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Performance of Concatenated Codes Using 8-Bit and 10-Bit Reed-Solomon Codes

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The performance improvement of concatenated coding systems using 10-bit instead of 8-bit Reed-Solomon codes is measured by simulation. Three inner convolutional codes are considered: (7,1/2), (15,1/4), and (15,1/6). It is shown that approximately 0.2 dB can be gained at a bit error rate of 10^{-6}. The loss due to nonideal interleaving is also evaluated. Performance comparisons at very low bit error rates may be relevant for systems using data compression.

I. Introduction

Concatenated codes consisting of an inner convolutional code and an outer Reed-Solomon (RS) code are used in several current and planned deep-space missions. The Voyager spacecraft, for example, employs a concatenated coding system based on a (7,1/2) convolutional code as the inner code and an 8-bit (255,223) RS code as the outer code. The Galileo mission will use an experimental (15,1/4) convolutional code [1] concatenated with the same 8-bit RS code. Future missions may use the recently discovered (15,1/6) convolutional code [2] together with a 10-bit (1023,959) RS code.

The plan to develop a switchable (8-bit and 10-bit) Reed-Solomon decoder[1] suggested a study of the performance improvement that can be obtained by using the 10-bit RS code in place of the 8-bit RS code. The results of this study are the main subject of this article.

Since maximum-likelihood decoding of convolutional codes generates errors in bursts, a block interleaver is used between the convolutional encoder and the RS encoder to randomize the symbol errors. The deinterleaving operation performed at the receiving station removes most of the dependency among errors that enter the RS decoder, given that the interleaving depth I is sufficiently large. It is important to assess the performance degradation resulting from interleaving at a given depth with respect to ideal interleaving that assumes totally independent errors.

The availability of a hardware Viterbi decoder built by C. Lahmeyer made it possible to generate enough error bursts to compute the performance degradation due to nonideal interleaving, as described in [3]. Simulation results have been stored on disk files in a compressed format which allows for

[1] R. Stevens, "Board Report for Risk Assessment Review for a Switchable Reed-Solomon Decoder," JPL Interoffice Memorandum RS-88-024 (internal document), Jet Propulsion Laboratory, Pasadena, California, December 30, 1988.

easy reconstruction of the actual decoded bit stream. The hardware decoder uses the traceback method to compute the decoded bits. Paths are traced back starting from a random state. Traceback is accomplished by three buffers, each 170 bits long, storing the results of comparisons of merging paths. Received symbols are quantized by a uniform 8-bit quantizer. Operations internal to the decoder are performed with 16-bit precision.

II. Results for Convolutional Codes

Three convolutional codes are considered: Voyager's (7,1/2) code, Galileo's (15,1/4) code, and the (15,1/6) code described in [2].

Figure 1 shows the bit error rate (BER) and the symbol error rates (8- to 16-bit symbols) for the (7,1/2) code as a function of the bit signal-to-noise ratio (bit SNR). These are the only results described in this article that were obtained by using software simulation instead of the hardware decoder, since the software decoding speed is reasonable for constraint length 7 codes. The software decoder operates with no quantization of the received symbols and uses a path truncation length of 64 bits. The survivors are updated by the register exchange method, and the decoded bits are taken from the survivor with the lowest accumulated metric. Each data point in Fig. 1 is the result of a simulation run of at least 10 million information bits. Figure 1 has been included primarily for comparison with other more powerful codes.

Figure 2 shows the same performance results for the (15,1/4) code, obtained by hardware simulation. The data points below the bit error rate curve are the results of software simulation [1] with no quantization and a path truncation length of 128 bits.

Similar results are shown in Fig. 3 for the (15,1/6) code. The performance of the (15,1/6) code found by hardware simulation is slightly worse than that previously found in [2] by software simulation. To facilitate the comparison of the (15,1/4) and the (15,1/6) codes, their bit error rates are shown together in Fig. 4.

III. Results for Concatenated Codes

Figures 5 to 13 show the performance of concatenated codes with ideal and nonideal interleaving, and point out the difference in performance between 8-bit and 10-bit RS codes. The bit SNR shown in these figures is the bit signal-to-noise ratio of the concatenated system, which includes a penalty of 0.58 dB due to the rate of the 8-bit RS code, or 0.28 dB for the 10-bit RS code. The bit error rate at the RS decoder output is computed from the bit and symbol error rates of the Viterbi decoder.[2]

Figures 5 and 6 show the bit error rate of the (7,1/2) code concatenated with the 8-bit and 10-bit RS codes, respectively. Interleaving depths $I = 2$ and $I = 4$ are shown along with ideal interleaving. Higher values of I had nonmeasurable performance degradation relative to ideal interleaving. Larger constraint length codes suffer more from shallow interleaving since the average length of bursts grows with the constraint length. It must be observed that results for nonideal interleaving need very large amounts of data (decoded bits) and are not very accurate even with runs of 10 million or more bits. The 1σ statistical uncertainty for BER estimates lower than 10^{-5} is more than 100 percent for nonideal interleaving, but only approximately 20 percent for ideal interleaving. Figure 7 shows a comparison of the bit error rates of the 8-bit and 10-bit codes concatenated with the (7,1/2) code and ideal interleaving. The advantage of the 10-bit RS code becomes apparent only at very low bit error rates. A different 10-bit RS code specifically optimized for concatenation with the (7,1/2) code could offer a larger improvement over the 8-bit RS code than the (1023,959) RS code, which was optimized for the (15,1/6) code.

Figures 8 and 9 show the bit error rate of the (15,1/4) code concatenated with the 8-bit and 10-bit RS codes, respectively. Figures 5 and 8 appeared in [3], and are repeated for comparison. Figure 10 shows a comparison of the bit error rates of the 8-bit and 10-bit codes concatenated with the (15,1/4) code and ideal interleaving. Now the advantage of the 10-bit RS code over the 8-bit RS code is approximately 0.2 dB at BER = 10^{-6}. This advantage grows to approximately 0.3 dB at BER = 10^{-12}.

Finally, Figs. 11 and 12 show the bit error rate of the (15,1/6) code concatenated with the 8-bit and 10-bit RS codes, respectively. Figure 13 shows a comparison of the bit error rates of the 8-bit and 10-bit codes concatenated with the (15,1/6) code and ideal interleaving. The advantage of the 10-bit code is slightly less than 0.2 dB at BER = 10^{-6}, and approximately 0.3 dB at BER = 10^{-12}.

IV. Conclusions

In summary, the improvement offered by the 10-bit RS code over the 8-bit RS code is approximately 0.2 dB at BER =

[2] F. Pollara and S. Dolinar, "Concatenated Codes Performance at Low Bit Error Rates," JPL Interoffice Memorandum 331-88.2-043 (internal document), Jet Propulsion Laboratory, Pasadena, California, July 13, 1988.

10^{-6} for both the Galileo system and the new (15,1/6) code proposed for future missions. For systems that transmit heavily compressed data and may have to operate at BER = 10^{-12}, the improvement increases to approximately 0.3 dB.

This improvement is quickly eroded by insufficient interleaving. From our limited results on interleaving losses it is nevertheless possible to conclude that interleaving depths of eight or higher will produce insignificant losses at BER $\geqslant 10^{-6}$.

A comparison of three concatenated systems is shown as a summary in Fig. 14. This figures includes Voyager's coding system, the Galileo experimental code, and a concatenated code available for future missions.

Acknowledgment

The authors acknowledge helpful suggestions by S. Dolinar, who also provided some of the data included in this article.

References

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[2] J. H. Yuen and Q. D. Vo, "In Search of a 2-dB Coding Gain," *TDA Progress Report 42-83*, vol. July–September 1985, Jet Propulsion Laboratory, Pasadena, California, pp. 26–33, November 15, 1985.

[3] K.-M. Cheung and S. Dolinar, "Performance of Galileo's Concatenated Codes with Nonideal Interleaving," *TDA Progress Report 42-95*, vol. July–September 1988, pp. 148–152, November 15, 1988.

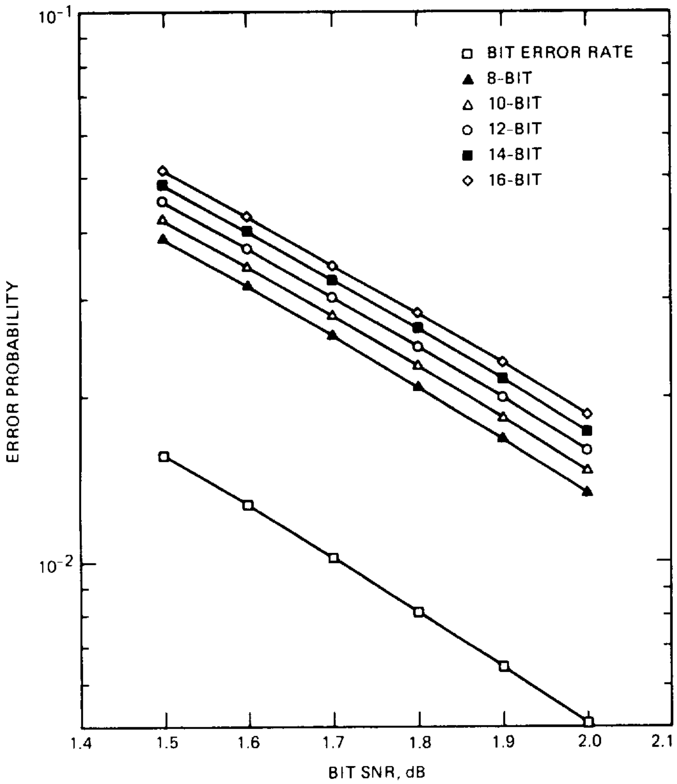

Fig. 1. Convolutional code performance: (7,1/2) code.

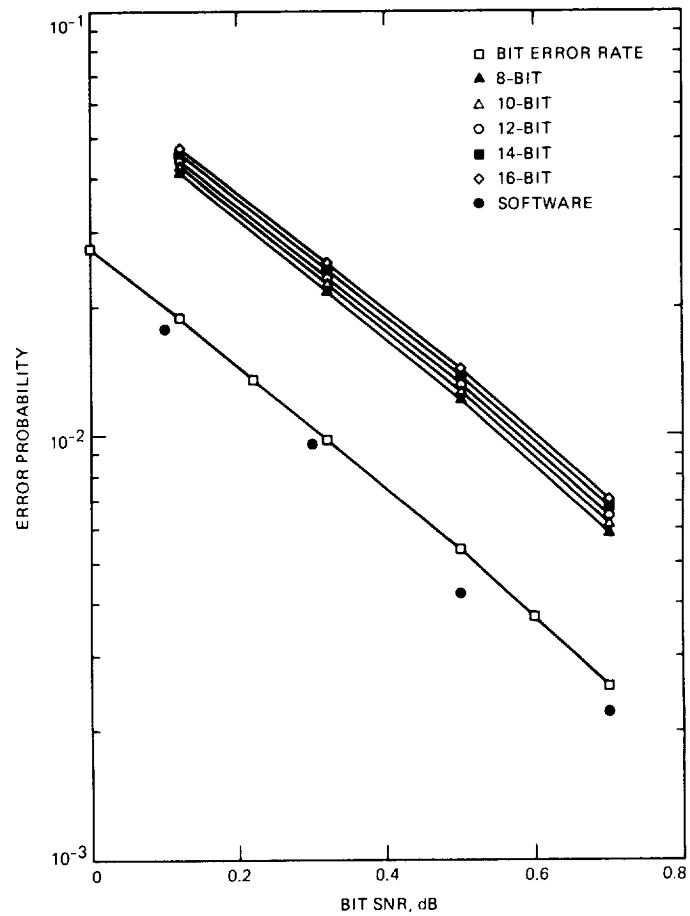

Fig. 2. Convolutional code performance: (15,1/4) code.

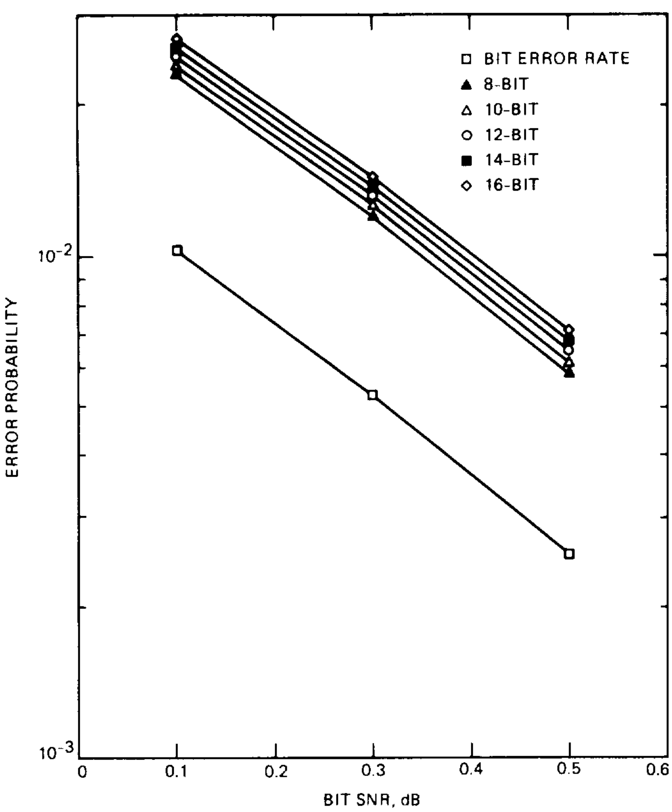

Fig. 3. Convolutional code performance: (15,1/6) code.

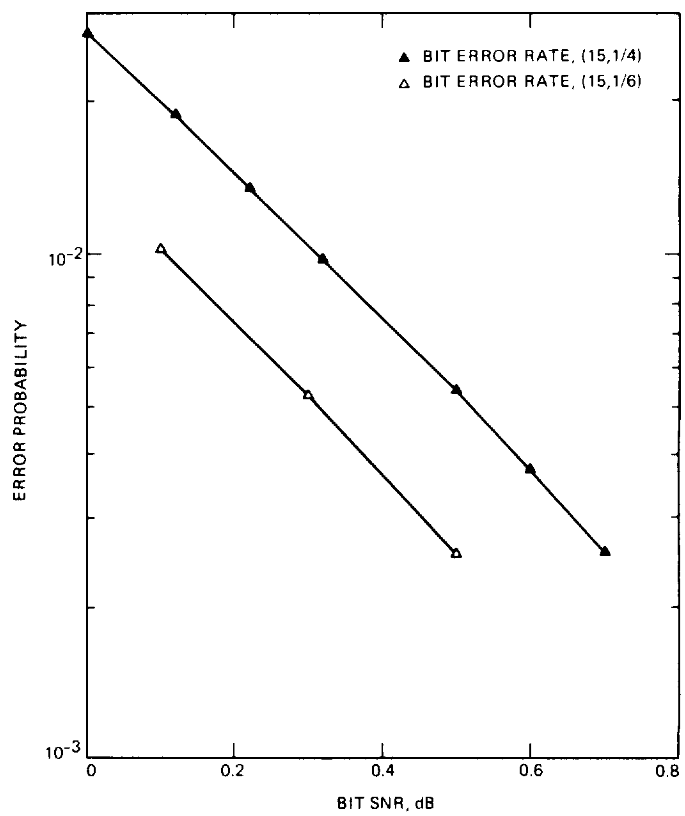

Fig. 4. Convolutional code performance comparison: (15,1/4) and (15,1/6) codes.

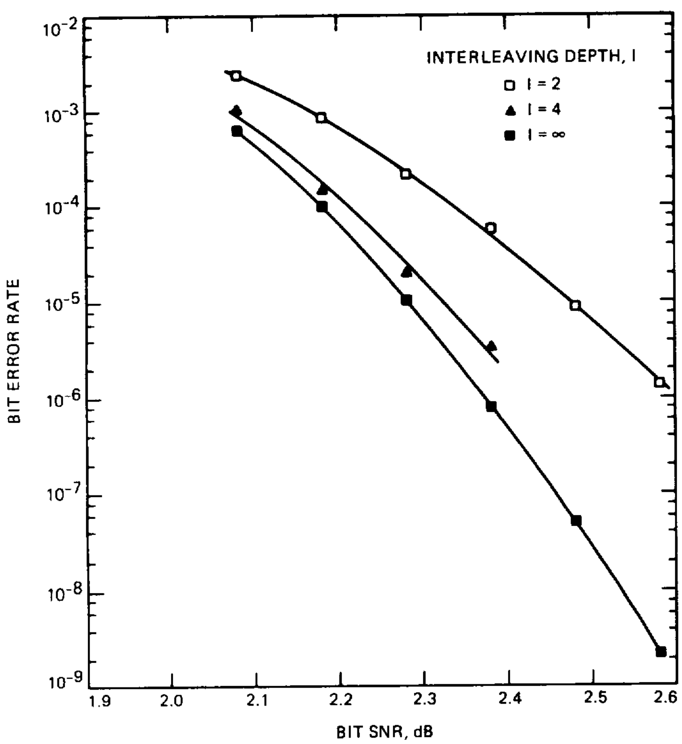

Fig. 5. Concatenated code performance: (7,1/2) convolutional and 8-bit (255,223) RS.

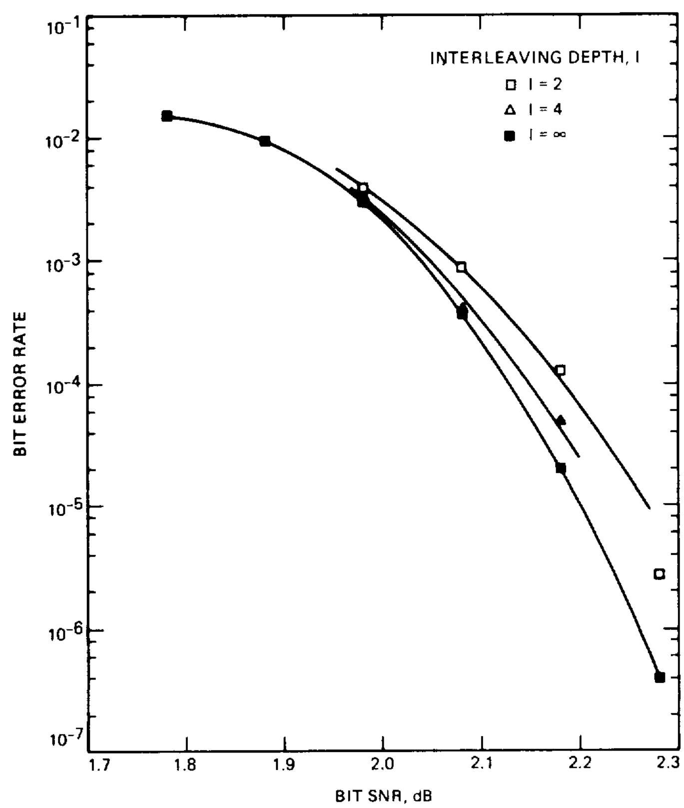

Fig. 6. Concatenated code performance: (7,1/2) convolutional and 10-bit (1023,959) RS.

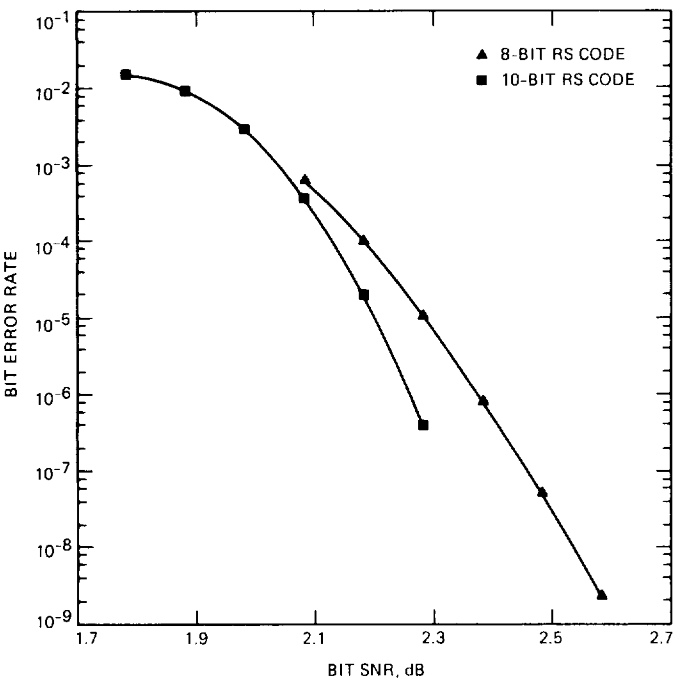

Fig. 7. Concatenated code performance comparison: 8-bit (255,223) and 10-bit (1023,959) RS concatenated with (7,1/2) convolutional.

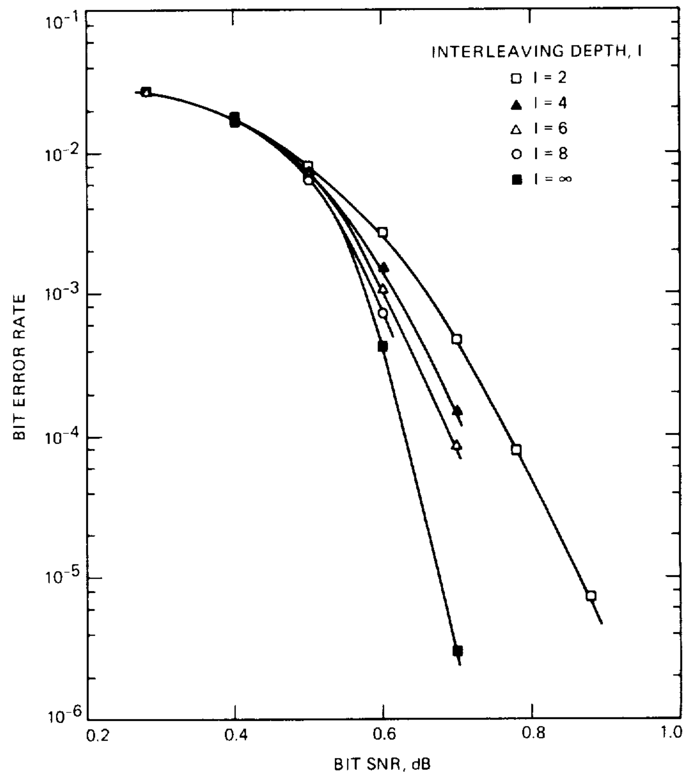

Fig. 9. Concatenated code performance: (15,1/4) convolutional and 10-bit (1023,959) RS.

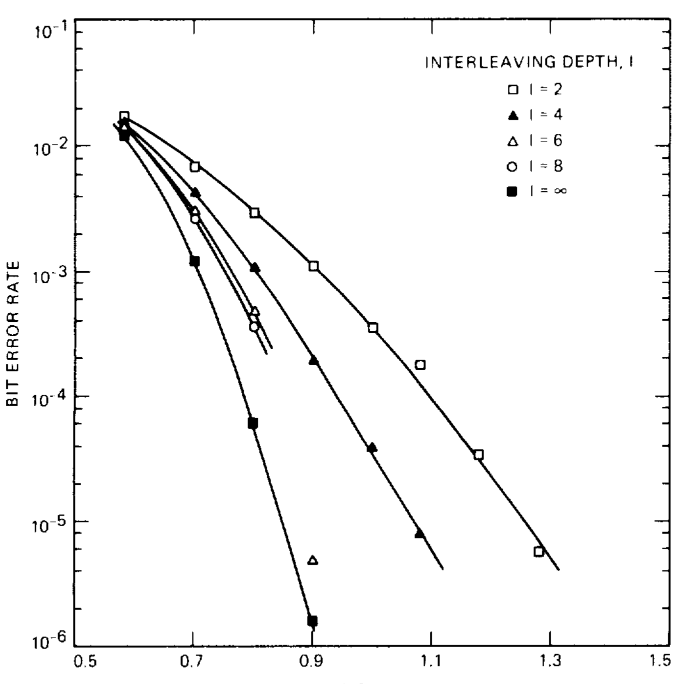

***Fig. 8. Concatenated code performance:* (15,1/4) convolutional and 8-bit (255,223) RS.**

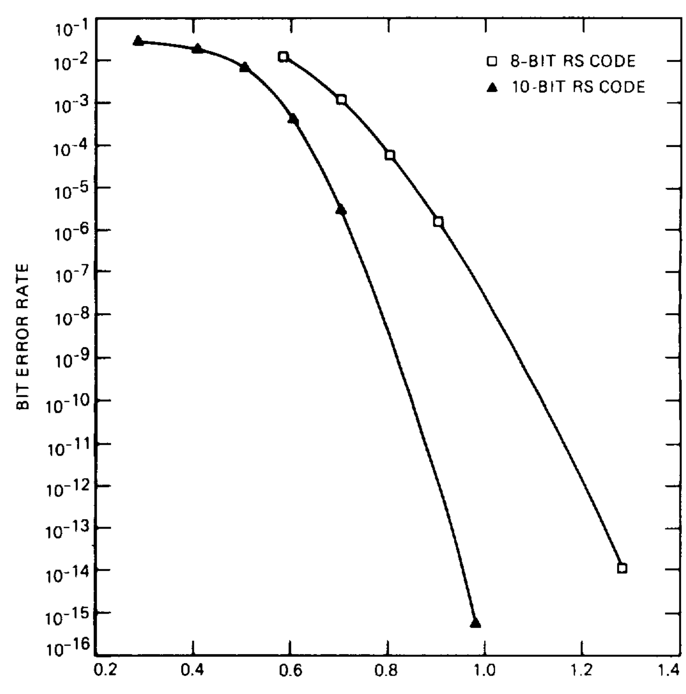

Fig. 10. Concatenated code performance comparison: 8-bit (255,223) and 10-bit (1023,959) RS concatenated with (15,1/4) convolutional.

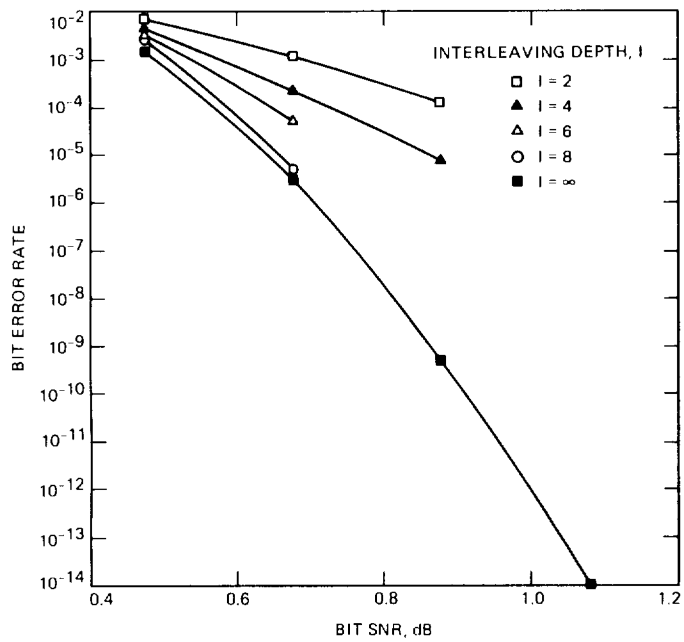

Fig. 11. Concatenated code performance: (15,1/6) convolutional and 8-bit (255,223) RS.

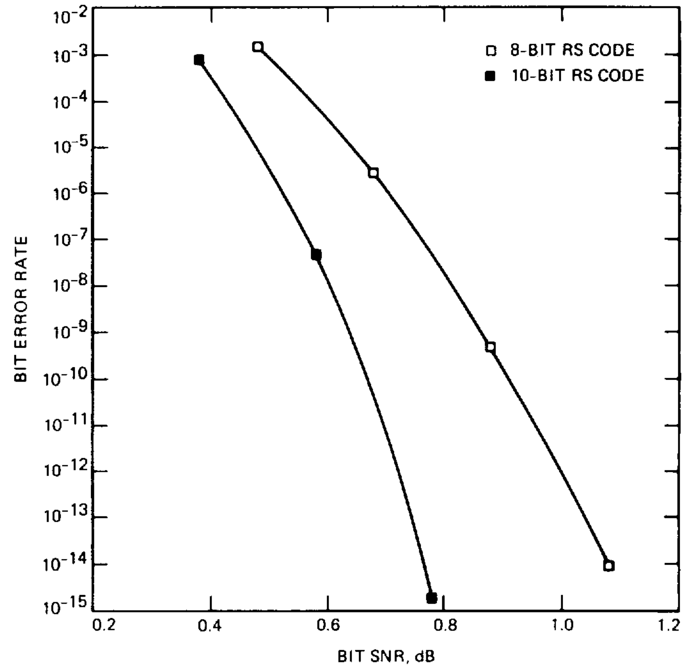

Fig. 13. Concatenated code performance comparison: 8-bit (255,223) and 10-bit (1023,959) RS concatenated with (15,1/6) convolutional.

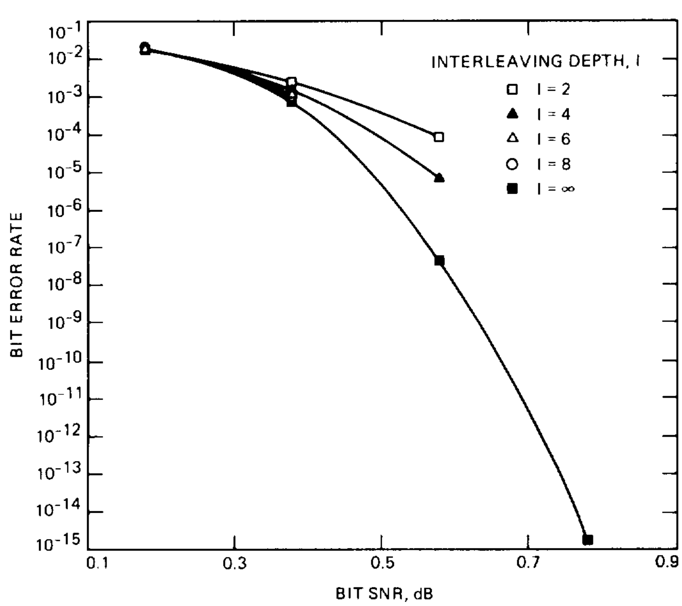

Fig. 12. Concatenated code performance: (15,1/6) convolutional and 10-bit (1023,959) RS.

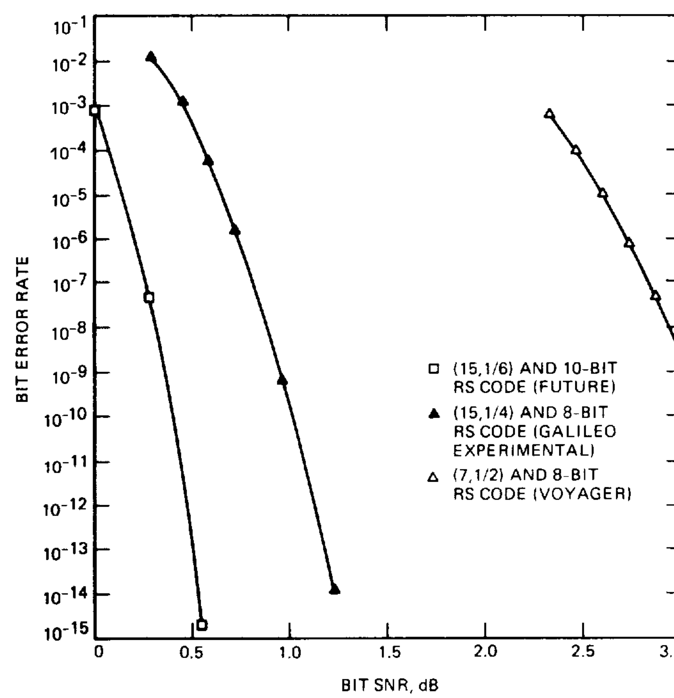

Fig. 14. Comparison of three concatenated codes.