NASA Technical Memorandum 102011

Numerical Studies of Convective Heat Transfer in an Inclined Semiannular Enclosure

Lin-Wen Wang
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio

Chain-Nan Yung
University of Toledo
Toledo, Ohio

An-Ti Chai
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio

and

Nasser Rashidnia
Sverdrup Technology, Inc.
NASA Lewis Research Center Group
Cleveland, Ohio

Prepared for the
1989 National Heat Transfer Conference
sponsored by the American Society of Mechanical Engineers
Philadelphia, Pennsylvania, August 6–9, 1989

NASA

Numerical Studies of Convective Heat Transfer in an Inclined Semiannular Enclosure

Lin-Wen Wang*
NASA Lewis Research Center
Cleveland, Ohio 44135

Chain-Nan Yung†
University of Toledo
Toledo, Ohio 43606

An-Ti Chai
NASA Lewis Research Center
Cleveland, Ohio 44135

Nasser Rashidnia
Sverdrup Technology, Inc.
NASA Lewis Research Center Group
Cleveland, Ohio 44135

Abstract

Natural convection heat transfer in a two-dimensional differentially heated semiannular enclosure was studied. The enclosure is isothermally heated and cooled at the inner and outer walls, respectively. A commercial software based on the SIMPLER algorithm was used to simulate the velocity and temperature profiles. Various parameters that affect the momentum and heat transfer processes were examined. These parameters include the Rayleigh number, Prandtl number, radius ratio, and the angle of inclination. The study covers a flow regime extending from conduction-dominated to convection-dominated flow. The computed results of heat transfer are presented as a function of flow parameters and geometric factors. It is found that the heat-transfer rate attains a minimum when the enclosure is tilted about $+50°$ with respect to the gravitational direction.

Introduction

Natural convection in a differentially heated cavity is an important characteristic in material science experiments, such as the growth of single crystals from melts, the solidification of castings, and the purification of materials. The study is also valuable to applications such as thermal energy storage, building energy use, nuclear reactor safety, cooling of electronic components, and analysis of smoke and fire spread in buildings.

The classical problems of natural convection in an annular enclosure with differentially heated surfaces have been the subject of numerous investigations. For example, Kuehn and Goldstein (refs. 1 and 2) conducted experimental and numerical analyses to determine the heat-transfer rate for Rayleigh numbers to 10^5. However, little is known regarding the free convection in an inclined semiannular enclosure. Recently, heat transfer in partially divided enclosures has received attention because of its application in the design of energy efficient buildings, electronic cooling systems, and materials processing. It is the purpose of this research to analyze flow and heat-transfer behavior in a semiannular enclosure.

The basic flow structure in a rectangular enclosure can be classified into three types, each governed by a different heat-transfer process. The first type of flow is induced by heating the enclosure from the top surface, sometimes called the heated-from-top flow phenomenon. Since the lighter fluid is always in the upper portion of the enclosure, there is no convective flow inside the cell. The second flow phenomenon is caused by a temperature gradient across the enclosure. The direction of heat transfer is perpendicular to the gravitational direction. In this situation free convection always exists (ref. 3). In the third type, a heated-from-below flow phenomenon, the heat is transferred in a direction parallel but opposite to gravity. The flow phenomenon in this case is closely related to the Bernard cell problems (ref. 4).

In a semiannular enclosure, all the three types of heat transfer may exist simultaneously and interact with each other. As a result, the flow pattern is more complicated than that in a rectangular enclosure.

In the present work, a numerical study of two-dimensional free convection in a semiannular enclosure is carried out. The computation uses the software, microCOMPACT, for velocity and temperature predictions. Because of the limited capa-

*National Research Council—NASA Research Associate.
†NASA Resident Research Associate.

bility of a personal computer, the grid nodes used are relatively coarse, and a highly accurate solution cannot be expected. However, it was believed that the study would help the understanding of the transport processes in inclined semiannular enclosures.

Formulations and Numerical Approach

The geometry of a semiannular enclosure is illustrated in figure 1. The radii of the enclosure for the inner and outer cylinder are R_{in} and R_{out}, respectively. The inner wall is kept at a higher temperature T_{in} and the outer wall, at a lower temperature T_{out}. The symmetrical planes at $\theta = 0$ and π are insulated. By applying the Boussinesq approximation for free convection flow, the governing equations for the steady-state velocity and temperature distributions in this geometry can be written in a vector form as

Continuity

$$\nabla \cdot V = 0 \tag{1}$$

Momentum

$$\rho(V \cdot \nabla)V = - \nabla P + F - \mu \nabla \times (\nabla \times V) \tag{2}$$

Energy

$$k(\nabla \cdot \nabla)T = \rho c_p (V \cdot \nabla)T \tag{3}$$

where F, the body force, is given by

$$F = - \rho g(\cos \theta)e_R + \rho g(\sin \theta)e_\theta \tag{4}$$

Based on these equations and the consideration of the geometry, the dimensionless quantities, Rayleigh number (Ra), Prandtl number (Pr), and radius ratio (R_R), and the angle of inclination ϕ are found to be important in the present study. These quantities are defined as

$$\mathrm{Ra} = \frac{g\beta \Delta T L^3}{\nu\alpha}$$

$$\mathrm{Pr} = \frac{\nu}{\alpha}$$

$$R_R = \frac{R_{out}}{R_{in}}$$

where L and ΔT are the differences of the radius and temperature, respectively, between the outer and inner cylinder, g is the gravitational acceleration, β is the thermal expansion coefficient, and α is the thermal diffusivity of the fluid. The physical properties, ρ, ν, k and c_p are, respectively, the fluid density, kinematic viscosity, thermal conductivity, and specific heat.

The analyses were conducted over various flow regions and geometrical conditions: The Rayleigh number varied from 10^3 to 10^5, the diameter ratio from 1.5 to 2.5, and inclination angles from $-90°$ to $+90°$. The results of velocity and temperature profiles were examined. The heat-transfer rate, expressed as the Nusselt number for the inner cylinder, is presented as a function of the Rayleigh number, the radius ratio, and the tilt angle.

The computation uses a control volume method and a staggered grid to discretize the equations and employs the SIMPLER procedure for the predictions and corrections of the velocity field. A description of the SIMPLER algorithm can be found in the book (ref. 5) by Patankar, who also developed, based on this method, a computer code for two-dimensional elliptic flow problems. This computer software, called microCOMPACT (ref. 6), can be run on a personal computer. The present work takes advantage of this program to simulate the flow in a semiannular enclosure.

Results and Discussions

Because of the limited working space in a personal computer, most calculations used a 20 by 20 grid; however, some of them are based on a grid of 12 by 14 (12 in circumferential direction and 14 in radial direction). The Nusselt numbers obtained from these two mesh systems differ by 2 percent.

The predicted temperature distribution, streamline, and velocity vector are presented in figures 2 to 4, for a flow condition of Pr = 0.7, Ra = 0.7×10^5, and $R_R = 2.0$ and for various inclinations (or tilt angles). When the enclosure is placed in a horizontal position, the tilt angle is either $-90°$ or $+90°$, and the thermal gradient produces a pair of convective streams. These circulated streams are symmetric with the central axis; they move upward in a path parallel to the axis and return along the solid surface to join the rising fluid in the lower portion of the test cell. Although the flow patterns are similar, the heat transfer that induces the flow is different.

In the $-90°$ enclosure a high temperature region (red) is located in the vertical axis (fig. 2(a)); the fluid rises solely because of the buoyancy force. On the other hand, in the $+90°$ enclosure the motion along the symmetrical plane is due to the fluid in that region being pushed upward by the heavier fluid around the outer cylinder, which tends to descend to the bottom of the enclosure. This phenomenon can also be interpreted from the temperature and velocity distributions of figures 2(e) and 3(e). A large part of the semiannulus remains at low temperature, and the fluid in the region is almost stagnant. In fact, a comparison of figures 3(a) and (e) shows that the velocity along the centerline in a $+90°$ enclosure is much smaller than that in a $-90°$ enclosure. The slightly asymmetric temperature and streamline patterns in these two cases (with respect to the central axis) result from the coarse, 12 by 14 grid used in calculations. Using the 20 by 20 grid eliminates the problem.

As the angle of inclination increases, the heat-transfer direction shifts from upward to downward. Consequently, the influence of buoyant convection becomes smaller. This effect changes the heat-transfer pattern from convection dominant to conduction dominant. Thus, the heat-transfer rate was expected to decrease as the angle varied. The isotherms and streamlines for a semiannulus inclined $-50°$, $0°$, and $+50°$ (parts (b) to (d) of figs. 2 and 3) show that the cold region expands as the angle changes from $-50°$ to $+50°$. For the $+50°$ case, stratification is strong; the fluid in the lower portion of the semiannulus is relatively stagnant and stays cold.

The temperature and velocity patterns are presented for a wide range of flow parameters, in an accompanying video tape. The variables for each picture are listed in table I.

The heat-transfer rates, in terms of Nusselt number, for various radius ratios, are presented as a function of the inclination angle in figure 5. The Nusselt number obtained in this study varies from 2.30 to 6.47 depending upon the geometrical conditions and the Rayleigh number. These values are the same order of magnitude as computed by Kumar (ref. 7) for horizontal annuli. The Nusselt number increases with increasing diameter ratio. However, when the enclosure is tilted in a positive direction, the Nusselt number decreases and reaches a minimum at angles around 30° to 50°, depending on the radius ratio. Further increasing the tilt angle causes the Nusselt number to increase by a small value due to an increase of the convective flow.

For a fixed radius ratio ($R_R = 2.0$), the changes in heat transfer for the two Rayleigh numbers vary with the angle of inclination (fig. 6). The heat-transfer rate increase is proportional to the Rayleigh number.

Summary of Results

The two-dimensional Navier-Stokes and energy equations are numerically solved to study the free convective momentum and heat-transfer processes in a differentially heated semiannular enclosure. The calculations are performed on a personal computer and use a relatively coarse grid. Although, very accurate results were not expected, the predicted velocity and temperature patterns are good enough to explain the physical phenomena inside the enclosure. The computed results indicate that the heat-transfer rate increases with increasing Rayleigh number and radius ratio and varies strongly with the angle of inclination.

References

1. Kuehn, T.H.; and Goldstein, R.J.: An Experimental and Theoretical Study of Natural Convection in the Annulus Between Horizontal Concentric Cylinders. J. Fluid Mech., vol. 74, pt. 4, 1976, pp. 695–719.
2. Kuehn, T.H.; and Goldstein, R.J.: A Parametric Study of Prandtl Number and Diameter Ratio Effects on Natural Convection Heat Transfer in Horizontal Cylindrical Annuli. J. Heat Transfer, vol. 102, 1980, pp. 768–770.
3. Kamotani, Y.; Wang, L.W.; and Ostrach, S.: Experiments on Natural Convection Heat Transfer in Low Aspect Ratio Enclosures. AIAA J., vol. 21, no. 2, 1983, pp. 290-294.
4. Wang, L.W.; Chai, A.T.; and Sun, D.J.: Convective Flows in Enclosures with Vertical Temperature or Concentration Gradients. AIAA Paper 89-0069, 1989 (also NASA TM-101373).
5. Patankar, S.V.: Numerical Heat Transfer and Fluid Flow. Hemisphere, New York, 1980.
6. Reference Manual for "MicroCOMPACT," Version 1.1: A General-Purpose Computer Program for Two-Dimensional Fluid Flow and Heat Transfer. Innovative Research, Inc., Maple Grove, MN, 1988.
7. Kumar, R.: Study of Natural Convection in Horizontal Annuli. Intern. J. Heat Mass Transfer, vol. 31, no. 6, pp. 1137-1148, 1988.

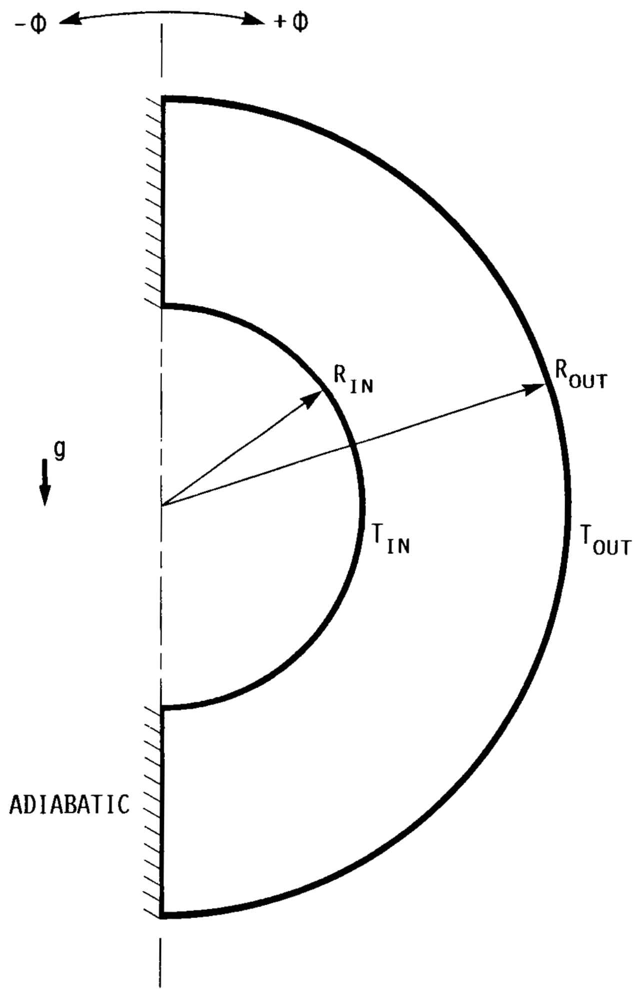
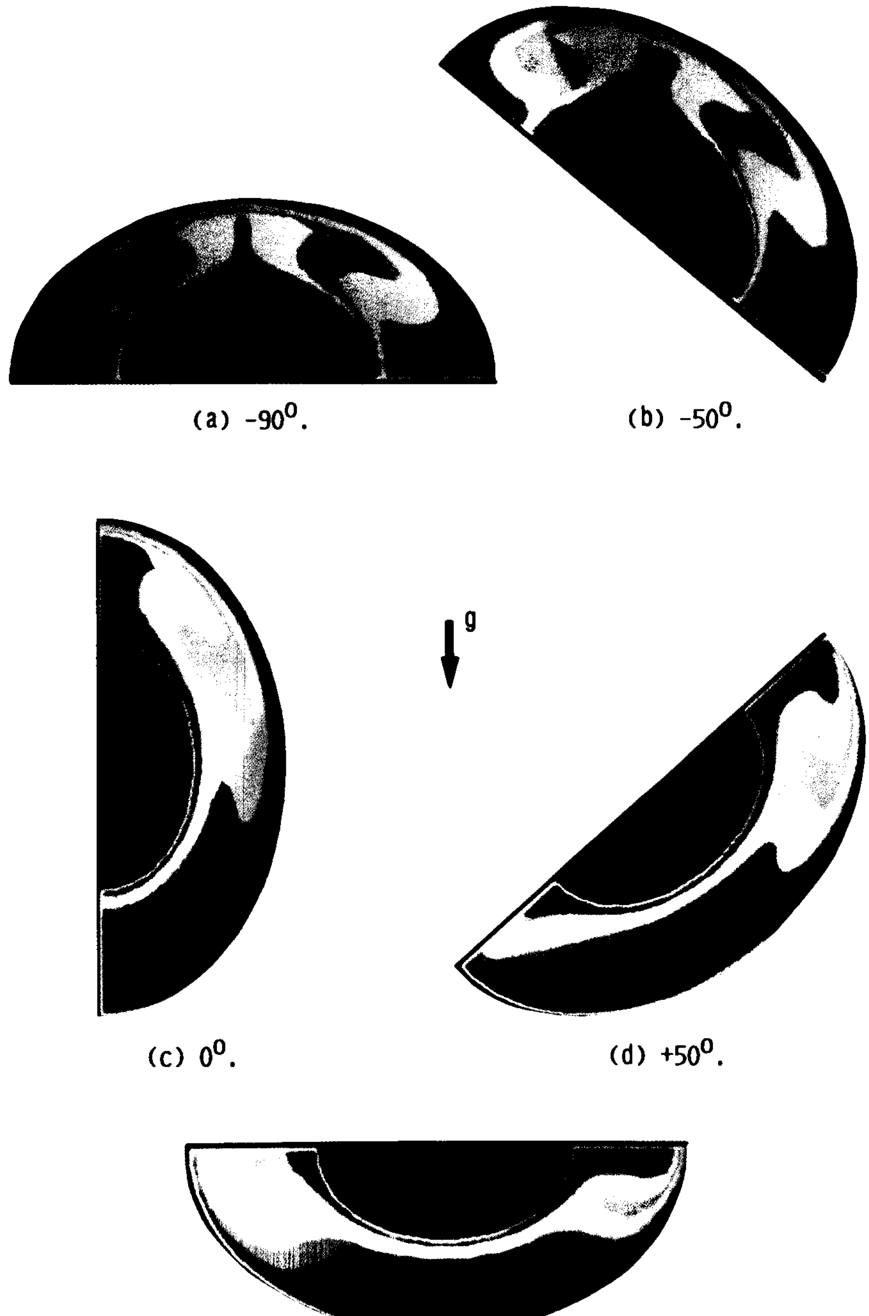


Figure 1.—Semiannular enclosure. Lift angle, 0°.

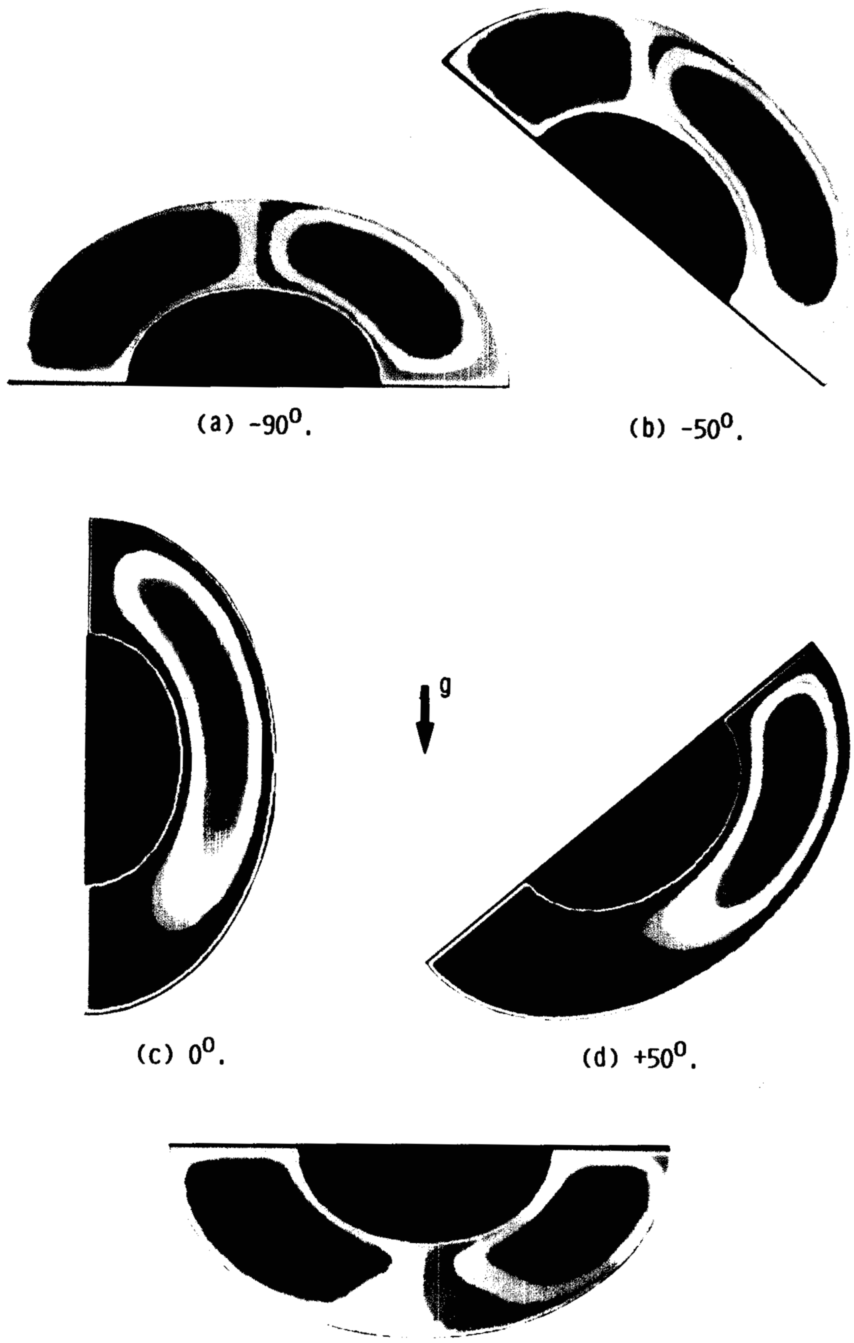


(a) -90°. (b) -50°.

(c) 0°. (d) +50°.

(e) +90°.

Figure 2.—Isotherm plots. Pr = 0.7; Ra = 0.7×10^5; and R_R = 2.0.



(a) -90^0.

(b) -50^0.

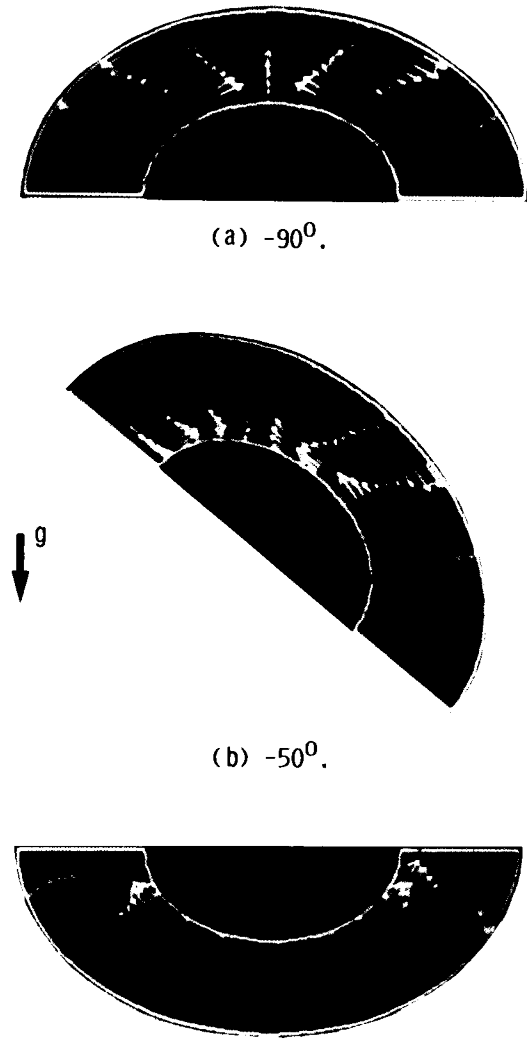
(c) 0^0.

(d) $+50^0$.

(e) $+90^0$.

Figure 3.—Streamlines. Pr = 0.7; Ra = 0.7×10^5; and R_R = 2.0.

ORIGINAL PAGE
COLOR PHOTOGRAPH



(a) -90^{0}.

(b) -50^{0}.

(c) $+90^{0}$.

Figure 4.—Velocity vectors. Pr = 0.7; Ra = 0.7×10^5; and R_R = 2.0.

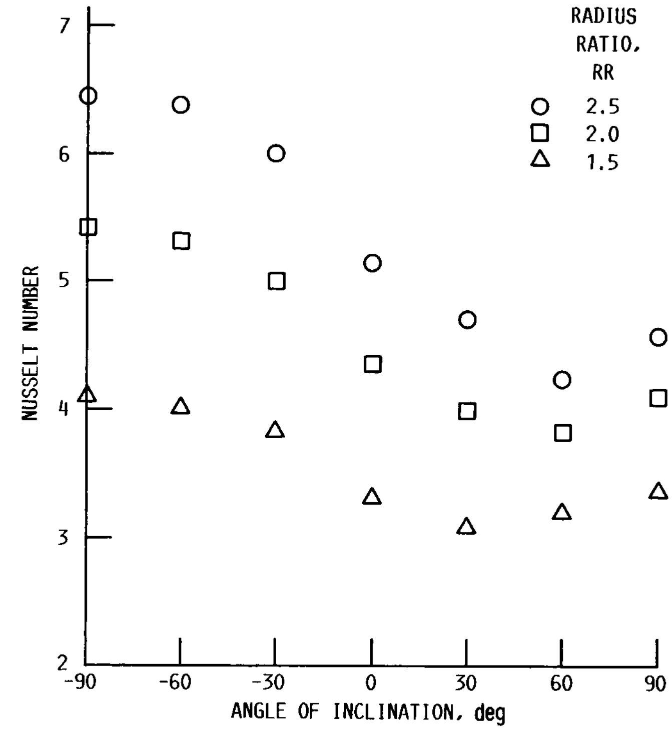


Figure 5.—Comparison of heat-transfer rates for various radius ratios and inclination angles. Pr = 0.7 and Ra = 0.7×10^5.

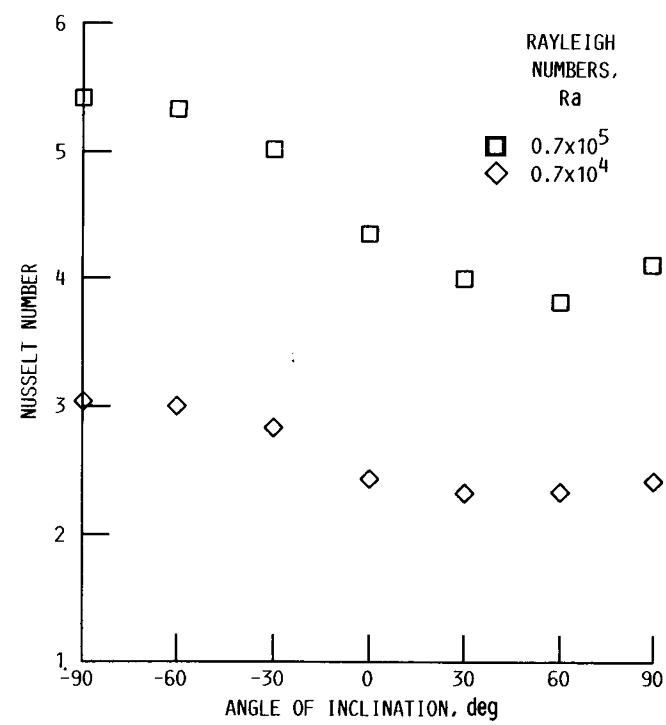


Figure 6.—Comparison of heat-transfer rates for various Rayleigh numbers and inclination angles. Pr = 0.7 and R_R = 2.0.

NASA
National Aeronautics and Space Administration

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-102011		

4. Title and Subtitle

Numerical Studies of Convective Heat Transfer in an Inclined Semiannular Enclosure

5. Report Date

6. Performing Organization Code

7. Author(s)

Lin-Wen Wang, Chain-Nan Yung, An-Ti Chai, and Nasser Rashidnia

8. Performing Organization Report No.

E-4723

10. Work Unit No.

674-25-05-08

9. Performing Organization Name and Address

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

11. Contract or Grant No.

13. Type of Report and Period Covered

Technical Memorandum

12. Sponsoring Agency Name and Address

National Aeronautics and Space Administration
Washington, D.C. 20546-0001

14. Sponsoring Agency Code

15. Supplementary Notes

Prepared for the 1989 National Heat Transfer Conference sponsored by the American Society of Mechanical Engineers, Philadelphia, Pennsylvania, August 6-9, 1989. Lin-Wen Wang, National Research Council—NASA Research Associate; Chain-Nan Yung, University of Toledo, Toledo, Ohio 43606 and NASA Resident Research Associate; An-Ti Chai, NASA Lewis Research Center; and Nasser Rashidnia, Sverdrup Technology, Inc., NASA Lewis Research Center Group, Cleveland, Ohio 44135.

16. Abstract

Natural convection heat transfer in a two-dimensional differentially heated semiannular enclosure is studied. The enclosure is isothermally heated and cooled at the inner and outer walls, respectively. A commercial software based on the SIMPLER algorithm was used to simulate the velocity and temperature profiles. Various parameters that affect the momentum and heat transfer processes were examined. These parameters include the Rayleigh number, Prandtl number, radius ratio, and the angle of inclination. The study covers a flow regime extending from conduction-dominated to convection-dominated flow. The computed results of heat transfer are presented as a function of flow parameter and geometric factors. It is found that the heat transfer rate attains a minimum when the enclosure is tilted about $+50^\circ$ with respect to the gravitational direction.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Natural convection Thermal convection	Unclassified – Unlimited Subject Category 29

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No of pages	22. Price*
Unclassified	Unclassified	8	A02

NASA FORM 1626 OCT 86

*For sale by the National Technical Information Service, Springfield, Virginia 22161