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FLEXIBLE MISSILE AUTOPILOT DESIGN STUDIES WITH PC-MATLAB/386

By

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ABSTRACT

Development of a responsive, high-bandwidth missile autopilot for airframes which have structural modes of unusually low frequency presents a challenging design task. Such systems are viable candidates for modern, state-space control design methods. The PC-MATLAB interactive software package provides an environment well-suited to the developement of candidate linear control laws for flexible missile autopilots. The strengths of MATLAB include: (1) Exceptionally high speed -- MATLAB's version for 80386-based PC's offers benchmarks approaching minicomputer and mainframe performance; (2) Ability to handle large design models of several hundred degrees of freedom, if necessary; and (3) Broad extensibility through user-defined functions. To characterize MATLAB capabilities, a simplified design example is presented. This involves interactive defintion of an observer-based state-space compensator for a flexible missile autopilot design task. MATLAB capabilities and limitations, in the context of this design task, are then summarized.

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PRESENTATION OVERVIEW

1. Introduction
2. MATLAB Background
3. Characteristics of MATLAB Environment
4. Classical Control Capabilities
5. Modern Control Design Example
6. Summary

INTRODUCTION

- JHU/APL acts as technical direction agent for US Navy weapon system programs

- A key task of APL's Guidance, Control, and Navigation Systems Group is the evaluation or conceptual design of missile guidance and control systems

- Analysis and design work requires a <u>flexible</u>, <u>interactive</u> linear modeling tool

- PC-MATLAB resident on 80386 engineering work-stations provides such a tool

- Work presented here shows general attributes of MATLAB, demonstrating use of PC-MATLAB/386 for linear design of a flexible missile autopilot

MATLAB BACKGROUND

- MATLAB (MATrix LABoratory) provides an interactive, matrix-oriented environment
- MATLAB is based on the EISPACK and LINPACK routines for matrix computations
- PC-MATLAB/386 is a high-performance MATLAB implementation for 80386-based workstations
- MATLAB built-in functions, plus higher-level functions developed for control system calculations, allow for effective controls design studies

HARDWARE AND SOFTWARE CONFIGURATION

- COMPAQ 386/20 computer
- Weitek 1167 numeric coprocessor
- PC-MATLAB/386 with Control Systems Toolbox

PC-MATLAB/386 ATTRIBUTES

- Interactive, high-level command environment

- Very high processing speed

- Easy extensibility via user-defined functions

A MATLAB INTERACTIVE COMMAND LINE EXAMPLE

```
>> k = lqr(a,b,q,rho*r); eig(a-b*k), y = step(a-b*k,b,c,d,1,t); plot(t,y);
```

- The <u>single line</u> above, typed at the MATLAB command line prompt, does several things:
  - Computes a quadratic regulator gain vector
  - Displays the closed-loop eigenvalues -- often useful for confirming that actuator bandwidth requirements are not excessive
  - Computes and plots a unit step response
- By varying the control cost (rho) above, a very large family of compensators may quickly be considered
- The above command line suggests the power and utility available from a high-level, interactive matrix language

PC-MATLAB/386 PROCESSING SPEED

- MATLAB's LINPACK Benchmark: 460 double precision KFLOPS

- This processing speed is:
  - 25 x faster than standard PC/AT
  - 6 x faster than Mac II
  - 3 x faster than MicroVax II

- Implication: the fast response time resulting from such performance allows for truly interactive design iterations on complex control laws

MATLAB EXTENSIBILITY

- User-defined functions may be developed through creation of simple text files

- Some typical user-defined functions:
  - Frequency-response plotting routines
  - Application-specific linear transformations
  - Multivariable Nyquist criterion

- Complex state-space or transfer-function models also defined through user text files

AN EXAMPLE OF A USER-DEFINED COMMAND FILE

- **Below command set calculates and plots the maximum and minimum singular values of a plant and observer-based compensator, for a loop broken at plant input**

```
function [smin,smax] = svdinput(a,b,c,kcon,kobs,w);
%
jay = sqrt(-1);
[nn,xx]=size(a); i2=eye(nn); [ng,xx]=size(c*a*b); phi = '(s*i2-a)';
for i = 1:nc;
   s = w(i)*jay; phieval = eval(phi);
   gs = c/phieval*b; ks = kcon / (phieval+b*kcon+kobs*c) * kobs;
   xx=svd(ks*gs); smin(i)=xx(ng); smax(i)=xx(1);
end;
%
%
% convert to decibels and plot output
%
smin=20*log10(smin); smax=20*log10(smax);
semilogx(w,smin,w,smax,'r--'); grid;
title('Max and Min Singular Values; Loop Broken at Plant Input ');
xlabel('Frequency  (rad/sec)'); ylabel('Magnitude  (db)');
```

- **Procedure requires only eleven lines of executable MATLAB code**

CLASSICAL CONTROL CAPABILITIES

- Frequency response
- Root locus
- Nyquist plots
- Development of dynamic compensators (lead-lag, notch filters, etc)

MODERN CONTROL DESIGN EXAMPLE

- Design plant describes tactical missile at a high-altitude flight condition
- Design plant includes single-plane rigid-body dynamics and effect of first flexible mode on sensed pitch rate
- Objective is to develop an autopilot to track commanded accelerations
- Design challenge is to achieve high closed-loop bandwidth in presence of low-frequency bending modes

DESIGN APPROACH

- Establish design goals for closed-loop responsiveness and stability
- Develop full-state feedback (LQR) gains for design plant
- Define linear observer to reconstruct full state vector
  - Use "robust observer" design (Doyle and Stein, 1979 IEEE Transactions on Automatic Control)
  - Adjust observer gains to recover original LQR loop transfer in desired frequency range

DESIGN PLANT MODEL

- Fifth-order state vector $\underline{x}$; $\dot{x} = \underline{A}\underline{x} + \underline{b}u$

- $\underline{x} = [q_r \quad q_r/s \quad a/s \quad q_f/s \quad q_f]$

- First three state variables are associated with rigid-body airframe; the last two describe flexible mode dynamics

- Rate gyro measurement: $[1\ 0\ 0\ 0\ 1] * \underline{x}$

- (Integrated) accelerometer measurement: $[0\ 0\ 1\ 0\ 0] * \underline{x}$

$$\mathbf{A} = \begin{bmatrix} 0 & -2.3557e{+}02 & 1.7967e{+}02 & 0 & 0 \\ 1.0000e{+}00 & 0 & 0 & 0 & 0 \\ 0 & 2.6158e{+}00 & -1.9951e{+}00 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0000e{+}00 \\ 0 & 0 & 0 & -2.4649e{+}04 & -3.1400e{+}00 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} -2.8031e{+}02 \\ 0 \\ 9.2587e{-}02 \\ 0 \\ 3.0723e{+}02 \end{bmatrix}$$

SOME OBSERVATIONS ON DESIGN PLANT MODEL

- Feedback of the first three states describes a very standard (rigid-body) autopilot topology, used by tactical missiles since 1950's

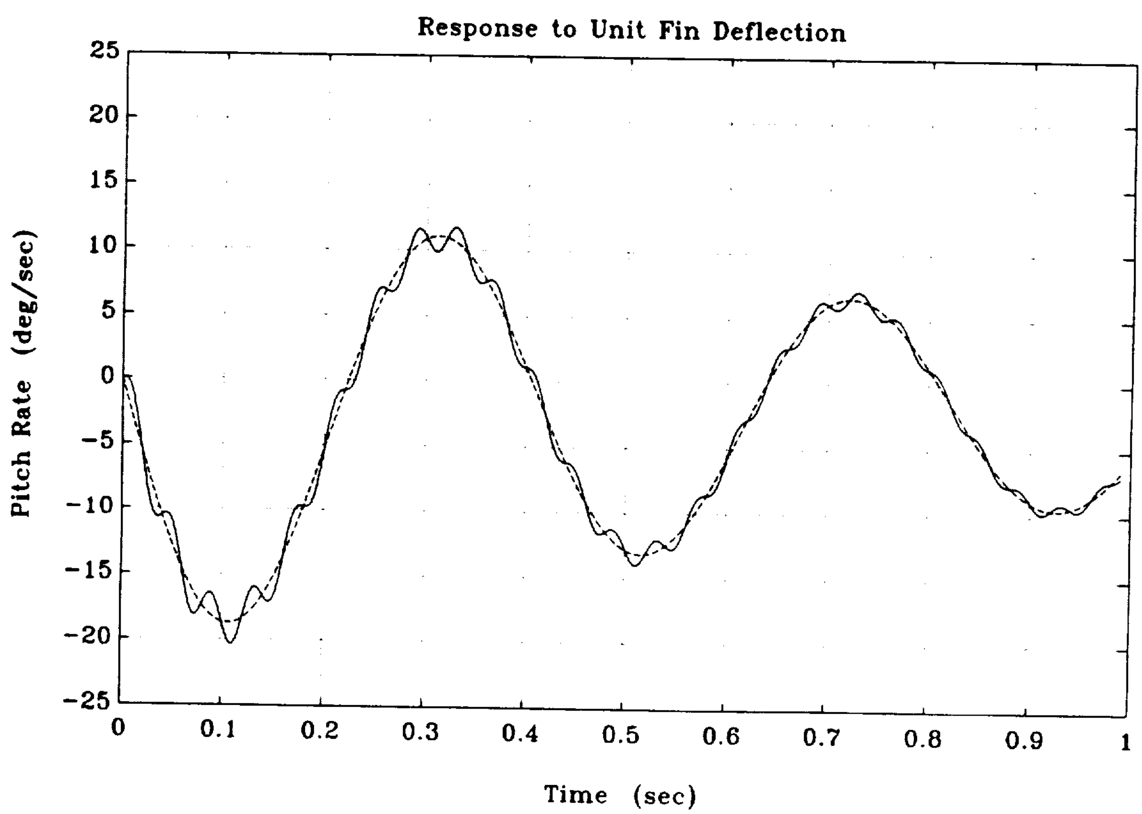
- Open-loop plant is characterized by lightly damped airframe (weathercock) poles, and by bending mode poles

  - Airframe pole frequency lies at nominal 2.5 Hz

  - Bending mode has nominal 25 Hz natural frequency

- Desired autopilot crossover frequency here will lie near the bending mode frequency

EFFECT OF STRUCTURAL MODE ON SENSED PITCH RATE (RATE GYRO MEASUREMENT)



CONTROLLABILITY AND OBSERVABILITY PROPERTIES OF PLANT

- System ($\underline{A}$,$\underline{b}$) is controllable

- System is unobservable if rate gyro alone, or accelerometer alone, is used as the measurement to reconstruct state vector

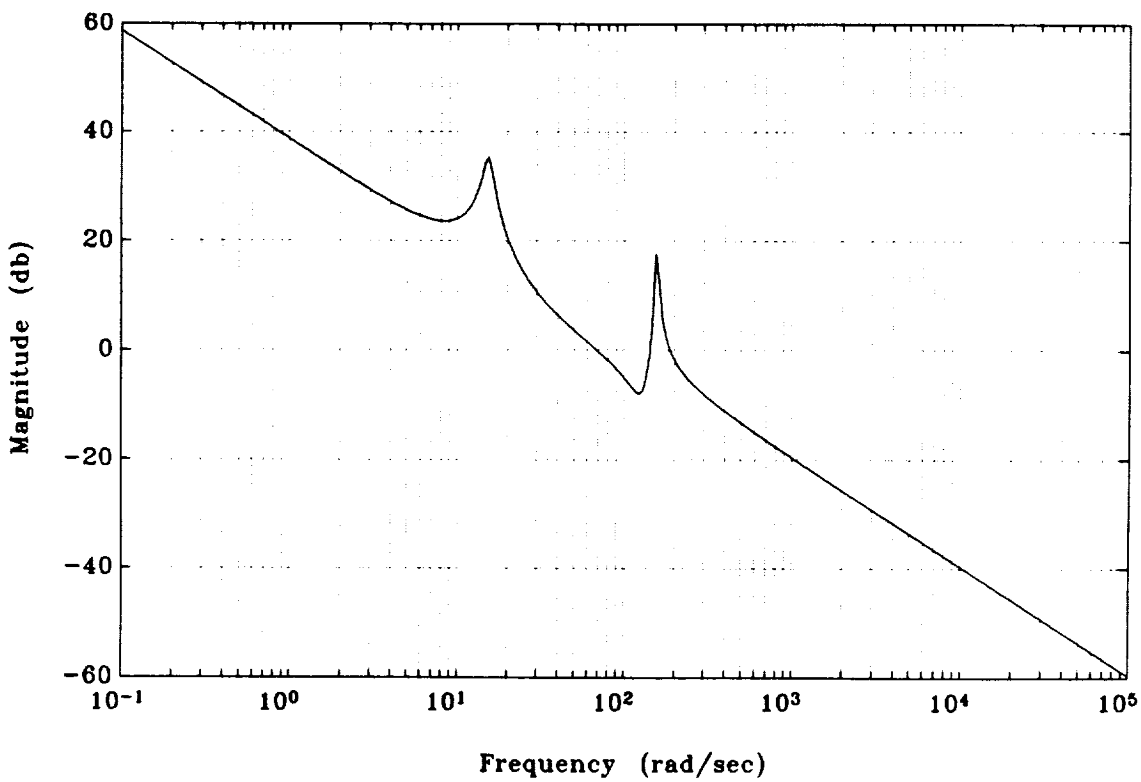
- Both sensor outputs thus should be used in the observer design

- Approach taken for this application:

  - Define a (non-square) design plant having one input (fin deflection) and two independent outputs (gyro and accelerometer)

  - Use extensions of loop transfer recovery (Williams and Madiwale, 1985 ACC) valid for non-square systems

FREQUENCY RESPONSE OF FULL-STATE FEEDBACK (LQR) SYSTEM (LOOP BROKEN AT PLANT INPUT)



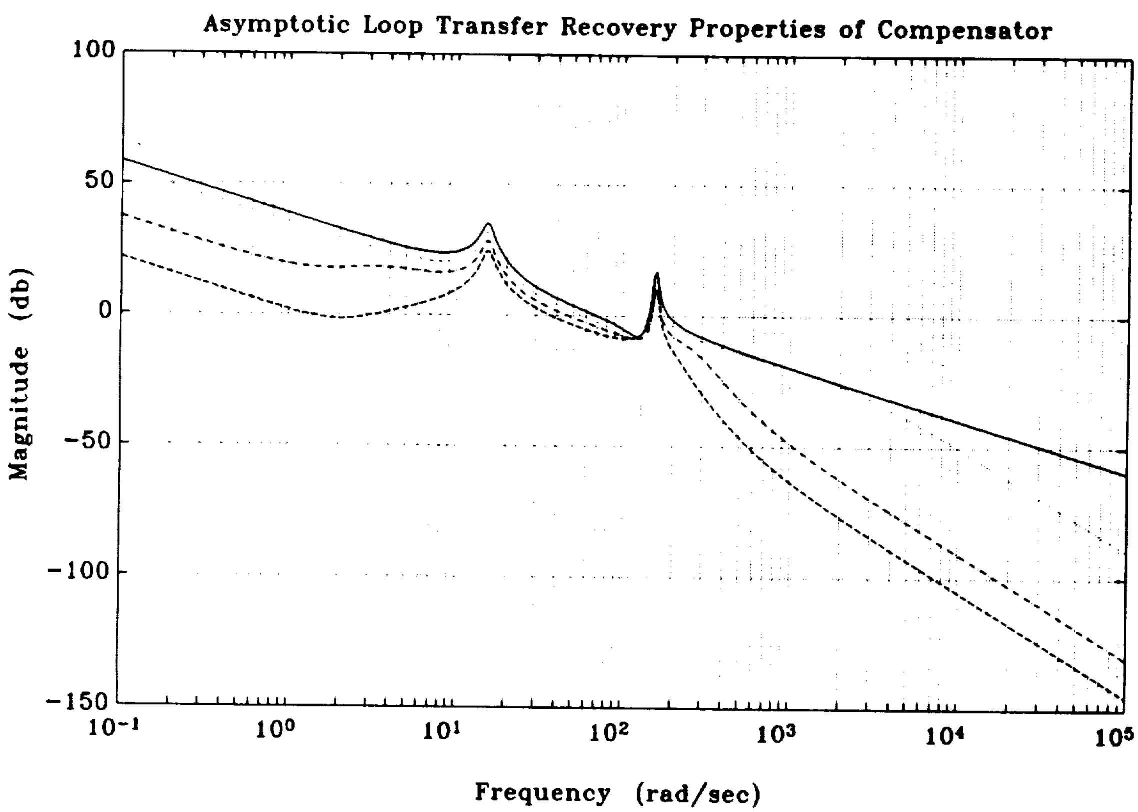
OBSERVATIONS ON LOOP TRANSFER RECOVERY PROCEDURE

- For this application, recovery at both the (rigid-body) airframe and bending mode frequencies may only be achieved with very high observer gains

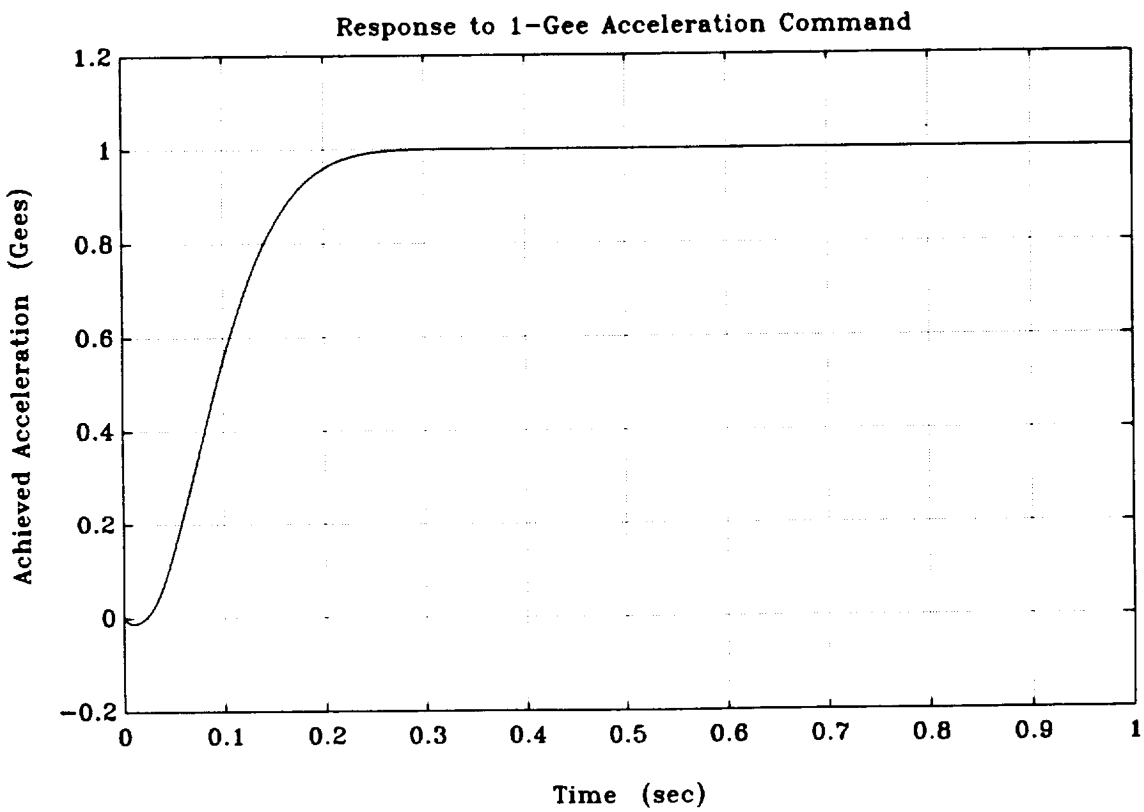
- For practical ranges of observer gains, recovery at airframe frequencies is obtained at the cost of lessened robustness in the structural mode frequency range

- Use of a set of user-defined MATLAB files, to implement a range of observer gain calculations, makes evaluation of this robustness tradeoff straightforward

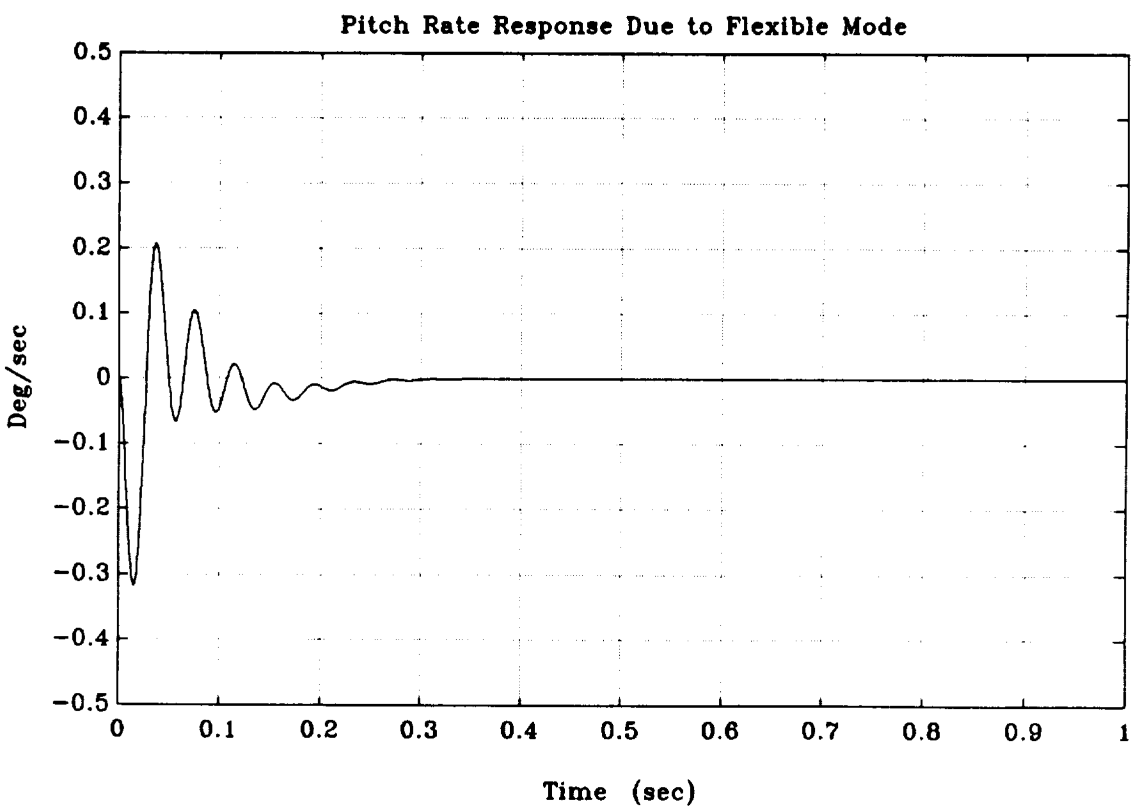
RECOVERY OF DESIRED FULL-STATE FEEDBACK SYSTEM WITH MODEL-BASED COMPENSATOR



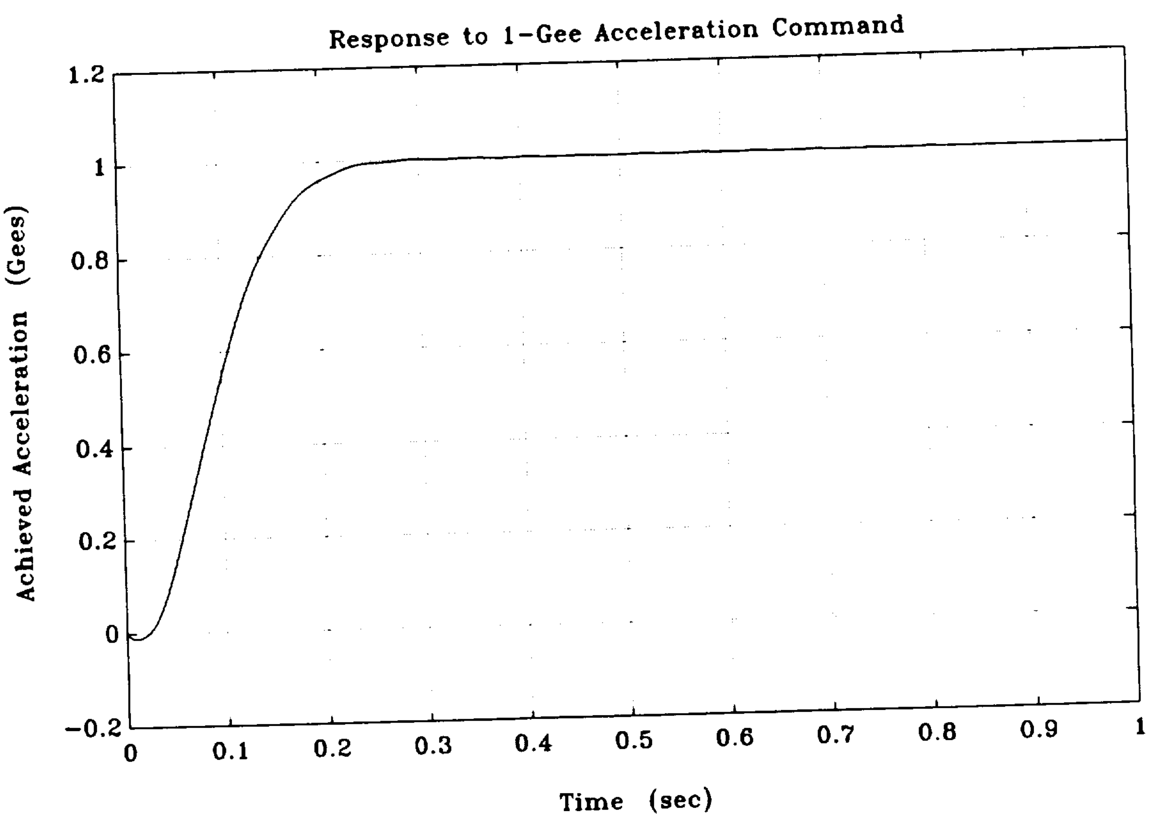
ACCELERATION STEP RESPONSE OF FINAL COMPENSATOR DESIGN



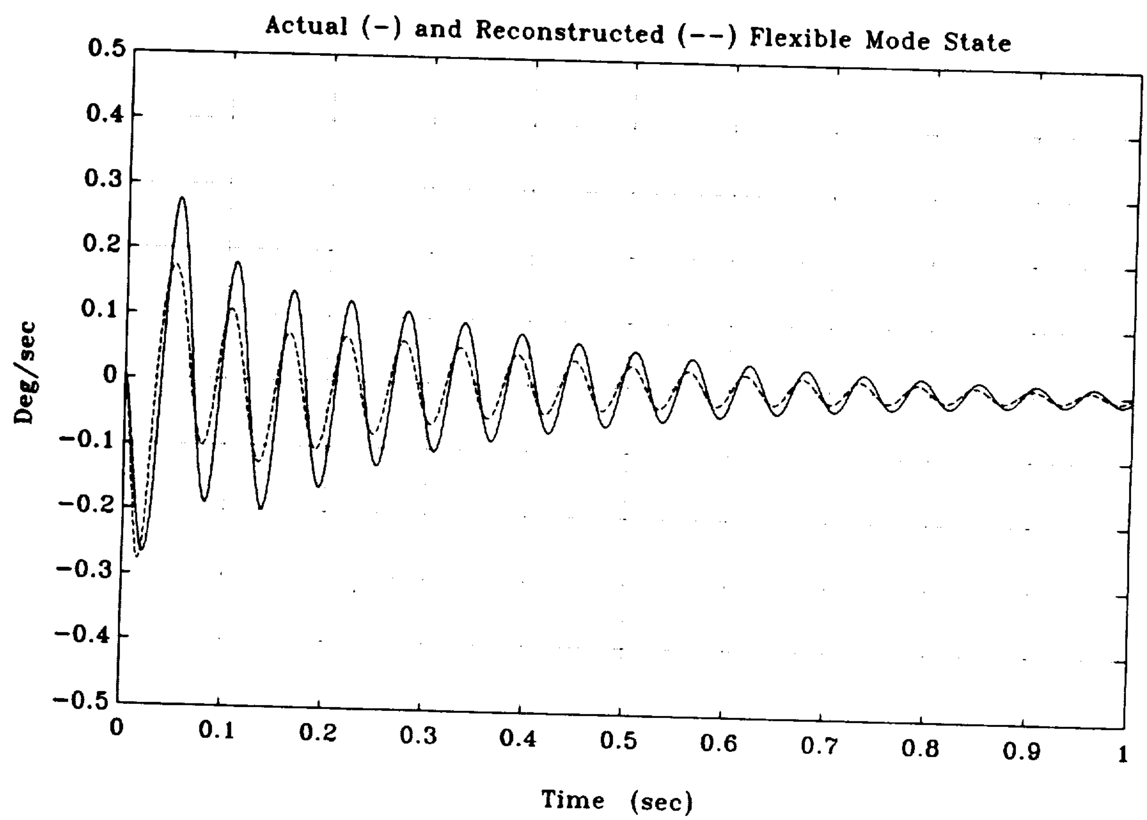
RESPONSE OF FLEXIBLE MODE STATE DURING ACCELERATION STEP RESPONSE



ACCELERATION STEP RESPONSE FOR CASE WHEN BENDING MODE IS PERTURBED TO 25 % LOWER VALUE



COMPARISON OF ACTUAL AND RECONSTRUCTED FLEXIBLE MODE STATE DURING STEP RESPONSE -- BENDING MODE PERTURBED TO 25 % LOWER VALUE



SUMMARY OF DESIGN RESULTS

- Model-based compensator yields a high-bandwidth autopilot, which is robust to at least a 25% perturbation in bending mode frequency

- A number of issues still not addressed:
  - Detailed noise sensitivity assessment
  - Effect of higher-frequency structural modes
  - Phase lag from actuator dynamics
  - Effect of structural modes on accelerometer measurement
  - Tolerance to uncertainties in aerodynamics

- Above concerns could also be addressed using MATLAB

SUMMARY: MATLAB APPLICABILITY FOR CONTROL DESIGN OF FLEXIBLE SYSTEMS

- MATLAB provides the necessary tools for a variety of control system design techniques

- Extensibility of MATLAB allows development of tools to implement recent modern control design methods, including loop transfer recovery

- Implementation for 80386-based machines (PC-MATLAB/386) has very high performance, allowing for interactive control design of complex systems such as flexible structures

- Any flexible structures control problem which can be cast into a state-space framework may benefit from design work with MATLAB