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THE RADIAL DEPENDENCE OF THE SOLAR ENERGETIC PARTICLE FLUX

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ABSTRACT

We discuss the radial dependence of the peak flux and the fluence of solar flare produced energetic particles under the assumption that they propagate diffusively in the heliosphere.

MODEL

There is considerable evidence that in many cases the propagation of solar energetic particles can be described by Parker's spherically symmetric transport equation which includes the effects of diffusion, convection, and adiabatic energy loss in the expanding solar wind (Parker, 1965).

$$\frac{\partial U}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 VU - r^2 K_r \frac{\partial U}{\partial r}\right) - \frac{2V}{3r}\frac{\partial}{\partial T}\left(\alpha TU\right) = 0 \tag{1}$$

where $U(r,T,t)$ = differential number density
V = solar wind speed
K_r = radial diffusion coefficient
T = particle kinetic energy
$\alpha = (T + 2m_oc^2)/(T + m_oc^2)$

The omnidirectional flux is then

$$j = vU/4\pi \tag{2}$$

where v = particle speed.

Equation (1) has generally been solved under the assumptions that

$$K_r = K_o r^b \tag{3}$$

where K_o = diffusion coefficient at 1 AU and
r = radial distance in AU.

When the differential number density is expressed as a power law in kinetic energy, $U = U_oT^{-\gamma}$, the explicit energy dependence of (1) can be eliminated. Even with these assumptions, analytic solutions to (1) have been found only for special cases of the radial dependence of K_r:

b = 1 (Fisk and Axford, 1968) and b = 0 (Lupton and Stone, 1973). Therefore (1) is usually solved numerically (Webb and Quenby, 1973; Ng and Gleeson, 1975; Hamilton, 1977; Zwickl and Webber, 1977). Examples of numerical solutions to (1) are shown in Fig. 1.

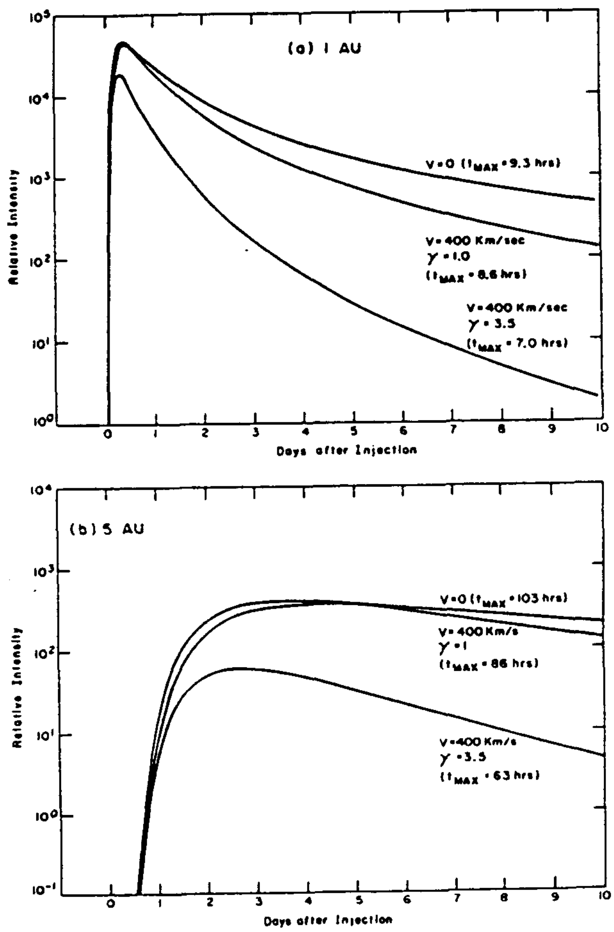

Fig. 1. Intensity-time profiles at (a) 1 AU and (b) 5 AU from a numerical solution of (1) for $K_o = 1.5 \times 10^{21}$ cm^2/s and b=0.5, values typical for 10 MeV protons. The three curves correspond to diffusion only (V=0), diffusion + convection (V=400 km/s, γ=1.0), and diffusion + convection + adiabatic deceleration (V=400 km/s, γ=3.5). (From Hamilton, 1981).

If the effects of convection and energy loss are removed from (1) (by setting V = 0, for example), then a pure diffusion equation results, and a solution has been given by Parker (1963) for any value of the radial index b. For the case of 3-dimensional space,

$$U(r,t) \propto \left(\frac{1}{K_o t} \right)^{\frac{3}{2-b}} \exp\left(- \frac{r^{2-b}}{(2-b)^2 K_o t} \right) \tag{5}$$

Of interest are the radial dependences of the time to maximum intensity, the maximum flux, and the fluence.

$$t_{max} = \frac{r^{2-b}}{3K_o(2-b)} \tag{6}$$

$$j_{max} \propto r^{-3} \qquad \text{(independent of } K_o \text{ or b)} \tag{7}$$

$$F = \int_0^\infty j(r,t)\, dt \qquad \text{(fluence)}$$

$$F \propto r^{-(b+1)} \tag{8}$$

These analytic solutions to the pure diffusion equation are useful for comparison with numerical solutions to the complete transport equation, but generally they do not agree well with observations of solar energetic particles at moderate energies (< 100 MeV). At these energies the effects of convection and energy loss become important. Including these additional terms reduces t_{max} and produces a more rapid decrease of j_{max} and F with increasing r.

OBSERVATIONS

Observations of solar energetic particles beyond 1 AU have been made with the Pioneer 10/11 and Voyager 1/2 spacecraft. Observations inside 1 AU have been reported from Helios 1/2. We review here the two studies which used simultaneous observations at two or more radial distances to deduce radial propagation parameters.

The power law index b of the radial diffusion coefficient has been determined for several events. Hamilton (1977) analyzed 11-67 MeV protons in solar particle events covering the radial range 1-6 AU. His values for b ranged from 0.3 to 0.5 with the trend towards the smaller value at larger radial distances. Beeck et al. (1987) studied protons and heavier ions over a somewhat lower energy range (0.4-27 MeV/nuc) for two particle events covering the radial range 0.65-1.9 AU. Their value for b ranged from 0.5 to 0.7. In both of these studies, b was deduced by fitting t_{max} at two or three radial distances.

To summarize these results, a value of b = 0.5 ± 0.2 covers all six events studied, with a trend to vary from b = 0.7 near 1 AU to b = 0.3 near 5 AU. Other observations at larger radial distances indicate a further reduction to $b \approx 0$ or somewhat less by 10 AU (Webber and Goeman, 1979). This variation of b with r of course implies that K_r is not really a simple power law in r. Nevertheless, it is a useful approximation over limited radial ranges.

From (6), we expect $t_{max} \propto r^{1.5}$ for $b = 0.5$ in the pure diffusion approximation, and this is close to what is observed. The radial dependence of t_{max} is only slightly weaker than this (Hamilton, 1977). Equation (7) predicts an r^{-3} dependence for j_{max}. The solution to (1), on the other hand, gives an $r^{-3.2}$ dependence for parameters typical for 30-67 MeV protons and $r^{-3.3}$ to $r^{-3.5}$ for 11-20 MeV protons depending on the spectral index γ. The more rapid decrease of j_{max} with r results largely from energy loss and thus is more important at lower energies and for larger values of γ.

Hamilton(1977) also studied two events for which the maximum flux fell off more rapidly with radial distance ($r^{-3.8}$ to $r^{-4.0}$). These events were observed in solar wind rarefaction regions in which the flux tube cross section increases more rapidly with r than $\propto r^2$ as is appropriate for 3-dimensional isotropic space. Parker (1963) has shown this more rapid flux tube expansion results in a more rapid decrease of j_{max} with increasing r.

To my knowledge, no observations of the radial dependence of the particle fluence have been reported. To make an estimate we may be guided once again by the pure diffusion approximation. Equation (8) would then suggest a $r^{-1.5}$ dependence for $b = 0.5$. Including effects of convection and energy loss causes a more rapid decrease. A resonable estimate is $F \propto r^{-2}$ to $r^{-2.5}$.

APPLICABILITY OF RESULTS

The model discussed above applies to a situation of spherical symmetry with istropic diffusion. Ng and Gleeson (1971) have shown, however, that the model (Eq. 1) applies anywhere within the flux tube connected to the flare site even though diffusion parallel to the interplanetary magnetic field is much more rapid than that perpendicular to it.

The particle events selected for the two studies cited above were some of the very few observed when two or more spacecraft at different radial distances have simultaneously been well-connected to a flare site. In most events, the rapid particle intensity decrease away from the best connected field line causes large flux differences at separated spacecraft in addition to any radial dependence. Thus the radial dependences in j_{max} and F cited above will rarely be observed in individual events except for fortuitously located spacecraft. On the other hand, these predictions may be useful on a statistical basis in extrapolating from the large data base collected at 1 AU.

Finally, we note that there are particle events in which there is very little interplanetary scattering, particularly inside of 1 AU (e.g. Bieber et al., 1980). These "scatter free" events are not described by (1). However for most of the large events in the 1-100 MeV energy range, (1) generally appears to be a good approximation.

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