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**THE INVARIANCE OF CLASSICAL ELECTROMAGNETISM UNDER
CHARGE-CONJUGATION, PARITY AND TIME-REVERSAL (CPT)
TRANSFORMATIONS**

by

John W. Norbury
Physics Department
Washington State University
Pullman, WA 99164

Abstract

The invariance of Classical Electromagnetism under CPT is studied by considering the motion of a charged particle in electric and magnetic fields. Upon applying CPT transformations to various physical quantities and noting that the motion still behaves physically demonstrates invariance.

1. Introduction

In teaching a recent course in electromagnetism I discussed the transformation properties of the electric \vec{E} , and magnetic fields \vec{B} under parity (P) time reversal (T) and charge conjugation (C) transformations. I was amazed to find that a unified discussion of CPT transformations in classical electromagnetism does not exist in any of the standard text books. The best I could find was a discussion of time-reversal and axial and polar vectors in (Jackson 1975). The aim of the present paper is purely pedagogical. I wish to discuss CPT transformations, not from a mathematical point of view (Rosen 1973) but from a completely physical point of view by considering how the transformations affect the motion of a charged particle in an electromagnetic field. This approach was very successful with the students; they came away with an excellent physical understanding of these transformations.

Our procedure will be as follows: we shall look at the motion of a charged particle in either \vec{E} or \vec{B} fields and then apply one of the C, P or T transformations to the whole situation. Each quantity, such as force, charge, etc. will transform in their own particular way. Because Maxwell's equations are invariant however we know that our transformed situation shouldn't be any different. In particular, by looking simply at trajectory of the particle, we shouldn't be able to tell that a transformation has taken place. Examining these "pictorial" transformations will give us a very clear physical understanding of the corresponding mathematical transformations.

Before proceeding we note the transformation properties of some of the quantities we shall need. Details can be found in (Jackson 1975, Perkins 1987, Rosen 1973).

Electric charge q is a scalar under P and T, but a pseudoscalar under C. ($Pq = q$, $Tq = q$,

$Cq = -q$) Velocity \vec{v} changes sign (vector) under T because of its form as $d\vec{x}/dt$.

Otherwise it is a vector under P and a pseudovector under C. ($P\vec{v} = -\vec{v}$, $T\vec{v} = -\vec{v}$, $C\vec{v} = \vec{v}$)

Force \vec{F} or acceleration is a pseudovector under T because of the form $d^2\vec{x}/dt^2$. Otherwise it transforms the same as \vec{v} . ($P\vec{F} = -\vec{F}$, $T\vec{F} = \vec{F}$, $C\vec{F} = \vec{F}$).

2. Electric Field

The transformation properties of the electric field are $\vec{P}\vec{E} = -\vec{E}$ (vector), $\vec{T}\vec{E} = \vec{E}$ (pseudo-vector), $\vec{C}\vec{E} = -\vec{E}$ (vector). Let us study these in turn by considering the motion of a positively charged particle in an electric field, as shown in Fig. 1.

2.1 Parity

Parity transformations are most easily visualized by reflection in a mirror. Imagine that the charge in Fig. 1 is moving to the right at right angles toward a mirror. Let us now reflect this whole situation in the mirror. Under reflection charge q is a scalar and electric field \vec{E} , force \vec{F} and velocity \vec{v} are vectors and so reverse sign as shown in Fig. 2. Thus the relative orientation of \vec{F} and \vec{E} are still the same. Thus we still have acceleration towards the mirror: we cannot tell that we are now in a reflected world.

2.2 Charge Conjugation

Under charge conjugation \vec{F} and \vec{v} remains the same, but \vec{E} and q change sign, as shown in Figure 3. Thus we still have acceleration of the charge to the right, so we can't tell that a C transformation has been made.

2.3 Time-Reversal

Now q and \vec{E} remain the same but \vec{v} ($d\vec{x}/dt$) changes direction, and \vec{a} ($d^2\vec{x}/dt^2$) or \vec{F} remains the same also. This time \vec{F} and \vec{v} become opposite to each other. As depicted in Fig. 4, the motion is like a movie of Fig. 1 being run backwards. Thus the particle starts at the far right hand side with velocity $-\vec{v}$ but slows down because the force is to the right, as it travels into the past. The motion still makes physical sense and we cannot distinguish travel into the future from travel into the past. (The position of the particle in Fig. 4 can be superposed directly onto Fig. 1)

3. Magnetic Field

The magnetic field \vec{B} transforms as $P\vec{B} = \vec{B}$ (pseudovector), $T\vec{B} = -\vec{B}$ (vector), $C\vec{B} = -\vec{B}$ (vector). We consider a positive charge moving counterclockwise in a magnetic field as shown in Fig. 5.

3.1 Parity

We now imagine a mirror parallel to the plane of the paper. Because \vec{F} and \vec{v} are in the same plane they will not be affected this time by a parity transformation. Any vector perpendicular to the plane of the paper will change sign. However, \vec{B} (which is perpendicular) is a pseudo-vector under P and will not change sign. Thus the mirror-reflected version of Fig. 5 is identical to that figure. Obviously then the motion is invariant under reflection.

3.2 Charge Conjugation

Figure 6 represents a charge conjugated version of Fig. 5. As can be seen the motion is still counterclockwise, so we couldn't tell that a transformation has been made.

3.3 Time-Reversal

The time-reversed situation of Fig. 5 is shown in Fig. 7. The motion here is identical to what we would see if we took a movie of the motion of Fig. 5 and then ran it backwards.

Summary

Note the importance of what has been done in the 6 transformations presented above. We have transformed each quantity separately, thus making it quite possible to obtain a resultant unphysical trajectory, ie. a trajectory violating Maxwell's equations and the Lorentz force law. (An example of an unphysical trajectory is a positive charged particle moving clockwise in a magnetic field directed into the page.) However, none of our transformed trajectories have been unphysical indicating invariance of Classical Electromagnetism and thus preventing us from assigning an absolute value to C, P, or T. That is, by examining the physical motion we cannot tell if a C, P or T transformation has been made. It is hoped that the ideas presented in this paper will provide a useful physical supplement to the study of classical electromagnetism.

Acknowledgements

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References

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Perkins, D. H., 1987, Introduction to High Energy Physics, 3rd ed., Addison-Wesley,
(section 3.7)

Rosen, J., 1973, American J. Phys. 41 586.

Figure Captions

Fig. 1. Motion of a Positive Charge moving towards a mirror with an initial velocity \vec{v} in a Uniform Electric Field. The position of the charge at various times is indicated with a symbol \oplus , at time t_i .

Fig. 2 Motion of Fig. 1 reflected in a mirror

$$(\vec{PE} = -\vec{E}, Pq = q, P\vec{v} = -\vec{v}, P\vec{F} = -\vec{F})$$

Fig. 3 Charge - conjugated version of Fig. 1. The positive charge becomes a negative charge indicated with a symbol \ominus .

$$(\vec{CE} = -\vec{E}, Cq = -q, C\vec{v} = \vec{v}, C\vec{F} = \vec{F})$$

Fig. 4 The charge of Fig. 1 travelling into the past.

$$(\vec{TE} = +\vec{E}, Tq = q, T\vec{v} = -\vec{v}, T\vec{F} = \vec{F})$$

Fig. 5 Motion of a Positive Charge with a velocity \vec{v} moving counterclockwise in a Uniform Magnetic Field directed into the page.

Fig. 6 Charge-conjugated version of Fig. 5 (\vec{B} points out of the page) ($\vec{CB} = -\vec{B}$)

Fig. 7 The charge of Fig. 5 travelling into the past.

$$(\vec{TB} = -\vec{B})$$

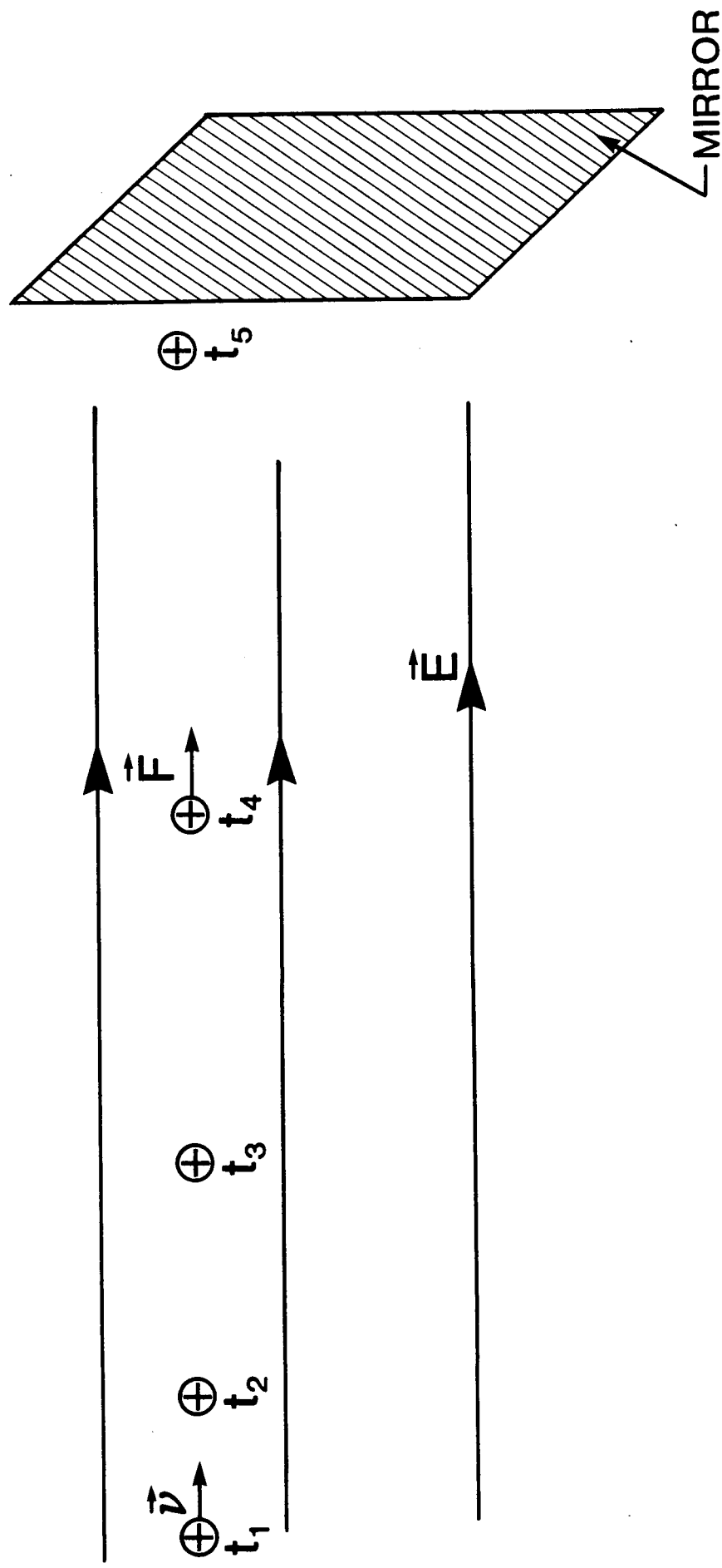


Fig. 1

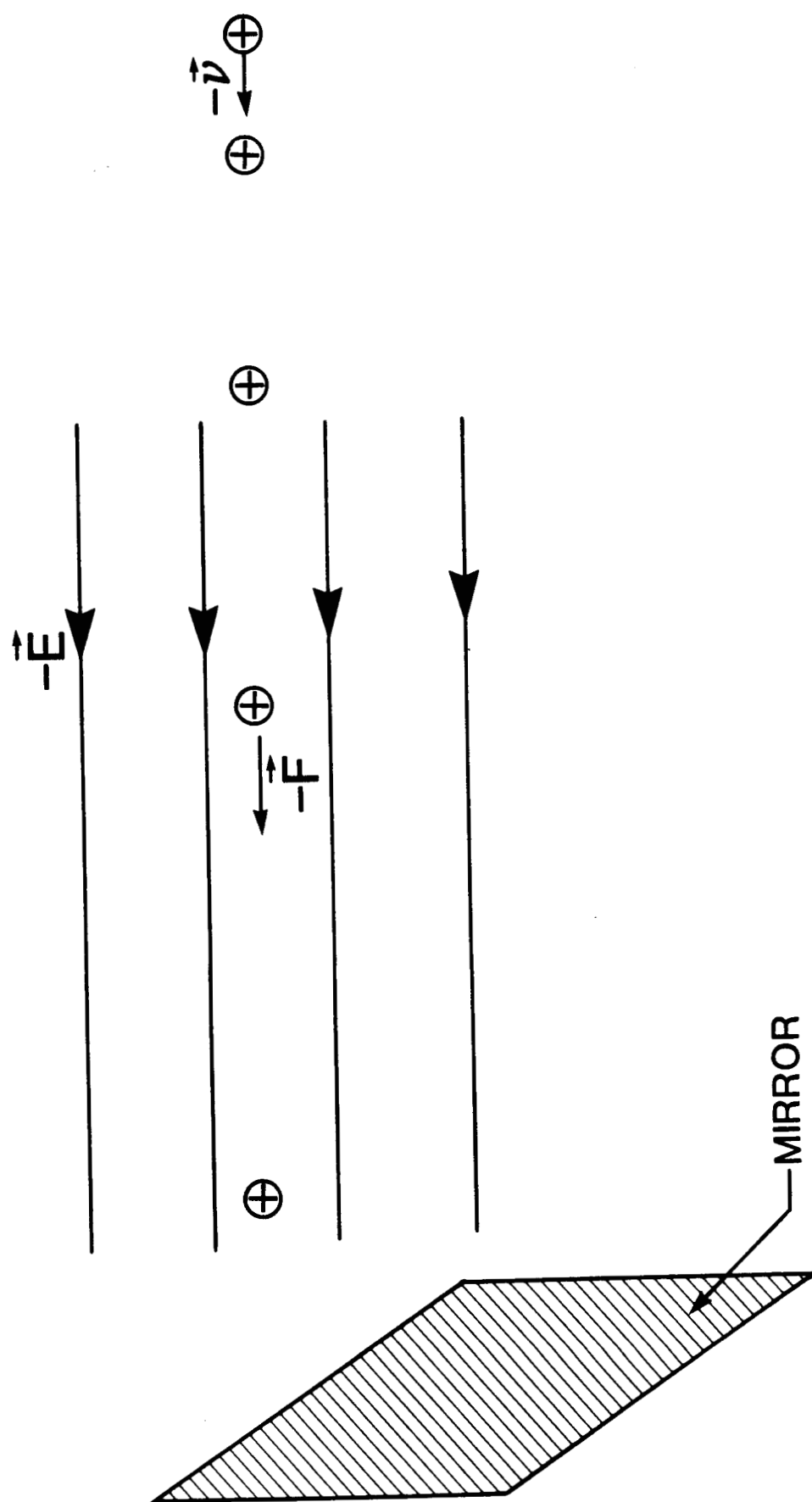
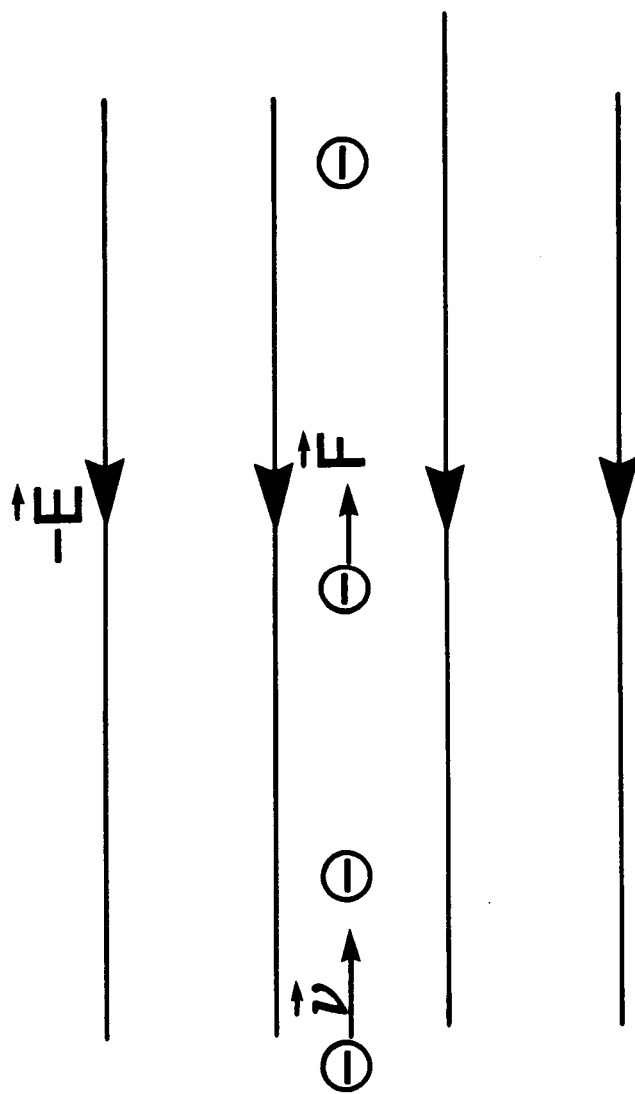


Fig. 2



\ominus

Fig. 3

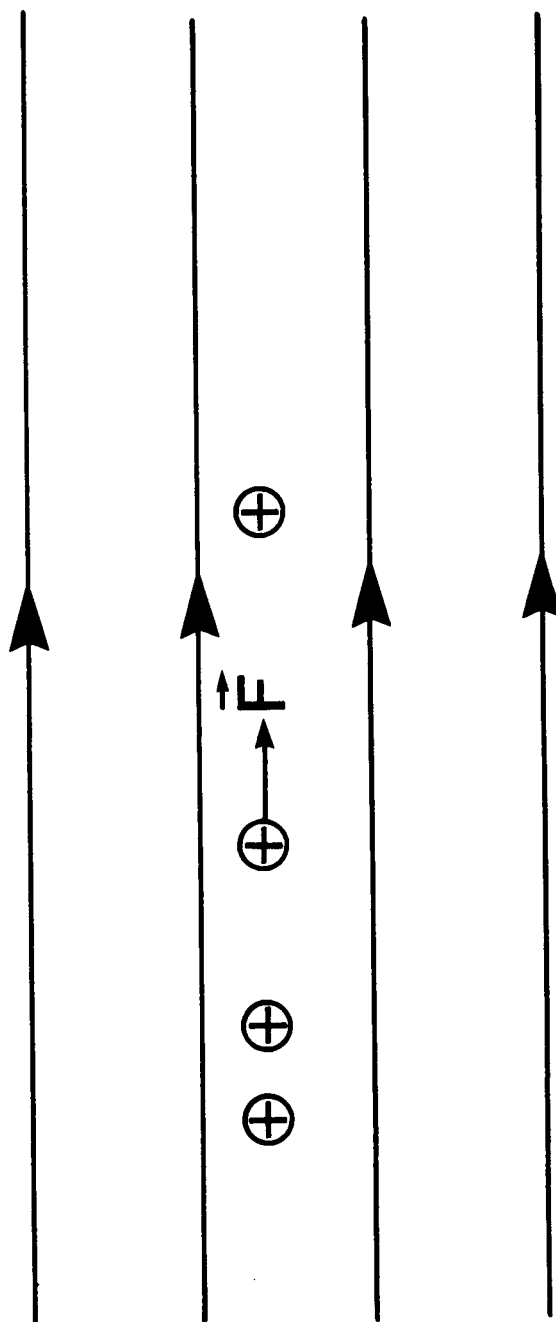


Fig 1

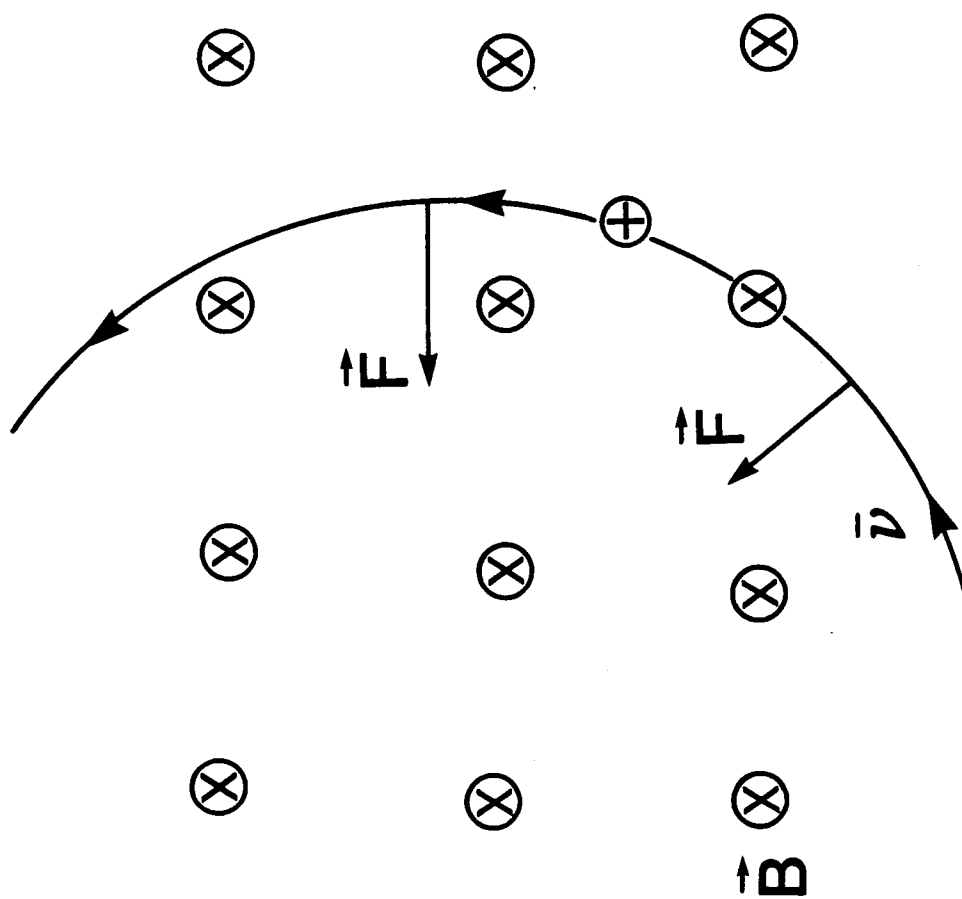


Fig 5

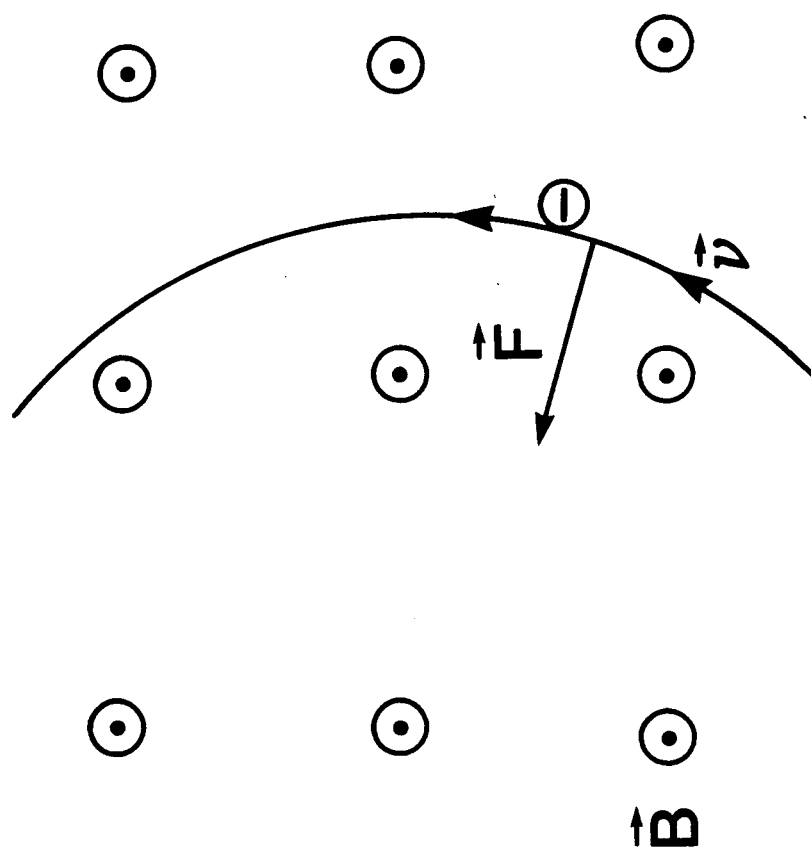


Fig. 6

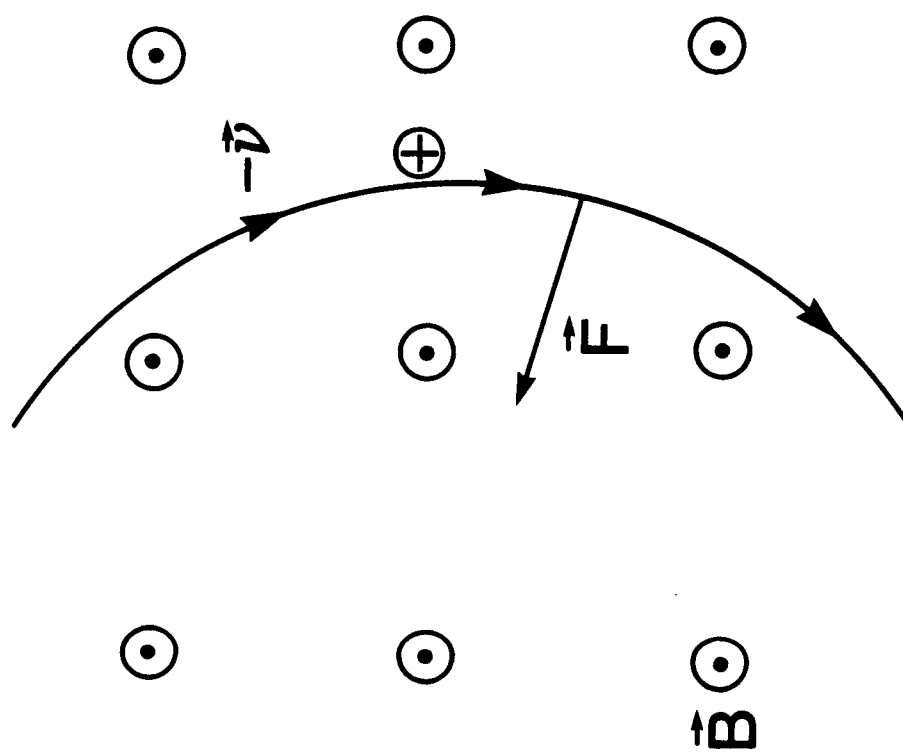


Fig. 7