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A Preliminary Compressible Second-Order Closure Model for High Speed Flows

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A preliminary version of the compressible second-order closure model that will be considered for use in the National Aero-Space Plane Project (NASP) is presented on the pages to follow. The proposed model requires the solution of modeled transport equations for the Favre-averaged Reynolds stress tensor and the turbulent dissipation rate in addition to the corresponding mean continuity, momentum and energy equations. In this preliminary model, the Reynolds heat flux and the mass flux terms are modeled by a simple gradient transport hypothesis (the same is true for the turbulent diffusion terms in the Reynolds stress transport equation). A model recently developed by Speziale, Sarkar and Gatski (1989) for the pressure-strain correlation (which more properly accounts for rotational strains) is used for the deviatoric part of the pressure gradient-velocity correlation. This early version of the model neglects correlations involving fluctuating dilatational terms. The modeled dissipation rate transport equation has been formulated to be consistent with Rapid Distortion Theory for a compressed isotropic turbulence. In order to facilitate the early implementation of the model into existing computer codes, Van Driest damping is proposed for integration to the wall. Ultimately, this will be supplanted by asymptotically correct low Reynolds number corrections to the models in order to obtain a more accurate calculation of wall flow properties. It is envisioned that as this modeling effort progresses, transport equations for the Reynolds heat flux, the mass flux, and possibly the rms density and temperature fluctuations may be added. Likewise, the modeling of dilatational effects (e.g., a model for the pressure-dilatation correlation) and the possible addition of an alternate specification for the turbulence length scale are being investigated with J. L. Lumley

(Cornell University) and T. B. Gatski (NASA Langley Research Center). A systematic program for the testing of this model which makes use of the results of both physical and numerical experiments is currently being developed in collaboration with T. B. Gatski and A. Kumar (NASA Langley Research Center).

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Preliminary Compressible Second-Order Closure Model

Nomenclature

$\rho \equiv$ mass density

$v_i \equiv$ velocity vector

$p \equiv$ pressure

$T \equiv$ temperature

$C_v \equiv$ specific heat (constant volume)

$Q_i \equiv$ Favre-averaged Reynolds heat flux

$\tau_{ij} \equiv$ Favre-averaged kinematic Reynolds stress tensor

$k \equiv$ turbulent kinetic energy

$\sigma_{ij} \equiv$ viscous stress tensor

$\mu \equiv$ dynamic viscosity

$R \equiv$ ideal gas constant

$\Phi \equiv$ viscous dissipation rate

$\epsilon \equiv$ turbulent dissipation rate

$\kappa \equiv$ thermal conductivity

$\partial_t(\,) \equiv \partial(\,)/\partial t$

$(\,)_{,i} \equiv \partial(\,)/\partial x_i$

For any flow variable f :

$$f = \bar{f} + f'' \quad , \quad (\,\bar{f} = \lim_{T\to\infty} \frac{1}{T}\int_0^T f(\mathbf{x}, t)\, dt\,)$$

$$f = \tilde{f} + f' \quad , \quad (\,\tilde{f} = \frac{\overline{\rho f}}{\bar{\rho}} \equiv \text{Favre average}\,)$$

Mean Continuity equation:

$$\partial_t(\bar{\rho}) + (\bar{\rho}\tilde{v}_k)_{,k} = 0 \tag{1}$$

Mean Momentum equation:

$$\partial_t(\overline{\rho}\tilde{v}_i) + (\overline{\rho}\tilde{v}_i\tilde{v}_j)_{,j} = -\overline{p}_{,i} + \overline{\sigma}_{ij,j} - (\overline{\rho}\tau_{ij})_{,j} \tag{2}$$

where

$$\begin{aligned}
\overline{\sigma}_{ij} &= -\frac{2}{3}\overline{\mu v_{k,k}}\delta_{ij} + \overline{\mu(v_{i,j} + v_{j,i})} \\
&\simeq -\frac{2}{3}\overline{\mu}\tilde{v}_{k,k}\delta_{ij} + \overline{\mu}(\tilde{v}_{i,j} + \tilde{v}_{j,i}) \\
\tau_{ij} &\equiv \widetilde{v_i'v_j'} \quad , \quad \overline{p} = \overline{\rho}R\tilde{T}
\end{aligned}$$

Mean Energy equation:

$$\partial_t(\overline{\rho}C_v\tilde{T}) + (\overline{\rho}\tilde{v}_kC_v\tilde{T})_{,k} = -\overline{pv_{k,k}} + \overline{\Phi} + (\overline{\kappa T_{,k}})_{,k} - Q_{k,k} \tag{3}$$

where

$$\begin{aligned}
\overline{\Phi} &= \overline{\sigma}_{ij}\overline{v}_{i,j} + \overline{\sigma''_{ij}v''_{i,j}} \\
&\simeq \overline{\sigma}_{ij}\tilde{v}_{i,j} + \overline{\rho}\epsilon \\
\overline{pv_{k,k}} &\simeq \overline{p}\tilde{v}_{k,k} \\
\overline{\kappa T_{,k}} &= \overline{\kappa}\tilde{T}_{,k} \\
Q_k &= \overline{\rho}C_v\widetilde{v_k'T'} \simeq -\overline{\rho}C_vC_\mu\frac{k^2}{\epsilon\sigma_T}\tilde{T}_{,k}
\end{aligned}$$

The model parameters required in the above expressions are

$$\sigma_T = 0.7$$

and

$$\begin{aligned}
C_\mu &= 0.09[1 - \exp(-y^+/25)] \\
&\quad y^+ = \overline{\rho}u_\tau y/\overline{\mu}
\end{aligned}$$

Favre-averaged Reynolds stress transport equation:

The exact transport equation for the Favre-averaged Reynolds stress τ_{ij} is as follows

$$\begin{aligned}\partial_t(\bar{\rho}\tau_{ij}) + (\tilde{v}_k\bar{\rho}\tau_{ij})_{,k} &= -\bar{\rho}\tau_{ik}\tilde{v}_{j,k} - \bar{\rho}\tau_{jk}\tilde{v}_{i,k} - C_{ijk,k} - \Pi_{ij} - \epsilon_{ij} + \frac{2}{3}\overline{p''v''_{k,k}}\delta_{ij} \\ &\quad -\overline{v'_i}\bar{p}_{,j} - \overline{v'_j}\bar{p}_{,i} + \overline{v'_i}\bar{\sigma}_{jk,k} + \overline{v'_j}\bar{\sigma}_{ik,k} + (\overline{v''_i\sigma''_{jk}} + \overline{v''_j\sigma''_{ik}})_{,k} \qquad (4)\end{aligned}$$

where

$$\begin{aligned}C_{ijk} &= \bar{\rho}\widetilde{v'_iv'_jv'_k} + \frac{2}{3}\overline{p''v''_k}\delta_{ij} \\ \Pi_{ij} &= \overline{v''_ip''_{,j}} + \overline{v''_jp''_{,i}} - \frac{2}{3}\overline{v''_kp''_{,k}}\delta_{ij} \\ \epsilon_{ij} &= \overline{\sigma''_{ik}v''_{j,k}} + \overline{\sigma''_{jk}v''_{i,k}}\end{aligned}$$

The models that are proposed in order to close (4) are as follows

$$\begin{aligned}\widetilde{v'_iv'_jv'_k} &\simeq -\frac{2}{3}C_s\frac{k^2}{\epsilon}(\tau_{ij,k} + \tau_{ik,j} + \tau_{jk,i}) \\ \overline{p''v''_k} &\equiv -\bar{\rho}R\tilde{T}\overline{v'_k} + \bar{\rho}R\widetilde{v'_kT'} \\ \overline{p''v''_{k,k}} &\simeq 0 \\ \overline{v'_k} &= \bar{v}_k - \tilde{v}_k = -\frac{\overline{\rho''v''_k}}{\bar{\rho}} \simeq \frac{C_\mu k^2}{\bar{\rho}\epsilon\sigma_\rho}\bar{\rho}_{,k} \\ \overline{v''_i\sigma''_{jk}} + \overline{v''_j\sigma''_{ik}} &\simeq \bar{\mu}(\tau_{ij,k} + \tau_{ik,j} + \tau_{jk,i})\end{aligned}$$

where

$$k = \frac{1}{2}\tau_{ii}\ ,\ C_s = 0.11[1 - \exp(-y^+/25)]\ ,\ \sigma_\rho = 0.5$$

In addition, we will use the model for Π_{ij} recently developed by Speziale, Sarkar and Gatski (1989),

$$\begin{aligned}\Pi_{ij} &\simeq \bar{\rho}[(C_1\epsilon + C_5\mathcal{P})b_{ij} - C_2\epsilon(b_{ik}b_{kj} - \frac{1}{3}II\delta_{ij}) \\ &- C_3k(b_{ik}\tilde{S}_{jk} + b_{jk}\tilde{S}_{ik} - \frac{2}{3}b_{mn}\tilde{S}_{mn}\delta_{ij}) \\ &- C_4k(b_{ik}\tilde{W}_{jk} + b_{jk}\tilde{W}_{ik}) - \frac{4}{5}(1 - C^*II^{1/2})k\tilde{S}_{ij}]\end{aligned}$$

where we have used the following notation

$$\mathcal{P} = -\tau_{ij}\tilde{v}_{i,j} \quad , \quad b_{ij} = \frac{\tau_{ij}}{2k} - \frac{1}{3}\delta_{ij} \; , \; II = b_{ij}b_{ij}$$
$$\tilde{S}_{ij} = \frac{1}{2}(\tilde{v}_{i,j} + \tilde{v}_{j,i}) \quad , \quad \tilde{W}_{ij} = \frac{1}{2}(\tilde{v}_{i,j} - \tilde{v}_{j,i})$$

The model constants for the term Π_{ij} are as follows:

$$C_1 = 3.4 \quad , \quad C_2 = 4.2 \quad , \quad C_3 = 1.25$$
$$C_4 = 0.40 \quad , \quad C_5 = 1.80 \quad , \quad C^* = 1.62$$

Finally, the model for the dissipation rate ϵ_{ij} is given by

$$\epsilon_{ij} = \frac{2}{3}\overline{\rho}\epsilon\delta_{ij}$$

Turbulent dissipation rate transport equation:

The modelled transport equation for the dissipation rate ϵ is as follows

$$\begin{aligned} \partial_t(\overline{\rho}\epsilon) + (\overline{\rho}\tilde{v}_k\epsilon)_{,k} &= \frac{\epsilon}{k}[-C_{\epsilon 1}\overline{\rho}\tau_{ij}(\tilde{v}_{i,j} - \frac{1}{3}\tilde{v}_{k,k}\delta_{ij}) - C_{\epsilon 0}\overline{v_i'}\overline{p}_{,i}] - C_{\epsilon 2}\overline{\rho}\frac{\epsilon^2}{k} \\ &- \frac{4}{3}\overline{\rho}\epsilon\tilde{v}_{k,k} + (C_\epsilon\frac{\overline{\rho}k}{\epsilon}\tau_{kl}\epsilon_{,l})_{,k} + (\overline{\mu}\epsilon_{,l})_{,l} \end{aligned} \tag{5}$$

The model parameters in (5) are the following:

$$C_{\epsilon 0} = 1.0 \quad , \quad C_{\epsilon 1} = 1.44$$
$$C_{\epsilon 2} = 1.83[1 - \exp(-R_T^2)] \quad \text{where} \quad R_T = \frac{\overline{\rho}k^2}{\overline{\mu}\epsilon}$$
$$C_\epsilon = 0.15[1 - \exp(-y^+/25)]$$

Incompressible (Isothermal) limit:

The Reynolds stress transport and dissipation rate equations reduce to

$$
\begin{aligned}
\partial_t(\tau_{ij}) + (\bar{v}_k\tau_{ij})_{,k} &= -\tau_{ik}\bar{v}_{j,k} - \tau_{jk}\bar{v}_{i,k} - \Pi^*_{ij} - \frac{2}{3}\epsilon\delta_{ij} \\
&+ \frac{2}{3}C_s[\frac{k^2}{\epsilon}(\tau_{ij,k} + \tau_{ik,j} + \tau_{jk,i})]_{,k} + [\nu(\tau_{ij,k} + \tau_{ik,j} + \tau_{jk,i})]_{,k} \\
\partial_t(\epsilon) + (\bar{v}_k\epsilon)_{,k} &= -C_{\epsilon 1}\frac{\epsilon}{k}\tau_{ij}\bar{v}_{i,j} - C_{\epsilon 2}\frac{\epsilon^2}{k} + (C_\epsilon\frac{k}{\epsilon}\tau_{kl}\epsilon_{,l})_{,k} + (\nu\epsilon_{,l})_{,l}
\end{aligned}
$$

where

$$
\begin{aligned}
\Pi^*_{ij} &= (C_1\epsilon + C_5\mathcal{P})b_{ij} - C_2\epsilon(b_{ik}b_{kj} - \frac{1}{3}II\delta_{ij}) \\
&- C_3k(b_{ik}\bar{S}_{jk} + b_{jk}\bar{S}_{ik} - \frac{2}{3}b_{mn}\bar{S}_{mn}\delta_{ij}) \\
&- C_4k(b_{ik}\bar{W}_{jk} + b_{jk}\bar{W}_{ik}) - \frac{4}{5}(1 - C^*II^{1/2})k\bar{S}_{ij}
\end{aligned}
$$

and the coefficients are as defined before with $y^+ = u_\tau y/\nu$.

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16. Abstract

A preliminary version of a compressible second-order closure model that has been developed in connecton with the National Aero-Space Plane Project is presented. The model requires the solution of transport equations for the Favre-averaged Reynolds stress tensor and dissipaton rate. Gradient transport hypotheses are used for the Reynolds heat flux, mass flux, and turbulent diffusion terms. Some brief remarks are made about the direction of future research to generalize the model.

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