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BOUNDARY ELEMENTS
FOR
STRUCTURAL ANALYSIS

Work carried ~~out in part~~
Under Contract NAS3-23697
NASA Program Manager ~~– C. C.~~ Chamis

Pratt & Whitney
Commercial Engine Business

This talk is intended to discuss the status of the boundary element method (BEM) for structural analysis, both in terms of the present and anticipated capabilities of the method and in terms of the incorporation of the method in the design/analysis process, particularly for gas turbine engine components.

The three-dimensional work discussed was carried out largely under the support of a National Aeronautics and Space Administration contract (NAS3-23697, "3D Inelastic Analysis Methods for Hot Section Components") with Dr. C. C. Chamis of NASA-Lewis Research Center as program manager. The BEST3D (Boundary Element Stress Technology - 3-Dimensional) code was developed jointly by Pratt & Whitney (R. B. Wilson and N. M. Miller) and the Civil Engineering Department of the State University of New York at Buffalo (P. K. Banerjee, S. Ahmad, D. Henry, G. Dargush, S. Raveendra). The two-dimensional work discussed represents a long period of development at Pratt & Whitney, originating in the work of Dr. T. A. Cruse and D. W. Snow in the mid-1970's, and more recently including the participation of Dr. Banerjee and his colleagues.

TOPICS

- Current 2D/axisymmetric capabilities
- BEST3D development effort
- Selected BEST3D analyses
- BEM incorporation in design/analysis process

It is not possible, in a brief talk, to give a comprehensive review of the BEM. After an extremely short review of the basis of the method, the talk will focus on the topics listed on this slide.

REVIEW

FOR A HOMOGENEOUS ELASTIC STRUCTURE

$$\mu\frac{\partial^2 u_i}{\partial x_j^2} + (\lambda + \mu)\frac{\partial^2 u_j}{\partial x_i \partial x_j} + f = 0$$

CAN BE MANIPULATED USING:

- Reciprocal Work Theorem
- Point Force (Kelvin) Solutions
- Limiting Operations

TO OBTAIN

The BEM is based on the application of the Reciprocal Work Theorem with (for isotropic materials) the Kelvin point load solution and its derived traction solution integrated against the desired boundary displacements and tractions. Suitable limiting operations as the point of load application is moved to the boundary allow derivation of the boundary integral equation (BIE), a boundary constraint equation relating the surface displacements and tractions for any well-posed elasticity problem. Since this (singular) integral equation can not generally be solved in closed form, the practical application of the BEM is based on the solution of a numerical approximation to this equation. In currently available general purpose codes it is usual to model both the part geometry and the displacement and traction variation using isoparametric interpolation functions.

1. **INTERIOR DISPLACEMENT EQUATION**

$$\begin{aligned} u_j(\xi) &= \int_S (t_i(x) G_{ij}(x,\xi) - F_{ij}(x,\xi) u_i)\, dS \\ &\quad + \int_V G_{ij}(x,\xi) f_i(x)\, dV \end{aligned}$$

2. **BOUNDARY INTEGRAL EQUATION**

$$\begin{aligned} (\delta_{ij} - c_{ij}) u_j(\xi_0) &= \int_S (t_i(x) G_{ij}(x,\xi_0) - F_{ij}(x,\xi_0) u_i(x))\, dS \\ &\quad + \int_V G_{ij}(x,\xi_0) f_i(x)\, dV \end{aligned}$$

3. **INTERIOR STRESS (STRAIN) EQUATION**

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	F/E	BEM
Primitive Variables	Displacements only	Displacements and tractions
Geometry Approximation	Throughout structure	Surface only
Dominant Cost	Equation solution	Surface integration

The computing and modelling characteristics of the BEM are fundamentally different from those of the finite element method. For elastic analysis the BEM models both displacements and tractions, on the surface of the body only, so that no interior geometric discretization is required. The dominant analysis cost for the BEM is the surface integration. This is due to the pairwise nature of the BIE - a complete surface integration must be done for each point which will contribute degrees of freedom to the final system equations. Typically, in a 3D BEM structural analysis, surface integration will require 60% to 75% of analysis time, while the solution of the system equations will consume only about 10% of the time.

The development of parallel processing computing environments can be expected to have a major impact on BEM analysis. Each substructure in a BEM analysis uses a distinct set of input data and generates a distinct set of output data. Questions of memory contention and/or data transfer conflict can be expected to be relatively minor - for either shared or local memory configurations. Exploitation of parallel computing is also possible within each substructure, although more code modification would be required.

2D/AXISYMMETRIC BEM CAPABILITIES

- o PRODUCTION

  elastic, substructured analysis for isotropic, anisotropic, composite materials

  arbitrary geometry

  general boundary conditions (mixed displacement/traction, springs)

  plane strain/stress fracture mechanics capability

  body forces

The two-dimensional (plane stress and plane strain) and axisymmetric capabilities of the BEM codes presently in use at Pratt & Whitney are listed on this chart. The code is highly integrated within a graphics pre-/post-processing environment. Commercial packages are also available which possess many, but not all of the capabilities outlined.

thermal stress

- DEVELOPMENT/RESEARCH

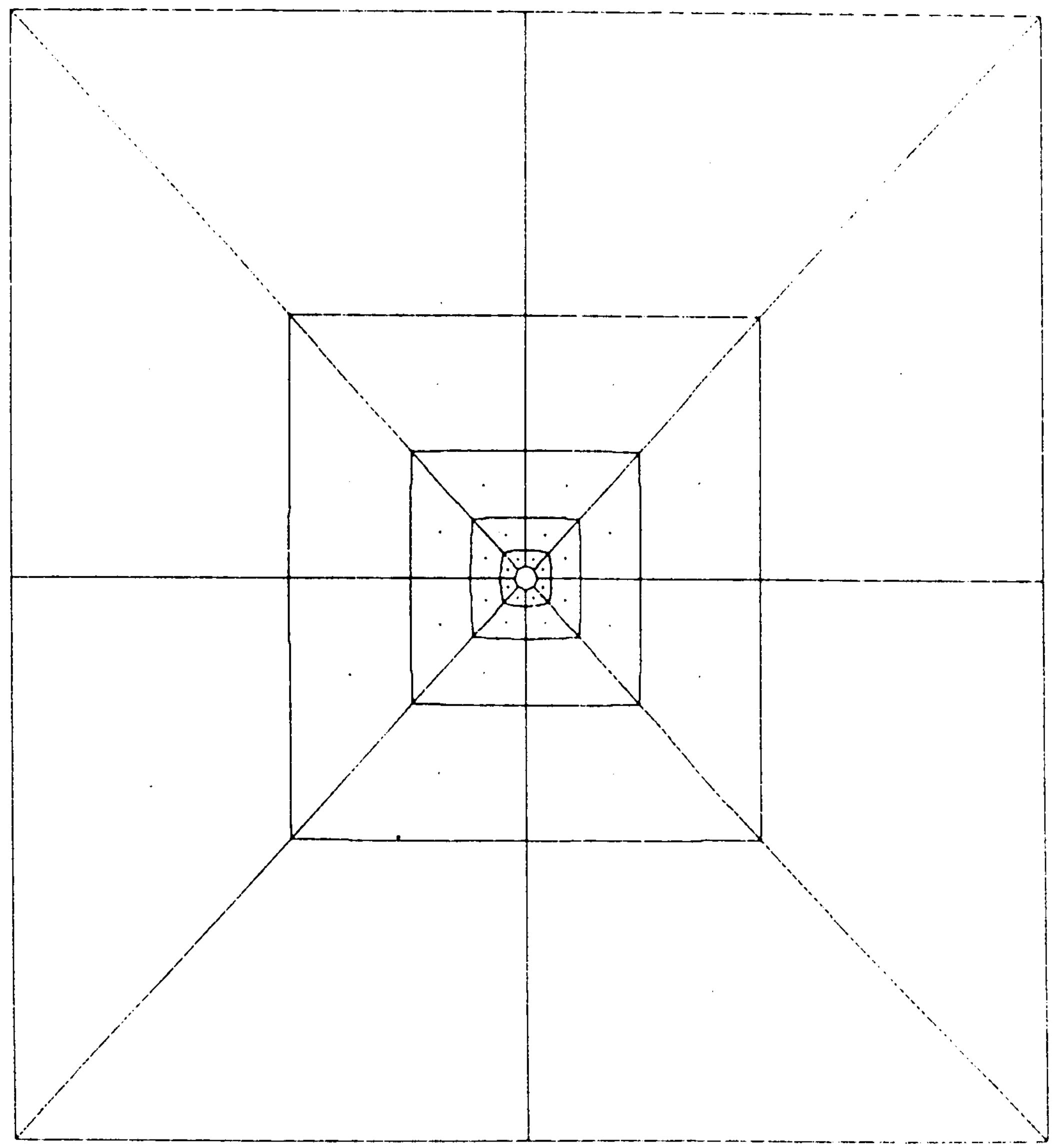
  plasticity, creep

  natural frequency/mode shape determination

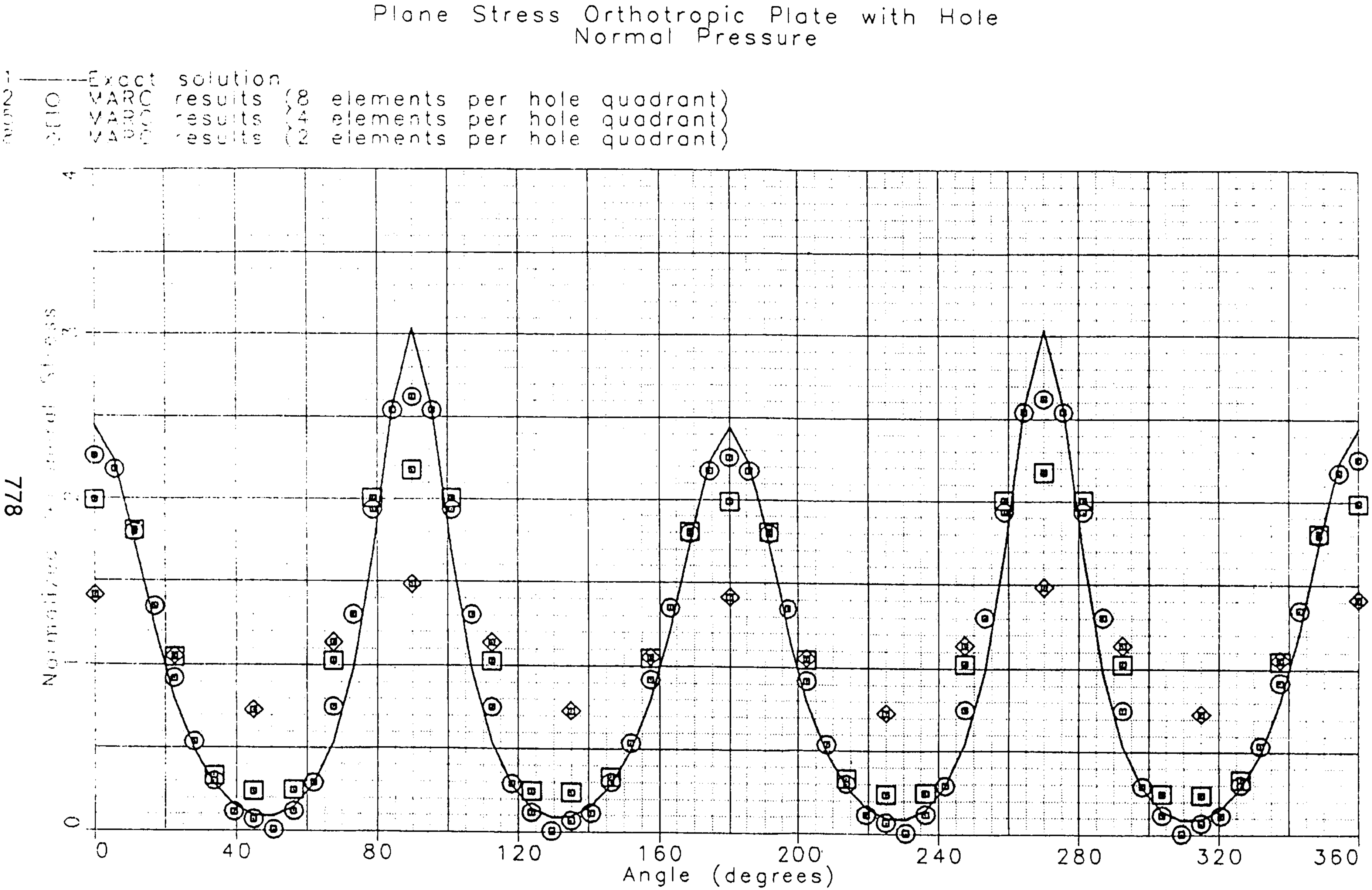
  material inhomogeneity

Advanced capabilities are presently being incorporated in the Pratt & Whitney code, and will become available in the production version over the next year.

2D STRESS CONCENTRATION CALIBRATION

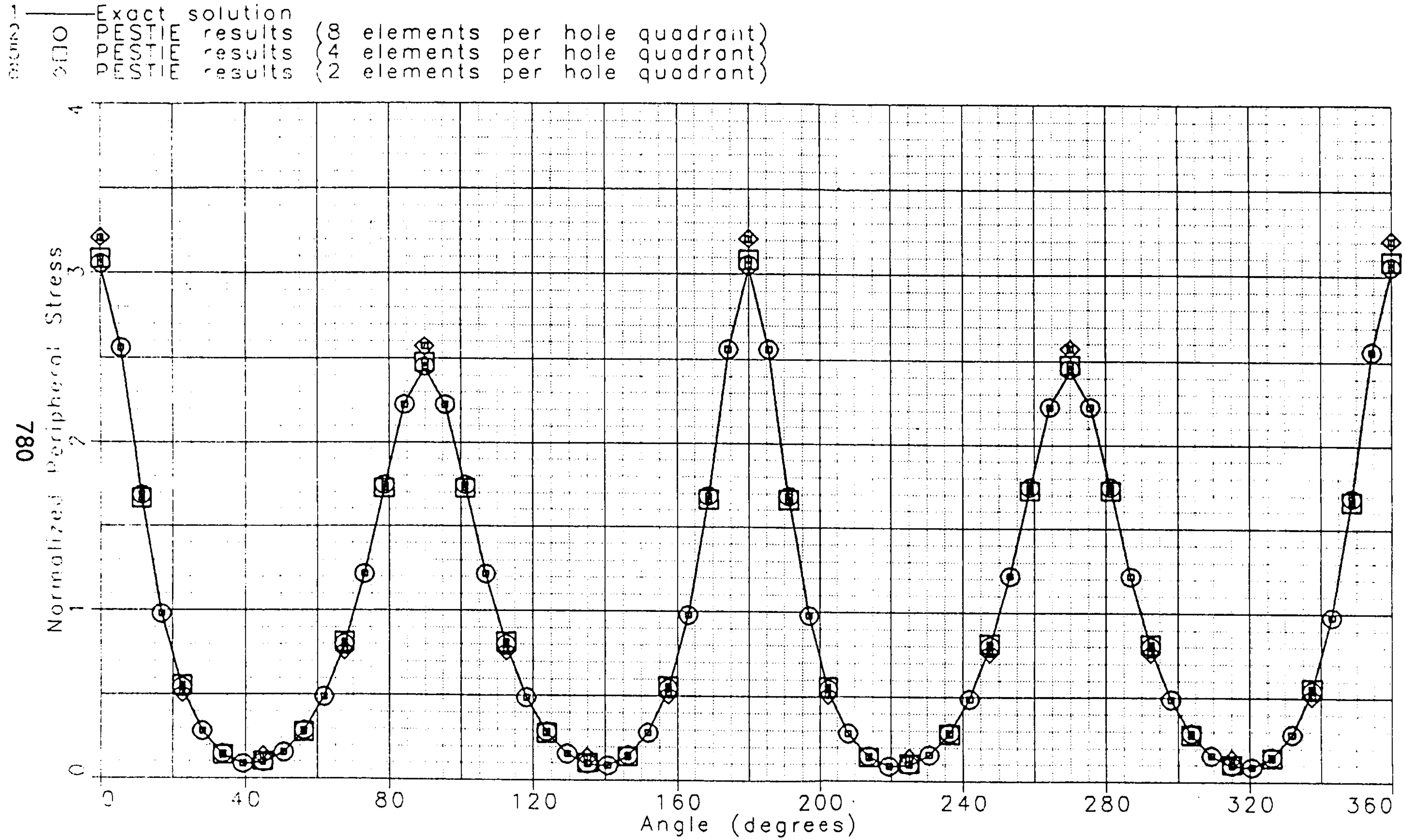


As an illustration of the advantages obtained through BEM use, for suitable problem types, consider a calibration study recently completed for the Pratt & Whitney code. An effectively infinite plate with a circular hole was subjected to a variety of loads, both remote and on the hole surface. Analyses were carried out using the P&W BEM code and the MARC finite element code (8 noded isoparametric plane stress elements). The meshes used two, four and eight elements per quadrant. Isotropic, orthotropic and layered composite materials were considered.

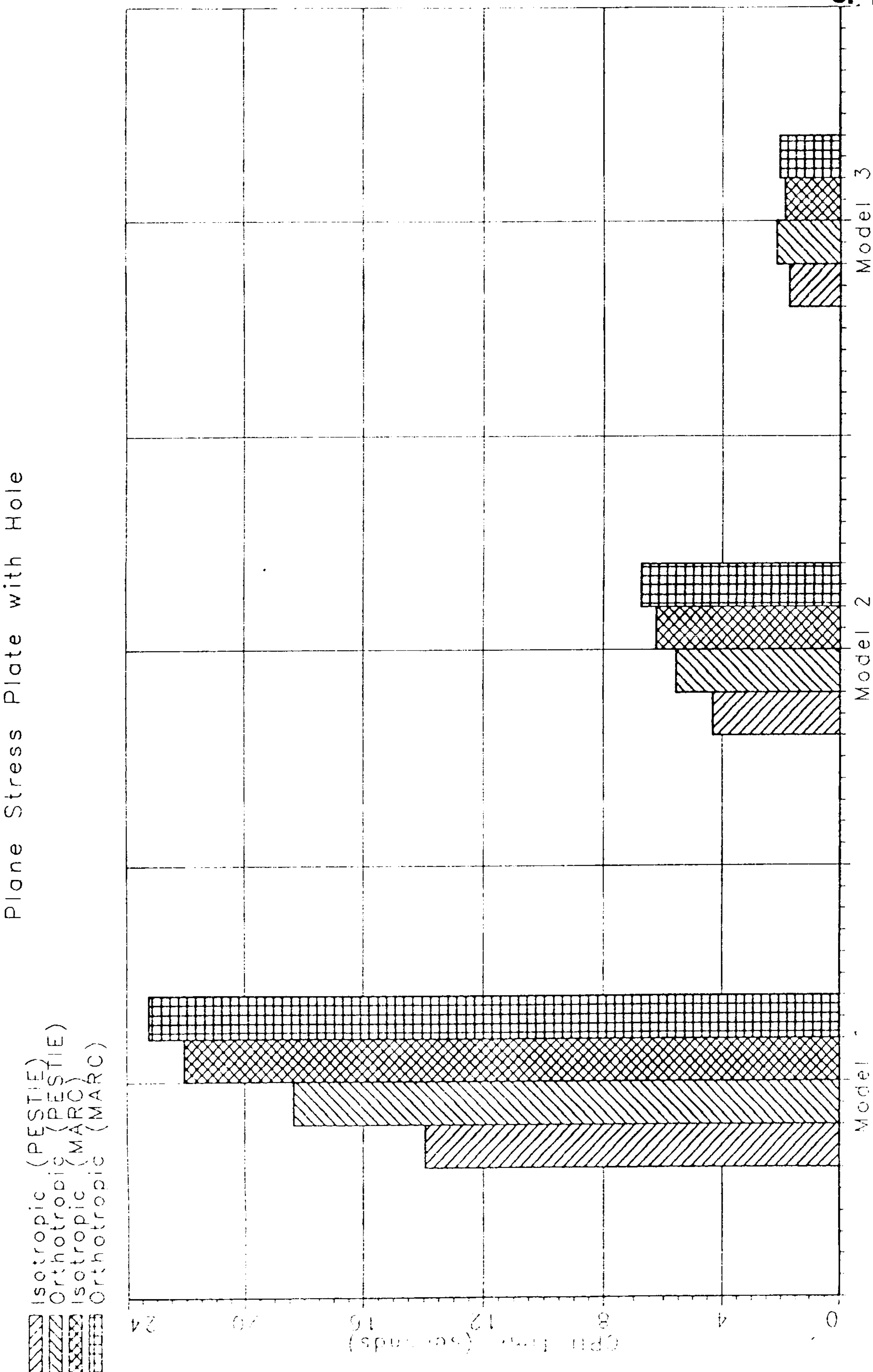

Plane Stress Orthotropic Plate with Hole
Normal Pressure
Exact solution
MARC results (8 elements per hole quadrant)
MARC results (4 elements per hole quadrant)
MARC results (2 elements per hole quadrant)
0
4
0
40
80
120
160
200
240
280
320
360
Angle (degrees)

For an orthotropic material subject to a normal pressure on the hole surface, The MARC analyses underpredict the peak tangential stress by 50%, 20% and 12% respectively, compared to the exact solution in Lekhnitski. For an isotropic material the same models give somewhat better accuracies of 20%, 5% and 2% respectively. For this problem the anisotropic material produces a significant change in the result, since for an isotropic material the tangential stress on the hole is unity at all locations.

Plane Stress Orthotropic Plate with Hole
Normal Pressure



The corresponding BEM models give accuracies of 7%, 3% and 1%, respectively (extremely close to the 6%, 1.5% and .4% obtained for the isotropic case). Significantly, no modification of the BEM model is required to maintain accuracy as the complexity of the material changes.


Plane Stress Plate with Hole
Isotropic (PESTIE)
Orthotropic (PESTIE)
Isotropic (MARC)
Orthotropic (MARC)
Model 1
Model 2
Model 3
0
4
8
12
16
20
24
CPU time (seconds)

Computing times for the coarse model were essentially equal for the BEM and finite element analyses. For the finest mesh the finite element orthotropic anslysis took 1.3 times as long as the BEM analysis. More importantly, the coarse BEM model gave better accuracy than the finest finite element model, yielding a CPU time ratio of better than 11 to 1 for (not quite) equivalent accuracy.

This problem illustrates the significant advantages obtainable with the BEM, both in analysis time and, equally significantly, in the fact that the BEM analysis gave consistent results for all materials.

BEST3D GOALS

- Develop structural analysis tool distinct from, and complementary to, the finite element method.

- Address wide range of problem types – elastic, inelastic, dynamic, vibration.

BOUNDARY ELEMENT METHOD (BEM) SELECTION

- Applicability to various problem types established.

- BEM suitable for complex geometries.

- Attractive for resolution of high gradients.

BEST3D is a general purpose BEM structural analysis code developed by Pratt & Whitney and the State University of New York at Buffalo under NASA contract NAS3-23697.

Major goals of the program were:

1. Development of a general purpose structural analysis tool applicable to problems not amenable to finite element solution

2. Development of analysis calibration capability for problems in which experimental data is lacking.

Previous experience with the BEM indicated that it was the most promising basis for developing this alternative method.

BEST3D (Boundary Element Stress Technology - 3D)

STRATEGY

- o Consider general geometry and boundary conditions
- o Organize code to accommodate all problem types within one structure
- o Make code expandable
- o Aim for machine independence
- o Avoid competition with professionals in pre-/post-processing and linear algebra

BEST3D development was planned to produce a single general purpose structural analysis code. Major emphasis was placed on the treatment of general geometry and boundary conditions and on the development of fundamental analysis capabilities. No major work was undertaken in the areas of pre-/post-processing.

BEST3D OVERVIEW

- 42,000 lines of new source code (FORTRAN 77)
- Implemented on HP, VAX, CRAY and IBM systems
- Major capabilities include:
  - elastic and inelastic analysis
  - forced response analysis
  - transient elastodynamics
  - natural frequency/mode shape calculations
  - substructuring for all problem types

BEST3D is a large code, but is written in standard Fortran 77. It has been successfully implemented on a variety of computing systems. Major capabilities are summarized on the chart.

MAJOR ADVANCES

- Time embedded dynamic formulation
- Complex variable forced response calculation
- 3D, real variable eigenvalue calculation
- Application of particular integrals to thermal stress, plasticity
- BEM variable stiffness plasticity

Major advances have been made during the development of BEST3D, both in the creation of new BEM analysis capabilities and in the incorporation of available capabilities (for the first time) in a general purpose code. Of particular importance is the fact that the all capabilities are made available for substructured analysis, a necessity for practical utilization of the program.

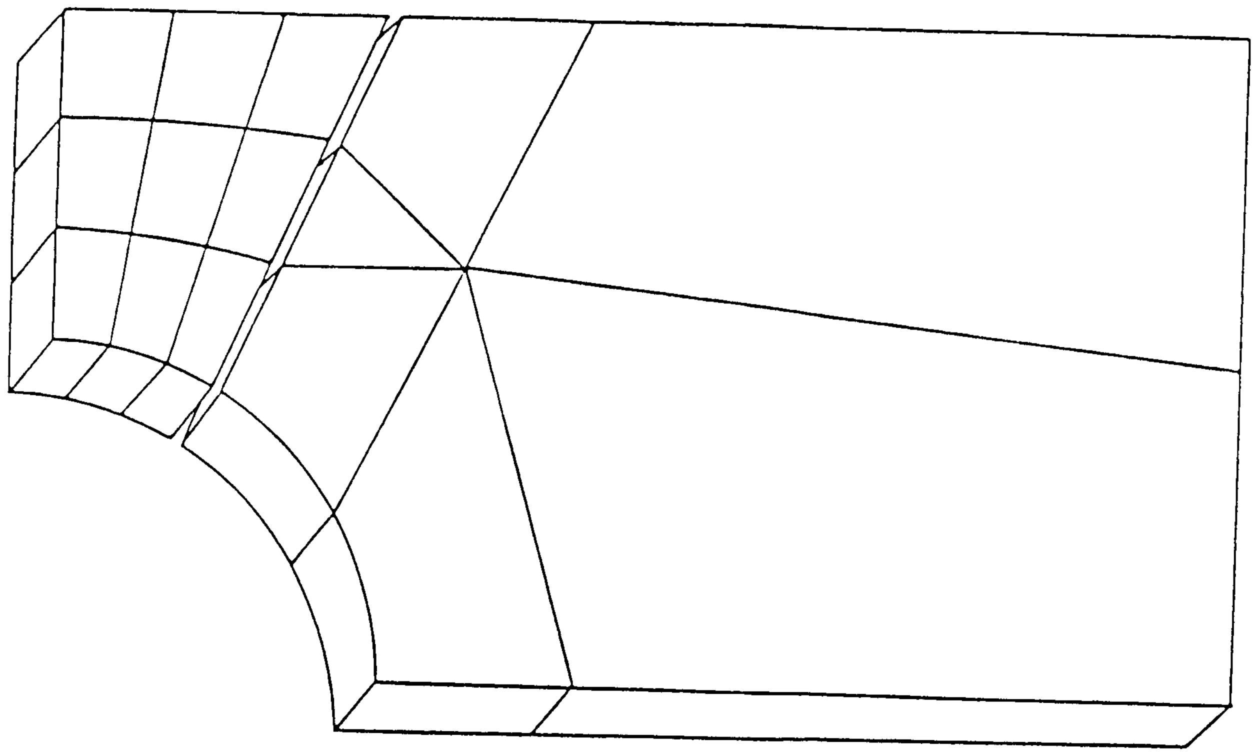
BEST3D PLASTICITY

- Representation of initial stress/strain
  - Volume cells
  - Particular solutions
- Solution algorithms
  - Iterative
  - Variable stiffness

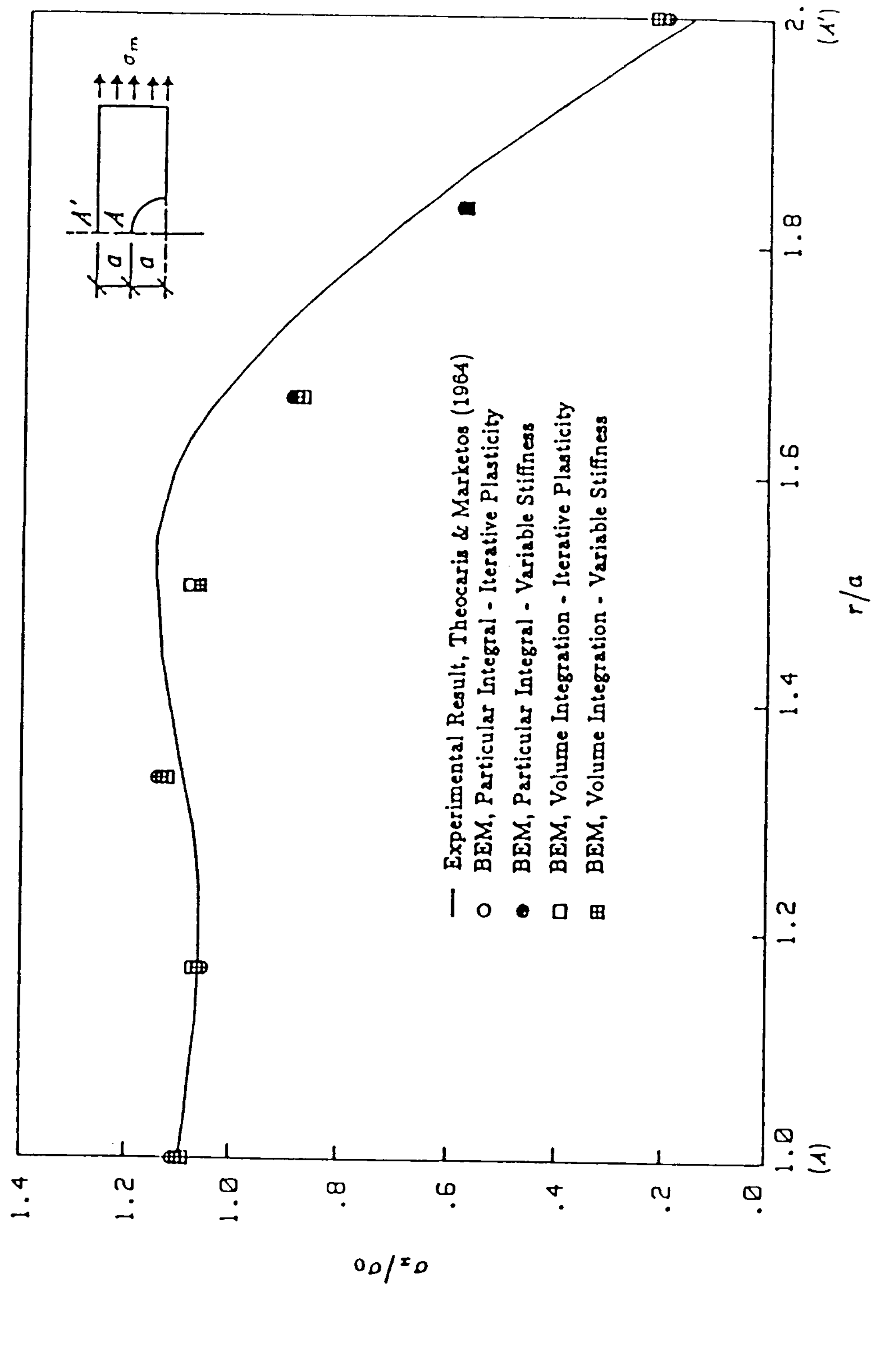
A major feature of BEST3D is the provision of a very complete plasticity capability for substructured analysis. In BEM analysis of problems with material nonlinearity it is necessary to provide a description of the variation of plastic strain in the interior of the part. Two different techniques for this description are available in BEST3D, as are two different solution algorithms. Both the use of particular solutions in plasticity and the variable stiffness solution algorithm are new developments carried out as part of the NASA contract referred to previously.

The availability of a variety of independent plasticity algorithms within BEST3D allows calibration of the code by running the same problem using different solution sequences.

PERFORATED PLATE

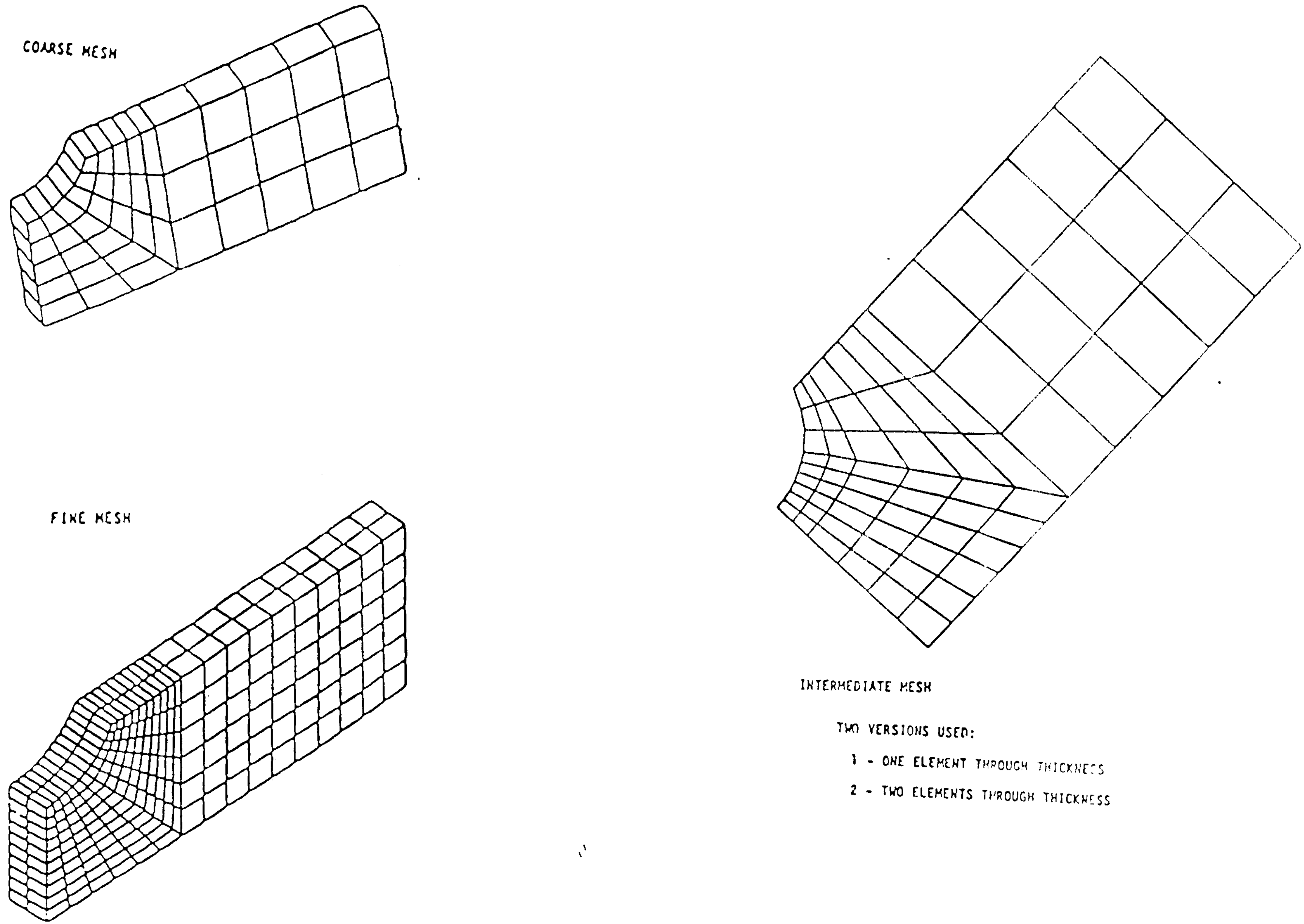


The classical problem of a tension loaded perforated plate was analyzed using the two region BEST3D model shown. Plastic strain distribution was modelled only in the region near the root of the notch, since the other region remains elastic. The model shown represents one quarter of the physical specimen because of symmetry considerations.



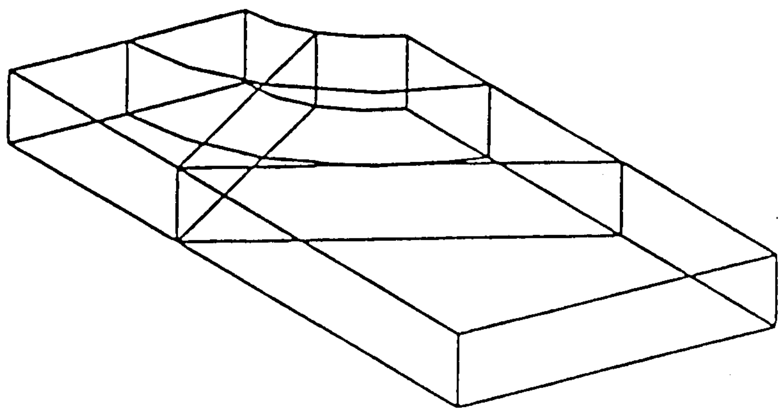
Experimental Result, Theocaris & Marketos (1964)
BEM, Particular Integral - Iterative Plasticity
BEM, Particular Integral - Variable Stiffness
BEM, Volume Integration - Iterative Plasticity
BEM, Volume Integration - Variable Stiffness
σ_m
λ'
λ
a
a
1.4
1.2
1.0
.8
.6
.4
.2
.0
σ_x/σ_0
1.0
(λ)
1.2
1.4
1.6
1.8
2.0
(λ')
r/a

The stress distribution at the root of the notch is shown for all four possible combinations of plastic strain representation and solution strategy. For the load applied the plastic front is 60% of the way from the hole surface to the free surface of the specimen. There is excellent agreement among all four solutions. Departures from the experimental data are due primarily to lack of detailed knowledge of the plastic properties of the actual material used. The BEM results are essentially identical to finite element results obtained using a variable stiffness algorithm and the same material properties as those used in the BEM analyses.

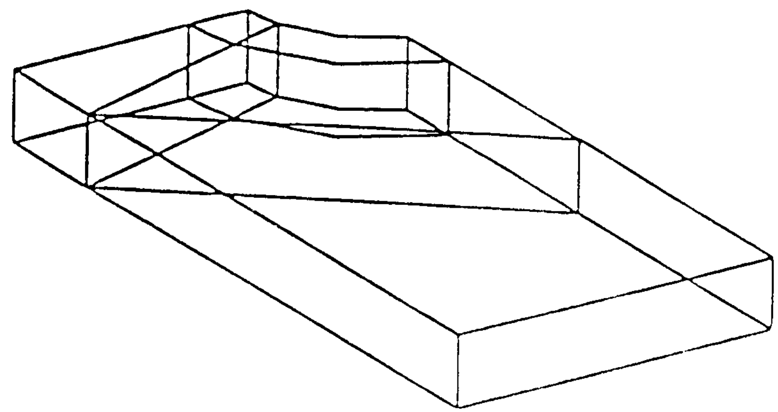


Finite Element Models for Benchmark Notch Analysis

A variety of finite element models were used for the nonlinear (MARC and MHOST) analysis of the NASA Benchmark Notch specimen. The specimen notch is typical of those found in turbine attachments. Use of the finest model shown was required to obtain good agreement with cyclic experimental data.



EQUALLY SPACED MESH

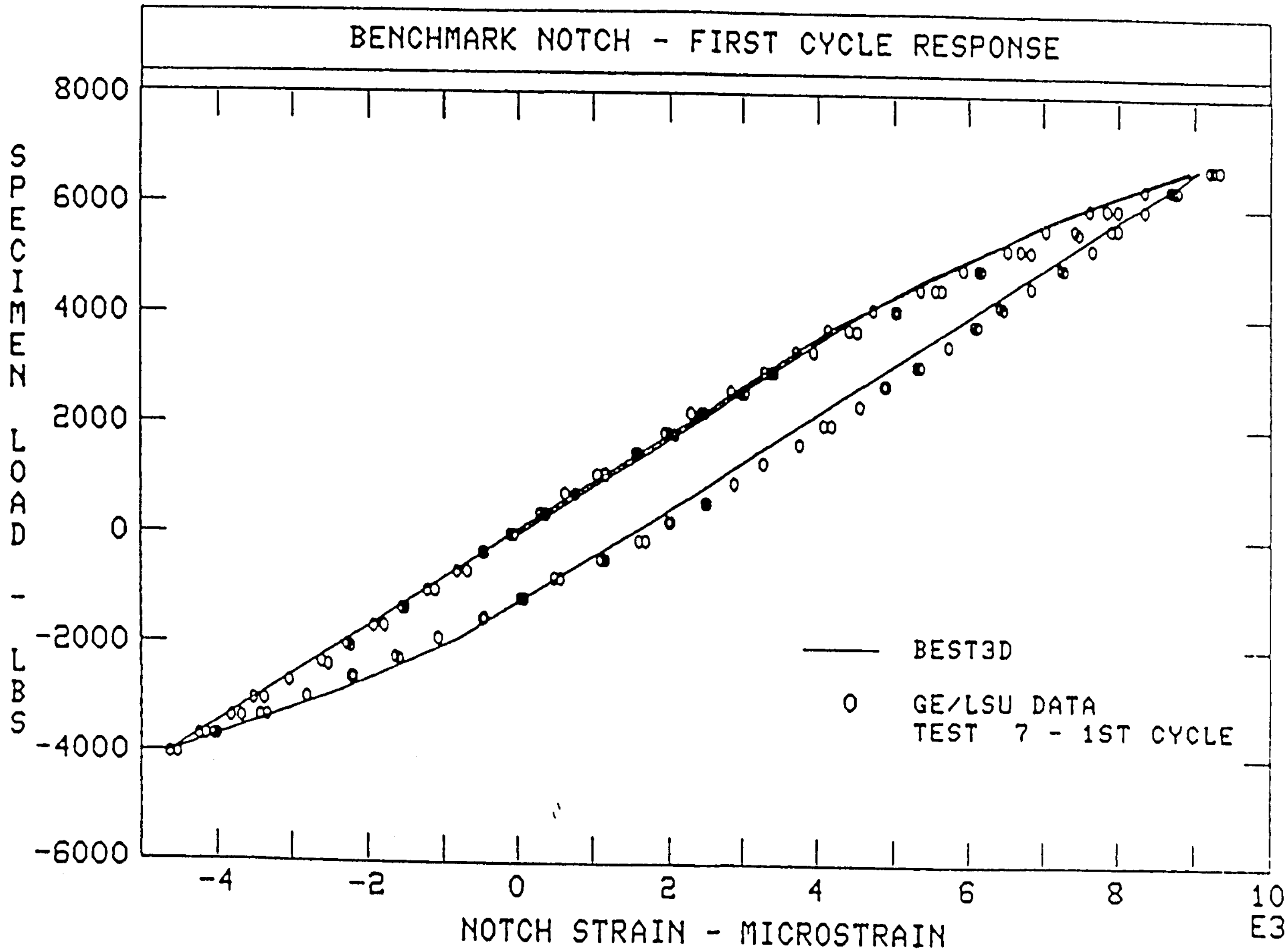


WEIGHTED MESH

Boundary Element Models for Benchmark Notch Analysis

Several BEM models were also used for the analysis of the same geometry. Proper weighting of element sizes in the region of expected stress concentrations is very effective in improving accuracy for a given mesh.

MANUAL MODE

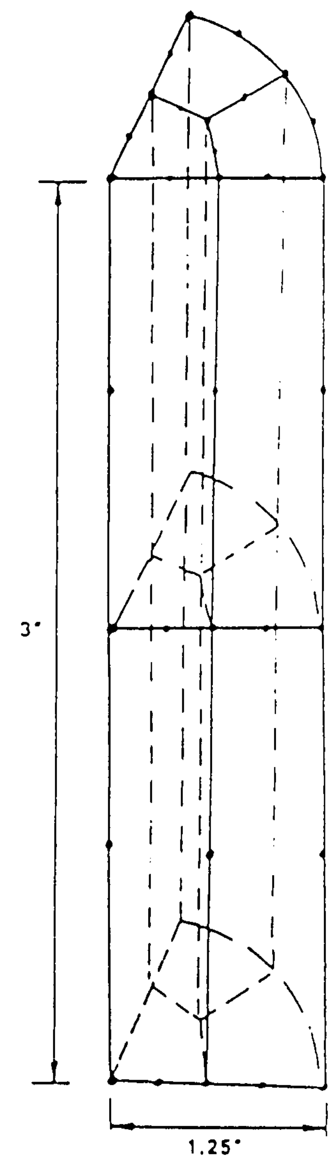


BENCHMARK NOTCH - FIRST CYCLE RESPONSE
SPECIMEN LOAD - LBS
8000
6000
4000
2000
0
-2000
-4000
-6000
-4
-2
0
2
4
6
8
10
E3
NOTCH STRAIN - MICROSTRAIN
BEST3D
O GE/LSU DATA
TEST 7 - 1ST CYCLE

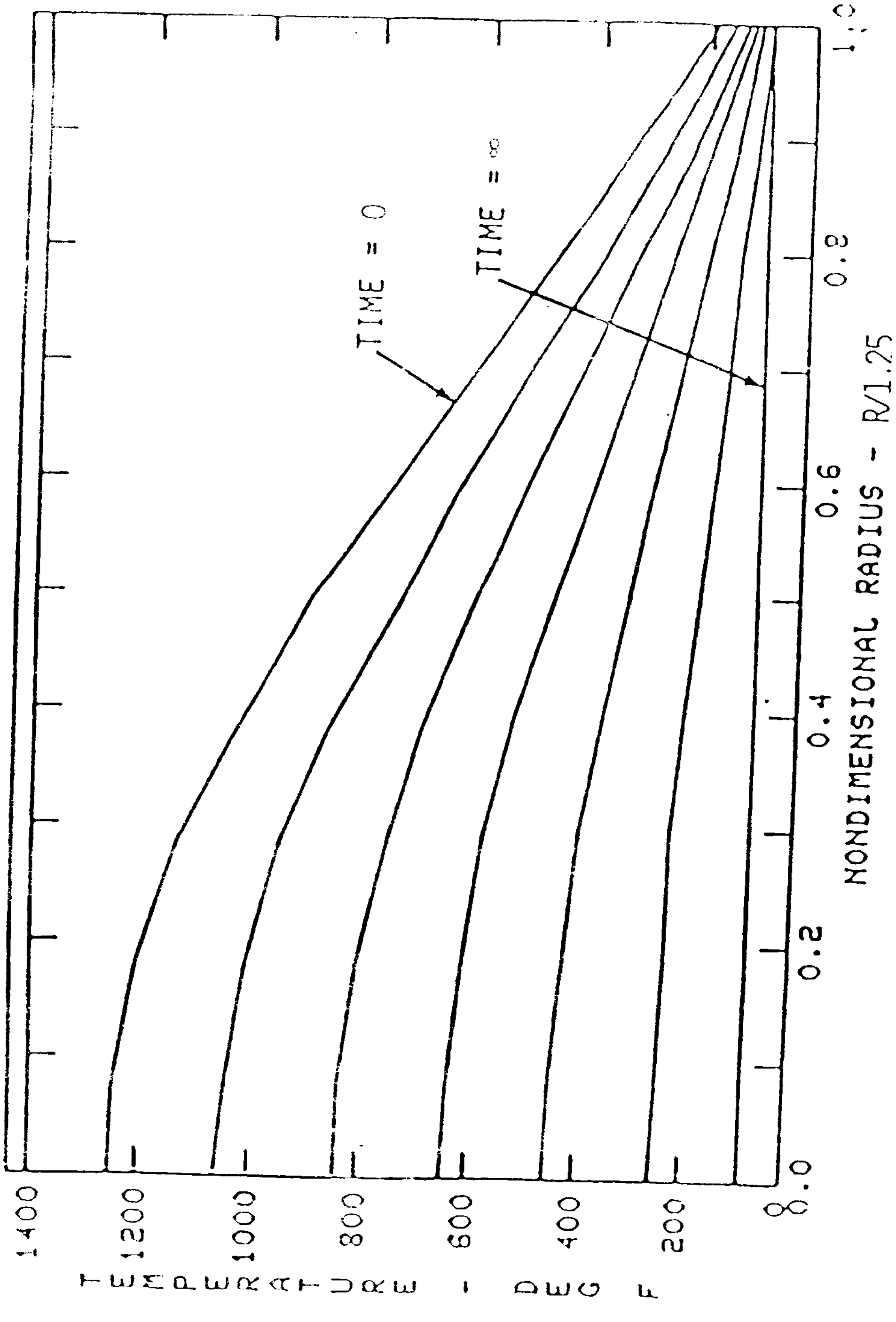
Use of the weighted mesh shown on the previous chart gave good agreement with experimentally measured strains at the notch root, over two and one-quarter load cycles. Use of the finest finite element model gave equivalent results, in about twice the CPU time.

RESIDUAL STRESS IN QUENCHED BAR

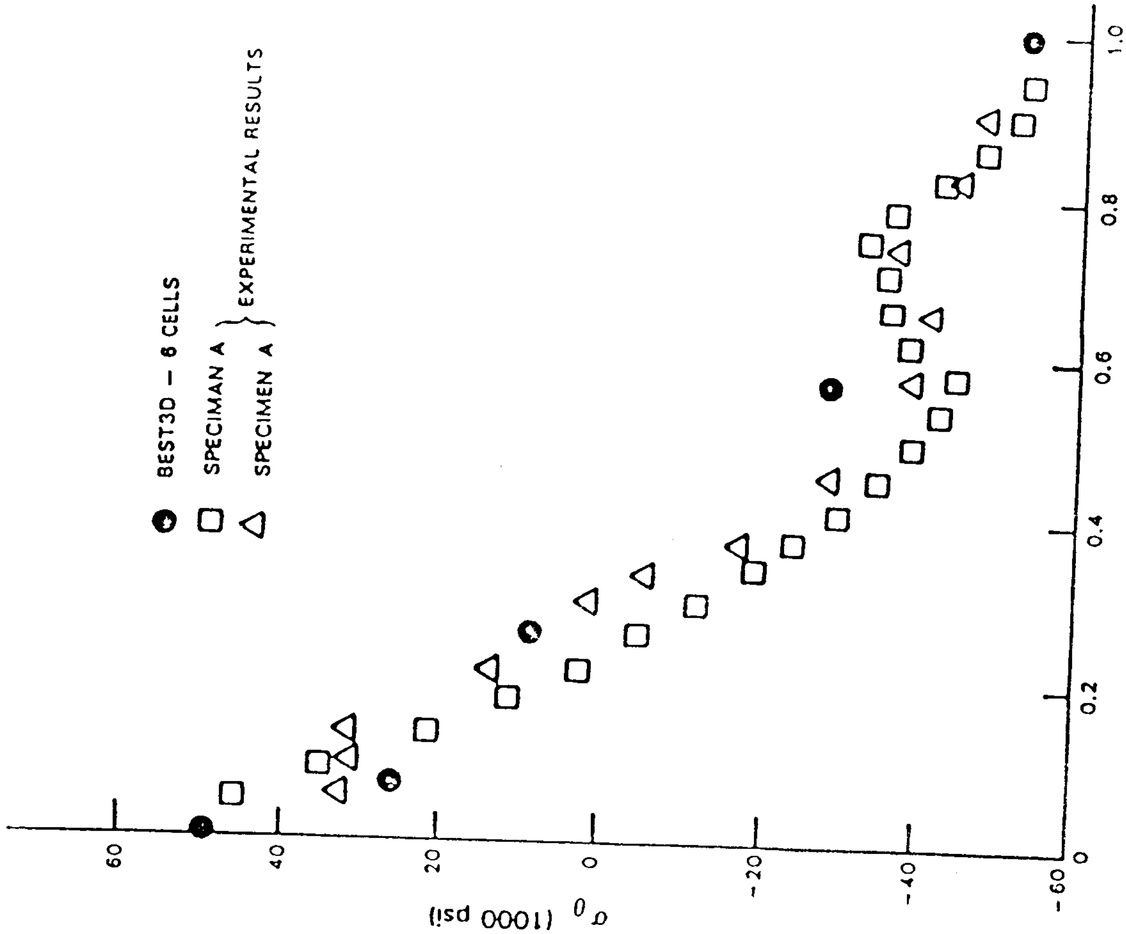
- Entire load thermal
- Steep plastic gradients
- Data available
- Axisymmetric analysis
- Volume cells, iteration used



A problem of significant interest in the gas turbine industry is the development of residual stress in large parts during forming and heat treatment. BEST3D was used to analyze a test in which a cylindrical bar was rapidly quenched from 1250° F and the residual stresses subsequently measured. Two nominally identical specimens were used in the experimental work.


TIME = 0
TIME = ∞
NONDIMENSIONAL RADIUS - R/1.25
0.0
0.2
0.4
0.6
0.8
1.0
TEMPERATURE - DEG F
0
200
400
600
800
1000
1200
1400

The temperature gradient was initially very severe (over 1000° F between the surface and center of the specimen), dropping to essentially zero at large times.

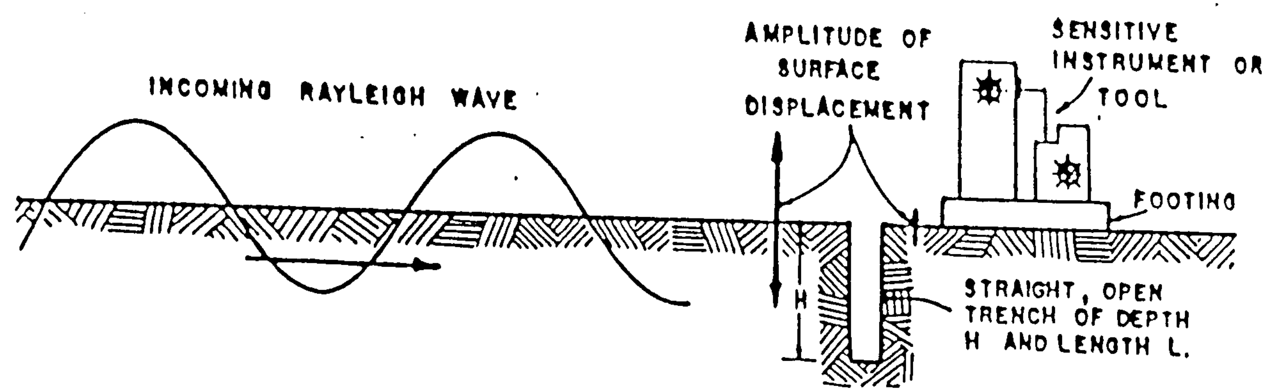

BEST3D – 6 CELLS
SPECIMAN A
SPECIMEN A
EXPERIMENTAL RESULTS
σθ (1000 psi)
60
40
20
0
-20
-40
-60
0
0.2
0.4
0.6
0.8
1.0
r2

The BEST3D analysis shows good agreement with the experimentally measured stresses, even for the relatively coarse model used.

VIBRATION ISOLATION

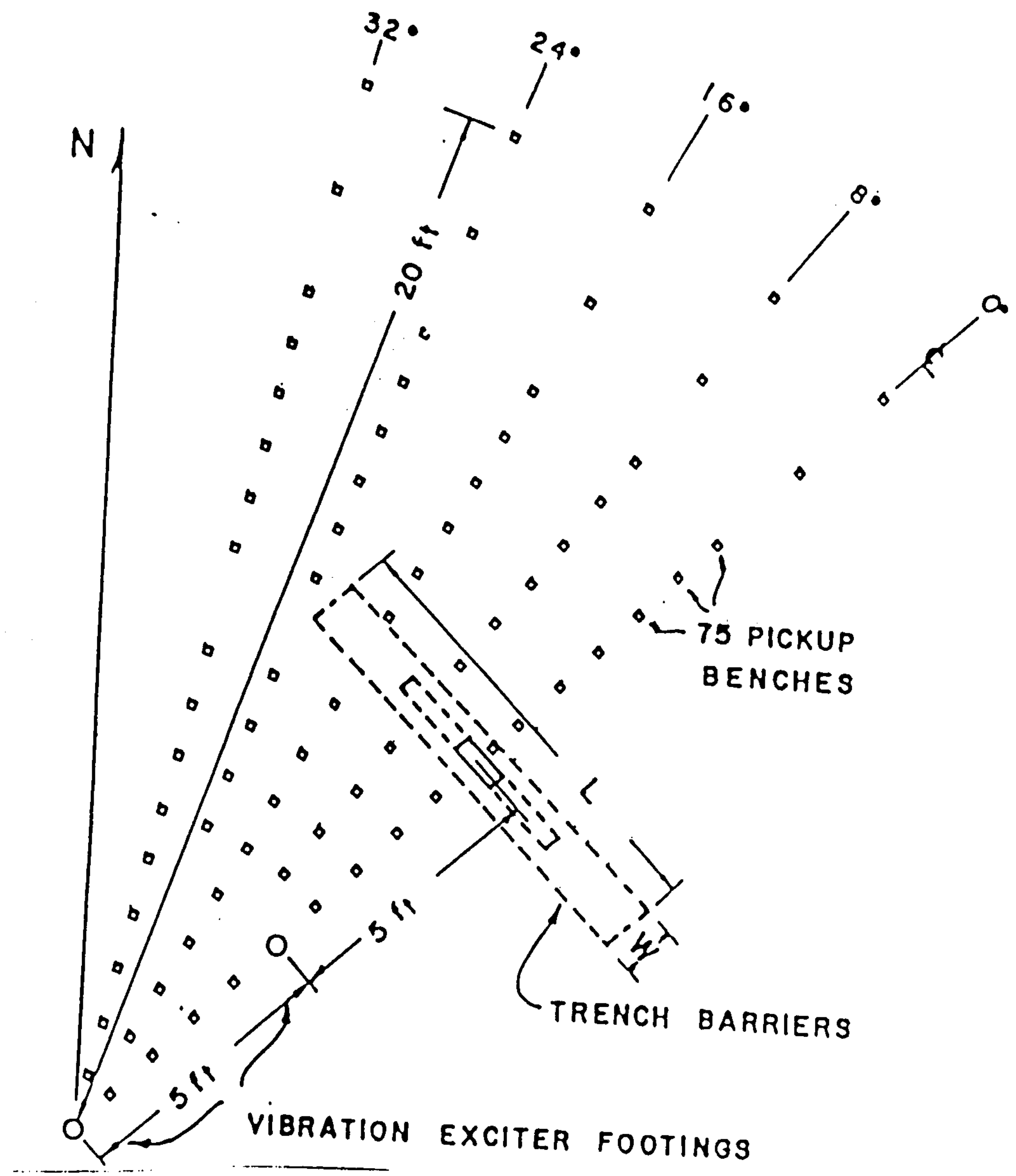
- Goal is to calculate effectiveness of trenches for isolation of sensitive equipment

- Time harmonic analysis used



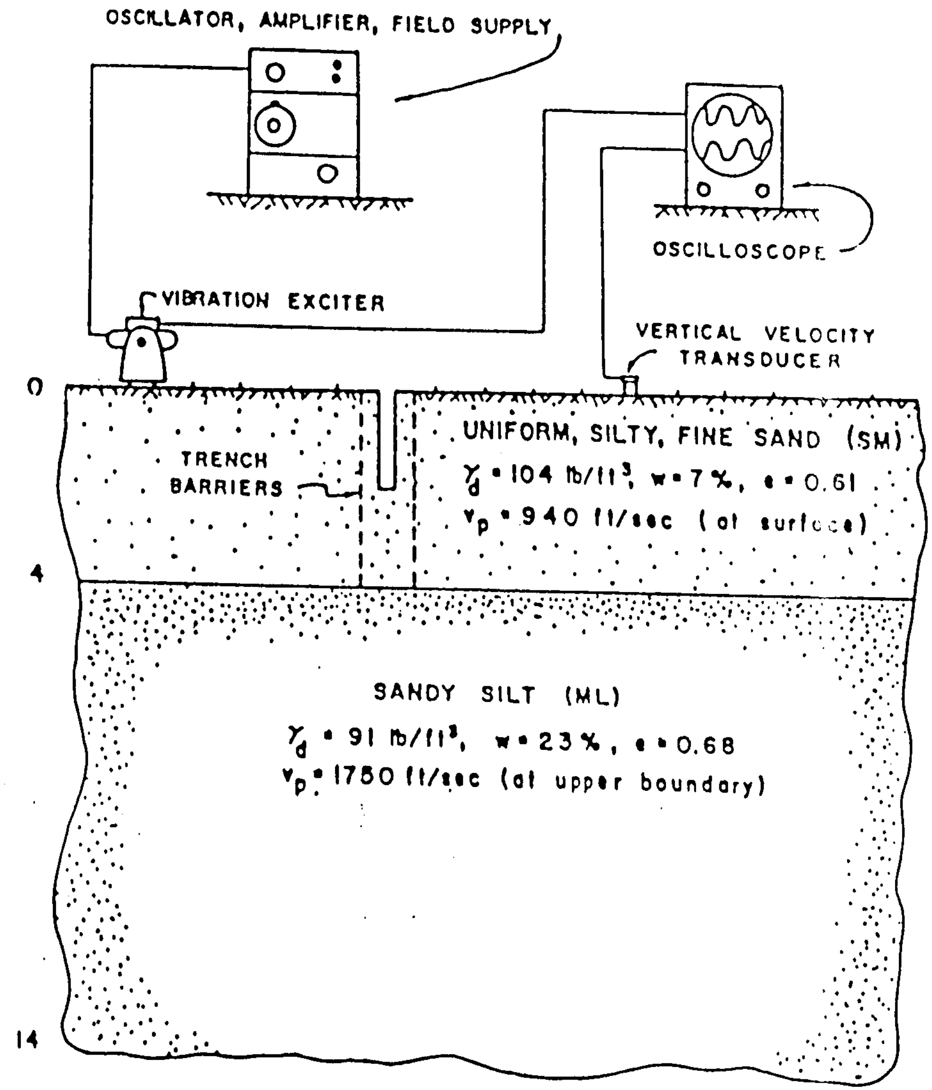
To calibrate the dynamic capabilities of BEST3D a BEM model was constructed to evaluate the effectiveness of trenches for the vibration isolation of sensitive equipment.

Passive Isolation Testing



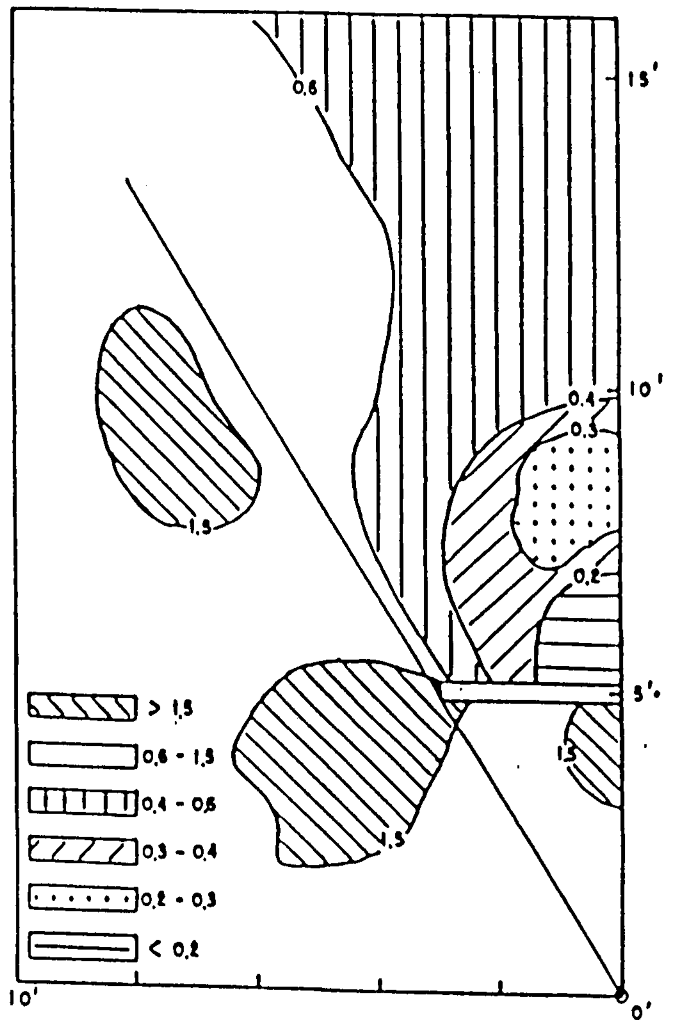
Extensive test data is available for this problem. The relative locations of the exciter, trench and vibration pickups are shown on this chart.

Substructured BEST3D Analysis Required

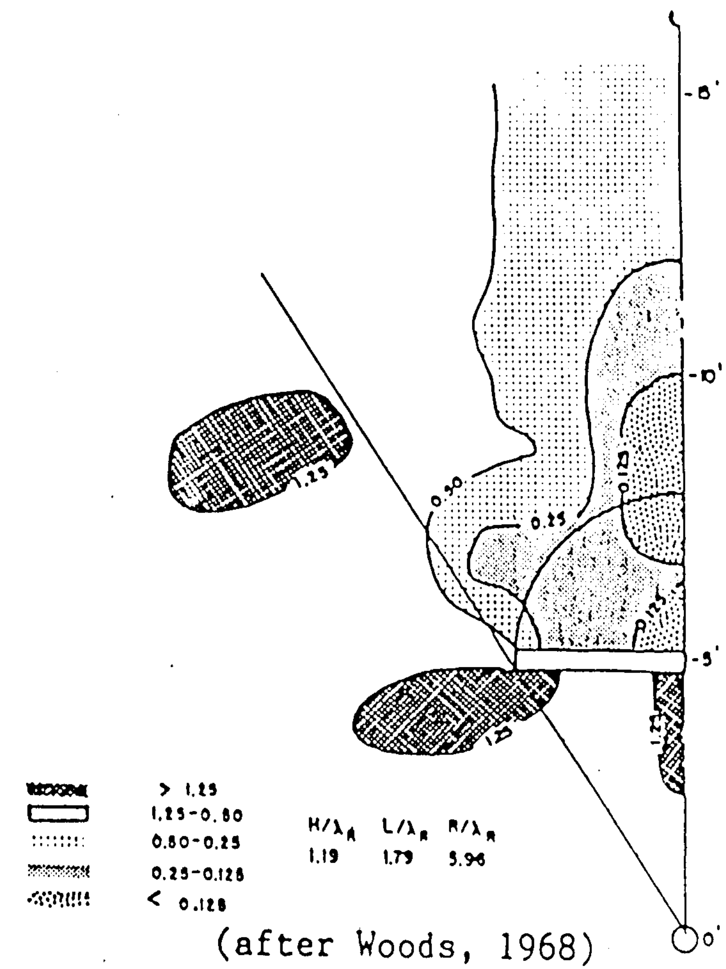


The inhomogeneous nature of the soil required the use of a substructured BEST3D analysis. The use of infinite boundary elements, automatically incorporating the radiation condition at infinity, eliminated the need for any modelling of far field boundaries.

Amplitude Reduction Factor Contours



BEST3D



(after Woods, 1968)

experimental

BEST3D predictions of amplitude reductions and amplifications show good agreement with the experimental results.

BEM USE IN DESIGN/ANALYSIS

- o Communication with existing geometry and pre-/post-processing codes required

- o Rapid job turnaround required - for elastic analyses

  .5 to 2 days for 2D (with iterations)

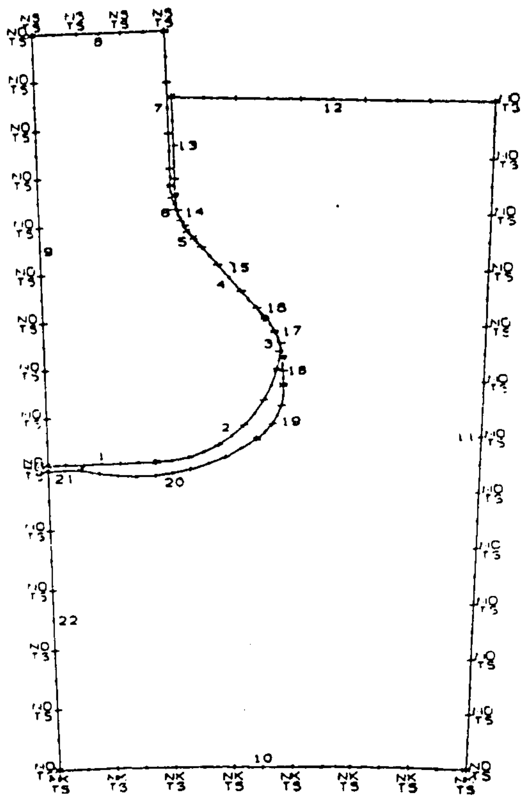
  2 to 10 days for 3D

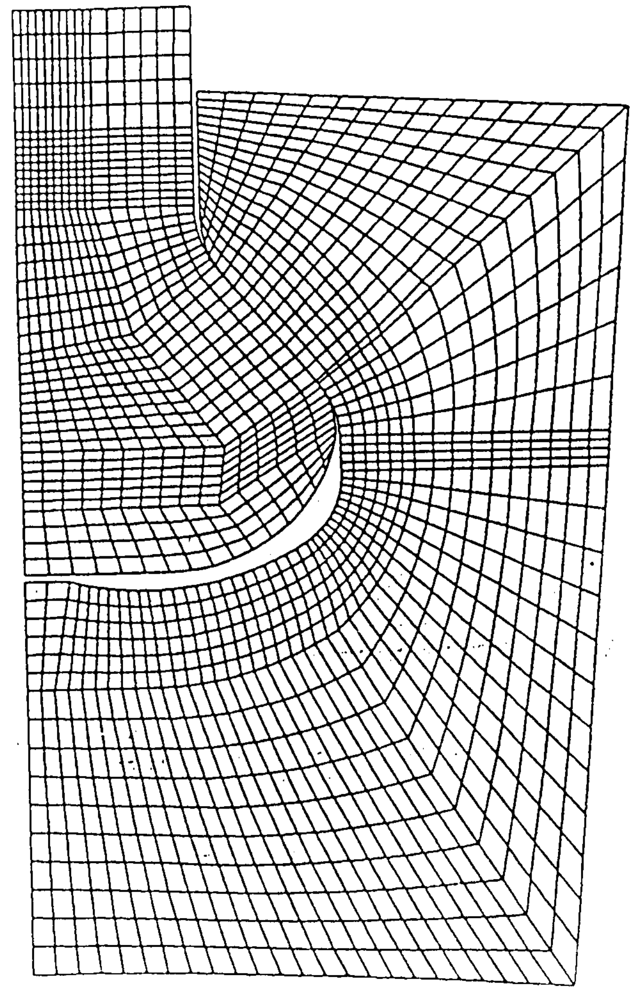
- o Calibration sufficient for intended use

The fundamental requirement for the incorporation of the BEM in practical design/analysis is the ability to produce credible results fast enough to impact the design or development process. The analysis turnaround time required is highly dependent on the type and complexity of the analysis involved. The degree of calibration required is also dependent on the intended use of the BEM results. Replacing an existing design system requires a much more thorough calibration process than does the use of a new tool simply for the ranking of a number of candidate designs.

COMPRESSOR DOVETAIL ANALYSIS

- o BEM replacing FEM
- o Calibration done
- o Interactive system

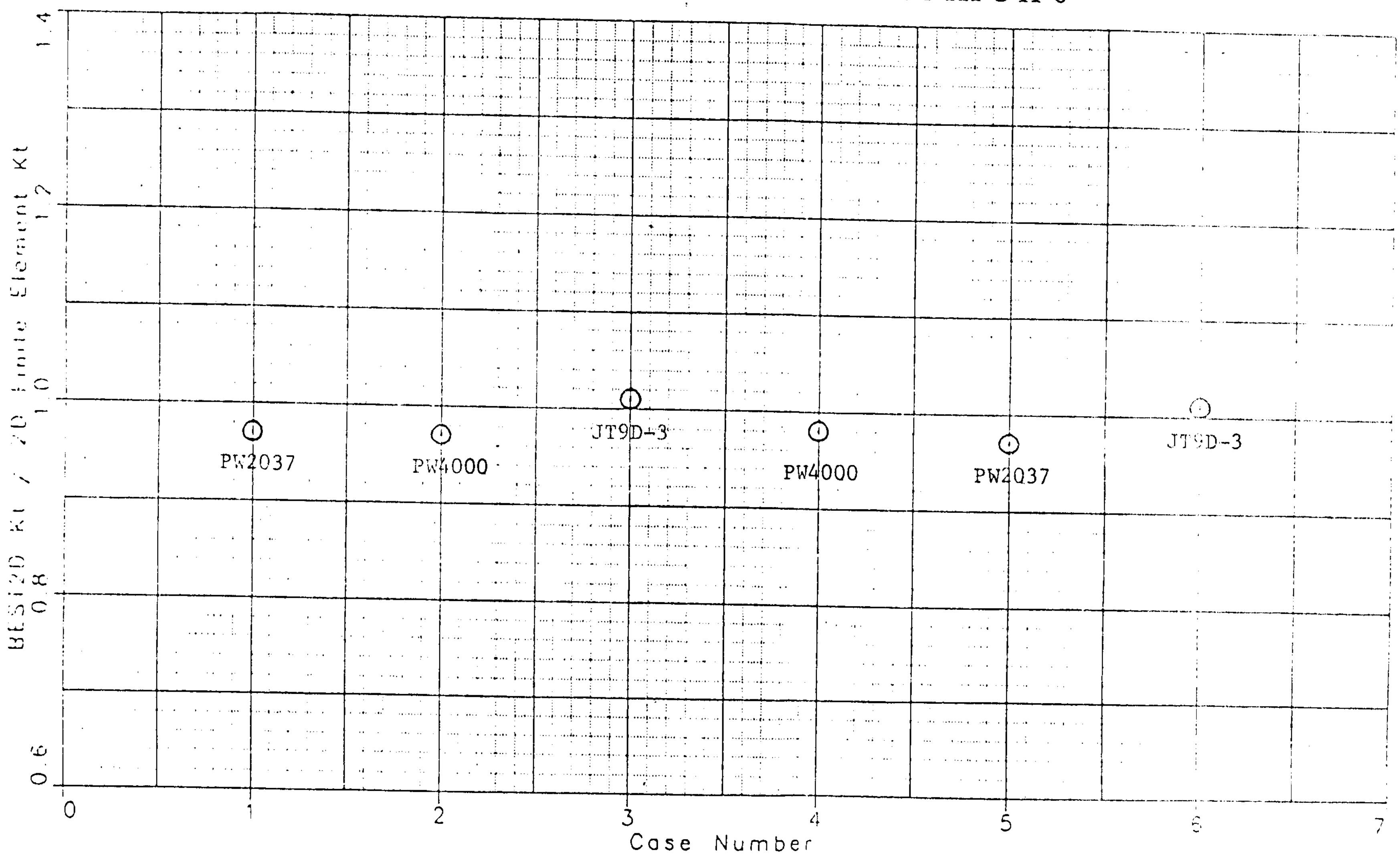




Pratt & Whitney is presently in the process of applying its interactive two-dimensional BEM system to the determination of stress concentrations in compressor dovetails. A typical finite element model previously used for these studies is shown, together with a BEM model of the same geometry. The BEM analysis is interactive, while the finite element analysis requires a batch (usually overnight) run.

Compressor Blade Dovetail Fillet Stress Kt Study

BEST2D vs 2D Finite Element



The new BEM system shows excellent agreement with the old system for a variety of actual engine geometries.

PRE/POST PROCESSING FOR BEST3D

- Solid modeller output available as

  IGES files

  (faceted) surface definitions

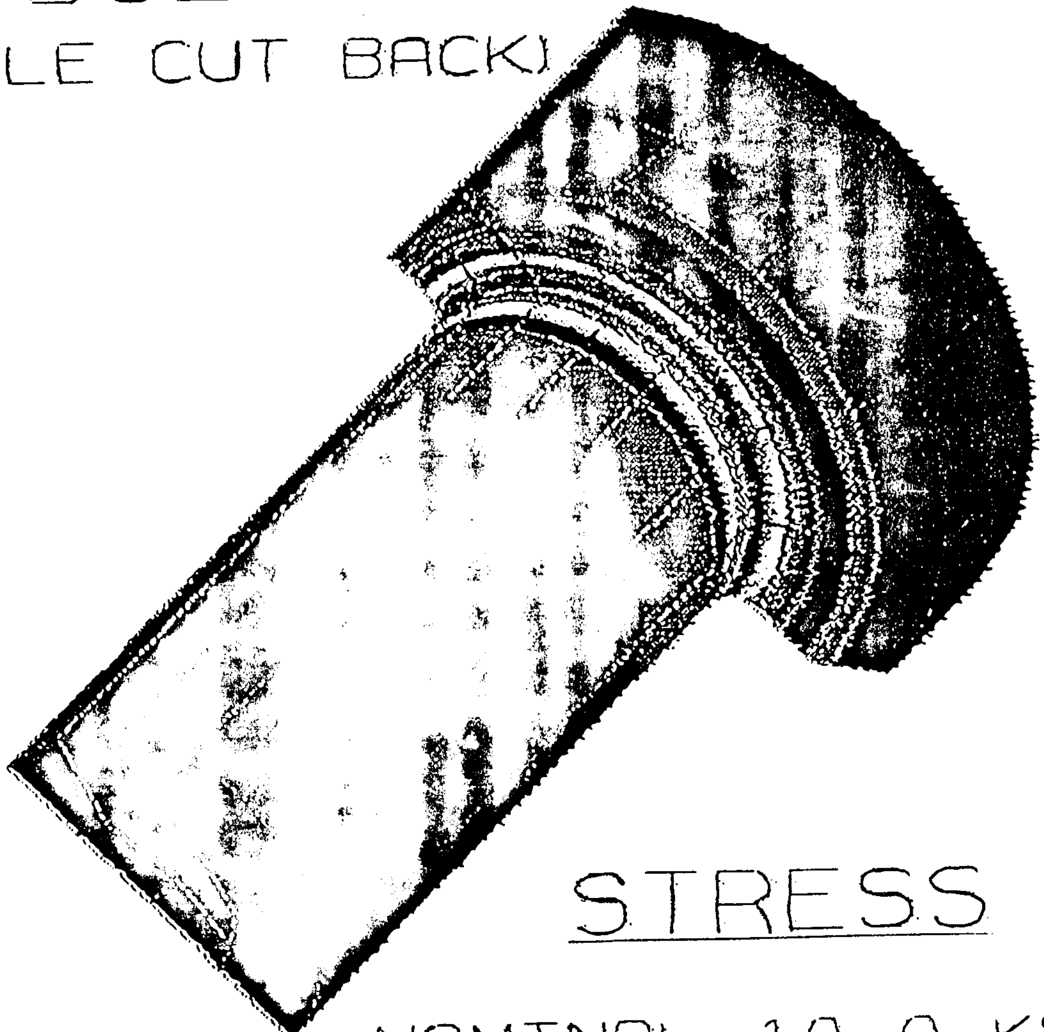
- These files can be input to existing FE preprocessors

- BEST3D models generated in terms of 6/8 node plate elements

- Local implementation should be a 2-4 week process

Practical utilization of BEST3D is heavily dependent on the ability to use existing geometry definitions and to provide rapid graphics access to results. BEST3D can be rapidly linked to many existing pre-/post- processing systems.

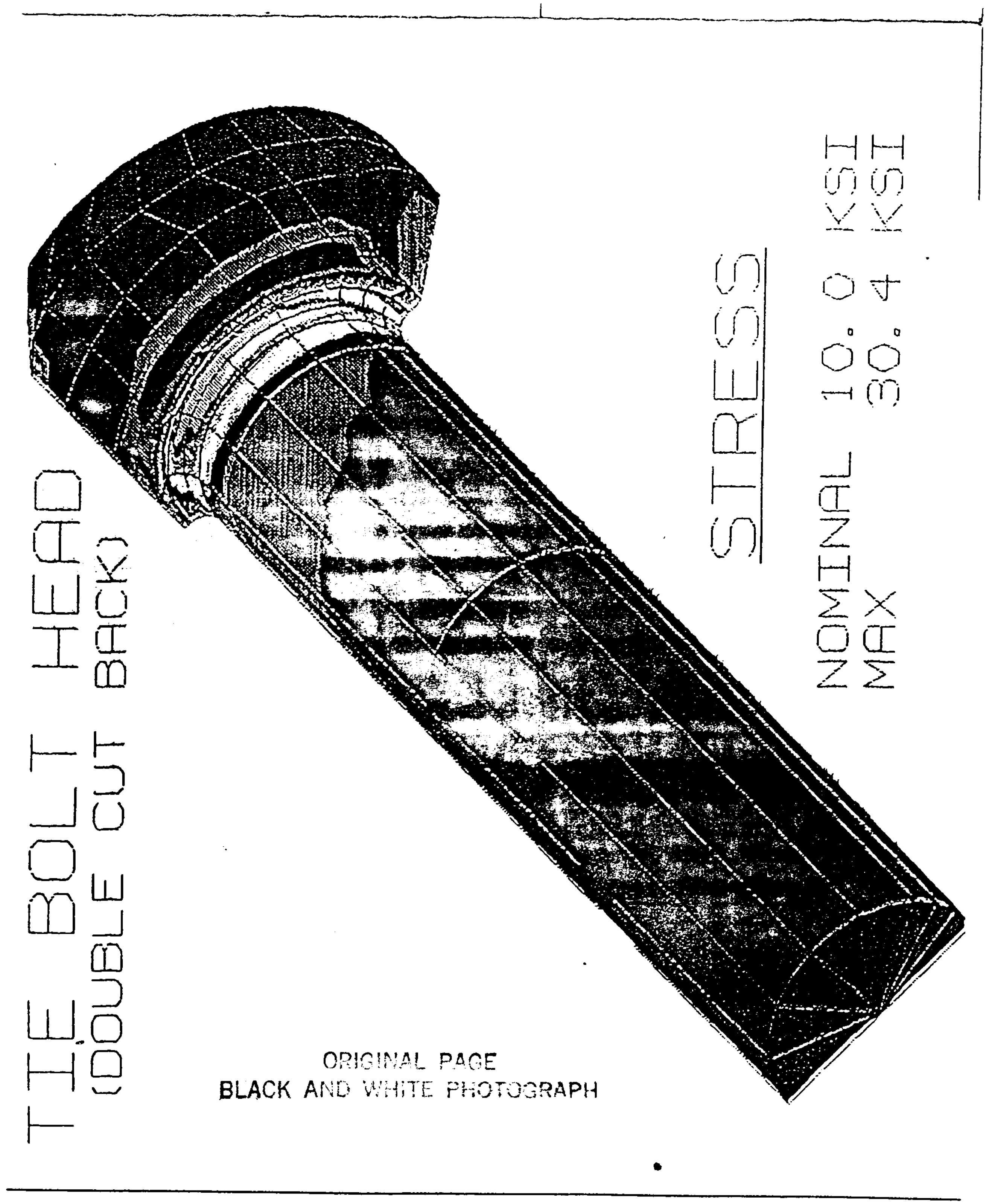
TIE BOLT HEAD
(SINGLE CUT BACK)



STRESS

NOMINAL 10.0 KSI
MAX 27.9 KSI

Both of the tie bolt geometries shown were input directly from a solid modeller into a P&W pre-processor for generation of BEST3D models. The same system allowed immediate evaluation of the results. It was possible to complete the entire model building process with a part-time effort over three days. Use of a CRAY computer to run the analyses allowed daytime turnaround of iterations on loads and boundary conditions.


TIE BOLT HEAD
(DOUBLE CUT BACK)
STRESS
NOMINAL 10.0 KSI
MAX 30.4 KSI

It was possible to provide a 3D comparison between the baseline geometry and a proposed modification rapidly enough to guide the design process.

Present supercomputer capabilities will allow fairly routine elastic analysis of 3D geometries, with daytime turnaround easily available for the less complex analyses. With proper attention to effective modelling, nonlinear analysis of structures such as turbine airfoils will also be feasible, using present computing capabilities, although probably on an overnight basis. Routine nonlinear analysis of very complex structures, such as an entire turbine blade (including platform and firtree) will probably require the exploitation of parallel processing capabilities.