANL
250   FORMAT(1X'CAUTION:  The dielectric loss tangent for the layered'/
     A11X'region must not be zero.  Erroneous results could also be'/
     B11X'obtained for very small values of dielectric loss.'/
     C11X'The loss tangent for layer 'I2' is 'E11.4/)
260   CE(I)=DCMPLX(EPR,EPI)
270   CONTINUE
      WRITE(6,101)
280   CONTINUE
290   LCON=.FALSE.
      CEUP=CEP*CUP
      IF(CDABS(CEUP).GE.1.0D+9) LCON=.TRUE.
      ERC=DREAL(CEP)
      EIC=DIMAG(CEP)
      NSIGN=1
      IF(ERC.LT.0.0)NSIGN=-1
```

```
      ERCP=NSIGN*ERC
      IF(ERCP.GT.1.0D+10)ERCP=1.0D+10
      ERC=NSIGN*ERCP
      CEP=DCMPLX(ERC,EIC)
      IF(NLAY.EQ.0.AND.(.NOT.LCON).AND.(.NOT.LPLAZ))WRITE(6,300)
300   FORMAT(1X'RADIATING INTO HALF-SPACE'/)
      IF(LCON)WRITE(6,320)
310   CONTINUE
320   FORMAT(1X'****  PERFECT CONDUCTOR OUTSIDE  ****'/)
      IF(.NOT.LCON)WRITE(6,330)CEP,CUP
330   FORMAT(1X'OUTSIDE HALF-SPACE EPSILON = '2F18.5/
     +       1X'OUTSIDE HALF-SPACE MU      = '2F18.5/)
      IF(FCON.EQ.2.54)WRITE(6,340)
340   FORMAT(1X'****  INPUT DIMENSIONS IN INCHES  ****')
      IF((FCON.EQ.1.0).AND.(F.EQ.3.0E10))WRITE(6,350)
350   FORMAT(1X'****  INPUT DIMENSIONS IN WAVELENGTHS  ****')
      IF((FCON.EQ.1.0).AND.(F.NE.3.0E10))WRITE(6,360)
360   FORMAT(1X'****  INPUT DIMENSIONS IN CENTIMETERS  ****')
      IF((FCON.NE.1.0).AND.(FCON.NE.2.54))WRITE(6,370)FCON
370   FORMAT(1X'****  INPUT CONVERSION FACTOR =  'F10.5'  ****')
      IF((FCON.NE.1.0).OR.(F.NE.3.0E10))WRITE(6,380)F
380   FORMAT(1X'****  FREQUENCY = 'E11.6' HERTZ  ****')
      WRITE(6,390)
390   FORMAT(/1X'++++++++ APERTURE ARRAY GEOMETRY +++++++++'/
     A1X'HOLE',3X'RADIUS',6X,'X',8X,'Y',7X,'POL.')
      LPOL=.TRUE.
      LSIZE=.TRUE.
      DIA=2.0*AIJ(1)
      DO 410 I=1,NHOLE
      WRITE(6,400)I,AIJ(I),XIJ(I),YIJ(I),PHIJP(I)
400   FORMAT(1X,I3,1X,F9.4,1X,F8.3,1X,F8.3,1X,F8.3)
      AIJ(I)=AIJ(I)/XLAM
      XIJ(I)=XIJ(I)/XLAM
      YIJ(I)=YIJ(I)/XLAM
      PHIJP(I)=PHIJP(I)*DTOR
      IF(DABS(AIJ(I)-AIJ(1)).GT.0.0001)LSIZE=.FALSE.
      IF(DABS(PHIJP(I)-PHIJP(1)).GT.0.0001)LPOL=.FALSE.
410   CONTINUE
      LSAME=.FALSE.
      IF(LSIZE.AND.LPOL)LSAME=.TRUE.
      WRITE(6,101)
      IF(NUMTE.EQ.0) GO TO 440
      DO 430 I=1,NUMTE
      WRITE(6,420)I,MIJ(I),NIJ(I)
420   FORMAT(1X'MODE'I2' = TE-'I1,I1)
      IF((1+MIJ(I)).GT.9) GO TO 480
      IF(NIJ(I).GT.6) GO TO 480
      IF(MIJ(I).LT.0) GO TO 480
      IF(NIJ(I).LT.1) GO TO 480
430   CONTINUE
```

```
440   IF(NUMTM.EQ.0) GO TO 470
      DO 460 I=1,NUMTM
      IP=I+NUMTE
      WRITE(6,450)IP,MIJP(I),NIJP(I)
450   FORMAT(1X'MODE'I2' = TM-'I1,I1)
      IF((1+MIJP(I)).GT.9) GO TO 480
      IF(NIJP(I).GT.6) GO TO 480
      IF(MIJP(I).LT.0) GO TO 480
      IF(NIJP(I).LT.1) GO TO 480
460   CONTINUE
470   WRITE(6,101)
      RETURN
480   WRITE(6,490)
490   FORMAT(1X'RANGE OF MODE INDICES EXCEEDED'/
     A1X'(M,N) VALUES ALLOWED:   (0,1) THROUGH (8,6)')
      STOP
500   WRITE(6,510)NHOLE,NMODE,NUMTE,NUMTM
510   FORMAT(1X'NUMBER OF MODES EXCEEDS ARRAY DIMENSIONS'//
     A1X'NHOLE='I3,5X'NMODE='I3,5X'NUMTE='I3,5X'NUMTM='I3)
      STOP
9999  WRITE(6,102)
      STOP
      END
```

```
      SUBROUTINE YMAT
c.......................................................................
c:  Calculates coefficients CY(II,JJ) of complex admittance matrix [Y] :
c.......................................................................:
      IMPLICIT REAL*8 (A,B,D-H,O-Z)
      IMPLICIT COMPLEX*16 (C)
      LOGICAL TEI,TEJ,TMI,TMJ,LPOL,LSIZE,LSAME,LCON,LPLAZ
      DIMENSION Y(2),YA(2)
      COMMON DIA,MMM,NHOLE,NMODE,NUMTE,NUMTM,F,FCON,ER,LPOL,LSIZE,LSAME
      COMMON /APERS/ IT,MI,NI,MJ,NJ,AI,AJ,TEI,TEJ,TMI,TMJ,PHI,PHIP,R
      COMMON /CONST/ CJ,PI,TWOPI,DTOR,RTOD,BIG,SMALL,BMAX,Y0,MAX,LAYM
      COMMON /LAYER/ CEP,CUP,NLAY,D(601),CE(601),CU(601),CEUP,LCON,LPLAZ
      COMMON /ARRYS/ MIJ(20),NIJ(20),MIJP(20),NIJP(20),
     A               CY(40,40),CA(40,40),CB(40,40),IPIV(40),INDX(40,2),
     B               AIJ(40),XIJ(40),YIJ(40),PHIJP(40)
      EXTERNAL FCTCWG
101   FORMAT(1X)
      DO 1000 IHOLE=1,NHOLE
      DO 1000 JHOLE=1,NHOLE
      DO 1000 IMODE=1,NMODE
      DO 1000 JMODE=1,NMODE
      II=(IHOLE-1)*NMODE+IMODE
      JJ=(JHOLE-1)*NMODE+JMODE
      IF(II.LE.JJ) GO TO 100
      CY(II,JJ)=CY(JJ,II)
      GO TO 1000
100   CONTINUE
      IF(IHOLE.EQ.1) GO TO 110
      IF(IHOLE.NE.JHOLE) GO TO 110
      IF(.NOT.LSIZE) GO TO 110
      CY(II,JJ)=CY(IMODE,JMODE)
      GO TO 1000
110   TEI=.TRUE.
      TEJ=.TRUE.
      TMI=.FALSE.
      TMJ=.FALSE.
      IF(IMODE.LE.NUMTE) GO TO 120
      TEI=.FALSE.
      TMI=.TRUE.
120   IF(JMODE.LE.NUMTE) GO TO 130
      TEJ=.FALSE.
      TMJ=.TRUE.
130   CONTINUE
      IT=0
      IF(TEI.AND.TEJ)IT=1
      IF(TMI.AND.TMJ)IT=2
      IF(TEI.AND.TMJ)IT=3
      IF(TMI.AND.TEJ)IT=4
      AI=AIJ(IHOLE)
      AJ=AIJ(JHOLE)
```

```
      PHPI=PHIJP(IHOLE)
      PHPJ=PHIJP(JHOLE)
      IF(TMI)PHPI=PHPI-0.5*PI
      IF(TMJ)PHPJ=PHPJ-0.5*PI
      PHIP=PHPJ-PHPI
      XJI=XIJ(JHOLE)-XIJ(IHOLE)
      YJI=YIJ(JHOLE)-YIJ(IHOLE)
      R=DSQRT(XJI*XJI+YJI*YJI)
      IF(IHOLE.EQ.JHOLE)R=0.0
      IF(DABS(R).LT.1.0D-20)R=0.0
      IF(DABS(PHIP).LT.1.0D-04)PHIP=0.0
      PHI=0.0
      IF(R.LT.1.0D-04) GO TO 160
      IF(DABS(XJI).LT.1.0D-04) GO TO 140
      PHI=DATAN2(YJI,XJI)-PHPI
      GO TO 150
140   PHI=0.5*PI
      IF(YJI.LT.0.0)PHI=PHI+PI
      PHI=PHI-PHPI
150   CONTINUE
160   IF(TMI) GO TO 170
      IF(.NOT.TEI) STOP 1111
      MI=MIJ(IMODE)
      NI=NIJ(IMODE)
      GO TO 180
170   IDEM=IMODE-NUMTE
      MI=MIJP(IDEM)
      NI=NIJP(IDEM)
180   IF(TMJ) GO TO 190
      IF(.NOT.TEJ) STOP 2222
      MJ=MIJ(JMODE)
      NJ=NIJ(JMODE)
      GO TO 200
190   JDEM=JMODE-NUMTE
      MJ=MIJP(JDEM)
      NJ=NIJP(JDEM)
200   CONTINUE
      IF(IHOLE.NE.JHOLE) GO TO 210
      IF(MJ.EQ.MI) GO TO 210
      CY(II,JJ)=(0.0,0.0)
      GO TO 1000
210   CONTINUE
      IF(IHOLE.EQ.JHOLE) GO TO 230
      IF(IHOLE.EQ.1) GO TO 230
      IF(.NOT.LSIZE) GO TO 230
      IF(.NOT.LPOL)  GO TO 230
      IHM1=IHOLE-1
      JHM1=JHOLE-1
      DO 220 IKH=1,IHM1
      DO 220 JKH=1,JHM1
```

```
      IF(DABS(AI-AIJ(IKH)).GT.0.001) GO TO 220
      IF(DABS(AJ-AIJ(JKH)).GT.0.001) GO TO 220
      TX=DABS(DABS(XJI)-DABS(XIJ(IKH)-XIJ(JKH)))
      IF(TX.GT.0.001) GO TO 220
      TY=DABS(DABS(YJI)-DABS(YIJ(IKH)-YIJ(JKH)))
      IF(TY.GT.0.001) GO TO 220
      PT=DABS(DABS(PHIP)-DABS(PHIJP(IKH)-PHIJP(JKH)))
      IF(PT.GT.0.001) GO TO 220
      IK=(IKH-1)*NMODE+IMODE
      JK=(JKH-1)*NMODE+JMODE
      CY(II,JJ)=CY(IK,JK)
      GO TO 1000
220   CONTINUE
230   CONTINUE
      EMI=2.0
      EMJ=2.0
      IF(MI.EQ.0)EMI=1.0
      IF(MJ.EQ.0)EMJ=1.0
      CALL FACS(MI,NI,MJ,NJ,AI,AJ,R,PHI,PHIP,EMI)
      YTEST=0.0001
      Y(1)=0.0
      Y(2)=0.0
      NDEL=50
      BDEL=0.25
      AA=0.0001
      BB=BDEL
      TOL=0.0001
240   CALL INTG2(AA,BB,FCTCWG,NDEL,TOL,YA,Y)
      Y(1)=Y(1)+YA(1)
      Y(2)=Y(2)+YA(2)
      IF(NLAY.EQ.0) GO TO 270
      DO 250 N=1,NLAY
      EUREAL=DREAL(CE(N)*CU(N))
      IF(EUREAL.LT.0.0)EUREAL=-EUREAL
      IF((BB*BB).LE.EUREAL) GO TO 260
250   CONTINUE
      GO TO 270
260   CONTINUE
      IF(DABS(BB-1.0).LT.0.01)BB=1.0
      AA=BB
      BB=BB+BDEL
      IF(DABS(AA-1.0).LT.0.01)AA=1.0001
      IF(DABS(BB-1.0).LT.0.01)BB=0.9999
      GO TO 240
270   IF(BB.LT.2.0) GO TO 260
      BDEL=1.0
      NBDEL=0
      AA=BB
      BB=BB+BDEL
280   CALL INTG2(AA,BB,FCTCWG,NDEL,TOL,YA,Y)
```

```
      Y(1)=Y(1)+YA(1)
      Y(2)=Y(2)+YA(2)
      IF(DABS(Y(1)).LE.YTEST) GO TO 290
      IF(DABS(YA(1)/Y(1)).GT.YTEST) GO TO 310
290   IF(DABS(Y(2)).LE.YTEST) GO TO 300
      IF(DABS(YA(2)/Y(2)).GT.YTEST) GO TO 310
300   NBDEL=NBDEL+1
      IF(NBDEL.GE.5) GO TO 330
      GO TO 320
310   NBDEL=0
320   IF(BB.GE.BMAX) GO TO 330
      IF(BB.GE.20.0)BDEL=5.0
      AA=BB
      BB=BB+BDEL
      GO TO 280
330   CYIJ=-Y0*DSQRT(EMI*EMJ)*DCMPLX(Y(1),Y(2))
      CY(II,JJ)=CYIJ
      WRITE(6,340)II,JJ,CY(II,JJ),BB,IHOLE,IMODE,JHOLE,JMODE
340   FORMAT(1X'CY('I2','I2')=('E11.4','E11.4')'3X'BB='F6.2,
     A4X'IH='I2,2X'IM='I2,3X'JH='I2,2X'JM='I2)
1000  CONTINUE
      WRITE(6,101)
      RETURN
      END
```

```
      SUBROUTINE SMAT
c.........................................................................
c:  Calculates coefficients of complex scattering matrix [S] (see eq.1):
c........................................................................:
      IMPLICIT REAL*8 (A,B,D-H,O-Z)
      IMPLICIT COMPLEX*16 (C)
      LOGICAL TEI,TEJ,TMI,TMJ
      LOGICAL LPOL,LSIZE,LSAME
      COMMON DIA,MMM,NHOLE,NMODE,NUMTE,NUMTM,F,FCON,ER,LPOL,LSIZE,LSAME
      COMMON /APERS/ IT,MI,NI,MJ,NJ,AI,AJ,TEI,TEJ,TMI,TMJ,PHI,PHIP,R
      COMMON /ZEROS/ XMNP(9,6),XMN(9,6)
      COMMON /CONST/ CJ,PI,TWOPI,DTOR,RTOD,BIG,SMALL,BMAX,YO,MAX,LAYM
      COMMON /ARRYS/ MIJ(20),NIJ(20),MIJP(20),NIJP(20),
     A               CY(40,40),CA(40,40),CB(40,40),IPIV(40),INDX(40,2),
     B               AIJ(40),XIJ(40),YIJ(40),PHIJP(40)
      DO 130 IHOLE=1,NHOLE
      DO 130 JHOLE=1,NHOLE
      DO 130 IMODE=1,NMODE
      DO 130 JMODE=1,NMODE
      TMI=.FALSE.
      IF(IMODE.GT.NUMTE)TMI=.TRUE.
      II=(IHOLE-1)*NMODE+IMODE
      JJ=(JHOLE-1)*NMODE+JMODE
      AKOI=AIJ(IHOLE)*TWOPI
      IF(TMI)GO TO 100
      NI=NIJ(IMODE)
      MIP=MIJ(IMODE)+1
      FSQI=(XMNP(MIP,NI)/AKOI)**2
      FTSQ=ER-FSQI
      FCON=DABS(FTSQ)
      IF(FCON.LT.SMALL)FCON=0.0
      IF(FCON.GT.BIG)FCON=BIG
      IF(FTSQ.GE.0.0)CYMNO=YO*DSQRT(FCON)
      IF(FTSQ.LT.0.0)CYMNO=-CJ*YO*DSQRT(FCON)
      GO TO 110
100   IDEM=IMODE-NUMTE
      NI=NIJP(IDEM)
      MIP=MIJP(IDEM)+1
      FSQI=(XMN(MIP,NI)/AKOI)**2
      FTSQ=ER-FSQI
      FCON=DABS(FTSQ)
      IF(FCON.LT.SMALL)FCON=SMALL
      IF(FCON.GT.BIG)FCON=BIG
      IF(FTSQ.GE.0.0)CYMNO=YO/DSQRT(FCON)
      IF(FTSQ.LT.0.0)CYMNO=YO/(-CJ*DSQRT(FCON))
110   CONTINUE
      CA(II,JJ)=CY(II,JJ)
      CB(II,JJ)=-CY(II,JJ)
      IF(II.EQ.JJ)CA(II,JJ)=CYMNO+CY(II,JJ)
      IF(II.EQ.JJ)CB(II,JJ)=CYMNO-CY(II,JJ)
```

```
      CY(II,JJ)=CB(II,JJ)
      IF(II.EQ.JJ)WRITE(6,120)II,CYMNO,IHOLE,IMODE
120   FORMAT(1X'YMN('I2')=('E11.4','E11.4')'3X'IHOLE='I2,3X'IMODE='I2)
130   CONTINUE
      WRITE(6,140)
140   FORMAT(/10X'SCATTERING MATRIX')
      CALL CINV(CA,MMM,IPIV,INDX,MAX)
      DO 160 I=1,MMM
      DO 160 J=1,MMM
      CB(I,J)=(0.0,0.0)
      DO 150 K=1,MMM
      CB(I,J)=CB(I,J)+CY(I,K)*CA(K,J)
150   CONTINUE
160   CONTINUE
      DO 200 I=1,MMM
      DO 200 J=1,MMM
      XISOL=-300.0
      XDX=CDABS(CB(I,J))
      IF(XDX.LT.1.0D-15)GO TO 180
      XISOL=20.*DLOG10(XDX)
      XX1=DIMAG(CB(I,J))
      XX2=DREAL(CB(I,J))
      PHASE=RTOD*DATAN2(XX1,XX2)
      WRITE(6,170)I,J,CB(I,J),XISOL,PHASE
170   FORMAT(1X'S('I2','I2')=('E11.4','E11.4')'3XF9.4' DB'3XF9.4' DEG')
      GO TO 200
180   WRITE(6,190)I,J,CB(I,J)
190   FORMAT(1X'S('I2','I2')=('E11.4','E11.4')'3X'BELOW -300 DB')
200   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CINV(CA,N,IPIV,INDX,MAX)
c.....................................................................
c:   Performs matrix inversion of (N by N) complex matrix CA.         :
c:   On return, CA contains inverted matrix.  CA must be dimensioned at :
c:   least N by N in calling routine.  IPIV and INDX are used internally:
c:   by CINV and must be dimensioned at least N and N by 2 in calling  :
c:   routine.  MAX is row dimension of CA and INDX in calling routine.  :
c....................................................................:
      IMPLICIT REAL*8 (A,B,D-H,O-Z)
      IMPLICIT COMPLEX*16 (C)
      DIMENSION CA(MAX,N),IPIV(N),INDX(MAX,2)
      DATA C0,C1 /(0.0D0,0.0D0),(1.0D0,0.0D0)/
      DATA BIG,SMALL /1.0D+38,1.0D-38/
      ISCALE=0
      RL=BIG
      RS=SMALL
      CDET=C1
      ADM=1.0
      DO 100 J=1,N
100   IPIV(J)=0
      DO 220 I=1,N
      AVM=0.0
      DO 130 J=1,N
      IF(IPIV(J).EQ.1) GO TO 130
      DO 120 K=1,N
      IF(IPIV(K)-1)110,120,260
110   CONTINUE
      AVA=CDABS(CA(J,K))
      IF(AVM.GE.AVA) GO TO 120
      IROW=J
      ICOL=K
      AVM=AVA
120   CONTINUE
130   CONTINUE
      IF(AVM.EQ.0.0) GO TO 250
      IPIV(ICOL)=IPIV(ICOL)+1
      IF(IROW.EQ.ICOL) GO TO 150
      CDET=-CDET
      DO 140 L=1,N
      CSWAP=CA(IROW,L)
      CA(IROW,L)=CA(ICOL,L)
      CA(ICOL,L)=CSWAP
140   CONTINUE
150   CONTINUE
      INDX(I,1)=IROW
      INDX(I,2)=ICOL
      CPIV=CA(ICOL,ICOL)
      APV=CDABS(CPIV)
      IF(APV.EQ.0.0) GO TO 250
      CPIVI=CPIV
```

```
      ADM=CDABS(CDET)
      IF(ADM.LT.RL) GO TO 160
      CDET=CDET/RL
      ADM=CDABS(CDET)
      ISCALE=ISCALE+1
      IF(ADM.LT.RL) GO TO 170
      CDET=CDET/RL
      ISCALE=ISCALE+1
      GO TO 170
160   CONTINUE
      IF(ADM.GT.RS) GO TO 170
      CDET=CDET*RL
      ADM=CDABS(CDET)
      ISCALE=ISCALE-1
      IF(ADM.GT.RS) GO TO 170
      CDET=CDET*RL
      ISCALE=ISCALE-1
170   CONTINUE
      APV=CDABS(CPIVI)
      IF(APV.LT.RL) GO TO 180
      CPIVI=CPIVI/RL
      APV=CDABS(CPIVI)
      ISCALE=ISCALE+1
      IF(APV.LT.RL) GO TO 190
      CPIVI=CPIVI/RL
      ISCALE=ISCALE+1
      GO TO 190
180   CONTINUE
      IF(APV.GT.RS) GO TO 190
      CPIVI=CPIVI*RL
      APV=CDABS(CPIVI)
      ISCALE=ISCALE-1
      IF(APV.GT.RS) GO TO 190
      CPIVI=CPIVI*RL
      ISCALE=ISCALE-1
190   CONTINUE
      CDET=CDET*CPIVI
      CA(ICOL,ICOL)=C1
      DO 200 L=1,N
200   CA(ICOL,L)=CA(ICOL,L)/CPIV
      DO 220 L1=1,N
      IF(L1.EQ.ICOL) GO TO 220
      CSWAP=CA(L1,ICOL)
      CA(L1,ICOL)=C0
      DO 210 L=1,N
210   CA(L1,L)=CA(L1,L)-CA(ICOL,L)*CSWAP
220   CONTINUE
      DO 240 I=1,N
      L=N+1-I
      IF(INDX(L,1).EQ.INDX(L,2)) GO TO 240
```

```
      IROW=INDX(L,1)
      ICOL=INDX(L,2)
      DO 230 K=1,N
      CSWAP=CA(K,IROW)
      CA(K,IROW)=CA(K,ICOL)
      CA(K,ICOL)=CSWAP
230   CONTINUE
240   CONTINUE
      GO TO 260
250   CDET=C0
      ISCALE=0
260   RETURN
      END
```

```
      SUBROUTINE INTG2(AA,BB,FX,NT,TOL,SUM,Y)
c.......................................................................
c:  Integrates 2 real functions (may be real and imaginary parts of    :
c:  of a complex function) over limits of AA to BB by using a Simpson  :
c:  integration procedure with interval halving until relative error   :
c:  TOL is reached on each interval.  NT is number of intervals.       :
c:  SUM is a 2 element array of the resultant integrals over AA to BB. :
c:  Y is the cumulative values of SUM from previous calls to INTG2.    :
c:  Y must be set to zero prior to first call to INTG2                :
c:  and is used when integrating FX over wide limits     ..............:
c:  by successive calls to INTG2.  FX(X,WK) is user      : May 1989    :
c:  supplied subroutine to evaluate integrands WK.       : M.C.Bailey  :
c:  FX must be declared EXTERNAL in calling routine.     : Hampton, VA :
c:.....................................................:..............:
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION SUM(2),Y(2),WK(2),SUMA(2),SUMB(2),ESUM(2),BASE(2)
      DIMENSION A(401,2),B(401,2)
      DATA NMAX,NIMAX/400,2/
      IF(NT.GT.NMAX) STOP
      DEL=(BB-AA)/NT
      DELO2=0.5D0*DEL
      DO 100 I=1,NT+1
      X=AA+(I-1)*DEL
      CALL FX(X,WK)
      A(I,1)=WK(1)
      A(I,2)=WK(2)
100   CONTINUE
      BASE(1)=DELO2*(A(1,1)+A(NT+1,1))
      BASE(2)=DELO2*(A(1,2)+A(NT+1,2))
      DO 110 I=2,NT
      BASE(1)=BASE(1)+DEL*A(I,1)
      BASE(2)=BASE(2)+DEL*A(I,2)
110   CONTINUE
      DENOM=1.D-28
      TEST=DABS(BASE(1)+Y(1))
      IF(TEST.GT.DENOM)DENOM=TEST
      TEST=DABS(BASE(2)+Y(2))
      IF(TEST.GT.DENOM)DENOM=TEST
      SUM(1)=0.0D0
      SUM(2)=0.0D0
      DO 300 K=1,NT
      ND=1
      FF=AA+(K-1)*DEL
      X=FF+DEL
      CALL FX(X,WK)
      A(2,1)=WK(1)
      A(2,2)=WK(2)
      SUMA(1)=DELO2*(A(1,1)+A(2,1))
      SUMA(2)=DELO2*(A(1,2)+A(2,2))
200   ND=ND+ND
```

```
      IF(ND.GT.NMAX) GO TO 270
      NB=ND/2
      NA=NB+1
      NP=ND+1
      DELN=DEL/ND
      IF(DELN.LT.1.D-6) GO TO 270
      DO 210 J=1,NB
      X=FF+DELN*(J+J-1)
      CALL FX(X,WK)
      B(J,1)=WK(1)
      B(J,2)=WK(2)
210   CONTINUE
      NAP=NA+1
      NBP=NB+1
      DO 220 J=1,NA
      JL=NAP-J
      JP=JL+JL-1
      A(JP,1)=A(JL,1)
      A(JP,2)=A(JL,2)
220   CONTINUE
      DO 230 J=1,NB
      JL=NBP-J
      JP=JL+JL
      A(JP,1)=B(JL,1)
      A(JP,2)=B(JL,2)
230   CONTINUE
      SUMB(1)=0.5D0*(A(1,1)+A(NP,1))
      SUMB(2)=0.5D0*(A(1,2)+A(NP,2))
      DO 240 NS=2,ND
      SUMB(1)=SUMB(1)+A(NS,1)
      SUMB(2)=SUMB(2)+A(NS,2)
240   CONTINUE
      SUMB(1)=DELN*SUMB(1)
      SUMB(2)=DELN*SUMB(2)
      ESUM(1)=DABS(SUMB(1)-SUMA(1))/DENOM
      ESUM(2)=DABS(SUMB(2)-SUMA(2))/DENOM
      SUMA(1)=SUMB(1)
      SUMA(2)=SUMB(2)
      IF(ESUM(1).GT.TOL) GO TO 200
      IF(ESUM(2).GT.TOL) GO TO 200
      GO TO 280
270   ND=ND/2
280   CONTINUE
      SUM(1)=SUM(1)+SUMA(1)
      SUM(2)=SUM(2)+SUMA(2)
      A(1,1)=A(NP,1)
      A(1,2)=A(NP,2)
300   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FCTCWG(BETA,FI)
c.....................................................................
c:  Evaluates integrand (real and imaginary parts) of equation 2.     :
c:....................................................................:
      IMPLICIT REAL*8 (A,B,D-H,O-Z)
      IMPLICIT COMPLEX*16 (C)
      LOGICAL TEI,TEJ,TMI,TMJ,LCON,LPLAZ
      DIMENSION FI(2)
      COMMON /APERS/ IT,MI,NI,MJ,NJ,AI,AJ,TEI,TEJ,TMI,TMJ,PHI,PHIP,R
      COMMON /LAYER/ CEP,CUP,NLAY,D(601),CE(601),CU(601),CEUP,LCON,LPLAZ
      CALL MLAYER(BETA,CW1,CW2)
      CALL UVIJ(BETA,UIJ,VIJ,IT)
      GO TO (10,20,30,40)IT
      GO TO 900
10    CALL STEI(BETA,SSI,TTI)
      IF(MI.NE.MJ) GO TO 15
      IF(NI.NE.NJ) GO TO 15
      IF(DABS(AI-AJ).GT.1.D-4) GO TO 15
      SSJ=SSI
      TTJ=TTI
      GO TO 50
15    CALL STEJ(BETA,SSJ,TTJ)
      GO TO 50
20    CALL STMI(BETA,SSI,TTI)
      IF(MI.NE.MJ) GO TO 25
      IF(NI.NE.NJ) GO TO 25
      IF(DABS(AI-AJ).GT.1.D-4) GO TO 25
      SSJ=SSI
      TTJ=TTI
      GO TO 50
25    CALL STMJ(BETA,SSJ,TTJ)
      GO TO 50
30    CALL STEI(BETA,SSI,TTI)
      CALL STMJ(BETA,SSJ,TTJ)
      GO TO 50
40    CALL STMI(BETA,SSI,TTI)
      CALL STEJ(BETA,SSJ,TTJ)
50    F1=+SSI*SSJ*UIJ
      F2=-TTI*TTJ*VIJ
      CCW=(F1*CW1+F2*CW2)*BETA
      FI(1)=DREAL(CCW)
      FI(2)=DIMAG(CCW)
      RETURN
900   WRITE(6,901)IT
901   FORMAT(1X'### ERRORS IN FCTCWG ###'
     A/1X'(IT must be between 1 and 4)'
     B/1X'IT='I5)
      STOP
      END
```

```
      SUBROUTINE MLAYER(BETA,CW1,CW2)
c.......................................................................
c:  Evaluates complex functions in equations 5 and 6.                  :
c.......................................................................:
      IMPLICIT REAL*8 (A,B,D-H,O-Z)
      IMPLICIT COMPLEX*16 (C)
      LOGICAL LCON,LPLAZ
      COMMON /LAYER/ CEP,CUP,NLAY,D(601),CE(601),CU(601),CEUP,LCON,LPLAZ
      DATA XKO,BIG,CSMAL/6.283185307179586D0,1.0D+28,(1.0D-28,0.0D0)/
      DATA CO,C1,CJ/(0.0D0,0.0D0),(1.0D0,0.0D0),(0.0D0,1.0D0)/
      CU(NLAY+1)=CUP
      CE(NLAY+1)=CEP
      BSQ=BETA*BETA
      CPF=CO
      CGPOG=CO
      CFPOF=C1
      IF(LCON) GO TO 35
      CPF=C1
      CBSQP=CEUP-BSQ
      IF(LPLAZ)CBSQP=C1-BSQ
      IF(DREAL(CBSQP))20,10,10
10    CKZ=XKO*CDSQRT(CBSQP)
      GO TO 30
20    CKZ=-CJ*XKO*CDSQRT(-CBSQP)
30    CFPOF=-CJ*CKZ
      CGPOG=CFPOF
35    IF(NLAY.EQ.0) GO TO 200
      N=NLAY
40    CARG=CE(N)*CU(N)-BSQ
      IF(DREAL(CARG).LT.0.0) GO TO 50
      CKZ=XKO*CDSQRT(CARG)
      GO TO 60
50    CKZ=-CJ*XKO*CDSQRT(-CARG)
60    CKZD=CKZ*D(N)
      A=DREAL(CKZD)
      B=DIMAG(CKZD)
      DSINA=DSIN(A)
      DCOSA=DCOS(A)
      NSIGN=1
      IF(B.LT.0.0)NSIGN=-1
      BP=NSIGN*B
      IF(BP.GT.1.0D-10) GO TO 70
      CSINKD=DSINA
      CCOSKD=DCOSA
      GO TO 100
70    IF(BP.LT.65) GO TO 80
      DCOSHB=0.5*(1.6948892D+28)
      DSINHB=NSIGN*DCOSHB
      GO TO 90
80    EP=DEXP(BP)
```

```
      EM=1.0D0/EP
      DSINHB=0.5D0*(EP-EM)
      DCOSHB=0.5D0*(EP+EM)
      DSINHB=NSIGN*DSINHB
90    CSINKD=DSINA*DCOSHB+CJ*DCOSA*DSINHB
      CCOSKD=DCOSA*DCOSHB-CJ*DSINA*DSINHB
100   CONTINUE
      CCU=CFPOF*(CU(N)/CU(N+1))/CKZ
      CCE=CGPOG*(CE(N)/CE(N+1))/CKZ
      CFOF1=CPF*CCOSKD-CCU*CSINKD
      CGOG1=CCOSKD-CCE*CSINKD
      CFPOF1=CKZ*(CPF*CSINKD+CCU*CCOSKD)
      CGPOG1=CKZ*(CSINKD+CCE*CCOSKD)
      CFPOF=CFPOF1/CFOF1
      CGPOG=CGPOG1/CGOG1
      CPF=C1
      N=N-1
      IF(N.GT.0) GO TO 40
200   CW1=-CJ*XK0*CE(1)/CGPOG
      CW2=CJ*CFPOF/(XK0*CU(1))
      RETURN
      END
```

```
      SUBROUTINE FACS(MII,NI,MJJ,NJ,AI,AJ,R,PHI,PHIP,EMI)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /FACTS/ MI,MJ,MP,MM,MPP,MMP,MI1,MJ1,MXP,MPI,MPJ,MPPI,MPPJ,
     A               EM1,EM2,AIKO,AJKO,RKO,XEI,XEJ,XMI,XMJ,
     B               XESI,XESJ,XMSI,XMSJ,SFI,SFJ,TFI,TFJ,TFEI,TFEJ,
     C               XDS,XDC,XDSP,XDCP,XDSM,XDCM
      COMMON /ZEROS/ XMNP(9,6),XMN(9,6)
      PI=2.0D0*DASIN(1.0D0)
      TWOPI=2.0D0*PI
      MI=MII
      MJ=MJJ
      MP=MJ+MI
      MM=MJ-MI
      MPP=MP+1
      MMP=MM+1
      MI1=(-1)**MI
      MJ1=(-1)**MJ
      MXP=-MM+1
      MPI=MI+1
      MPJ=MJ+1
      MPPI=MPI+1
      MPPJ=MPJ+1
      EM1=EMI-1.0
      EM2=2.0/EMI
      AIKO=AI*TWOPI
      AJKO=AJ*TWOPI
      RKO=R*TWOPI
      PHIPMI=MI*PHIP
      PHIPMJ=MJ*PHIP
      XDS=DSIN(PHIPMI)
      XDC=DCOS(PHIPMI)
      AGMP=MP*PHI-PHIPMJ
      AGMM=MM*PHI-PHIPMJ
      XDSP=DSIN(AGMP)
      XDCP=DCOS(AGMP)
      XDSM=DSIN(AGMM)
      XDCM=DCOS(AGMM)
      XEI=XMNP(MPI,NI)
      XEJ=XMNP(MPJ,NJ)
      XMI=XMN(MPI,NI)
      XMJ=XMN(MPJ,NJ)
      XESI=XEI*XEI
      XESJ=XEJ*XEJ
      XMSI=XMI*XMI
      XMSJ=XMJ*XMJ
      DEI=XESI-MI*MI
      DEJ=XESJ-MJ*MJ
      FEI=AIKO/DSQRT(DEI)
      FEJ=AJKO/DSQRT(DEJ)
      SFI=MI*FEI
```

```
SFJ=MJ*FEJ
TFI=XESI*FEI
TFJ=XESJ*FEJ
TFEI=-0.5D0*MI*MI/(XEI*DEI)
TFEJ=-0.5D0*MJ*MJ/(XEJ*DEJ)
RETURN
END
```

```
      SUBROUTINE UVIJ(BETA,UIJ,VIJ,IT)
c.................................................................
c:  Evaluates functions in table I or table II.                  :
c................................................................:
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BJ(21)
      COMMON /FACTS/ MI,MJ,MP,MM,MPP,MMP,MI1,MJ1,MXP,MPI,MPJ,MPPI,MPPJ,
     A               EM1,EM2,AIKO,AJKO,RKO,XEI,XEJ,XMI,XMJ,
     B               XESI,XESJ,XMSI,XMSJ,SFI,SFJ,TFI,TFJ,TFEI,TFEJ,
     C               XDS,XDC,XDSP,XDCP,XDSM,XDCM
      UIJ=0.0D0
      VIJ=0.0D0
      IF(RKO.GT.1.0D-04) GO TO 150
      IF(MI.NE.MJ) RETURN
      GO TO (110,120,130,140)IT
110   UIJ=-EM1*XDC
      VIJ=EM2*XDC
      RETURN
120   UIJ=-EM2*XDC
      RETURN
130   UIJ=EM1*XDS
      RETURN
140   UIJ=EM1*XDS
      RETURN
150   ARG=RKO*BETA
      CALL BESJS(ARG,BJ,MPP)
      GO TO (160,190,220,250)IT
160   T1=BJ(MPP)*XDCP
      IF(MI.GT.MJ) GO TO 170
      T2=MI1*BJ(MMP)*XDCM
      GO TO 180
170   T2=MJ1*BJ(MXP)*XDCM
180   UIJ=MJ1*(T1-T2)
      VIJ=MJ1*(T1+T2)
      RETURN
190   T1=BJ(MPP)*XDCP
      IF(MI.GT.MJ) GO TO 200
      T2=MI1*BJ(MMP)*XDCM
      GO TO 210
200   T2=MJ1*BJ(MXP)*XDCM
210   UIJ=-MJ1*(T1+T2)
      RETURN
220   T1=BJ(MPP)*XDSP
      IF(MI.GT.MJ) GO TO 230
      T2=MI1*BJ(MMP)*XDSM
      GO TO 240
230   T2=MJ1*BJ(MXP)*XDSM
240   UIJ=MJ1*(T1-T2)
      RETURN
250   T1=BJ(MPP)*XDSP
```

```
      IF(MI.GT.MJ) GO TO 260
      T2=MI1*BJ(MMP)*XDSM
      GO TO 270
260   T2=MJ1*BJ(MXP)*XDSM
270   UIJ=MJ1*(T1-T2)
      RETURN
      END
```

```
      SUBROUTINE STEI(BETA,SS,TT)
c..............................................................
c:  Evaluates equations 12 and 13 for i-th aperture.           :
c:.............................................................:
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BJ(21)
      COMMON /FACTS/ MI,MJ,MP,MM,MPP,MMP,MI1,MJ1,MXP,MPI,MPJ,MPPI,MPPJ,
     A               EM1,EM2,AIKO,AJKO,RKO,XEI,XEJ,XMI,XMJ,
     B               XESI,XESJ,XMSI,XMSJ,SFI,SFJ,TFI,TFJ,TFEI,TFEJ,
     C               XDS,XDC,XDSP,XDCP,XDSM,XDCM
      ARG=AIKO*BETA
      CALL BESJS(ARG,BJ,MPPI)
      BJA=BJ(MPI)/ARG
      SS=SFI*BJA
      DN=XESI-ARG*ARG
      IF(DABS(DN).GT.1.0D-05) GO TO 10
      CALL BESJS(XEI,BJ,MPI)
      TT=TFEI*BJ(MPI)
      RETURN
10    TT=TFI*(MI*BJA-BJ(MPPI))/DN
      RETURN
      END


      SUBROUTINE STMI(BETA,SS,TT)
c..............................................................
c:  Evaluates equations 14 and 15 for i-th aperture.           :
c:.............................................................:
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BJ(21)
      COMMON /FACTS/ MI,MJ,MP,MM,MPP,MMP,MI1,MJ1,MXP,MPI,MPJ,MPPI,MPPJ,
     A               EM1,EM2,AIKO,AJKO,RKO,XEI,XEJ,XMI,XMJ,
     B               XESI,XESJ,XMSI,XMSJ,SFI,SFJ,TFI,TFJ,TFEI,TFEJ,
     C               XDS,XDC,XDSP,XDCP,XDSM,XDCM
      TT=0.0D0
      ARG=AIKO*BETA
      DN=XMSI-ARG*ARG
      IF(DABS(DN).GT.1.0D-05) GO TO 10
      CALL BESJS(XMI,BJ,MPPI)
      SS=AIKO*0.5D0*BJ(MPPI)
      RETURN
10    CALL BESJS(ARG,BJ,MPI)
      SS=AIKO*ARG*BJ(MPI)/DN
      RETURN
      END
```

```
      SUBROUTINE STEJ(BETA,SS,TT)
c.....................................................................
c:  Evaluates equations 12 and 13 for j-th aperture.                 :
c:....................................................................:
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BJ(21)
      COMMON /FACTS/ MI,MJ,MP,MM,MPP,MMP,MI1,MJ1,MXP,MPI,MPJ,MPPI,MPPJ,
     A               EM1,EM2,AIKO,AJKO,RKO,XEI,XEJ,XMI,XMJ,
     B               XESI,XESJ,XMSI,XMSJ,SFI,SFJ,TFI,TFJ,TFEI,TFEJ,
     C               XDS,XDC,XDSP,XDCP,XDSM,XDCM
      ARG=AJKO*BETA
      CALL BESJS(ARG,BJ,MPPJ)
      BJA=BJ(MPJ)/ARG
      SS=SFJ*BJA
      DN=XESJ-ARG*ARG
      IF(DABS(DN).GT.1.0D-05) GO TO 10
      CALL BESJS(XEJ,BJ,MPJ)
      TT=TFEJ*BJ(MPJ)
      RETURN
10    TT=TFJ*(MJ*BJA-BJ(MPPJ))/DN
      RETURN
      END


      SUBROUTINE STMJ(BETA,SS,TT)
c.....................................................................
c:  Evaluates equations 14 and 15 for j-th aperture.                 :
c:....................................................................:
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION BJ(21)
      COMMON /FACTS/ MI,MJ,MP,MM,MPP,MMP,MI1,MJ1,MXP,MPI,MPJ,MPPI,MPPJ,
     A               EM1,EM2,AIKO,AJKO,RKO,XEI,XEJ,XMI,XMJ,
     B               XESI,XESJ,XMSI,XMSJ,SFI,SFJ,TFI,TFJ,TFEI,TFEJ,
     C               XDS,XDC,XDSP,XDCP,XDSM,XDCM
      TT=0.0D0
      ARG=AJKO*BETA
      DN=XMSJ-ARG*ARG
      IF(DABS(DN).GT.1.0D-05) GO TO 10
      CALL BESJS(XMJ,BJ,MPPJ)
      SS=AJKO*0.5D0*BJ(MPPJ)
      RETURN
10    CALL BESJS(ARG,BJ,MPJ)
      SS=AJKO*ARG*BJ(MPJ)/DN
      RETURN
      END
```

```
      SUBROUTINE BESJS(XX,BJ,MP)
c...................................................................
c:  Evaluates Bessel function of first kind or orders 0 through MP-1  :
c:  for argument XX, and stores values in BJ(1) through BJ(MP).        :
c:  BJ must be dimensioned at least (MP+1) in calling program.         :
c:..................................................................:
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION BJ(1),B(130)
      DATA PI,PIO2 /3.14159265358979D0,1.57079632679490D0/
      DATA BIG,SMALL,PIO4 /1.0D+38,1.0D-38,0.78539816339745D0/
      DATA TWOTRD /0.666666666666667D0/
      X=DABS(XX)
      BJ(1)=1.0D0
      MT=MP-1
      N=MT+1
      IF(X.LT.80.)GO TO 110
      DO 100 I=1,N
100   BJ(I)=DSQRT(2.0D0/(PI*X))*DCOS(X-PIO4-PIO2*(I-1))
      GO TO 310
110   CONTINUE
      DO 120 I=2,N
120   BJ(I)=0.0D0
      IF(X.LT.1.D-06)RETURN
      IF(X-15.)130,130,140
130   NTEST=20.0D0+10.0D0*X-X**TWOTRD
      GO TO 150
140   NTEST=90.0D0+0.5D0*X
150   IF(MT-NTEST)170,160,160
160   N=NTEST-1
      GO TO 180
170   N=MT
180   BPREV=0.0
      N1=N+1
      F=2.0D0/X
      D=1.0D-06
      IF(X-5.)190,200,200
190   MA=X+6.0D0
      GO TO 210
200   MA=1.4D0*X+60.0D0/X
210   IFIXX=X
      MB=N+IFIXX/4+2
      MZERO=MAX0(MA,MB)
      MMAX=NTEST
      DO 280 M=MZERO,MMAX,3
      FM1=SMALL
      FM=0.0D0
      ALPHA=0.0D0
      IF(M-(M/2)*2)230,220,230
220   JT=-1
      GO TO 240
```

```
230     JT=1
240     M2=M-2
        DO 260 K=1,M2
        MK=M-K
        B(MK)=F*MK*FM1-FM
        IF(B(MK)-BIG)250,310,310
250     FM=FM1
        FM1=B(MK)
        JT=-JT
        S=1+JT
260     ALPHA=ALPHA+B(MK)*S
        B(1)=F*FM1-FM
        ALPHA=ALPHA+B(1)
        BTEST=B(N1)/ALPHA
        IF(DABS(BTEST-BPREV)-DABS(D*BTEST))290,290,270
270     BPREV=BTEST
280     CONTINUE
290     DO 300 I=1,N1
300     BJ(I)=B(I)/ALPHA
310     IF(XX.LT.0.0D0)GO TO 320
        RETURN
320     N=MT+1
        DO 330 I=1,N
        NN=I-1
330     BJ(I)=BJ(I)*(-1.0)**NN
        RETURN
        END
```

Report Documentation Page

NASA
National Aeronautics and Space Administration

1. Report No. NASA TM-101614	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle CWG - A Fortran Program for Mutual Coupling in a Planar Array of Circular Waveguide-fed Apertures		5. Report Date June 1989
		6. Performing Organization Code
7. Author(s) M. C. Bailey		8. Performing Organization Report No.
		10. Work Unit No. 506-44-21-03
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665-5225		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546-0001		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

This paper is a documentation of a Fortran program which calculates the mutual coupling between circular apertures in a conductive plane. The program is quite general in that the apertures do not have to be the same sizes, nor do they have to be polarized in the same direction. In addition, several waveguide modes (TE and/or TM) may be specified in the apertures and the mutual coupling between all combinations of apertures and modes will be calculated. The program also allows multiple layers of homogeneous dielectrics to be placed over the aperture array. Outside the layered region, one can specify either a homogeneous half-space, or a perfect reflecting surface.

17. Key Words (Suggested by Author(s)) antenna apertures arrays coupling		18. Distribution Statement Unclassified - Unlimited Subject Category 32	
19. Security Classif. (of this report) unclassified	20. Security Classif. (of this page) unclassified	21. No. of pages 45	22. Price A03

NASA FORM 1626 OCT 86