ASSESSMENT OF SPAR ELEMENTS AND FORMULATION OF SOME BASIC 2-D AND 3-D ELEMENTS FOR USE WITH TESTBED GENERIC ELEMENT PROCESSOR

MOHAMMAD A. AMINPOUR+

INTRODUCTION

The initial CSM Testbed was based on Level 13 of the SPAR finite element computer program. Until recently, the element library of the Testbed has been limited to those elements in Level 13 of SPAR. The development of a generic element processor has enabled element researchers to develop, implement and assess element formulations with relative ease. An assessment of new elements as well as the existing SPAR Level 13 elements has revealed some definite shortcomings with the SPAR Level 13 2-D and 3-D elements. The SPAR S81 solid element does not pass the patch test problem proposed by MacNeal-Harder[1]. These deficiencies are identified ~~below.~~ The 2-D elements, however, seem to perform well taking into account the limitations imposed by the theory used to formulate them, (i.e., thin plates only). Common deficiencies of the 2-D and 3-D elements in SPAR have to do with their adaptability to the nonlinear analysis utilities developed by Lockheed Palo Alto Research Lab. Also, the "EFIL" format of the SPAR element data does not conform to the standard format of the Testbed. Accordingly, even if the deficiencies of the 3-D elements concerning their performance are corrected, their use will still be limited in the Testbed environment. Therefore, a new set of some basic 2-D and 3-D elements have been formulated to overcome the performance deficiencies as well as the deficiencies concerning their adaptability to nonlinear analysis utilities in the Testbed and the conformity of their EFIL to the standard format of the Testbed. These elements are formulated and implemented into the Testbed as two element processors (processors ES3 and ES4). Finally, the performance and accuracy of these elements are studied.

+Research Scientist, Analytical Services and Materials, Inc., Hampton, VA.

OBJECTIVES

- ASSESS SPAR ELEMENTS AND IDENTIFY DEFICIENCIES
- FORMULATE NEW 2-D AND 3-D ELEMENTS
  - STANDARD EFIL FORMAT
  - PROVIDE EASY USE OF NONLINEAR ANALYSIS UTILITIES
- UTILIZE GENERIC ELEMENT PROCESSOR
- ASSESS NEW ELEMENTS

OBJECTIVES

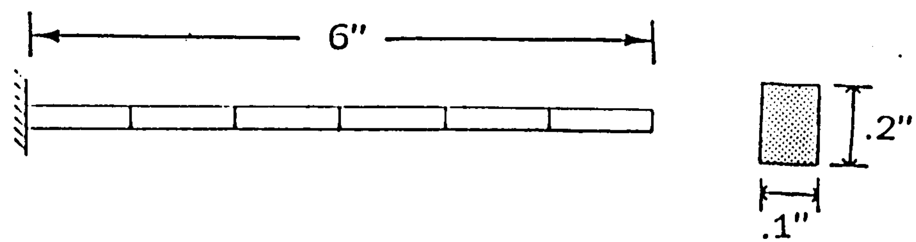
The initial objectives of this study were to assess the accuracy and performance of the 2-D and 3-D elements in SPAR Level 13 for linear stress analysis and to identify any deficiencies, and to modify the elements, if necessary. However, after studying the elements, it was decided to develop a new set of 2-D and 3-D elements in the Testbed using a different element formulation than the element formulation in SPAR. The elements in SPAR are based on the Pian hybrid stress formulation[2]. These new Testbed elements are developed based on the Reissner mixed formulation for reasons that will be discussed later. The new elements have a standard EFIL format, i.e., the EFIL format of the generic element processors developed by Lockheed Palo Alto. The new elements provide flexibility for finite element research and also provide flexibility for use of nonlinear analysis utilities developed by Lockheed Palo Alto. Finally, the performance and accuracy of new elements are assessed.

SPAR 1-D AND 2-D ELEMENTS RESULTS

MACNEAL-HARDER TEST CASE RESULTS

STRAIGHT CANTILEVERED BEAM

RECTANGULAR-ELEMENTS



Normalized Tip Displacement in Direction of Load †

Tip Loading Direction	SPAR element E21 (1-D)	SPAR element E43 (2-D)	QUAD 4 MSC/ NASTRAN
Extension	1.000	.996	.995
Inplane Shear	.998	.993	.904
Out of Plane Shear	.998	.986	.986
Twist	.938	.664	.941

† Exact solutions are given in Ref. 1.

SPAR 1-D AND 2-D ELEMENTS RESULTS

The test problem selected to verify the performance of the 1-D and 2-D elements in SPAR Level 13 is shown on the opposite page. This problem has been proposed by MacNeal-Harder[1] as a test case. It is a straight cantilever beam discretized into six finite elements. The beam is 6.0 in. long and 0.2 in. deep. The width of the beam is 0.1 in. These dimensions define a rather long beam and the discretization is quite coarse, making it difficult to achieve the correct solution. The beam is subjected to different types of tip loading, and the normalized displacements at the tip in the direction of the load are tabulated in the table shown on the opposite page. The finite element solutions are normalized by the exact solutions as presented in Ref. 1. The SPAR E21 beam element achieves good results with a maximum error of 6.2% for the case of twist (torsion) loading. In the case where the SPAR E43 flat shell element (class C^1 continuity) is used, the results are all satisfactory except for the case of twist. The reason for this discrepancy is that the SPAR 2-D elements are formulated according to Kirchhoff's thin plate theory in which the effect of transverse shear deformations are ignored. It is observed that this is a thick plate, the thickness being half the height of the beam. When the beam is loaded in bending the effect of shear deformation is very small (negligible) because of the long length of the beam as compared with the other dimensions. However, when the beam is loaded in torsion, the effect of the transverse shear deformation becomes important for a thick plate, like the one considered here. Therefore, the SPAR E43 element performed as expected, i.e., to perform well under bending type of loading and not nearly as well under torsion type of loading for this problem. Results using another 4-node quadrilateral element, namely the MSC/NASTRAN QUAD4 element are also presented in the table for comparison. Using the QUAD4 element, all results have a maximum error of less than 10%. These results are from Ref. 1, which also includes results using other MSC/NASTRAN elements.

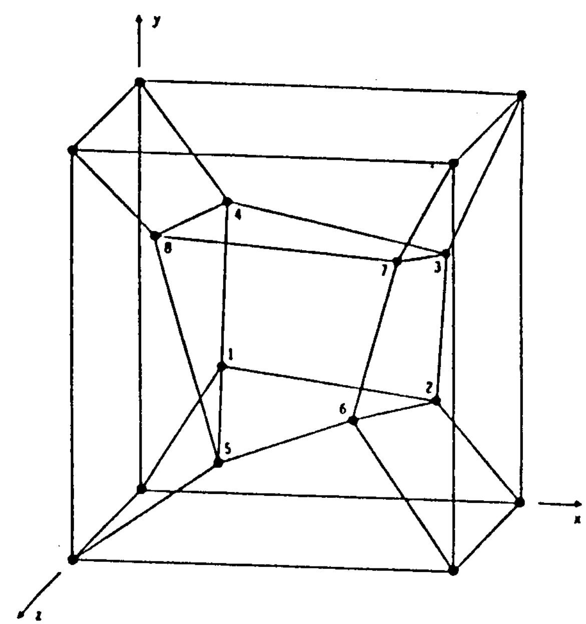
TEST PROBLEM FOR SPAR S81 ELEMENT

APPLIED DISPLACEMENTS

$$u = \frac{1}{2}(2x + y + z) \times 10^{-3}$$

$$v = \frac{1}{2}(x + 2y + z) \times 10^{-3}$$

$$w = \frac{1}{2}(x + y + 2z) \times 10^{-3}$$



$$E = 10^6$$

$$\nu = .25$$

RESULTING STRAINS AND STRESSES

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = 10^{-3}$$

$$\gamma_{xy} = \gamma_{yz} = \gamma_{xz} = 10^{-3}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 2000$$

$$\sigma_{xy} = \sigma_{yz} = \sigma_{xz} = 400$$

TEST PROBLEM FOR SPAR S81 ELEMENT

The test problem selected to verify the performance of the 3-D elements in SPAR Level 13 is shown on the opposite page. This problem has been proposed by MacNeal-Harder[1] as a patch test for solid elements. The problem is composed of seven hexahedron elements forming a unit cube. All the faces of the elements, except for the faces forming the external boundary, are warped (i.e., the four points of a face do not lie in one plane). The locations of the internal points are given in Ref. 1. The suggested displacement boundary conditions on the external boundary are stated on the opposite page. This set of displacement boundary conditions results in a constant strain loading condition with the resulting stresses as shown. The SPAR S81 solid element was used to solve this problem. Unfortunately the computer run failed to complete because the strain energy matrix turned out to be singular, at least, for some of the elements. When the faces of the middle hexahedron were made flat so that no warping existed on the internal element faces, the fate of the computer run did not change. Therefore, investigations were conducted regarding the formulation and implementation of the SPAR S81 solid element.

ASSUMED STRESS HYBRID FORMULATION

$$\Pi = \frac{1}{2} \int_V \sigma^T D^{-1} \sigma \, dv - \int_S (n^T \sigma)^T u \, ds + \int_{S_T} u^T t_o \, ds$$

ASSUMPTIONS: Equilibrium is satisfied
strains derived from stresses
continuous displacements

$$T = \int_S (n^T \sigma)^T u \, ds = \int_V \sigma^T L[u] \, dv + \underbrace{\int_V \left(L^T[\sigma]\right)^T u \, dv}_{= \, 0}$$

ASSUMED STRESS HYBRID FORMULATION

The SPAR finite element formulation is based on the principal of minimum complementary energy. The formulation is implemented in accordance with the Pian assumed stress hybrid formulation[2], the functional for which is given on the opposite page[3,4]. For this functional to be valid, the assumptions stated on the opposite page must hold true when choosing the form of the stresses, strains, and displacements[4]. The assumed stresses are treated as the field variables and the displacements are treated as the interface (hybrid) variables on the boundary of the elements. Furthermore, the assumed displacements must satisfy the displacement boundary conditions[4]. When these assumptions hold true, the minimization of the functional recovers, in an integral sense, the strain-displacement relationship, forces the tractions to be continuous across the boundaries of adjacent elements, and satisfies the traction boundary conditions[4]. The second integral in the functional, denoted by T, at the bottom of the opposite page is a surface integral representing the work done on the boundaries of the elements. This surface integral can be written in terms of two volume integrals using Gauss's Theorem which is often referred to as integration by parts. As denoted on the opposite page, the second volume integral vanishes for the Pian formulation because the stresses are chosen so that equilibrium is satisfied, i.e., $L^t[\sigma]=0$. Therefore, only the first volume integral needs to be evaluated for each element. This approach, in general, is more efficient than evaluating six surface integrals (one integral per face) for each element. However, the investigations revealed that SPAR Level 13 evaluates the surface integral. Furthermore, it was found that SPAR Level 13 evaluates the unit normal to each surface only once, i.e., SPAR assumes that the surfaces of the elements are flat planes and no warping exists. This feature may explain in part the reason for the failure of the SPAR S81 element for the problem with warped faces. Further studies revealed that the assumed displacements on the boundaries (faces) of the adjacent elements were not continuous, therefore, leaving "gaps" between the adjacent elements and violating the third assumption stated on the opposite page. This explains the failure of the SPAR S81 element for the test problem with flat faces. However, the assumed displacement functions for the S81 element are continuous across the element boundaries (faces) only when the elements are rectangular parallelopiped, and the more the deviation from a rectangular parallelopiped, the larger the error. Therefore, for slight distortions, say 10 degrees, the SPAR S81 element will work, but for MacNeal-Harder problem with distortions of the order 45 degrees the SPAR S81 element fails to work.

DEFICIENCIES NOTED FOR SPAR ELEMENTS

- INTERELEMENT DISPLACEMENT CONTINUITY REQUIREMENT VIOLATED
- SENSITIVE TO OUT-OF-PLANE WARPING
- DIFFICULT TO COUPLE WITH GENERIC ELEMENT PROCESSOR
- DIFFICULT TO COUPLE WITH NONLINEAR ANALYSIS UTILITIES
- COMPUTATIONALLY INEFFICIENT
- LACK OF IN-LINE DOCUMENTATION; DIFFICULT TO MAINTAIN OR UPGRADE
- NONSTANDARD EFIL FORMAT

DEFICIENCIES NOTED FOR SPAR ELEMENTS

The first two deficiencies listed on the opposite page are peculiar to the SPAR solid elements S81 and S61 and have already been discussed. The other deficiencies listed are common to both 2-D and 3-D elements in SPAR. These deficiencies are self explanatory and were discussed, to a limited extent, in the introduction.

PROPOSED SOLUTION TO DEFICIENCIES

- DEVELOP NEW 2-D AND 3-D ELEMENTS
  - NEW FORMULATION (REISSNER MIXED METHOD).
  - IMPROVED COMPUTATIONAL EFFICIENCY
  - EASY TO FOLLOW AND MAINTAIN FOR ELEMENT RESEARCH
- COMPATIBILITY WITH GENERIC ELEMENT PROCESSOR
- STANDARD EFIL FORMAT
- PROVIDES FOR NONLINEAR ANALYSIS CAPABILITY

PROPOSED SOLUTION TO DEFICIENCIES

It is desirable to modify the SPAR element architecture to overcome the deficiencies regarding their compatibility with generic element processor, standard EFIL format, and their easy adaptability for use with nonlinear utilities. It is desirable that the solid elements not be limited for use as rectangular parallelopiped elements and be able to perform well for elements with warped faces. With these considerations in mind, some basic 2-D and 3-D elements have been formulated based on the Reissner mixed method functional so that the above objectives are accomplished. Also, the new implementation would be computationally more efficient and easy to follow, maintain, and upgrade for future element research.

PRESENT FORMULATION BASED ON

REISSNER MIXED METHOD

$$\Pi = \frac{1}{2} \int_V \sigma^T D^{-1} \sigma \, dv - \int_V \sigma^T L[u] \, dv + \int_{S_T} u^T t_o \, ds$$

ASSUMPTIONS: Strains derived from stresses

Continuous displacements

PRESENT FORMULATION BASED ON REISSNER MIXED METHOD

As discussed earlier, in order to reach the stated objectives, some basic 2-D and 3-D elements are formulated using the Reissner mixed method. The functional for the Reissner mixed method, along with assumptions to make this functional valid, are given on the opposite page[4]. Two assumed independent field variables are in this formulation. One field is either the stresses or the strains, and the other field is the displacements. The assumed displacements must also satisfy the displacement boundary conditions[4]. The minimization of this functional recovers, in an integral sense, the equations of equilibrium, strain-displacement relations, forces the tractions to be continuous across the boundaries of adjacent elements, and finally satisfies the traction boundary conditions[4]. A quick look at the Pian hybrid stress formulation and the accompanied discussion reveals that had the surface integral, which was denoted by T, been replaced with its equivalent volume integral and had the displacements been treated as field variables instead of interface variables, the present formulation would have been obtained. When the assumed stresses in the present functional are chosen to satisfy the equilibrium equations, this functional will, as it must, reduce to that of the principal of minimum complementary energy, which is the basis for the hybrid stress formulation. The present formulation based on Reissner mixed method has certain advantages which will be discussed next.

ADVANTAGES OF REISSNER MIXED METHOD

- IF STRESS FUNCTIONS CHOSEN SATISFY EQUILIBRIUM, THEN FORMULATION REDUCES TO PIAN HYBRID-STRESS FORMULATION

- SINCE ASSUMED FIELD DOES NOT HAVE TO SATISFY EQUILIBRIUM, THEN FORMULATION MAY:
  - ASSUME STRESS OR STRAIN FIELDS
  - EXPRESS ASSUMED FIELD IN CARTESIAN OR NATURAL COORDINATES

- THIS FREEDOM OF CHOICE IS USEFUL FOR ELEMENT RESEARCH AND NONLINEAR ANALYSIS

ADVANTAGES OF REISSNER MIXED METHOD

One of the merits of this method is the one already discussed, i.e., if the assumed stresses are chosen so as to satisfy the equilibrium equations then the formulation will, as it must, reduce to the hybrid stress formulation. The other merit of this formulation is that since the assumed stresses now do not have to satisfy the equilibrium equations, one has more freedom in assuming the form of the stress functions. For example, strain functions can be assumed and the stresses can be calculated from the assumed strains. In general, these stresses will not satisfy the equilibrium equations, but in this formulation we have the freedom to do so. This freedom to assume the strains rather than the stresses is useful for material nonlinear analysis, e.g., plasticity. In addition, Pian points out[5] that for deformed elements it would be better if one chooses the stresses (or strains) in Lagrangian natural coordinates rather than the rectangular Cartesian coordinates as this will eliminate the directional bias in assuming the form of the stress or strain functions. Therefore, this implementation provides the freedom to choose the stresses (or strains) in either coordinates and to study their relative performance.

NEW ELEMENTS IN THE TESTBED

- SOLID ELEMENTS (PROCESSOR ES3)

8-NODE HEXAHEDRON	20-NODE HEXAHEDRON
6-NODE PENTAHEDRON	15-NODE PENTAHEDRON
4-NODE TETRAHEDRON	10-NODE TETRAHEDRON

- FLAT SHELL ELEMENTS (PROCESSOR ES4)

4-NODE MEMBRANE	3-NODE MEMBRANE
4-NODE PLATE (BENDING)	3-NODE PLATE (BENDING)
4-NODE MEMBRANE & BENDING	3-NODE MEMBRANE & BENDING
4-NODE SHEAR PANEL	3-NODE SHEAR PANEL

- ALL ELEMENTS ARE OF CLASS C^o CONTINUITY

NEW ELEMENTS IN THE TESTBED

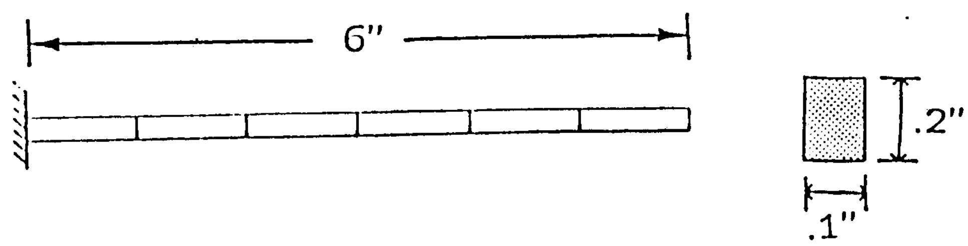
Based on the formulation just described, six solid elements and eight flat shell elements were implemented into the Testbed. The elements are all of class C^0 continuity. The 2-D elements account for the effects of transverse shear deformation, so that problems involving moderately thick plates and laminated composite plates, for which $G_{12}/E_1 < 1/10$, can be solved. This feature is in contrast to the SPAR 2-D elements which are of class C^1 continuity and do not account for the effects of transverse shear deformation, which limits their use to thin plates only. These new elements are implemented in the Testbed as two new processors, processors ES3 and ES4. Processor ES3 contains the six solid elements listed on the opposite page. Processor ES4 contains the eight 2-D flat shell elements listed on the opposite page. These new elements resolve the deficiencies noted for the elements in SPAR Level 13 and provide equivalent or greater capability than the elements they replace.

SHELL ELEMENTS (PROCESSOR ES4) RESULTS

MACNEAL-HARDER TEST CASE RESULTS

STRAIGHT CANTILEVERED BEAM

RECTANGULAR-ELEMENTS



NORMALIZED TIP DISPLACEMENT IN DIRECTION OF LOAD

Tip Loading Direction	Assumed Displ.	CARTESIAN COORDINATES Assumed Stress	CARTESIAN COORDINATES Assumed Strain	NATURAL COORDINATES Assumed Stress	NATURAL COORDINATES Assumed Strain	QUAD4 MSC/ NASTRAN
Extension	.995	.996	.995	.996	.995	.995
Inplane Shear	.0933	.993	.904	.993	.904	.904
Out of Plane Shear	.0302	.980	.980	.980	.980	.986
Twist	.932	.941	.940	1.089	.940	.941
Inplane Moment	.0933	1.000	.910	1.000	.910	—

SHELL ELEMENTS (PROCESSOR ES4) RESULTS

One problem selected to assess the new flat shell elements implemented in Testbed processor ES4 is the MacNeal-Harder[1] cantilever test problem discussed earlier. The solution to this problem is obtained using all the available options in the processor ES4 and the results are tabulated on the opposite page. The results from a conventional isoparametric assumed displacement finite element formulation (class C^0 continuity, also implemented in processor ES4 as an option) performs well for extension, relatively well for torsion, and very poorly for the other types of loading. The performance of the displacement based element is related to the assumed field. Deformation due to extension is represented by the assumed bilinear displacement shape functions exactly, and therefore even one element (instead of six) would provide an accurate result. For the case of deformation due to torsion, the assumed bilinear displacement shape functions represent a fairly good approximation to the deformation, and therefore even with this coarse mesh the result is satisfactory with 6.2% error. On the other hand, deformation due to out-of-plane and in-plane shear varies cubically along the length of the beam and the deformation due to in-plane moment varies quadratically along the length of the beam. Therefore, the assumed bilinear displacement shape functions cannot represent the deformation unless the mesh is refined considerably. The results from MSC/NASTRAN QUAD4 element are reproduced from Ref. 1 for comparison with the current ES4 results using 4-node quadrilateral elements. The QUAD4 element is also an isoparametric element but in addition it employs selective reduced order integration. To account for transverse shear deformations, the QUAD4 element uses string-net approximation and augmented shear flexibility[6]. The results from the implementation of the Reissner mixed method in Testbed processor ES4 (class C^0 continuity) are obtained by assuming the stresses in Cartesian coordinates, the strains in Cartesian coordinates, the stresses in natural coordinates and the strains in natural coordinates. The results are all within a maximum error of less than 10%. Satisfactory performance of these elements with such a coarse mesh is only achieved with rectangular elements. If the elements are distorted, the mesh has to be refined considerably to achieve comparable results (except for the case of extension). For a distorted mesh, the SPAR E43 flat shell elements perform poorly for membrane loading case (except for extension). The E43 element with a distorted mesh has approximately the same performance as the undistorted mesh for bending and torsion loading cases. These results are obtained since the formulation of the SPAR 2-D element for the in-plane deformation is of class C^0 continuity while for the out-of-plane deformation is of class C^1 continuity. However, as mentioned before, class C^1 has the limitation of having satisfactory performance only for thin plates.

SOLID ELEMENTS (PROCESSOR ES3) RESULTS

MACNEAL - HARDER PATCH TEST FOR SOLIDS

STRESS DISTRIBUTION

Element Type	CARTESIAN COORDINATES		NATURAL COORDINATES	
	Assumed Stress*	Assumed Strain	Assumed Stress	Assumed Strain
8-node	exact	exact	exact	exact
20-node	exact	exact	exact	exact

* assumed stresses satisfy equilibrium within each element.

C - 4

SOLID ELEMENTS (PROCESSOR ES3) RESULTS

The first test problem selected to assess the solid elements developed in the Testbed processor ES3 is the MacNeal-Harder[1] proposed patch test problem for solid elements discussed earlier. Both the 8-node and the 20-node hexahedron elements were used to obtain the solution to this problem. Also all the options available in the processor were used to obtain the solution. The stress functions chosen for these elements are the ones given in Ref. 4 except for the correction of typographical errors. The assumed stresses expressed in Cartesian coordinates satisfy the equilibrium equations, while the assumed strains in Cartesian coordinates, the assumed stresses and the assumed strains in Lagrangian natural coordinates do not satisfy the equilibrium equations. It is worth mentioning again that all the faces (except the ones representing the external boundary) of the elements are warped (i.e., the four corner points of a face do not lie in one plane). In addition, the midside nodes of the 20-node elements were chosen so as to make all the faces (except the ones representing the external boundary) of the elements to be curved surfaces. Therefore, the 8-node and the 20-node elements are being tested to their full capacity. This test would make the hidden errors more likely to show up. For all cases however, the exact theoretical stress distribution was recovered.

ANOTHER TEST PROBLEM FOR SOLID ELEMENTS

APPLIED STRESSES

$$\sigma_{xx} = +\ 1000x$$

$$\sigma_{xy} = -\ 1000y$$

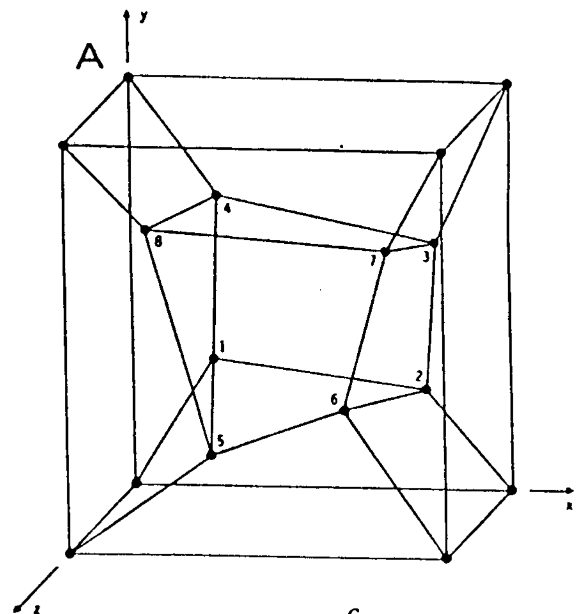
$$\sigma_{yy} = \sigma_{zz} = \sigma_{yz} = \sigma_{xz} = 0$$

RESULTING DISPLACEMENTS

$$u = \frac{1}{2}\left(x^2 - 2.25y^2 + .25z^2\right) \times 10^{-3}$$

$$v = -.25xy \times 10^{-3}$$

$$w = -.25xz \times 10^{-3}$$



$E = 10^6$

$\nu = .25$

ANOTHER TEST PROBLEM FOR SOLID ELEMENTS

As a second test to assess the performance of the solid elements in processor ES3 of the Testbed, the same mesh was chosen with a different set of boundary conditions. The tractions applied to the boundaries of the solid and the resulting theoretical displacements are stated on the opposite page. The maximum displacement will occur at point A (x=z=0 y=1) in the X-direction. This problem is different from the previous one in that it is not a constant strain problem. In fact, this problem presents a very severe boundary condition for the 8-node element. For this element, the assumed σ_{xx} is a function of a local element coordinate system y and z only[7], while the assumed σ_{xy} is a function of z only[7], of the local coordinate system. In other words, the assumed stress functions for the 8-node element do not represent the traction boundary conditions applied to this problem. Therefore, the 8-node element is not expected to have a superb performance for this problem, unless the mesh is refined. On the other hand, the 20-node element is expected to have a satisfactory performance because the assumed σ_{xx} and σ_{xy} functions[7] do represent the traction boundary conditions applied to this problem.

SOLID ELEMENTS (PROCESSOR ES3) RESULTS

NORMALIZED DISPLACEMENT AT POINT A IN THE x DIRECTION

Element Type	CARTESIAN COORDINATES		NATURAL COORDINATES	
	Assumed Stress*	Assumed Strain	Assumed Stress	Assumed Strain
8-node	.746	.760	.749	.763
20-node	1.005	1.027	1.008	1.066

* assumed stresses satisfy equilibrium within each element.

SOLID ELEMENTS (PROCESSOR ES3) RESULTS

From the resulting displacement functions on the previous page, it is deduced that maximum displacement will occur at point A in the X-direction and has a value of -1.125×10^{-3}. From the table on the opposite page, the 8-node element performs about the same (in error by about 25%) regardless of the assumed field and the coordinate system. However, following the discussion on the previous page, this is to be expected. On the other hand, the 20-node element has a much better performance as expected. A maximum error of 6.6% is observed for the case where the strain field is assumed and expressed in the natural coordinates, while the best performance (.5% error) is attributed to the case where the stress field is assumed and expressed in the Cartesian coordinates to satisfy the equilibrium equations.

SUMMARY

- NEW SOLID AND SHELL ELEMENT FAMILIES
- ELEMENT FORMULATION BASED ON REISSNER MIXED METHOD
- NEW ELEMENTS IMPLEMENTED AS NEW PROCESSORS ES3 (SOLID ELEMENTS) AND ES4 (SHELL ELEMENTS)
- STANDARD EFIL FORMAT FOR ALL ELEMENTS
- NEW ELEMENTS PROVIDE FLEXIBILITY FOR FINITE ELEMENT RESEARCH

SUMMARY

A family of solid elements and a family of flat shell elements have been developed and implemented in the Testbed. They are all of class C^0 continuity. The solid elements are implemented in the Testbed as processor ES3 and the flat shell elements are implemented as processor ES4. The formulation of these elements is based on the Reissner mixed method and the user has the option of choosing either assumed stress or assumed strain field each of which can be expressed in the Cartesian or the Lagrangian coordinates. These elements are implemented in conjunction with the Testbed generic element processor developed by Lockheed Palo Alto and have a standard EFIL format. These elements provide flexibility for the use of the nonlinear analysis utilities developed by Lockheed Palo Alto. These elements also provide flexibility for finite element research in that different functions can be assumed for the field variables (stress or strain) and their performance be studied. This capability is possible, because in this formulation, the assumed field (stress or strain) is not required to satisfy the equilibrium equations.

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