HQ GRANT
IN-90-CR
235171
41P

STATISTICAL AND PHYSICAL EVOLUTION OF QSO's

David Caditz and **Vahé Petrosian***

Center for Space Science and Astrophysics
Stanford University

CSSA-ASTRO-89-15
September 1989

*Also Department of Applied Physics.

Statistical and Physical Evolution of QSO's

David Caditz and Vahé Petrosian
Center for Space Science and Astrophysics
Stanford University

Abstract

We discuss the relationship between the physical evolution of discrete extragalactic sources, the statistical evolution of the observed population of sources, and the cosmological model. Three simple forms of statistical evolution - pure luminosity evolution (PLE), pure density evolution (PDE), and generalized luminosity evolution (GLE) - are considered in detail together with what these forms imply about the physical evolution of individual sources. Two methods are used to analyse the statistical evolution of the observed distribution of QSO's from combined flux limited samples. It is shown that both PLE and PDE are inconsistent with the data over the redshift range $0 < z < 2.2$, and that a more complicated form of evolution such as GLE is required, indepedent of the cosmological model. This result is important for physical models of AGN, and in particular, for the accretion disk model which recent results show may be inconsistent with PLE.

I. Introduction

Since the recognition of QSO's as a separate class of extragalactic objects, great emphasis has been placed on statistical analysis of the QSO population. It has been hoped that information contained in this population in the form of its distribution and general evolution would reveal the nature of the QSO energy source and help to constrain the values of some of the cosmological parameters. Much progress has been made in determining the general properties of the QSO population in terms of, for example, spectral shape, line widths, variability, etc., and physical models have been constructed which explain many of these observed features.

The most widely accepted model today - what has been referred to recently as the standard model - proposes mass accretion onto a massive black hole as the primary QSO energy source (Sheilds, 1977; Malkan, 1983). To conserve angular momentum, the infalling matter is expected to take the form of an accretion disk, and radiation from this disk is thought to be the primary source of QSO luminosity. Although the disk radiation may be reprocessed in a variety of ways, the total QSO luminosity is expected to scale with the energy production of the accretion disk. This model has gone a long way toward explaining features such as those described above, but much refinement of this model has yet to be done to determine whether ultimately it can explain QSO properties in detail.

An important test for any physical model concerns the formation and temporal evolution of the individual objects, which we call the physical evolution. Two different physical models may explain spectral features of the QSO population at a given era, for example, but each may produce different present day populations. Long lived QSO's all born at an early era and all evolving in a similar manner, the so called pure luminosity evolution model (PLE), may produce far different population evolution than would QSO's with relatively constant properties undergoing continuous formation and death, the pure density evolution model (PDE). A proper account of the constraints imposed by the observed statistical evolution of the QSO population should be made for any proposed physical evolution model of QSO's.

Early analyses of radio selected quasars by Schmidt (1968) and Lynds and Wills (1972) demonstrated dramatic evolution in the QSO population which appeared to be independent of source luminosity, and a PDE model was tentatively adopted. Longair and Scheuer (1970) showed that PDE and PLE were indistinguishable in the absence of features in the luminosity function and Lynds and Petrosian (1972) gave a more detailed analysis of the interplay between density and luminosity evolution. Petrosian (1973) analysed both radio and optically selected QSO's assuming PDE and found the evolution to depend upon the selection criterion. He proposed a two class model with radio loud sources undergoing strong density evolution and radio quiet sources showing little or no density evolution. Schmidt and Green (1978), in a preliminary analysis of the Palomar Bright Quasar Survey (BQS), showed the evolution to be consistent with PDE. Giaconni *et al.* (1979) and Tananbaum *et al.* (1979) showed that PDE would violate the limits on the X-ray background, and Marshall *et al.* (1983) concluded that PLE is consistent with both the X-ray background limits and the observed number counts in the AB and BF samples. Schmidt and Green (1983), in a thorough analysis of the BQS and five other optically selected samples, showed luminosity dependence to exist in QSO evolution and proposed the luminosity dependent density evolution model which is a generalization of the two class model of Petrosian. Marshall *et al.* (1984) and Marshall (1985) found PLE with a steep power law luminosity function to be consistent with the available optical data, but once again, due to the featureless shape of the luminosity function, could not rule out density evolution. Recent surveys by Boyle (1986) and Koo and Kron (1988) show a break in the power law luminosity function which has been fit by Marshall (1988) to a PLE model. Koo and Kron fit a luminosity dependent density evolution model similar to that of Schmidt and Green to their data. Boyle *et al.* (1988) use a PLE model, but cannot rule out density evolution at the low luminosity end of the luminosity function. In the present work, we find both PDE and PLE to be inconsistent with the available QSO data.

Until recently, little has been done to compare the physical evolutions with the statistical evolutions. Cavaliere *et al.* (1983a,b) and Cavaliere and Padovani (1988) discuss the relationship between physical parameters, such as the Eddington ratio, L/M, and the statistical evolution under the gravitational accretion hypothesis. Such a comparison between physical and statistical evolutions requires knowledge of the characteristic evolution of individual QSO's as a function of various population parameters. This physical evolution is not directly observable, but must be inferred from the evolution of the ensemble as a whole. Because of the observational selection effects inherent in any QSO survey, the ensemble evolution is itself not a well determined quantity. As described above, the availability of deeper QSO samples detecting intrinsically fainter objects, has helped to determine the ensemble evolution (Boyle *et al.*, 1987; Marshall, 1988). But, as we shall show below, characteristic physical evolution is not determined uniquely by the ensemble evolution - a proposed form of evolution may be shown only to be consistent with the observed ensemble averages as well as various other constraints such as X-ray background and present day density of galaxies which may be presumed to have once been active QSO's.

The problem of determining an appropriate law for physical evolution is further complicated by the ambiguity in the choice of cosmological model. A knowledge of the cosmological parameters is necessary to convert observed fluxes and distributions into absolute luminosities and object densities. As it has been stressed (e.g., Caditz and Petrosian (1988)) because of this ambiguity and other uncertainties such as intergalactic absorption, it is uncertain how accurately physical evolution can be determined from the QSO population.

The main purpose of this work is to describe the relationship between evolution of the QSO luminosity function, the physical evolution of individual objects, and the cosmological model, and to test popularly accepted views of physical and statistical evolution. In Section II we describe the mathematical framework relating the physical and statistical evolution of a population of extragalactic objects. In Section III we apply various statistical tests to determine the evolution of the luminosity function and to see which evolutionary models are consistent with the presently available QSO data. We determine that PLE, as proposed by the above mentioned authors, is *not* an adequate explanation of the evolution seen in the QSO population and the idea that QSO's are long lived phenomena may be questionable. Section IV gives a brief summary and discussion.

II. The Relationship Between Physical and Statistical Evolution

Two basic properties of any extragalactic source which may be related to observable quantities are its absolute luminosity, L, and its age, $t_a = t - t_i$, where t_i is the object's time of formation, time of brightening, or time at some other standard phase of evolution which may be applied to all objects. These quantities are related to the observed flux, l, and redshift, z, via:

$$\mathrm{L} = 4\pi \mathrm{d}_l^2 l \tag{1}$$

$$t = \int_z^\infty \frac{dz'}{(1+z')\mathrm{H}(z', \Omega_i)} \tag{2}$$

where $\mathrm{d}_l = \mathrm{d}_l(z, \Omega_i)$ is the luminosity distance, and $\mathrm{H} = \mathrm{H}(z, \Omega_i)$ is the Hubble constant at redshift z (e.g., Weinberg, 1972). Here Ω_i stand for the cosmological parameters $(\Omega, q_o, \Lambda, ...)$.

If the physical evolution of these objects, which we denote as $\dot{\mathrm{L}} \equiv \partial \mathrm{L}/\partial t$, is a function only of luminosity and time, then the objects must obey the continuity equation,

$$\frac{\partial}{\partial t}\Psi(\mathrm{L}, t) + \frac{\partial}{\partial \mathrm{L}}\left[\dot{\mathrm{L}}\Psi(\mathrm{L}, t)\right] = \mathrm{S}(\mathrm{L}, t) \tag{3}$$

where $\Psi(\mathrm{L}, t)$ is the luminosity function, or comoving density of objects at (L, t), and $\mathrm{S}(\mathrm{L}, t)$ is the source function which describes the difference between the instantaneous birth and death rates. $\dot{\mathrm{L}}$ and S together describe the physical evolution of the sample. The formal solution to equation (3) is given by,

$$\Psi(\mathrm{L}, t) = \Psi_i(\mathrm{L}_i, t_i)e^{-\tau} + e^{-\tau}\int_0^\tau e^{\tau'}\left(\mathrm{S}/\alpha\right) d\tau' \tag{4}$$

where we have defined the quantities:

$$\alpha(\mathrm{L}, t) = \frac{\partial \dot{\mathrm{L}}}{\partial \mathrm{L}} \tag{5}$$

and

$$\tau(t) = \int_{t_i}^t \alpha(\mathrm{L}(t'), t')\, dt'. \tag{6}$$

Here $\mathrm{L}_i(\mathrm{L}, t) = \mathrm{L} - \int_{t_i}^t \dot{\mathrm{L}}(t')\, dt'$, and Ψ_i is the initial source distribution at t_i. In general, we can expect the source density, $\rho(t) = \int \Psi(\mathrm{L}, t)\, d\mathrm{L}$ and the shape of the luminosity function to vary with time.

Notice that the $\dot{\mathrm{L}}$ and S can not be determined uniquely from the function Ψ, even if Ψ is known throughout the entire L-t parameter space. Therefore, it is necessary to make further assumptions about the physical evolution. In the following, we assume various functional forms for $\dot{\mathrm{L}}$ and S to obtain solutions to equation (3) which exemplify proposed forms of QSO statistical evolution.

a) Pure Luminosity Evolution

If we assume that the source formation occurred in a short burst at some initial time t_i (i.e., $\mathrm{S}(\mathrm{L},t) = \phi(\mathrm{L})\delta(t-t_i)$) then the form of the luminosity function is completely specified by the conditions at t_i and the physical evolution of the objects which must have ages comparable to the cosmological age. The solution (4) in this case simplifies to $\Psi(\mathrm{L},t) = \phi\big(\mathrm{L}_i(\mathrm{L},t)\big)e^{-\tau(\mathrm{L},t)}$. If we further assume that all objects brighten or dim in the same manner, so that the physical evolution is described by

$$\mathrm{L}(t) = \mathrm{L}_i g(t-t_i), \qquad \mathrm{S}(\mathrm{L},t) = \phi(\mathrm{L})\delta(t-t_i), \tag{7}$$

where L_i is the luminosity at formation, and $g(0) = 1$, then from equations (5) and (6) we find $\alpha = \dot{g}/g$ and $\tau = \ln\big(g(t-t_i)\big)$ so that

$$\Psi(\mathrm{L},t) = \frac{1}{g(t-t_i)}\phi\big(\mathrm{L}/g(t-t_i)\big). \tag{8}$$

This form of luminosity function is easily recognized as PLE because it maintains its shape on a $\log(\Psi)$ - $\log(\mathrm{L})$ plot and undergoes a simple translation in luminosity, determined by the function $g(t-t_i)$. Evidence of this kind of behavior for the luminosity function would suggest the physical evolution of equation (7).

Equation (7), however is not the only kind of physical evolution which can give rise to PLE. An extreme opposite is a model of physical evolution with continuous creation of short lived (compared to cosmological time) sources; $\mathrm{L}_i \propto \delta(t-t_i)$. In this case, the evolution of the luminosity is unimportant and we can set $\dot{\mathrm{L}} = 0$ in equation (3). However.

we can still conserve the total number of sources and obtain equation (8) if the physical evolution functions have the form

$$\mathrm{L}(t) = \mathrm{L}_i \delta(t - t_i), \qquad \mathrm{S} = \frac{\dot{g}}{g^2} \Sigma(\mathrm{L}/g), \tag{9}$$

with $\phi(x) = x^{-1} \int_x^{x_i} \Sigma(x')\, dx'$. Both models described by equations (7) and (9) are extreme and require special conditions. Models with intermediate conditions between these two extremes can also give rise to solution (8) and PLE if the function $g(t)$ appearing in equations (7) and (9) are identical. This is unlikely because $g(t)$ in (7) describes the evolution of the luminosity of sources which is most likely determined by the source environment, while $g(t)$ in equation (9) describes the rate of formation of the sources which may be coupled to global conditions. We may therefore conclude that in general PLE requires very special conditions.

b) Pure Density Evolution

If we assume that objects may form (and die) at any era, and that they undergo little or no evolution in luminosity, then we may set $\dot{\mathrm{L}} = 0$ in equation (3) and we find a solution of the form

$$\Psi(\mathrm{L}, t) = \int_0^t \mathrm{S}(\mathrm{L}, t')\, dt' \tag{10}$$

If we further assume that the source function is separable, $\big($i.e., $\mathrm{S}(\mathrm{L}, t) = \eta(t)\phi(\mathrm{L})$ $\big)$, then we obtain the solution

$$\Psi(\mathrm{L}, t) = \rho(t)\phi(\mathrm{L}) \tag{11}$$

where

$$\rho(t) = \int_0^t \eta(t)\, dt. \tag{12}$$

Equation (11) is the so called pure density evolution (PDE) form of the luminosity function. Under PDE, the luminosity function maintains its shape, but its overall normalization may vary with time, either increasing or decreasing depending on the sign of S. An observed luminosity function of the form of equation (11) suggests the physical evolution described above. Note, however, that for a constant or increasing density, one cannot differentiate between short and long lived sources.

PDE cannot be obtained from a δ - function source term (except in the trivial case of zero evolution) because, in that case, the total number of sources must be conserved. Over a limited range in luminosity, however, some general forms of physical evolution could produce a luminosity function of equation (11).

c) Generalized Luminosity Evolution

A third simplified luminosity function of interest is one with invariant shape,

$$\Psi(\mathrm{L}, t) = \rho(t)\phi(\mathrm{L}/g(t))/g(t) \tag{13}$$

where the functions $\rho(t)$ and $g(t)$ describe the density and luminosity evolution, respectively. This generalized luminosity evolution (GLE) lends itself to easy interpretation in the number-flux moment test to be discussed below. The physical evolutions which can give rise to such a statistical evolution are almost as restrictive as those discussed in the previous two cases. For example, if there exists a universal evolution of the luminosity of objects, independent of their initial luminosity and epoch of formation, $\mathrm{L} = \mathrm{L}_o g(t)$, and if the rate of formation is such that the luminosity distribution of newly created objects is the same as those which were created earlier and had evolved according to the universal law, then $\mathrm{S}(\mathrm{L}, t) = \phi(\mathrm{L}/g(t))\eta(t)$, and one obtains a luminosity function of invariant shape with $\rho(t) = \int_0^t g(t')\eta(t')\,dt'$. Note that for $\eta(t) \propto \delta(t)$ this reduces to the PLE case, and for $g(t) =$ constant it reduces to PDE.

Most of the assumptions involved in the cases described above are very restrictive and assume that the physical evolutions are determined by global conditions and not necessarily by the immediate environment of the object. In a realistic situation, both global and local conditions will be important and the physical evolutions will be more complicated giving rise to the general luminosity function described by equation (4). These models are, however, commonly assumed, and we therefore test them for consistency with available QSO data.

III. Analysis of the QSO Data

The above discussion is further complicated by the fact that the luminosity function, $\Psi(\mathrm{L}, t)$, is not directly observable, but must be estimated from the observed data which covers an incomplete portion of the luminosity-time luminosity-redshift plane, even if the cosmological model is known. In the following analyses, we use redshift, z, in place of cosmic time, t, used above. The relationship between z and t is given by equation (2), and the above equations carry over in a straight forward manner. For PDE the luminosity function, Ψ, is presumed to be a separable function of L and z and the behavior of high redshift, low luminosity objects (not seen in a flux limited survey) may be inferred from the luminosity function at low redshifts (where most of the luminosity function is seen). Various techniques have been developed (Schmidt, 1968; Lynden-Bell, 1971; Jackson, 1974; Felten, 1976; Turner, 1979; Petrosian, 1986) for estimating the PDE luminosity function based on the observed portion of the L - z plane. Constructing a PLE or GLE luminosity function from incomplete data, on the other hand, is more complicated. The luminosity function at high redshifts cannot be inferred from that at low redshifts without knowledge of the evolution, $g(z)$. In practice, a functional form for $g(z)$ is chosen, and it is assumed that in terms of the new parameter, $\mathrm{L}' = \mathrm{L}/g(z)$, and z the luminosity function is separable and the techniques described above for PDE may be employed. The luminosity function determined in this manner is then checked for consistency with the requirement for PLE or GLE - i.e., constant logarithmic shape of the luminosity function - which, if consistent, indicates a correct choice of $g(z)$.

The shape of the luminosity function at a given redshift (or time) is not greatly affected by uncertainty in the cosmological model. For standard models this uncertainty is usually within the random errors. However, we emphasize again that the evolutions of the luminosity function, described by $g(z)$ and $\rho(z)$, will depend on the choice of cosmological parameters, although acceptance or rejection of a PLE model will not. (Caditz and Petrosian, 1988).

a) Results from Previous Analyses

Previous analyses of QSO data derived from flux limited samples, have shown that the luminosity function at 2500 Åis of a power law form, $\Psi(\mathrm{L}, t) \propto \mathrm{L}^{-\beta}$ with $\beta \approx 3.5$ (Marshall 1985) lacking any significant features. It is therefore impossible to distinguish between PDE and PLE evolutionary forms. A horizontal shift in luminosity (PLE), for example, is completely equivalent to a vertical shift in density (PDE). Constraints such as overproduction of the X-ray background (Giaconni *et al.*, 1979; Tananbaum *et al.*, 1979; Morisawa and Takahara, 1988) or violation of the limits set by optical number counts (Marshall *et al.*, 1983) have been used to rule out PDE for such samples. However, various other forms of evolution such as PLE or GLE have not been shown to be uniquely correct. Two more recent surveys, (Boyle, 1986; Koo and Kron, 1988), have acheived fainter limiting blue magnitudes of B = 20.9 and B = 22.5 respectively. These deeper surveys allow examination of the $\mathrm{L} - z$ plane to lower luminosities, even at higher redshifts. These authors have found a break in the power law index of the luminosity function which can be used to determine the evolutionary form. Boyle *et al.* (1988) and Marshall (1988) claim to obtain self consistent PLE solutions with $g(z) \propto (1+z)^k$ where $k \approx 3.2$ $(\Omega = 1, \Lambda = 0)$. (We note that this form of evolution evolves the break essentially parallel to the flux limit over the redshift range of the surveys, though it is highly unlikely that the break is the result of incompleteness.) Marshall (1987) has shown that this evolutionary law, extrapolated to the present era, would give a QSO luminosity function consistent with that observed for Seyfert galaxies (see also Cavaliere *et al.*, 1985; Weedman, 1986a). Under the PLE model, then, Seyferts may be considered to be the present day remnants of once luminous QSO's.

It is tempting, therefore, to interpret this form of evolution of the luminosity function as the result of the common physical evolution of individual objects, all born at a given era, t_i, and all undergoing evolution according to a common law, $\mathrm{L} = \mathrm{L}_o(1+z)^{3.2}$. A smooth transition would then exist between bright, high redshift QSO's and low redshift Seyfert galaxies.

b) Nonparametric Analysis

To test these results we use the combined QSO data from various surveys described in Table 1. These are flux limited optical surveys from which we obtain luminosities at a standard wavelength of 2500Å assuming a spectral index of $\alpha = -1$ as is common practice (Marshall *et al.*, 1984; Weedman, 1986b) and a Friedman model ($\Omega = 1$, $\Lambda = 0$). We note, however, that in the accretion disk model the spectrum may be more complicated and the above k-correction may not be correct. We shall address this point in a future publication.

Our general analysis below tests the consistency of the QSO data with the more general GLE model which, as mentioned above, contains both PLE and PDE. A rejection of GLE therefore, rules out both PLE and PDE. According to GLE, the luminosity function evolves in density by the function $\rho(z)$, and in luminosity according to a particular law, $\mathrm{L} = \mathrm{L}_o g(z)$, yet it maintains its shape over the redshift range of interest. When the luminosity function $\phi(\mathrm{L}')$ is calculated using the transformation $\mathrm{L}' = \mathrm{L}/g(z)$, it should be independent of redshift, z. Likewise, the density function, $\rho(z)$, should be independent of L'. Using a nonparametric procedure (Petrosian 1986) appropriately modified for combined samples (see Appendix A), we have calculated the luminosity function for the data given in Table 1. We have considered the evolutionary laws $g(z) =$ constant, corresponding to PDE, $g(z) \propto (1+z)^{3.2}$ ($\Omega = 1$, $\Lambda = 0$), as given by Marshall (1988), and $g(z) \propto \exp(t_l/\tau)$ where $\tau = .73$ Gyr and $t_l = t_0 - t(z)$ is the cosmological lookback time as discussed below. In this section we assume a standard Friedman cosmology with density parameter $\Omega = 1$ and zero cosmological constant.

i) Test of PDE: $g(z)$ = constant. Nonparametric luminosity functions assuming no luminosity evolution are plotted in Figure 1. The characteristic power law form is seen at high luminosities with a break and a flatter slope at lower luminosities. The position of the break clearly evolves with redshift, being brighter in the high redshift bins. The two lowest redshift luminosity functions do not appear to have the same double power law shape. Figure 1a shows a clear evolution of the break luminosity with redshift, ruling out PDE directly from the observed data, and without recourse to the X-ray background or number count arguments described above.

The nonparametric cumulative density function, $\sigma(\mathrm{V}) = \int \rho(\mathrm{V})\, d\mathrm{V}$, is given by curve a in Figure 2. In the absense of evolution, $\sigma(\mathrm{V})$ should increase linearly with volume (dashed lines in Figure 2), but the calculated $\sigma(\mathrm{V})$ shows a dramatic increase in the comoving density of objects. This could be mainly the result of a general brightening of the characteristic or average luminosity with redshift, or a decrease in the number of objects with time $(\mathrm{S}(\mathrm{L}, t) < 0)$.

ii) Test of PLE: $g(z) = (1+z)^{3.2}$. Figure 1b shows the luminosity functions assuming luminosity evolution of the form $g(z) = (1+z)^{3.2}$ as given by Marshall (1988). The requirement of constant shape is fairly well met for redshifts, $z > 0.6$, but it fails at lower redshifts where the shape changes into a single power law form. The Kolmogorov-Smirnov Test applied to the lowest redshift bin in Figure 1b rejects the double power law as given by Marshall and Boyle *et al.* (1988) whereas a single power law is not rejected. The departure from the double power law shape for low redshifts indicates either that PLE is not correct over the entire range of redshift, or that the evolutionary law (or cosmology) is not correct. A more strongly evolving characteristic luminosity, for example, would put the break in slope beyond the observable limit in the low redshift bins. In this case, however, it may be hard to evolve the high redshift QSO luminosity function into the low redshift Seyfert luminosity function as suggested by Marshall (1987) and Weedman (1986a).

The nonparametric cumulative density function (curve *b* in Figure 2) increases with volume somewhat faster than linear as expected for constant comoving density. This could be either the result of a true increase in comoving density or luminosity evolution which is stronger, especially at lower redshifts, than $g(z) = (1+z)^{3.2}$ as assumed.

iii) Test of PLE: $g(z) = \exp(t_l/.73)$. In order to test the latter possibility mentioned above, we have repeated the above analysis assuming the exponential form for $g(z)$. This form of evolution is also commonly used and, because it increases more rapidly with z it may give a better fit to the QSO data. We will show below that this is indeed the case. Cumulative luminosity and density functions, assuming exponential evolution in luminosity are plotted in Figures 1c and curve c in Figure 2. In this case the break in the luminosity function has evolved out of the observable region of the $\mathrm{L}-z$ plane at redshifts $z < 0.6$, although the slope of the luminosity function is flattening at low z inconsistent with PLE or GLE. Figure 2 (curve *c*) shows a higher comoving density of objects at redshifts $z < 0.6$ ($\mathrm{V} < 8 \times 10^9 h^3 \quad \mathrm{Mpc}^3$) than expected for PLE ($\sigma(\mathrm{V}) \propto \mathrm{V}$), indicating either a true decrease in the number of objects with redshift or a luminosity evolution weaker than the assumed form.

iv) Test of GLE. The above result shows that GLE provides an approximate description of the data. However, it should be noted that if GLE is indeed a true description, then any departure of the luminosity function from a constant shape when evolved according to a particular physical evolutionary law could be attributed to an incorrect choice of evolutionary law or cosmological model. Conversely, if GLE is not the true form of evolution, then the luminosity function may be made to resemble GLE over a limited range of the $\mathrm{L}-z$ plane with the appropriate (incorrect) choice of physical evolution or cosmology. The evolutionary laws considered above give results consistent with GLE over only a limited range of redshift and luminosity and it is not certain whether a form of luminosity evolution can be found which produces GLE over the entire range. For these reasons we now apply a more rigorous test which does not require the assumption of the form of physical evolution to determine whether the data support GLE.

c) The Number - Flux Test

The number - flux test is described by Loh and Spillar (1986). The method of moments, which is a generalization of this test, is described by Caditz and Petrosian (1988). In Appendix B of this work we extend the method of moments for application to combinations of various flux (or magnitude) limited samples. The number-flux test can be used to determine cosmological parameters from a given set of data if certain aspects of the evolution of the luminosity function are known (e.g., comoving density or characteristic luminosity). Conversely, if the cosmological parameters are assumed, or known by independent means, this test can be used to determine the evolution of the luminosity function from a given data set. Unlike the above described nonparametric test, the number - flux test can be used to determine the luminosity function from a given set of data independently of an assumed form of physical evolution $g(z)$, and in fact, can be used to determine both the luminosity evolution, $g(z)$, and the density evolution, $\rho(z)$, of the sample for any assumed cosmological model.

We have applied the number-flux test to various combinations of the QSO data given in Table 1 assumung a double power law form of the luminosity function as given by Marshall (1988) with break luminosity $L_*(z)$. Our basic results are given in Figures 3 and 4. Figure 3 shows the variation with redshift of the number of objects contained within a comoving shell at redshift z, divided by z^2. This number is the product of the comoving density, which is determined by the luminosity function, and the comoving volume of the shell at z, which depends on the cosmological model. Curves expected for constant comoving density are shown for various values of Ω. Figure 3 indicates that constant density is not consistent with the data for reasonable values of Ω, and either an increase in the density or variation in the shape of the luminosity function occurs for $z < 0.6$, consistent with the nonparametric analysis above. Figure 4 shows the variation with z of the apparent break luminosity,

$L_*(z)/d_l^2(z)$. (The constant values of L_*/d_l^2 are due to the coincidence mentioned earlier that the luminosity evolution law seems to be parallel to the sample cut-off luminosity in a magnitude limited sample.) The values plotted in both Figure 3 and 4 are independent of the cosmological model. Given a cosmological model which determines the volume, $V(z)$, and the luminosity distance, $d_l(z)$, we may determine directly from Figures 3 and 4 the functions $\rho(z)$ and $g(z)$. These functions are plotted in Figures 5 and 6, respectively, for Friedman models with various values of density parameter, Ω, and $\Lambda = 0$. We find, as mentioned above, that there is a significant increase in the comoving density at $z < 0.6$, and also, a rapid evolution in characteristic luminosity, $g(z) = \exp(t_l/\tau)$, which we have plotted as a solid line along with the data in Figure 6 ($\tau \approx .73$ Gyr for $\Omega = 1$, $\Lambda = 0$, $H_o = 100$ km s^{-1} Mpc^{-1}). In agreement with our earlier results (Figures 1b and 2b), we find that the power law form of luminosity evolution, $g(z) = (1+z)^k$, fails for $z < 0.6$ although it fits well at higher redshifts, and that comoving density is not independent of redshift except perhaps for unreasonably large values of Ω.

The above analysis assumed a double power law luminosity function with constant slopes of 1.35 and 3.6 over the entire redshift range $0 < z < 2.2$. While this form of luminosity function is correct at high redshifts, the nonparametric method discussed above indicates that it may be incorrect for redshifts, $z < 0.6$, and that the luminosity function may indeed change shape. A more sophisticated application of the number-flux test, using higher moments of the data, can be used to determine the evolution of the shape of the luminosity function and thus reduce the number of assumptions (Caditz and Petrosian, 1988). Unfortunately, for reasons discussed in Appendix B, the present data are insufficient to perform such an analysis. In any case, it is clear that either the comoving density and characteristic luminosity vary as shown in Figures 5 and 6, or the luminosity function changes shape at $z \sim 0.6$, or both. We conclude, therefore, that both PDE and PLE are excluded in the redshift range $0 < z < 2.2$, although GLE may be correct.

IV. Discussion and Conclusions

We have described various forms of evolution of the QSO luminosity function and how these forms may be recognized in a given set of data. We stress that although certain forms of luminosity function evolution may suggest particular forms of physical evolution of individual objects, physical evolution cannot be uniquely determined. For example, in the most natural interpretation, PLE results from long lived objects, all born at a given era, and all undergoing similar evolution, yet it may be the result of short lived objects produced continuously by a source function with evolving parameters.

Recent work by Wandel and Petrosian (1988) on the accretion disk model for the AGN central engine suggests that these objects do not undergo the type of physical evolution expected to produce a PLE type luminosity function. In a subsequent paper we will discuss in detail the physical evolution predicted under the accretion disk model and whether such physical evolution can reproduce GLE type statistical evolution. Because there have been recent claims that the QSO luminosity function does obey PLE, we have analysed data from various QSO surveys to determine the validity of these claims. We have found that, for reasonable values of cosmological parameters, PLE is not consistent with the data over the redshift range $0 < z < 2.2$. This draws into question the claims of a continuous evolution from high redshift QSO's to low redshift Seyfert galaxies. We have, in addition, shown PDE to be inconsistent with the data independently of arguments concerning the X-ray background or number counts. We have found that a form of GLE which contains both luminosity evolution described by $g(z) = \exp(t_l/\tau)$ and density evolution may be an appropriate description of the evolution of the QSO luminosity function. Luminosity evolution of the form $g(z) = (1+z)^k$ appears to be inconsistent with the data at redshifts $z < 0.6$. In addition, the shape of the luminosity function may change at $z \sim 0.6$, indicating a more complicated form of evolution.

We note that no single survey combines enough low redshift data with deep, high redshift data to provide an unambiguous determination of the consistency of PLE. Our results are necessarily derived from a combined data set. It has been suggested (Cavaliere, Giallongo, and Vagnetti, 1989) that unforseen errors may be introduced by combining data from surveys, each with different uncertainties and biases. The low redshift AB and BF surveys which are on average closer to their flux limits could, for example be biased toward high comoving densities while the AAT may be biased in the opposite manner. It is hard to see, however, how this bias could cause an error large enough to make an $\Omega \leq 1$ universe appear as $\Omega > 3$ as suggested by Figures 5 and 6 if PLE is assumed to be correct. The number of low (high) redshift sources would have to be reduced (increased) by an order of magnitude to make the data consistent with both $\Omega \leq 1$ and PLE. In addition, uncertainties occurring near the flux limit, described by Cavaliere *et al.*, call into question the maximum likelihood analyses of Marshall (1987) and Boyle *et al.* (1988) which suggest PLE.

We wish to thank Dr. Wandel for useful discussions on the physical evolution of AGN, and Dr. Linder for helpful discussions of the manuscript. This work was supported by NASA grants NCC 2-322 and NGR 05-020-668.

Appendix A

Non-Parametric Method for Combined Flux Limited Surveys

The non-parametric method for a single flux limited survey is described by Lynden-Bell (1971) and Petrosian (1986). Here, we present a method for constructing a nonparametric luminosity function from combined samples, each with different flux limits, l_{lim}, as illustrated in Figure A.1. Objects denoted by circles, squares, and triangles are from surveys which correspond to flux limits 1, 2, and 3, respectively. In a procedure analogous to that of Lynden-Bell, we associate with each object a minimum absolute luminosity, L_{min}, determined by the intersection of a vertical line (constant z) through the object and the curve corresponding to the appropriate flux limit, below which the object would not have been detected. L_{min} depends on both the redshift of the source and the flux limit of the survey, but does not depend on the luminosity of the source itself. If the true distribution of sources, $\Psi(L, z)$, is a separable function of L and z, then L and L_{min} will also be independent variables, and the nonparametric method for single samples may be directly applied to the combined data on the L - L_{min} plane.

If we combine all objects on a plot of L *vs.* L_{min} they will appear as in Figure A.2 where we now make no distinction between objects taken from different surveys. Using the nonparametric method for single surveys, we find for the cumulative luminosity function,

$$\Phi(L_{i+1}) = \Phi(L_i)(1 + 1/N_i) \tag{A.1}$$

where the objects are ordered by increasing luminosity and N_i is the number of objects in the rectangular region indicated in Figure A.2. Iterating the above equation we find

$$\Phi(L) = \Phi_o \prod_{j=1}^{i-1}(1 + 1/N_j) \tag{A.2}$$

where Φ_o is an arbitrary constant of normalization and N_j is defined as the number of objects with $L > L_j$ and $L_{min} < L_j$. Here, L_j is the luminosity of the j^{th} brightest object of the combined sample.

For analysis of a single survey, or combined surveys with identical limits, the definition of N_j is completely equivalent to that given in the above references, namely, N_j consists of all objects with $L > L_j$ and $z < z_{max}(L_j)$ where $z_{max}(L_j)$ is the maximum redshift that an object of luminosity L_j can have and still be detected. Using the proscription $L_{min} < L_j$, however, in place of $z < z_{max}(L_j)$, allows generalization of the nonparametric method to any number of different samples. This method can in fact be used for surveys such as the Palomer Bright Quasar Survey in which each object is assigned a different value of limiting flux.

The determination of the cumulative density function $\sigma(z) = \int_0^z \rho(z')(dV/dz)\, dz'$ from combined samples is identical to the above with the replacement of the distribution in (L,L_{min}) space with that in the (V,V_{max}) space. Here, however, we define N_i as the number of objects with $V < V_i$ and $V_{max} > V_i$. This prescription is equivalent to counting N_i as the set of all objects, j, such that $V_j < V_i$ and $L_j > L_{min}(z_i, l_{lim,j})$, which for standard cosmological models provides computational advantages.

Appendix B

The Method of Moments for Combined Samples

The Method of Moments for a single sample is described by Caditz and Petrosian (1988). This method uses ratios of moments of the luminosity function to determine unknown cosmological parameters as well as parameters describing the shape and evolution of the luminosity function. All parameters of the luminosity function can be determined in this manner except one describing the luminosity evolution or one describing the density evolution of the luminosity function. These two unknown parameters are always found in combination with the cosmological parameters, and assumptions must be made about one set of parameters (e.g., cosmological) to determine the others (e.g., the density and luminosity evolution of the sources). For example, if we assume pure luminosity evolution, $\Psi(\mathrm{L}, z) = (1/g(z))\phi(\mathrm{L}/g(Z))$, then only two moments are needed, and the method of moments becomes identical to the number-flux test described by Loh and Spillar (1986). In this case the total number of objects and the total flux (divided by the limiting flux) within a redshift interval z to $z + \delta z$ are given by,

$$\mathrm{N} = \rho(z)\mathrm{V}'\delta\Omega\delta z \int_{x_o}^{\infty} \phi(x)\, dx \tag{B.1}$$

and

$$\mathrm{F}_1 = \rho(z)\mathrm{V}'\delta\Omega\delta z \int_{x_o}^{\infty} x\, \phi(x)\, dx \tag{B.2}$$

where $\mathrm{V}'\delta\Omega\delta z$ is the volume element within a redshift range δz and solid angle $\delta\Omega$, $\rho(z)$ is the total density (we have normalized $\Psi(\mathrm{L}, z)$ to unity), and we have defined the quantities

$$x \equiv \mathrm{L}/g(z) \tag{B.3}$$

and

$$x_o \equiv \mathrm{L}_o/g(z) = 4\pi d_l^2 l_o/g(z) \tag{B.4}$$

Here, l_o is the limiting flux and d_l is the luminosity distance which depends on cosmological parameters as well as redshift, and $\mathrm{V}' = d\mathrm{V}/dz$. Both N and F_1 depend on cosmological parameters through the volume element, $\mathrm{V}'\delta\Omega\delta z$, yet the quantity

$$\mathrm{C}_1(x_o) = \mathrm{F}_1/\mathrm{N} = \frac{\int_{x_o}^{\infty} x\phi(x)\, dx}{\int_{x_o}^{\infty} \phi(x)\, dx} \tag{B.5}$$

depends only on the parameter x_o and can be compared directly with the corresponding ratio of the observed data,

$$\mathrm{C}_1 = \frac{\mathrm{F}_1}{\mathrm{N}} = \frac{1}{\mathrm{N}l_o} \sum l_i, \tag{B.6}$$

to determine this unknown parameter. With x_o known, Equations B.1 and B.4 can be solved for the quantities

$$\rho(z)\mathrm{V}' = \frac{\mathrm{N}}{\delta\Omega\delta z \int_{x_o}^{\infty} \phi(x)\, dx} \tag{B.6}$$

and

$$d_l^2(z)/g(z) = \frac{x_o}{4\pi l_o}. \tag{B.7}$$

One further assumption must be made about the density evolution, the luminosity evolution, or the cosmological parameters. For example, if we assume constant density, $\rho(z) =$ constant, then Equation B.6 can be solved for the cosmological parameters (contained in $\mathrm{V}'(z)$) and then Equation B.7 determines $g(z)$. The assumption of cosmological parameters gives $\rho(z)$ and $g(z)$ directly, and the assumption of $g(z)$ in equation (B.7) gives the cosmological parameters in $d_l(z)$, and $\rho(z)$ from equation (B.6). Note that if the luminosity function obeys a simple power law, then C_1 will be independent of x_o, and the method of moments fails. However, for finite total flux and source counts, the power law must be broken at some luminosity, $\mathrm{L}_*(z)$. Equations B.5 and B.6 can then be used to find the variation of L_* with redshift which determines the luminosity evolution.

Clearly, higher moments can be used to determine unknown parameters of the luminosity function. We define the n^{th} flux moment as,

$$\mathrm{F}_n = \rho(z)\mathrm{V}'\delta\Omega\delta z \int_{\mathrm{L}_o}^{\infty} (\mathrm{L}/\mathrm{L}_o)^n \Psi(\mathrm{L}, z)\, d\mathrm{L} \tag{B.8}$$

and the corresponding ratio as

$$C_n(L_o) = \frac{F_n}{N}. \tag{B.9}$$

The variance of the n^{th} moment is given by, $\sigma_n = \frac{1}{n}(C_{2n} - C_n^2)$ (Kendall, 1963, p. 229), which increases with higher moments. More sophisticated applications of the method of moments, using higher moments to determine unknown parameters of the luminosity function are practical only for larger samples.

Different surveys, or different plates from the same survey, may have different sky coverage, $\delta\Omega$, and different limiting flux, l_o. We wish to combine these samples in a manner which preserves the separation of cosmological parameters and luminosity function parameters described above. We arbitrarily choose one survey ($\delta\Omega_0$, l_0) as a standard from the m + 1 surveys we wish to combine. For each survey, i = 0 to m, we define the quantities,

$$a_i = l_{o,i}/l_0 = L_{o,i}/L_0 \tag{B.10}$$

and

$$b_i = \delta\Omega_i/\delta\Omega_0. \tag{B.11}$$

The individual flux moments are now defined by,

$$F_{n,i} = \rho(z)V'\delta\Omega_0\delta z\frac{b_i}{a_i^n}\int_{a_iL_0}^{\infty}(L/L_0)^n\Psi(L)\,dL \tag{B.12}$$

The total number of objects in the combined sample is given by

$$N = \sum_{i=0}^{m} F_{o,i} = \sum_{i=0}^{m} N_i \tag{B.13}$$

and we have the quantities $C_{n,i}$ given by,

$$C_{n,i} = \frac{F_{n,i}}{N_i} = \frac{1}{a_i^n}C_n(a_iL_0) = \frac{1}{a_i^n}\frac{\int_{a_iL_0}^{\infty}(L/L_0)^n\Psi(L)\,dL}{\int_{a_iL_0}^{\infty}\Psi(L)\,dL} \tag{B.14}$$

The important question in how to combine the samples for the purpose of achieving highest statistical significance. In general, we can assign different statistical weights to the samples and define an average value for the $C_{n,i}$'s as

$$C_n = \frac{1}{\sum_{i=0}^{m} w_i} \sum_{i=0}^{m} w_i C_{n,i} \tag{B.15}$$

where w_i are statistical weights to be discussed below. As in the case of a single survey, the generalized C_n depend only on the parameters of the luminosity function and on L_o, and when compared with the appropriate observed ratios of moments, can be solved for these parameters.

The weights, w_i, defined in Equation B.15 are completely arbitrary and we may choose them in the manner which best suits our purposes. For example, for $w_i = 1$ we have,

$$C_n = \frac{1}{m+1} \sum_{i=0}^{m} C_{n,i} = \left\langle \frac{F_n}{N} \right\rangle. \tag{B.16}$$

C_n is thus the average value of that parameter obtained from the individual surveys. This choice of weights may not be optimal for combining surveys with widely different sky coverages and limiting fluxes because the average may be dominated by one survey, e.g., a small, deep survey, and not include fully the contribution of objects from surveys with larger sky coverage which will have average luminosities close to the brighter sample limits. For this reason we choose the weights, $w_i = N_i$ and obtain,

$$C_n = \frac{\sum_{i=0}^{m} F_{n,i}}{\sum_{i=0}^{m} N_i} = \frac{\langle F_n \rangle}{\langle N \rangle} = \frac{\sum \frac{b_i}{a_i^n} \int_{a_i L_o}^{\infty} (L/L_0)^n \Psi(L)\, dL}{\sum \frac{b_i}{a_i^n} \int_{a_i L_o}^{\infty} \Psi(L)\, dL}. \tag{B.17}$$

This choice of w_i gives adequate weight to both small, deep surveys with small $\delta\Omega_i$ as well as bright surveys with large $\delta\Omega_i$. We stress that the cosmological parameters as well as the parameters of the luminosity function are, in principle, independent of the choice of weights, and for large enough samples, different choices of weights should give the same results.

Table 1 QSO Surveys

Sample	N	$d\Omega\,(deg^2)$	B_{lim}	Reference
MBQS	32	87.65	17.65	*Ap. J.* **287**, 1984
		21.30	17.25	
AB	22	37.20	18.25	*Astr. Ap. Suppl.* **80**, 1980
BF	35	1.72	19.80	*Ap. J.* **269**, 1982
AAT	167	0.35	20.40	*M.N.R.A.S.* **227**, 1987
		0.70	20.65	
		3.15	20.90	
Marano	23	0.69	21.00	*M.N.R.A.S.* **232**, 1988
Koo & Kron	29	0.29	22.60	*Ap. J.* **325**, 1988

REFERENCES

Boyle, B. J., 1986, Ph.D. thesis. University of Durham

Boyle, B. J., Fong, R., Shanks, T., Peterson, B. A. 1987 *M.N.R.A.S.*, **227**, 717.

Boyle, B. J., Shanks, T., Peterson, B. A., 1988, M.N.R.A.S., in press.

Caditz, D., Petrosian, V. 1988 *Ap. J. (Letters)*, **337**, L65

Cavaliere, A., Giallongo, E., Messina, A., and Vagnetti, F. 1983a, *Ap. J.*, **269**, 57

——— 1983b, in *Quasars and Gravitational Lenses*, ed. J. P. Swings (Liege: Institut d'Astrophysique), 282

Cavaliere, A., Giallongo, E., and Vagnetti, F. 1985, *Ap. J.*, **296**, 402

Cavaliere, A. and Padovani, P. 1988, *Ap. J.*, **333**, L33

Felten, J. E. 1976, *Ap. J.*, **207**, 700

Giaconni *et al.* 1979, *Ap. J. (Letters)*, **234**, L1

Jackson, J. C. 1974, *M.N.R.A.S.*, **166**, 281

Kendall, E. 1963, *The Advanced Theory of Statistics, Vol. 1* (London: Charles Griffin)

Koo, D. C., Kron, R. G. 1988, *Ap. J*, **325**, 92

Longair, M. S. and Scheuer, P. A. G. 1970, M.N.R.A.S., **151**, 45

Loh, E. D., and Spillar, E. J. 1986, *Ap. J. (Letters)*, **307**, L1

Lynden-Bell, D. 1971, sl M.N.R.A.S., **155**, 95

Lynds, R. and Wills, D. 1972 *Ap.J.*, **172**, 531

Malkan, M. A. 1983, *Ap. J.*, **268**, 582

Marshall, H. L. 1985, *Ap. J.*, **299**, 109

Marshall, H. L. 1987, *Astron. J.*. **94**, 628

Marshall, H. L. 1988, *Evolution of Low Luminosity Quasars*, to appear in PASP.

Marshall, H. L., Avni, Y., Braccesi, A., Huchra, J. P., Tananbaum, H., Zamorani, G., and Zitelli, V. 1984, *Ap. J.*, **283**, 50

Marshall, H. L., Avni, Y., Tananbaum, H., and Zamorani, G. 1983, *Ap. J.*, **269**, 35

Morisawa, K., Takahara, F. 1988, submitted to *Publications of the Astronomical Society of Japan*

Petrosian, V. 1973, *Ap. J.*, **183**, 359

Petrosian, V. 1986, in *Structure and Evolution of Active Galactic Nuclei*, ed. G. Giuricin, F. Mardirossian, M. Mezzetti, and M. Ramella (Dordrecht: Reidel), 353

Schmidt, M. 1968, *Ap. J.*, **151**, 393

Schmidt, M., and Green, R. F. 1978, *Ap. J.*, **220**, L1

Schmidt, M., and Green, R. F. 1983, *Ap. J.*, **269**, 352

Sheilds, G. A. 1977, *Nature*, **272**, 706

Tananbaum *et al.* 1979, *Ap. J.*, **234**, L9

Turner, E. L. 1979, *Ap.J.*, **231**, 645

Wandel, A., Petrosian, V. 1988, *Ap. J.*, **329**, L11

Weedman, D. W. 1986a, in *Structure and Evolution of Active Galactic Nuclei*, ed. G. Giuricin, F. Mardirossian, M. Mezzetti, and M. Ramella (Dordrecht: Reidel), 215

——— 1986b *Quasar Astronomy* (Cambridge University Press), 140

Weinberg, S. 1972, *Gravitation and Cosmology* (New York: John Wiley and sons), 415

Figure 1a. - Nonparametric cumulative luminosity functions, $\Phi(> \mathrm{L})$, from the data listed in Table 1 for redshift ranges $0 < z < 0.3$, $0.3 < z < 0.6$, $0.6 < z < 1.0$, $1.0 < z < 1.4$, $1.4 < z < 1.8$, $1.8 < z < 2.2$, from top to bottom, respectively ($\Omega = 1$, $\mathrm{H}_o = 100$ km s^{-1} Mpc^{-1}, $\Lambda = 0$). (Normalization is arbitrary and curves have been shifted vertically for clarity.) The luminosity evolution, $g(z)$, has been assumed constant corresponding to pure density evolution. The luminosity function clearly evolves with redshift from a single power law at low z to a double power law at high z, and the break luminosity also increases with z. This shows that PDE is not a consistent description of the data.

Figure 1b. - Same as Figure 1a, but assuming a luminosity evolution of the form $g(z) = (1+z)^{3.2}$. The shape of the luminosity function remains constant for $z > 0.6$, but changes at lower redshifts inconsistent with GLE. The variation of the luminosity function could be the result of either a true change of shape, or an incorrect choice of $g(z)$ or cosmological model. A more strongly evolving $g(z)$ may shift the low redshift luminosity functions toward higher luminosities, and the shape may then remain invariant.

Figure 1c. - Same as Figure 1a, but assuming a luminosity evolution of the form $g(z) = \exp(t/\tau)$ with t being the lookback time and $\tau = .73$ Gyr. Here the break has evolved below the observable region at low redshifts, indicating perhaps that the exponential luminosity evolution gives a better description of the data, but the slope of the luminosity function appears to flatten at low z, inconsistent with PLE or GLE.

Figure 2. - Nonparametric cumulative density functions for the data given in Table 1 ($\Omega = 1$, $\mathrm{H}_o = 100$ km s^{-1} Mpc^{-1}, $\Lambda = 0$, normalization is arbitrary). Dotted lines indicated expected behavior for constant density $\big(\sigma(\mathrm{V}) \propto \mathrm{V}\big)$ expected for PLE. Curve a – assuming $g(z) = const.$ a dramatic increase in the density of objects with redshift is observed. This is mainly the result of an increase in the average luminosity with redshift (see Figure 1a). Curve b – $g(z) = (1+z)^{3.2}$ gives a small increase in the density of objects with redshift. This could be the result of a true increase in density with z, or an incorrect $g(z)$ or cosmological model. Curve c – $g(z) = \exp(t/\tau)$ shows a decrease in density at $z \sim 0.6$ ($\mathrm{V}\sim 10^{1}0\mathrm{Mpc}^{3}$). Evolution in $\rho(z)$ indicates that PLE is inconsistent with the data for all three curves.

Figure 3. - Results of the method of moments (see text): Product of comoving density and comoving volume within a shell at redshift z, divided by z^2 (arbitrary normalization). Error bars indicate 95% confidence regions. Calculated values are independent of cosmological model and any assumptions about $g(z)$, but it is assumed that the shape of the luminosity

function does not vary with redshift. Solid lines indicate expected behavior assuming constant comoving density for the indicated values of Ω and $\Lambda = 0$. Dotted line is the $\Omega = 1$ curve normalized to fit the high redshift data. An increase in density at $z \sim 0.6$ is indicated, although it may be the result of a change in the shape of the luminosity function. PLE, which requires constant density and luminosity function shape, is shown to be inconsistent with the data for reasonable values of Ω.

Figure 4. - Results of method of moments: The ratio of the break luminosity, $L_*(z)$, to the square of the luminosity distance, $d_l(z)$ (arbitrary normalization). Error bars indicate 95% confidence regions. Values are independent of cosmological model and assumptions about $g(z)$, but require constant shape of the luminosity function. The constancy of this ratio with redshift is the result of the apparent coincidence that the break luminosity evolves essentially parallel to the flux limit of the surveys over the observed redshift range.

Figure 5. - Comoving density as a function of redshift from the data given in Figure 3 assuming various values of Ω. The comoving density, $\rho(z)$, may be considered constant, and the evolution consistent with PLE for $\Omega \sim 3$, but for reasonable values of Ω, the comoving density or the shape of the luminosity function must evolve.

Figure 6. - Evolution of the break luminosity, $L_*(z)$, from the data given in Figure 4, assuming the indicated values of Ω. Solid lines indicate expected behavior for $g(z) = (1+z)^{3.2}$ (upper curve) and $g(z) = \exp(t/.73)$ (lower curve). While the evolution of L_* can be described by either law for $z > 0.6$, only the exponential law fits the data over the entire redshift range. It may be hard to fit either law to the data when high values of $\Omega \sim 3$ are assumed.

ADDRESSES

David Caditz
Center for Space Science and Astrophysics
ERL 301
Stanford University
Stanford, CA 94305

Vahé Petrosian
Center for Space Science and Astrophysics
ERL 304
Stanford University
Stanford, CA 94305

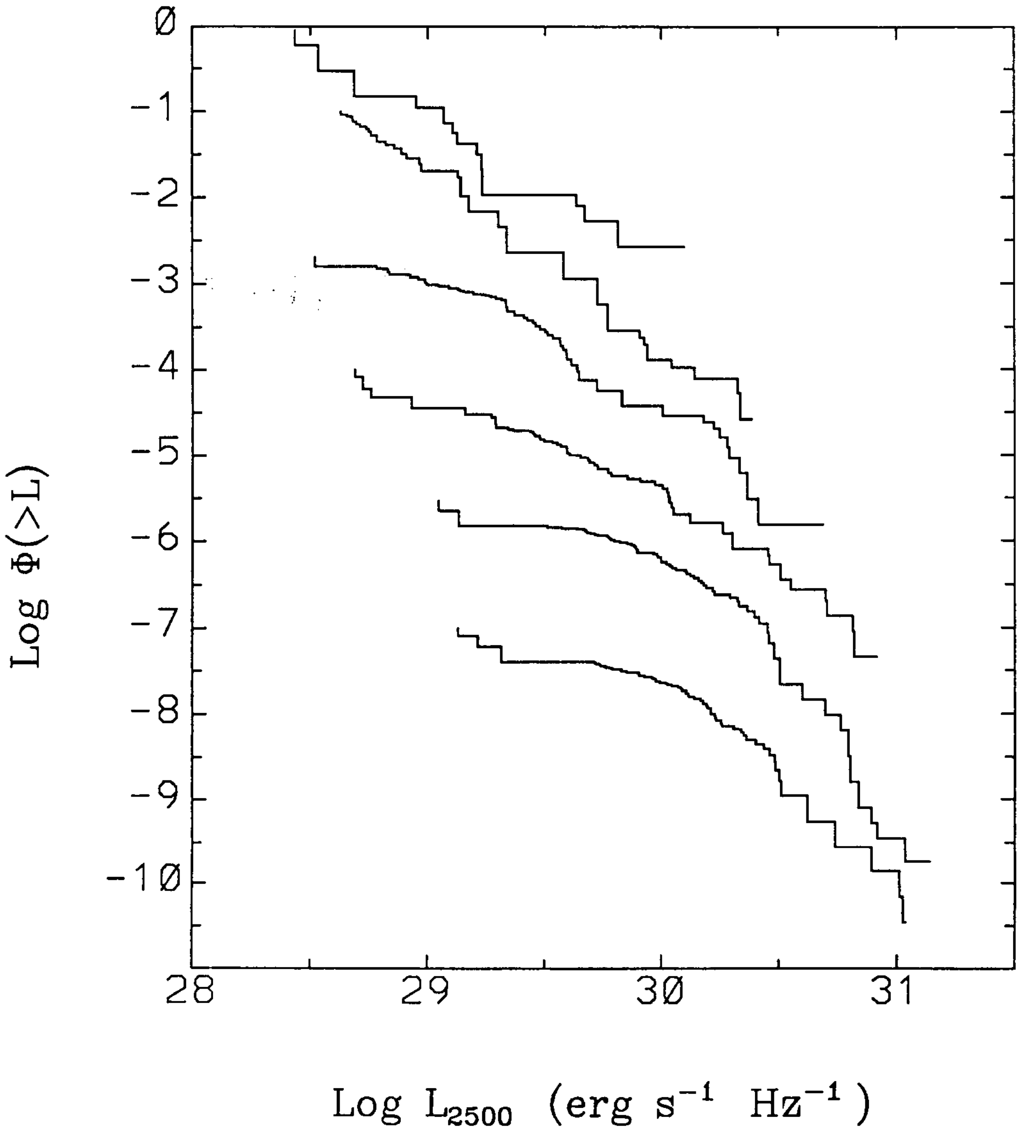

Figure 1a

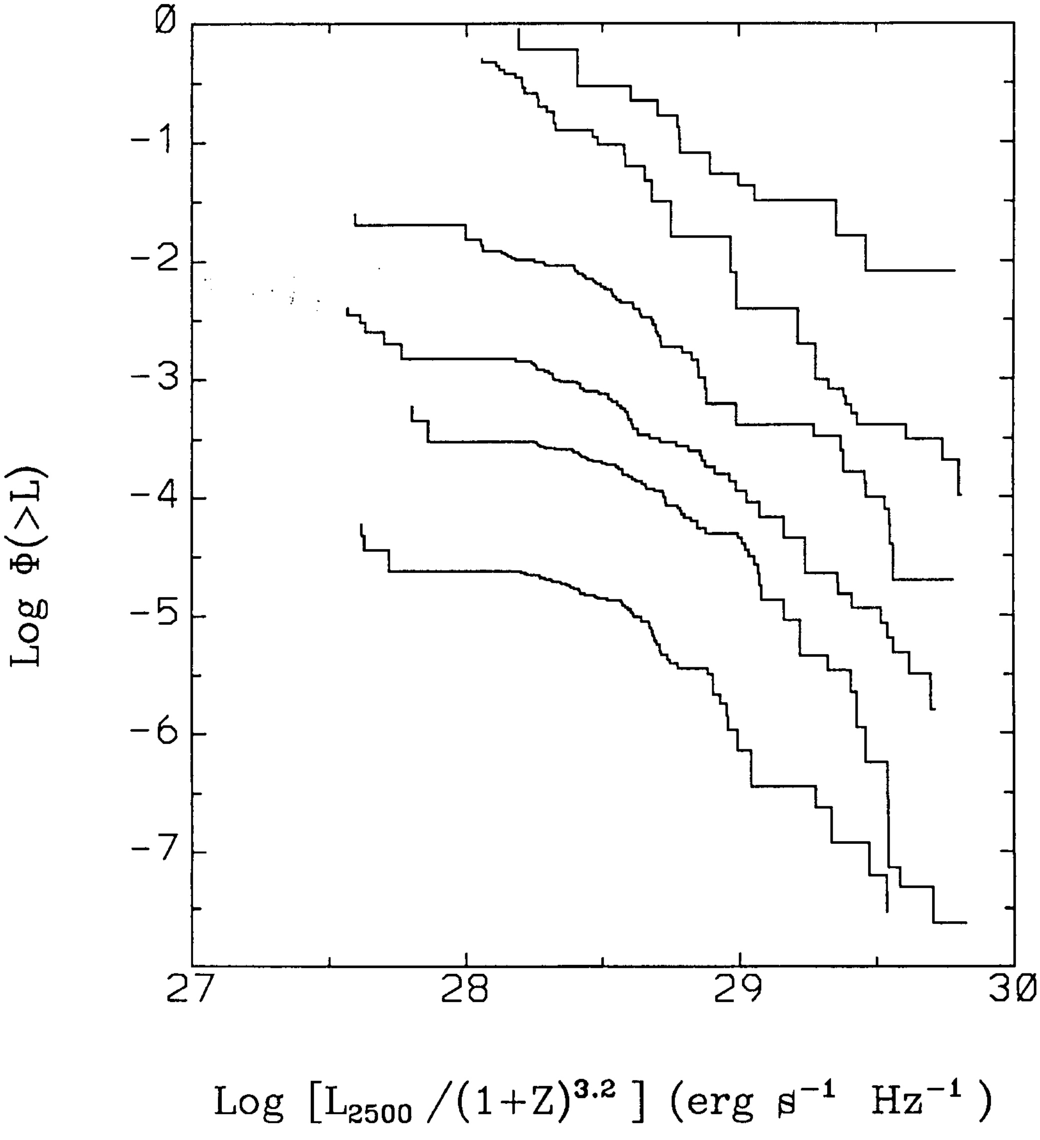

Figure 1b

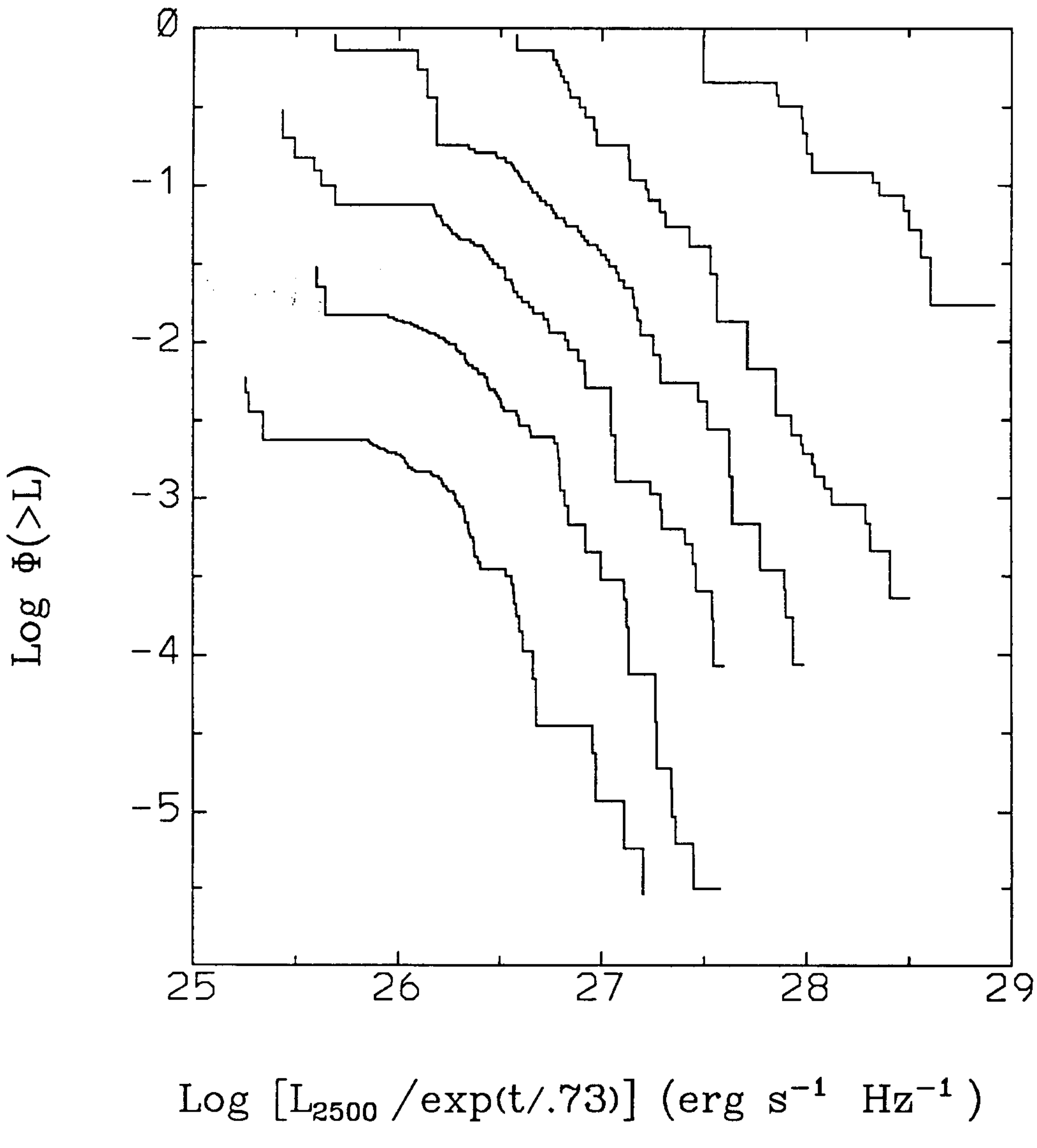

Figure 1c

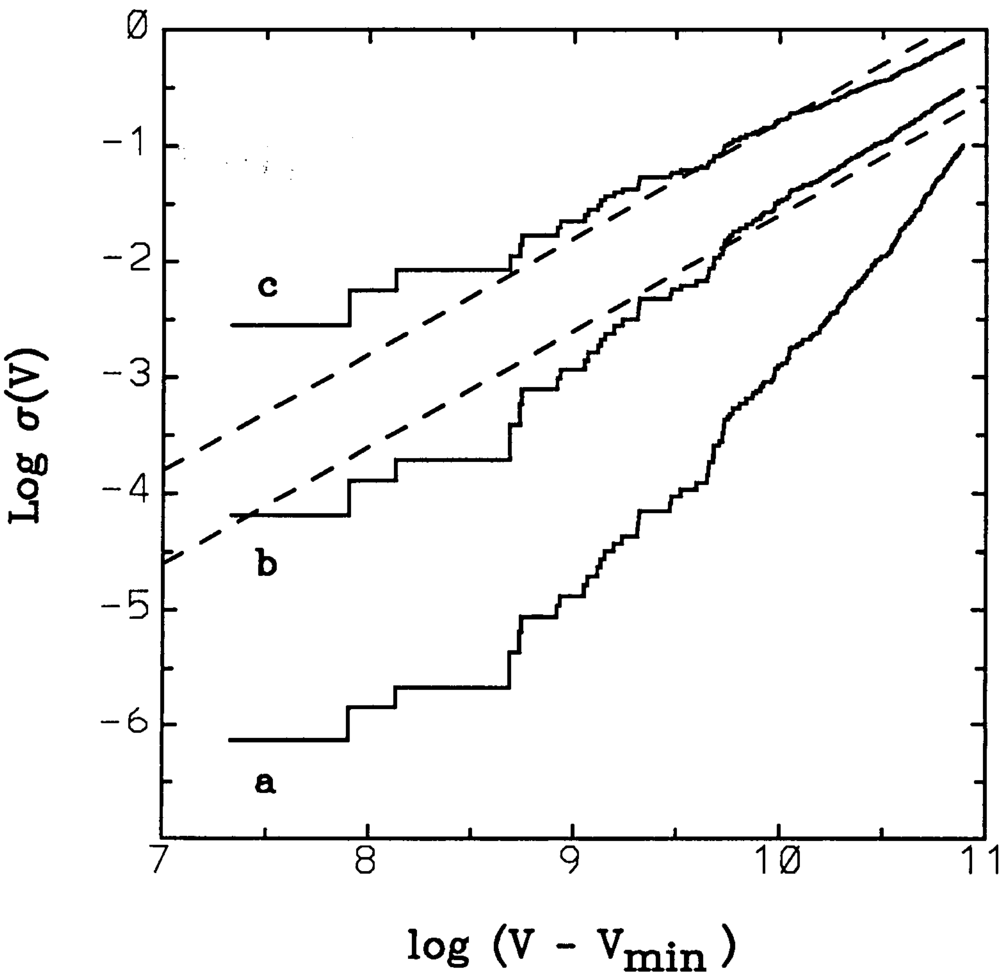

Figure 2

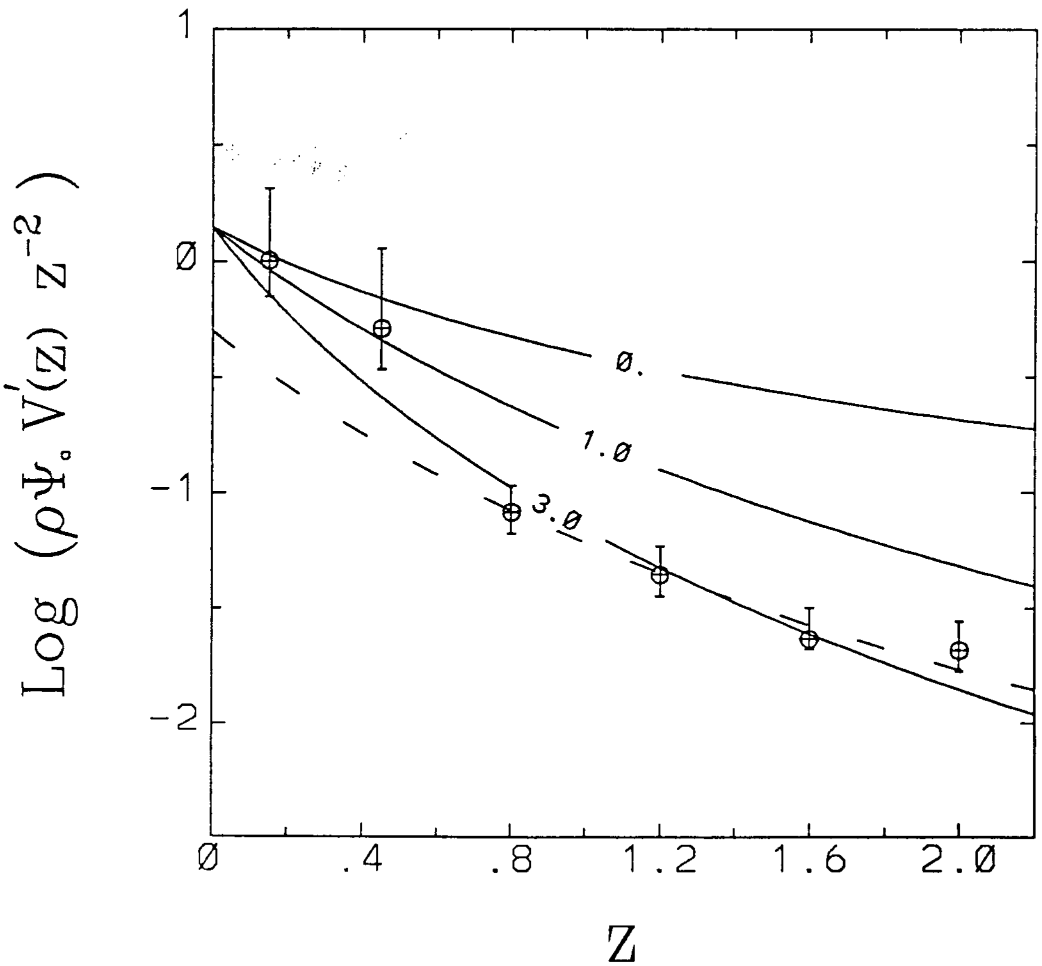

Figure 3

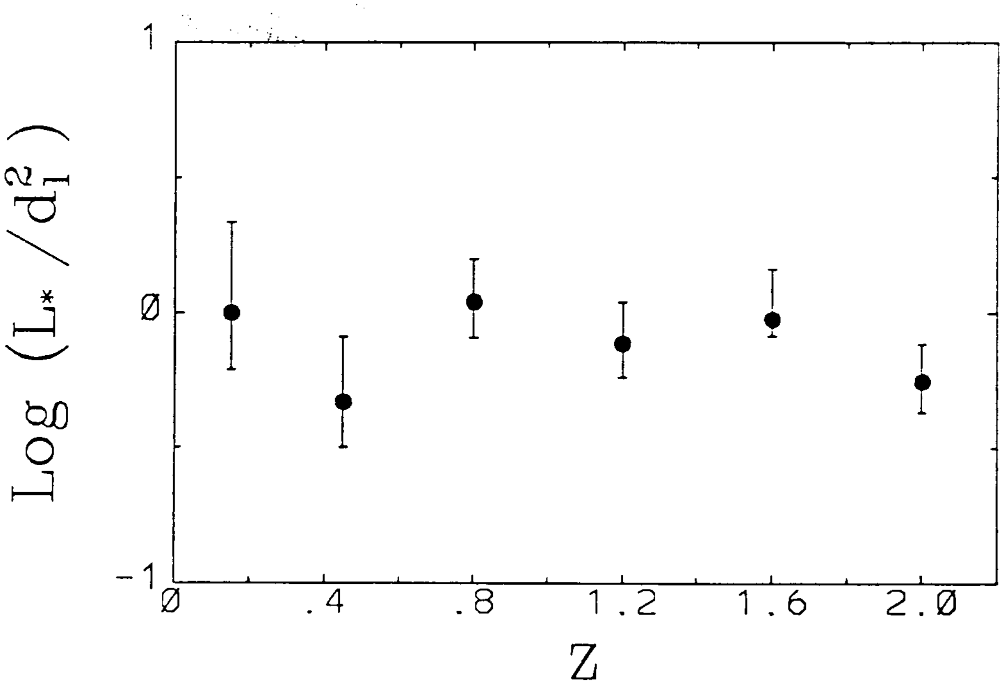

Figure 4

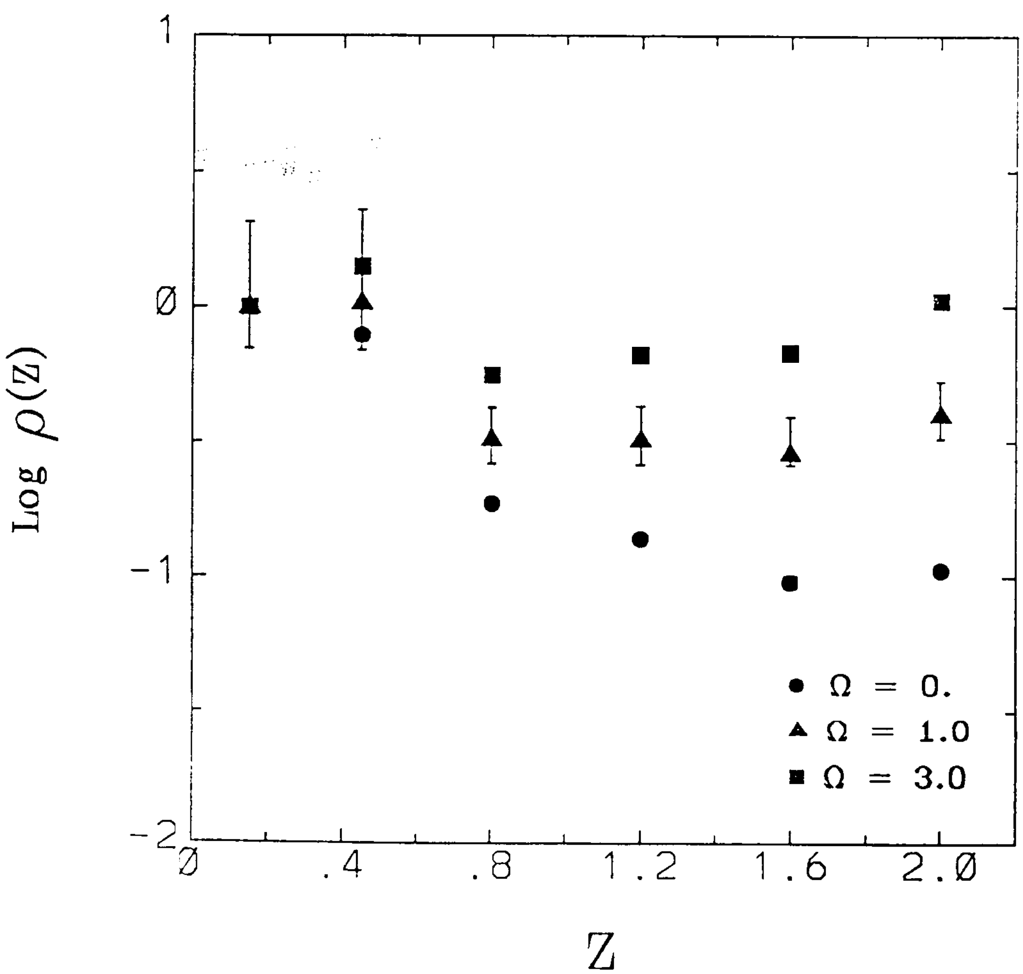

Figure 5

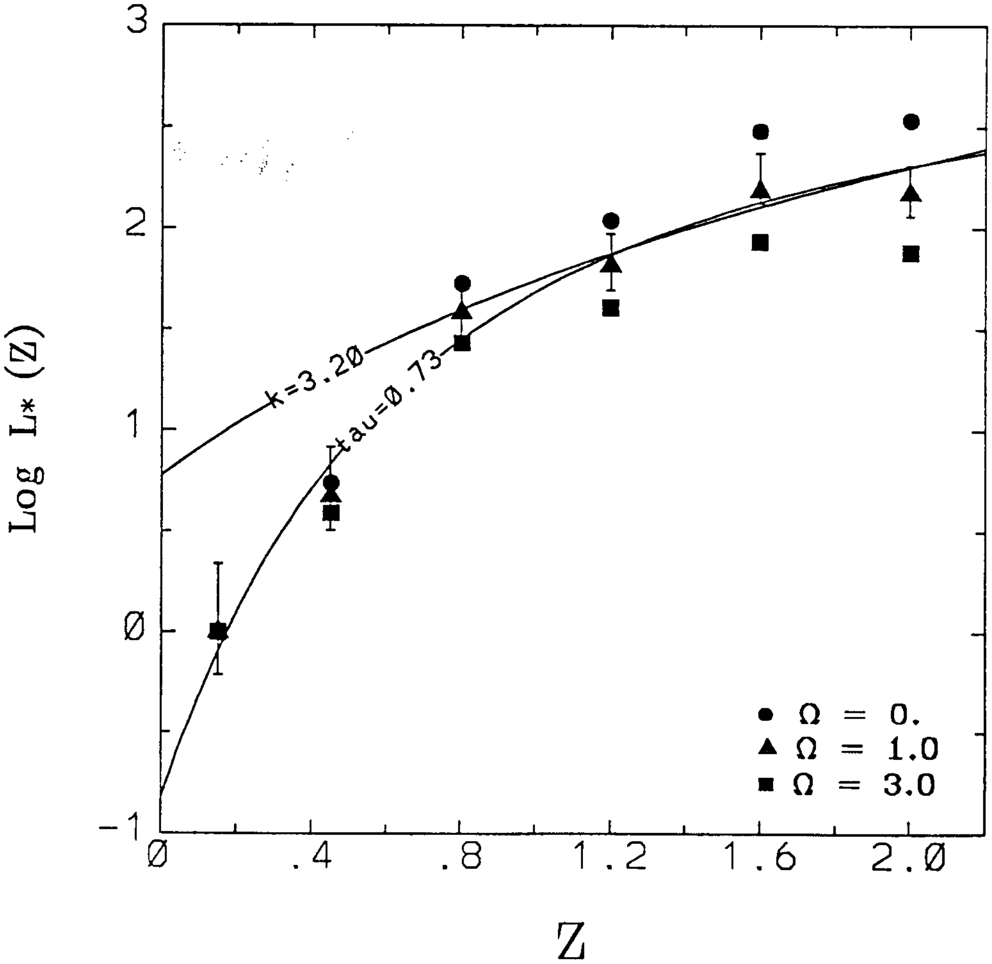

Figure 6

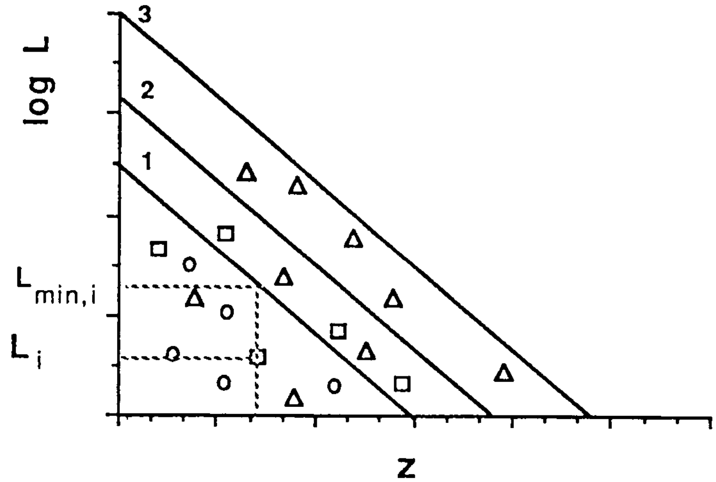

Figure A.1

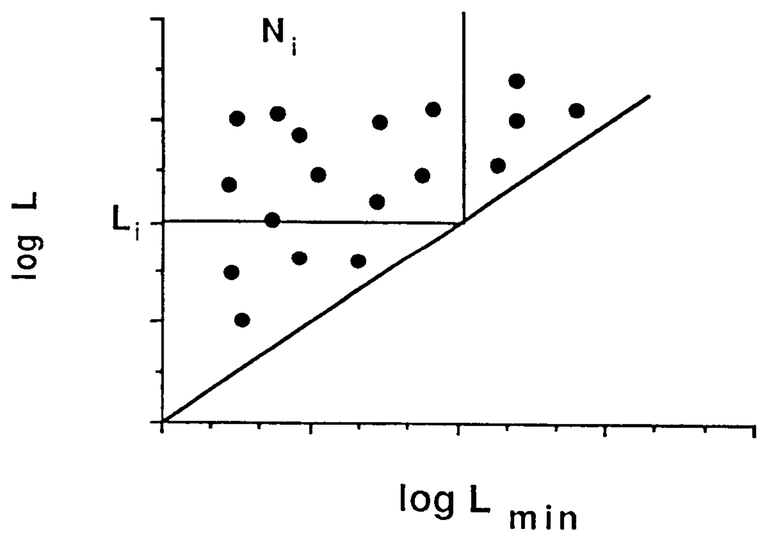

Figure A.2