P.5 A NUMERICAL STUDY OF SCALAR GRADIENTS IN KELVIN-HELMHOLTZ BILLOWS

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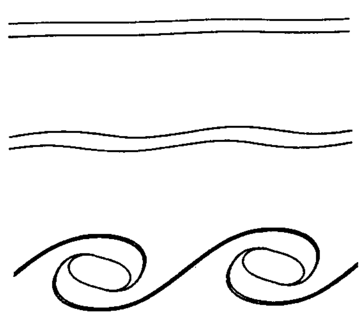
A high-resolution numerical technique is used to model the development of a periodically perturbed shear layer imbedded in an initially vertical gradient of a passive scalar. The technique follows the development of the vorticity through an initial linear growth stage and well into the nonlinear development of Kelvin-Helmholtz billows, in the zero-viscosity, zero-diffusion limit. The resulting scalar distribution rapidly develops regions of extremely sharp scalar gradients, which wind around the periodically spaced vortical low-gradient cores. Vertical cross sections through different parts of the billow structure are presented and compared with rocket measurements of electron density fine structure in the mesosphere. Gradient limits imposed by finite diffusion are calculated, and implications for atmospheric radar observations are discussed.

What is the Kelvin-Helmholtz Instability ?

A region of fluid shear is unstable.

Disturbances grow, billows form.



Rockets, radar observe advected scalar quantities (temperature, ionization).

What happens to scalar distribution?

What features should show up in rocket, radar data?

Linear perturbation of vortex layer:

$$y = \frac{1}{2}h + \varepsilon\, e^{\sigma t + i(kx - \frac{\phi}{2})}$$

$$y = -\frac{1}{2}h + \varepsilon\, e^{\sigma t + i(kx + \frac{\phi}{2})}$$

implies

$$\sigma = \pm\frac{\omega}{2}\sqrt{e^{-2kh} - (kh-1)^2}$$

$$\phi:\ e^{i\phi} = e^{kh}\left[1 - kh \pm i\sqrt{e^{-2kh} - (kh-1)^2}\right]$$

Thus, for this choice of phase angle, amplitude grows as

$$e^{\sigma t}$$

We may measure this amplitude as the vertical extent of the vorticity boundaries, less h.

A semilog plot of amplitude vs. time should show unit slope in this linear growth phase of development.

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Equations:

For 2-d incompressible, unbounded fluid, velocity may be found from the vorticity distribution:

$$du(x,y) = \frac{-\omega}{2\pi r^2}(y-y')\,dA$$

$$dv(x,y) = \frac{\omega}{2\pi r^2}(x-x')\,dA$$

With zero viscosity, a 2-d barotropic fluid is governed by

$$\frac{d\omega}{dt} + \mathbf{u}.\nabla\omega = 0$$

A moving fluid element cannot change its vorticity.
We will begin with a bounded region of constant vorticity.
Boundary determines flow.
We apply Green's theorem to obtain

$$u = -\,\omega\int_C \psi_0(x-x',y-y')\;dx'$$

$$v = -\,\omega\int_C \psi_0(x-x',y-y')\;dy'$$

$$\psi_0 = \frac{1}{2\pi}\ln\sqrt{(x-x')^2 + (y-y')^2}$$

is stream function for point vortex. We use periodic form.
In zero diffusion limit, may also use

$$\frac{dN}{dt} + \mathbf{u}\,\nabla N = 0$$

Numerical procedure:

1. Define contours by markers, interpolating function
2. Compute marker velocities based on vorticity boundaries.
3. Move markers one time step with integration technique.
4. Insert markers as needed to faithfully represent curves .

Limitations:

1. 2-d: no way to represent 3rd dimension instabilities, vortex stretching of fully developed turbulence.
2. No way to incorporate bouyancy, viscosity, or diffusion.
3. Complexity increases with time: n squared.

Advantages:

1. High resolution, only limited by accumulated error.
2. Sharp gradients may be represented faithfully
3. Higher order 2-d instabilities not artificially suppressed.
4. Vortex behavior of flow easily visualized.
5. Faster than other methods for simple boundaries.

Results:

1. Linear growth from small disturbance (method check).
2. Maximum instability billow in initial vertical scalar gradient.
3. Cuts across scalar map compared to rocket data.

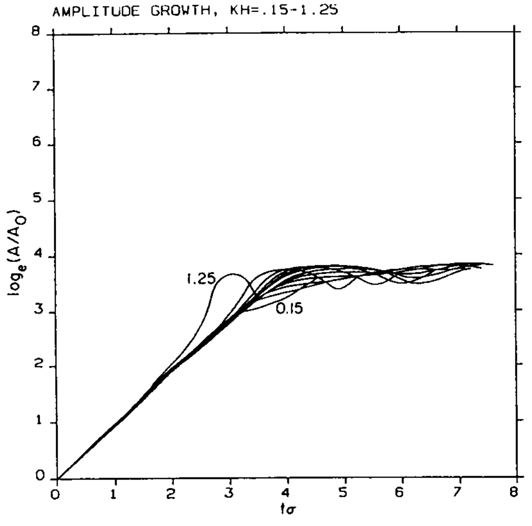

Figure 1. Billow amplitudes vs. time scaled by growth factor for kh = 0.15, 0.25, 0.35, 0.45, 055, 0.65, 0.75, 0.85, 0.95, 1.05, 1.15, and 1.25.

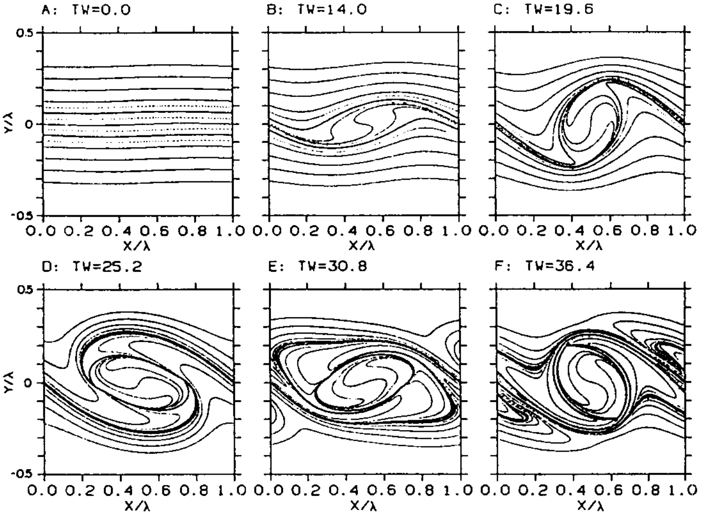

Figure 2. Scalar development in billow with time. (a) initial condition, with constant scalar vertical gradient. contours at y/λ = ± 0.06 are also boundaries of vorticity region. (b) tω = 14.0. (c) tω = 19.6. (d) tω = 25.2. (e) tω = 30.8. (f) tω = 36.4.

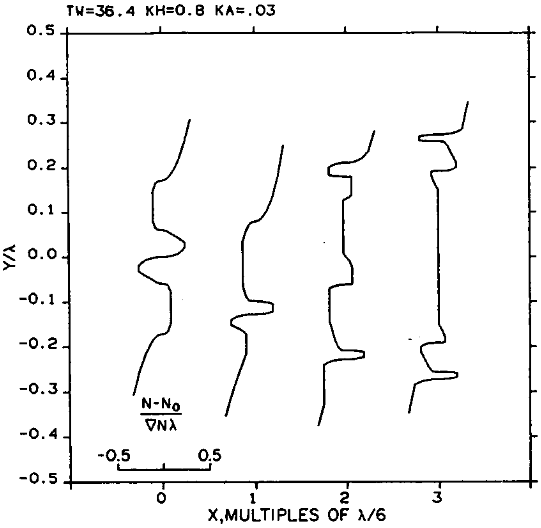

Figure 3. Vertical cross sections of scalar for pattern of Figure 2(f), through stagnation point, two intermediate locations between stagnation point and core, and core center.

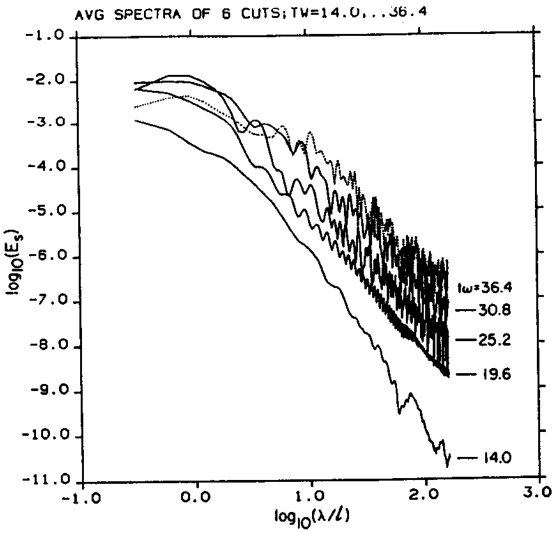

Figure 4. Log average of six energy spectral densities from cuts through scalar irregularities in simulated billows.

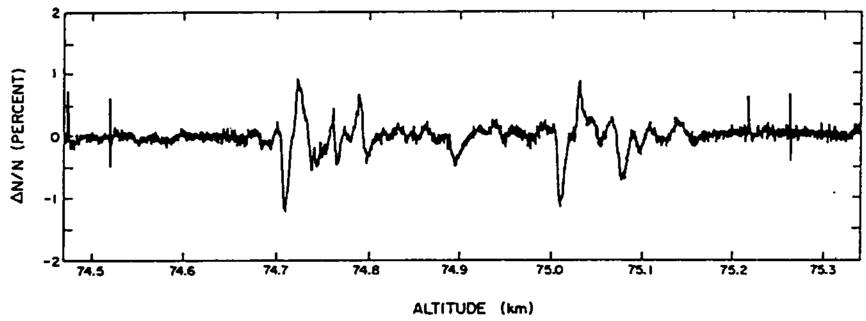

Figure 5. Reconstruction of digital electron density data for a portion of 28 May 1975 Peru rocket shot [Stoltzfus et al., *Adv. Space Res.*, *4*(6), 143, 1984].

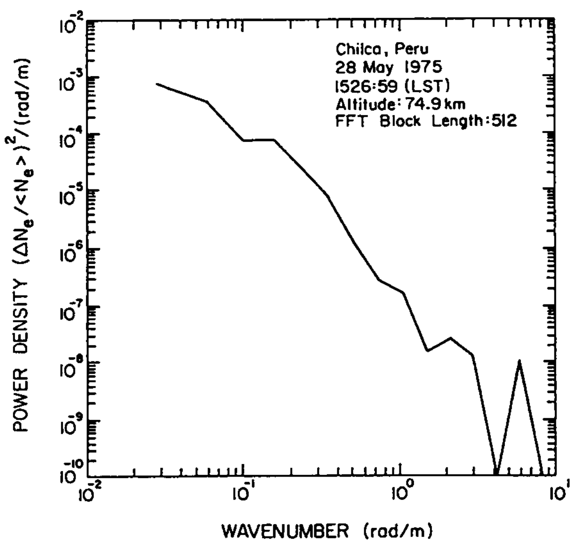

Figure 6. Power spectrum from 28 May 1975 Peru rocket shot. Spectrum is average of spectra from two parts of data of Figure 4, with noise estimate subtracted, and smoothing in wave number domain.