ENHANCED ELEMENT-SPECIFIC MODAL FORMULATIONS FOR FLEXIBLE MULTIBODY DYNAMICS

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ABSTRACT

The accuracy of current flexible multibody formalisms based on assumed modes is examined in the context of standard spacecraft motions involving structural components undergoing both slow and fast overall translational and rotational motions as well as small deformations. Limitations of current techniques in treating (1) element-specific coupling behavior of large motion and small deformation, and (2) motion-induced structural stiffness variations, are noted.

The roles of nonlinear and linear elastic structural theories in accurately predicting transient large-displacement dynamic behavior of flexible multibody systems are examined in detail. Coupling effects between deformation and overall motion are carefully scrutinized in the context of assumed-mode discretization techniques. Consistently linearized beam, plate, and shell formulations involving in-plane stretch variables are proposed and shown to yield very accurate simulation results and extremely fast modal convergence for most motions involving small strains. In some particular cases, however, in which membrane stiffness dominates bending stiffness, a nonlinear strain formulation is required in order to capture proper coupling between deformation and overall motion. Unfortunately, with standard component modes, algorithmic formalisms involving nonlinear strain-displacement expressions show very slow modal convergence. A procedure involving use of constraint modes is proposed to alleviate this problem.

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ENHANCED ELEMENT-SPECIFIC MODAL FORMULATIONS

FOR

FLEXIBLE MULTIBODY DYNAMICS

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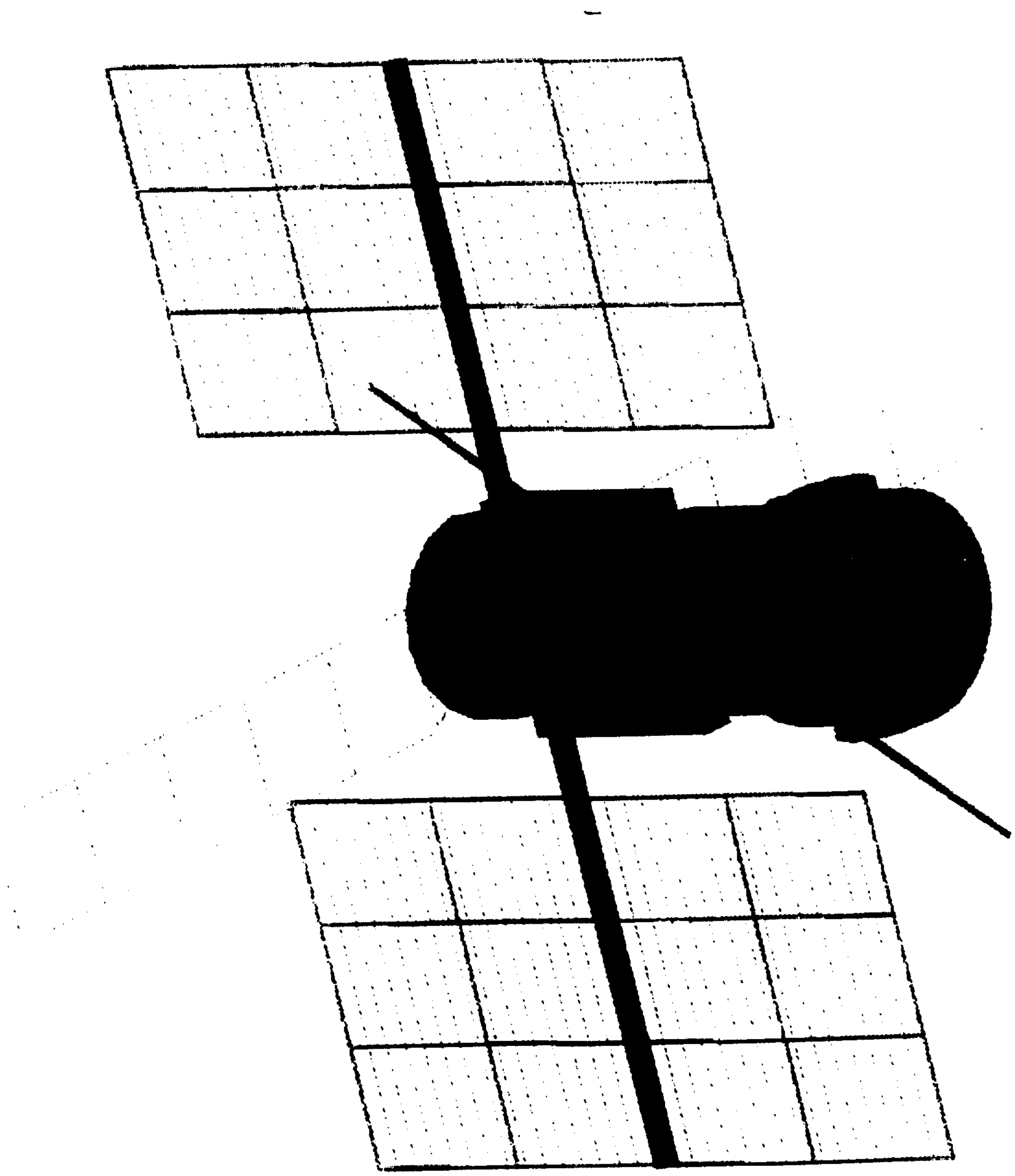


////////// Outline \\\\\\\\\

I. Limitations of Existing Flexible Multibody Formalisms

- Examples
- Verification

II. Linear and Nonlinear Element-Specific Formulations

- Consistently-Linearized Beam, Plate, Shell Multibody Models
- Second-Order Beam, Plate Models

III. Simulation Results

- Membrane/Bending Problems
- Convergence

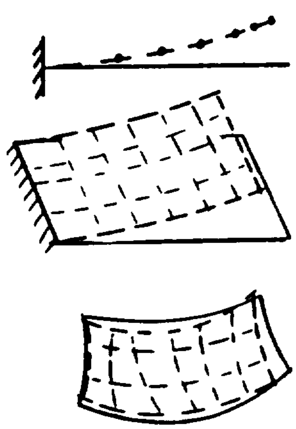
Current Flexible Multibody Formalisms - Modal Approach "Limitations"

- Do Not Account For Large-Displacement Element-Specific Behavior
- Inadequate Account of Motion-Induced Stiffness Variations

Finite Element Component Model

Structural Model

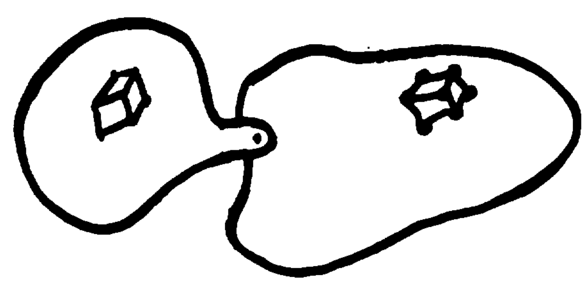
BEAMS, PLATES, SHELLS, AXISYMMETRIC SOLIDS, 3-D SOLIDS, PLANE STRESS, PLANE STRAIN, *etc.*



Flexible Multibody System Model

Generic Body

3-D CONTINUA

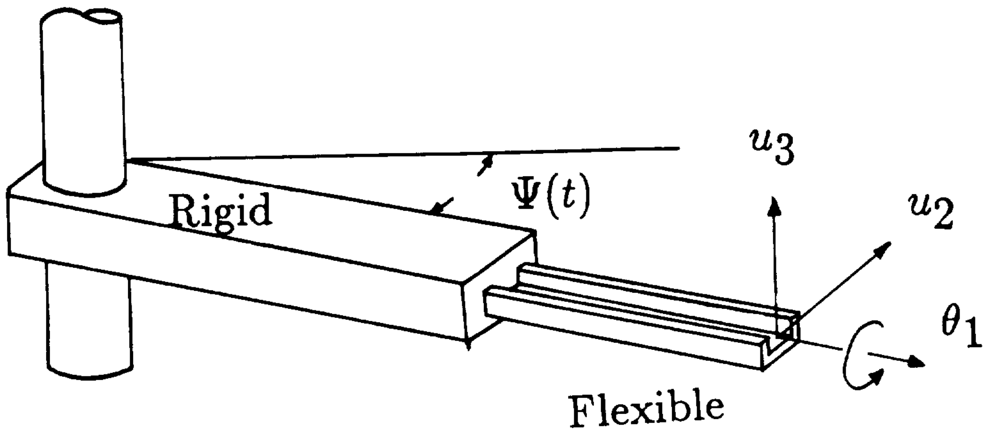


Eigensolution (Linear) → Modal Integral Processor → Simulation (Nonlinear) → *Solution*

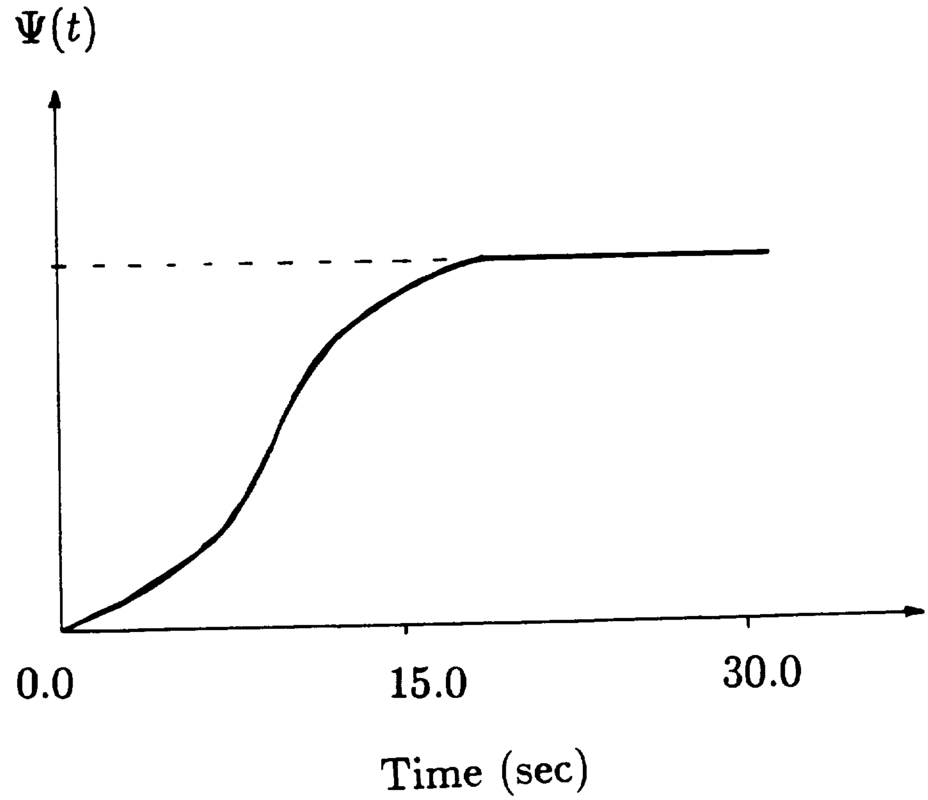
$\lambda_i, \phi_{k,i}$

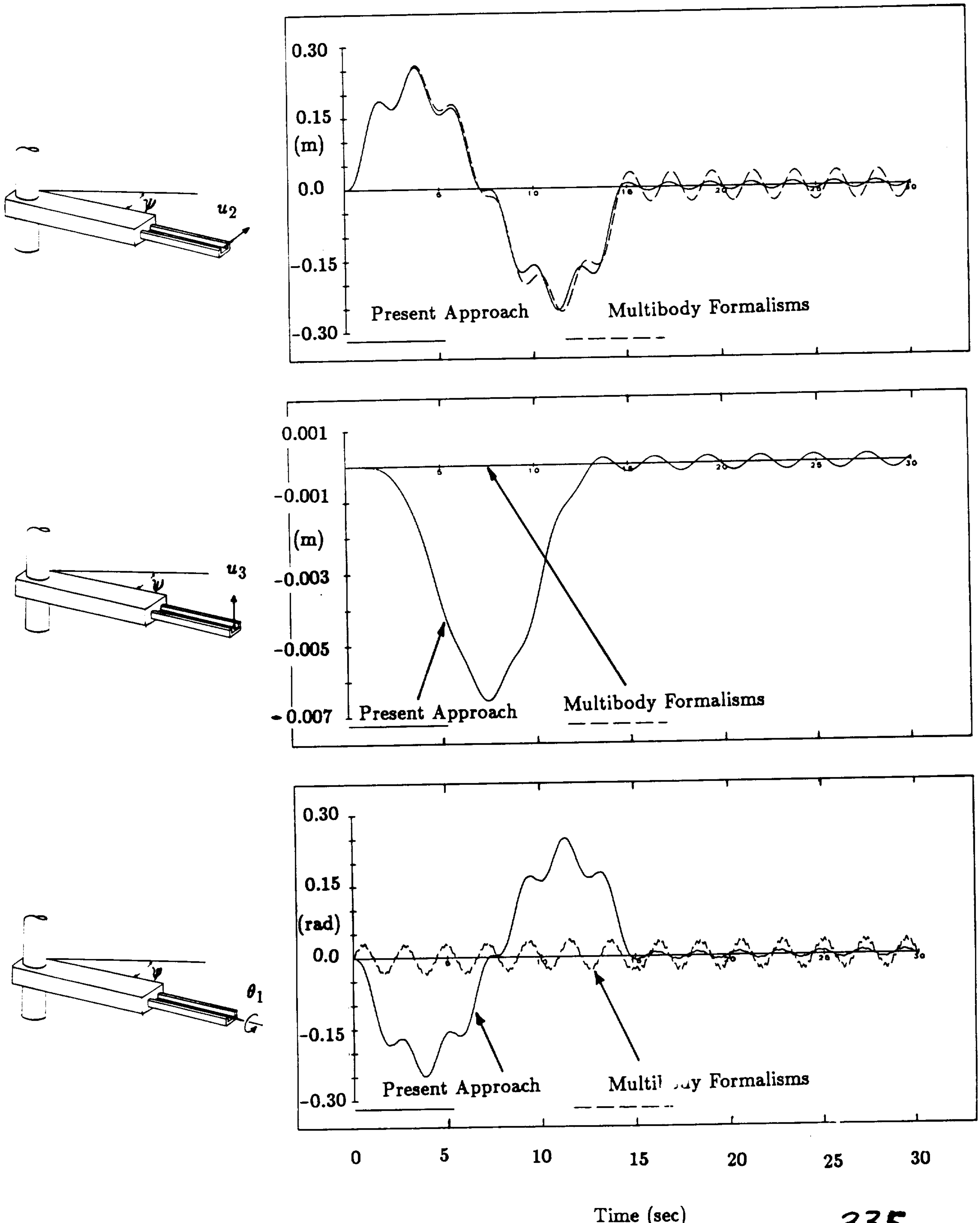
$$S_{ij} = \int_0^L \rho \phi_{1i} \phi_{1j} dx$$

Slow Repositional Maneuver of Channel Beam



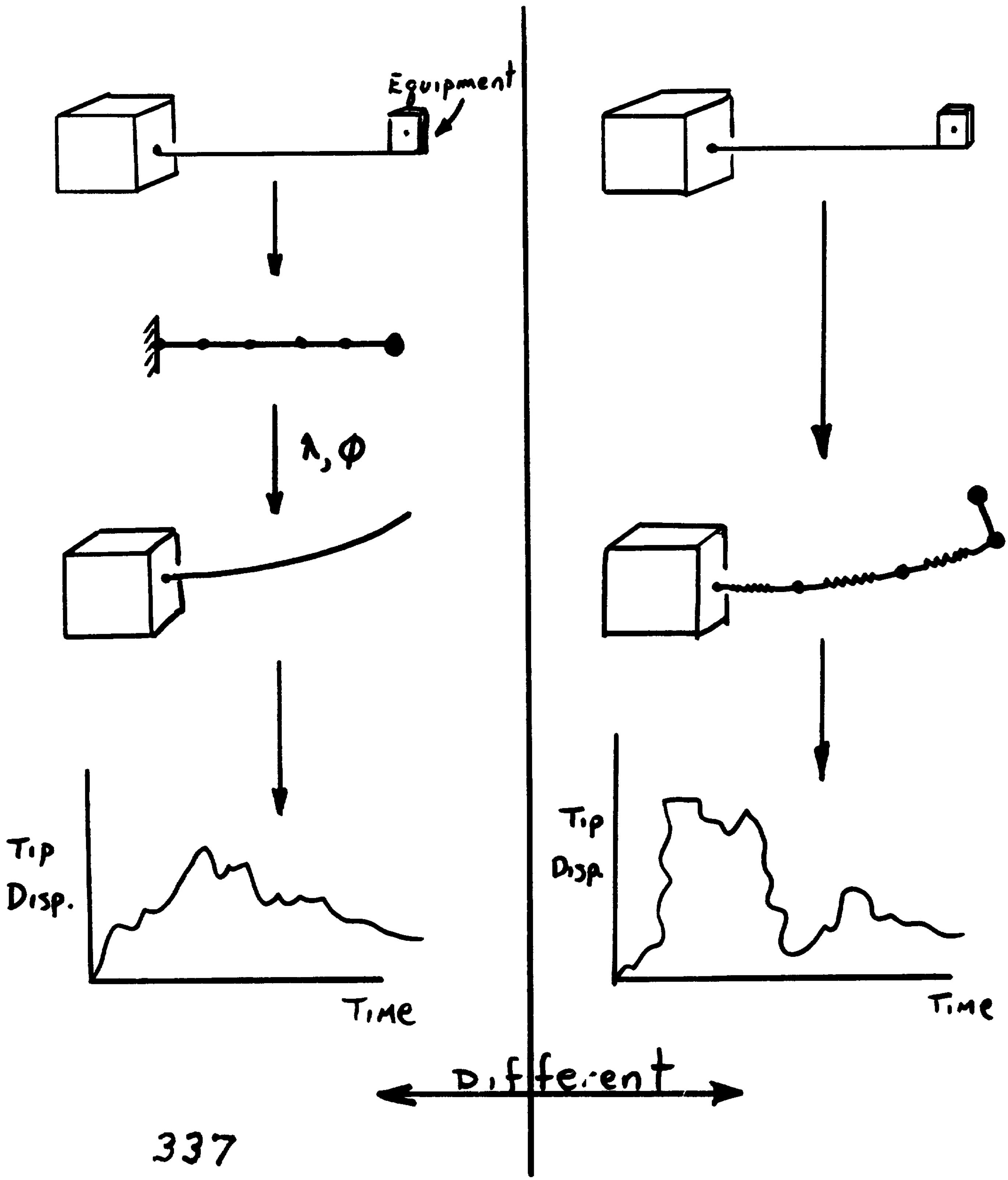
Repositional Maneuver Angle





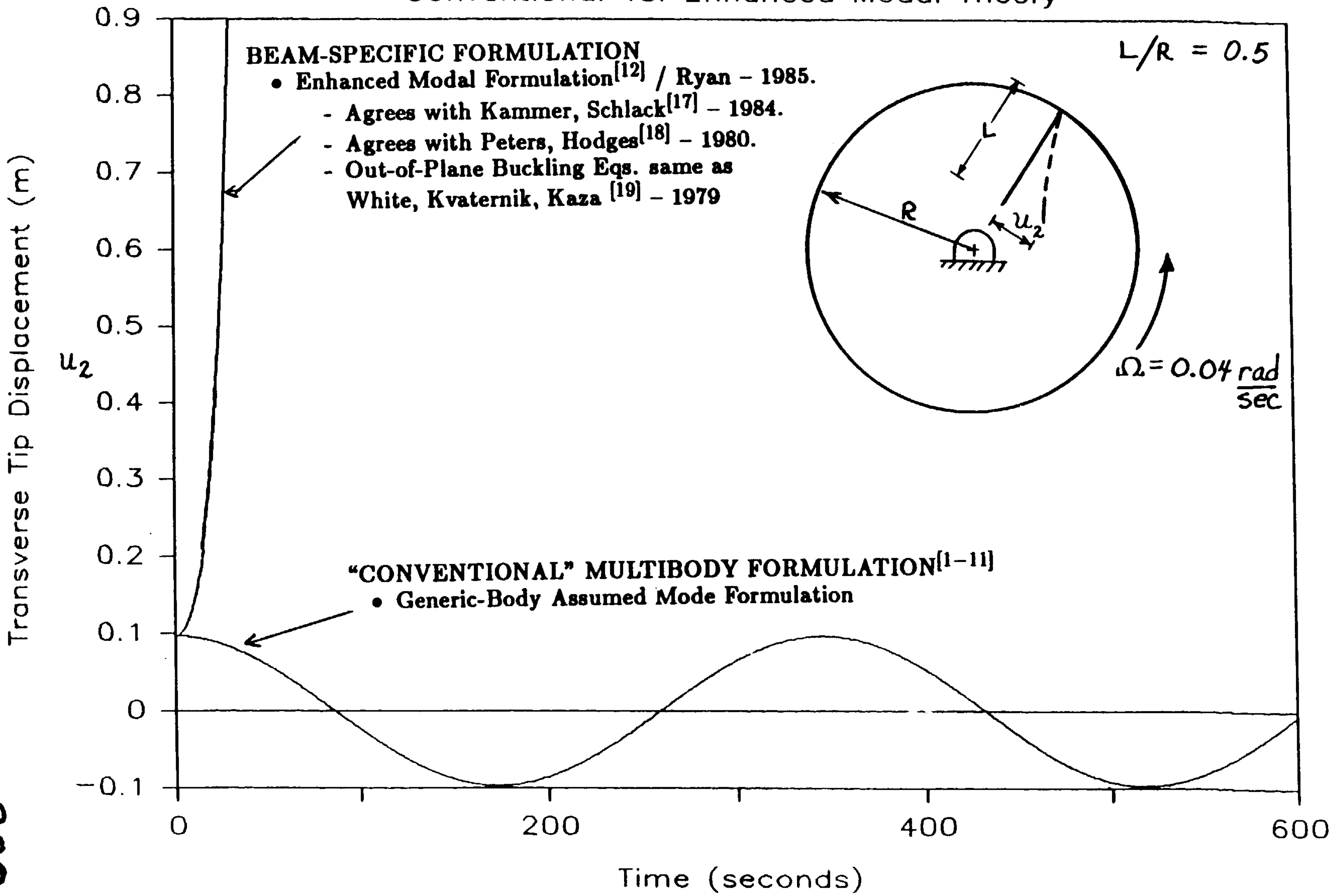
u_2
0.30
0.15
(m)
0.0
-0.15
-0.30
Present Approach
Multibody Formalisms
u_3
0.001
-0.001
(m)
-0.003
-0.005
-0.007
Present Approach
Multibody Formalisms
θ_1
0.30
0.15
(rad)
0.0
-0.15
-0.30
Present Approach
Multibody Formalisms
0
5
10
15
20
25
30
Time (sec)

BEAM WITH OFFSET TIP MASS



Constant Speed Spin — Buckling Analysis

Conventional Vs. Enhanced Modal Theory



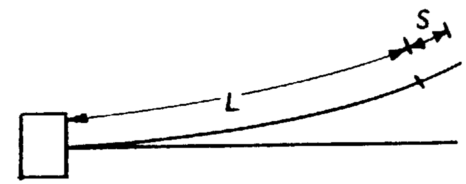
////////// Alternatives to Present \\\\\\\\\\\\\\\\\\\\
////////// Flexible Multibody Dynamic Formalisms \\\\\\\\\\\\\\\\\\\\

- Discrete Representations
- Nonlinear Finite Element Methods
- Linear and Nonlinear
  Enhanced Modal Approaches

Consistently-Linearized Multibody Structural Theories

- Beam

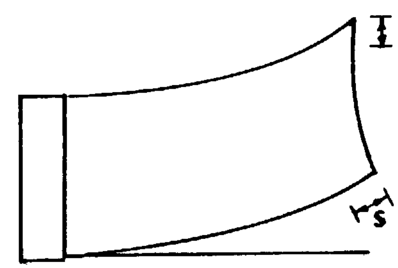
$$s \stackrel{\triangle}{=} u_1 + \frac{1}{2}\int_0^x [(\frac{\partial u_2}{\partial \sigma})^2 + (\frac{\partial u_3}{\partial \sigma})^2] d\sigma$$



- Plate

$$r \stackrel{\triangle}{=} u_2 + \frac{1}{2}\int_0^y (\frac{\partial u_3}{\partial \eta})^2 d\eta$$

$$s \stackrel{\triangle}{=} u_1 + \frac{1}{2}\int_0^x (\frac{\partial u_3}{\partial \xi})^2 d\xi$$



$$M\ddot{q} + G\dot{q} + (K_I + K_L + K_g)q = F$$

Advantages:
- Excellent Convergence
- Captures Motion-Induced Bending Stiffness Variation
- Ease of Modal Reduction/Controls
- Easily-Implemented – Linear in Deformation

Disadvantages:
- Doesn't Capture Motion-Induced Membrane Stiffness

Second-Order Structural Multibody Theories

- Beam

$$U_b = \frac{1}{2}\int_0^{\ell} E\left\{I_{zz}\left(\frac{\partial^2 u_2}{\partial x^2}\right)^2 + I_{yy}\left(\frac{\partial^2 u_3}{\partial x^2}\right)^2\right\}dx$$

$$U_s = \frac{1}{2}\int_0^{\ell} EA\left\{\left[\left(\frac{\partial u_1}{\partial x}\right) + \frac{1}{2}\left(\frac{\partial u_2}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial u_3}{\partial x}\right)^2\right]^2\right\}dx$$

- Thin Rectangular Plates:

$$U_b = \frac{1}{2}\int_0^{b}\int_0^{a} \beta\left\{\left(\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2}\right)^2 -2(1-\nu)\left[\left(\frac{\partial^2 u_3}{\partial x^2}\right)\left(\frac{\partial^2 u_3}{\partial y^2}\right) - \left(\frac{\partial^2 u_3}{\partial x \partial y}\right)^2\right]\right\}dxdy$$

$$\begin{aligned}U_s = \frac{1}{2}\int_0^{b}\int_0^{a} \gamma\Bigg\{&\left(\frac{\partial u_1}{\partial x}\right)^2 + \left(\frac{\partial u_1}{\partial x}\right)\left(\frac{\partial u_3}{\partial x}\right)^2 + \left(\frac{\partial u_2}{\partial y}\right)^2 \\ &+\left(\frac{\partial u_2}{\partial y}\right)\left(\frac{\partial u_3}{\partial y}\right)^2 + \frac{1}{4}\left[\left(\frac{\partial u_3}{\partial x}\right)^2 + \left(\frac{\partial u_3}{\partial y}\right)^2\right]^2 \\ &+2\nu\left[\left(\frac{\partial u_1}{\partial x}\right)\left(\frac{\partial u_2}{\partial y}\right) + \frac{1}{2}\left(\frac{\partial u_2}{\partial y}\right)\left(\frac{\partial u_3}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial u_1}{\partial x}\right)\left(\frac{\partial u_3}{\partial y}\right)^2\right] \\ &+\frac{(1-\nu)}{2}\Bigg[\left(\frac{\partial u_1}{\partial y}\right)^2 + 2\left(\frac{\partial u_1}{\partial y}\right)\left(\frac{\partial u_2}{\partial x}\right) + \left(\frac{\partial u_2}{\partial x}\right)^2 \\ &+2\left(\frac{\partial u_1}{\partial y}\right)\left(\frac{\partial u_3}{\partial x}\right)\left(\frac{\partial u_3}{\partial y}\right) + 2\left(\frac{\partial u_2}{\partial x}\right)\left(\frac{\partial u_3}{\partial x}\right)\left(\frac{\partial u_3}{\partial y}\right)\Bigg]\Bigg\}dxdy\end{aligned}$$

$$M\ddot{q} + G\dot{q} + (K_I + K_L + K_n)q = F$$

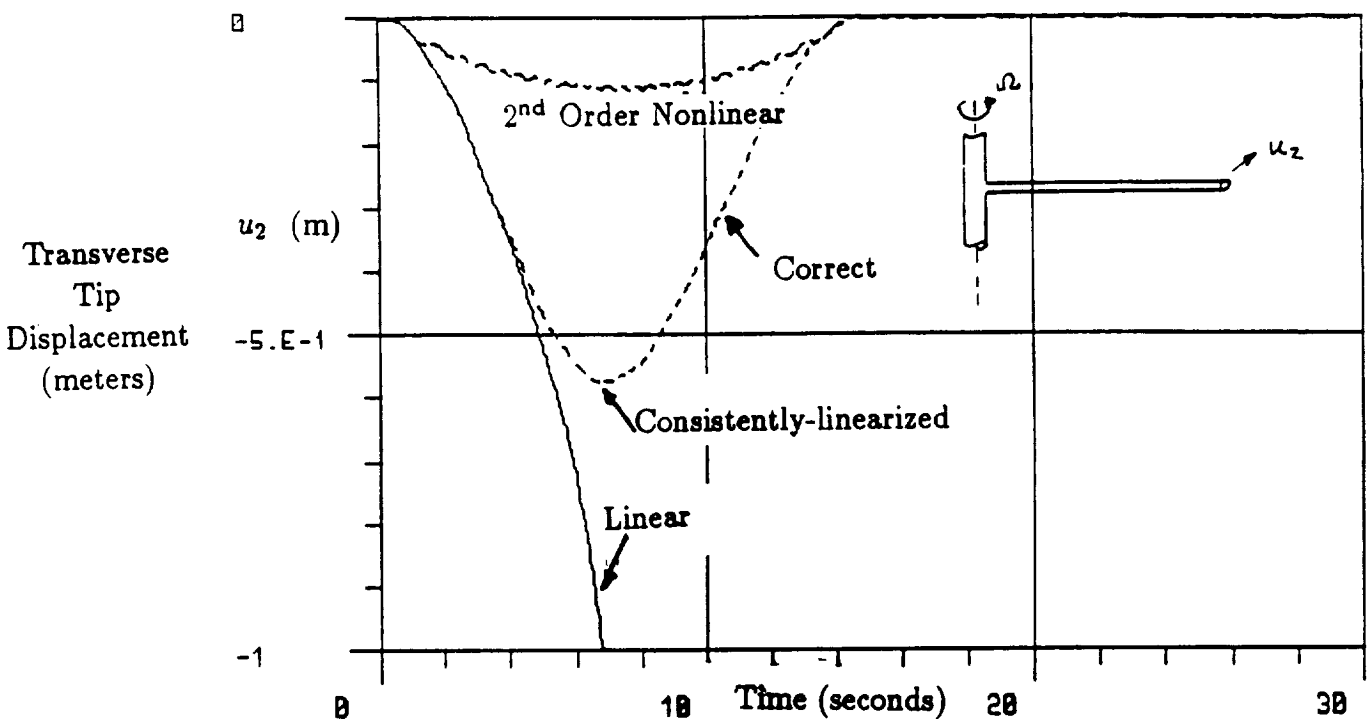
Advantages:

- Captures Important Motion-Induced Bending AND Membrane Stiffness Variations for Small Strain

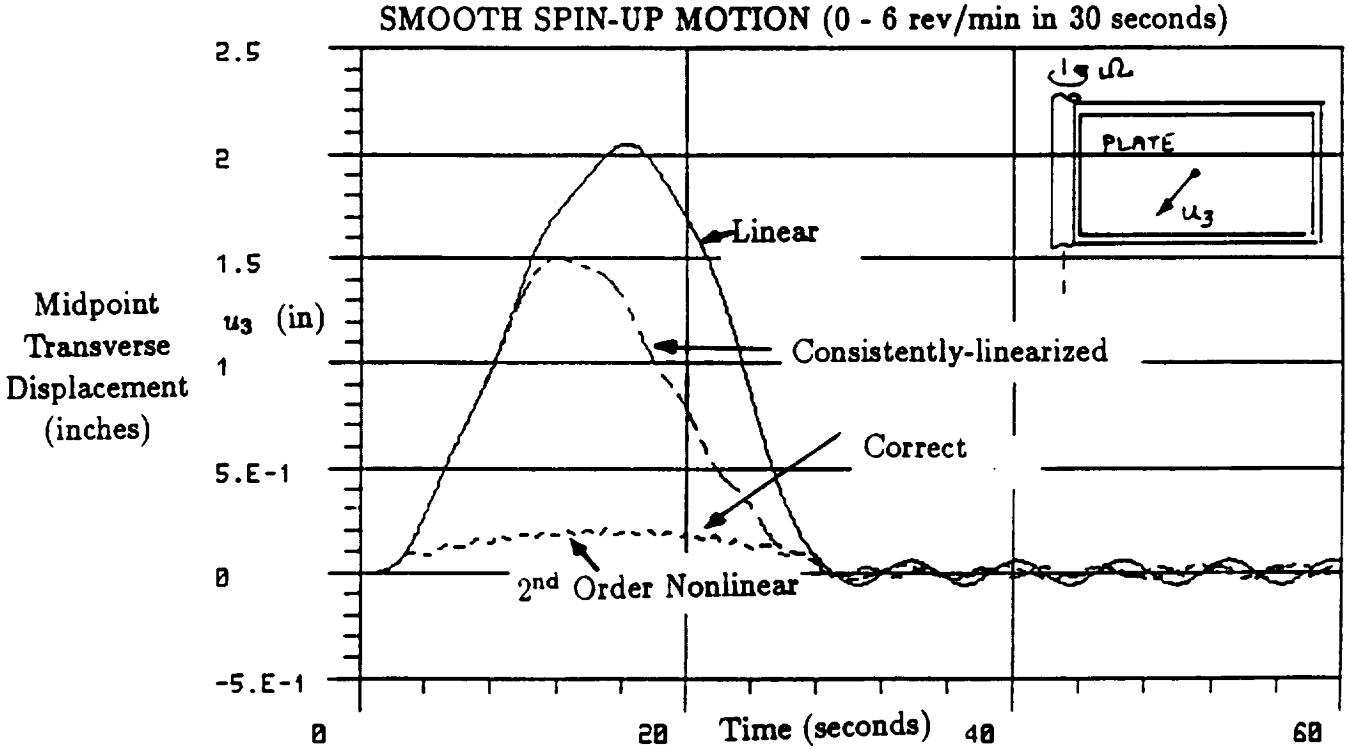
Disadvantages:

- Poor Convergence With Standard Modes
- Order Reduction Results in Very Inaccurate Models
- Very Costly to Incorporate

SMOOTH SPIN-UP MOTION (0 - 6 rad/sec in 15 seconds)

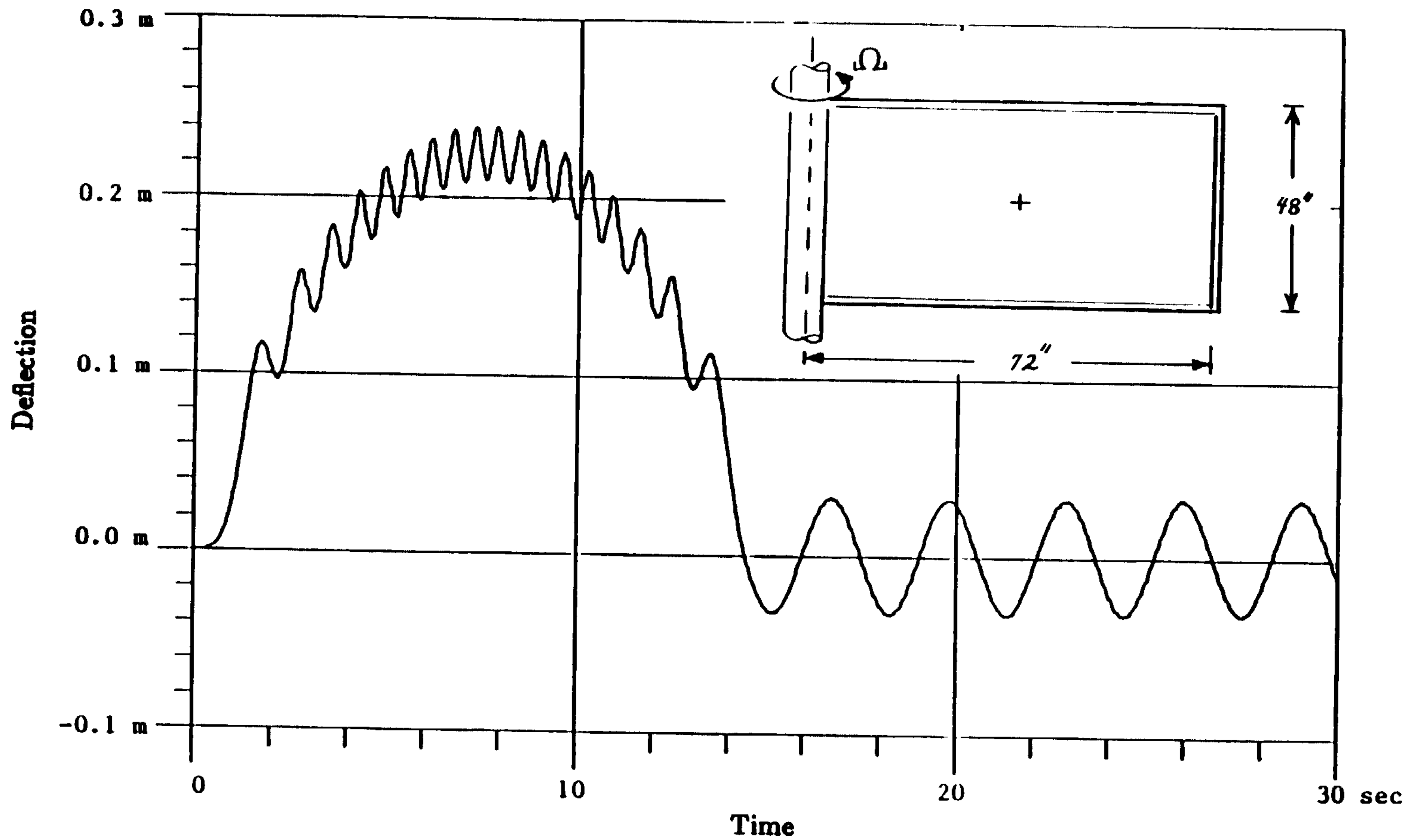


SMOOTH SPIN-UP MOTION (0 - 6 rev/min in 30 seconds)



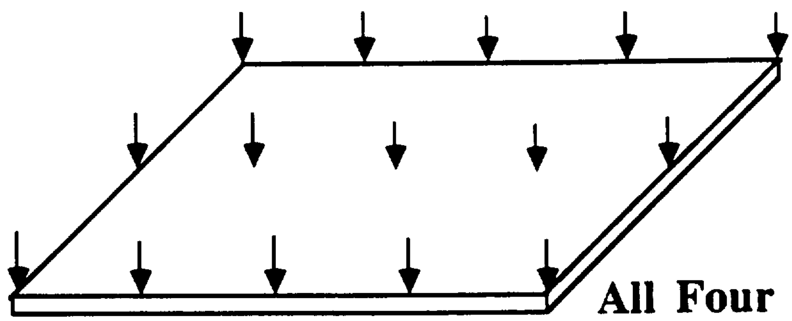
SIMPLY SUPPORTED RECTANGULAR PLATE SPIN-UP MOTION
Assumed Mode Approach with Nonlinear Strain Expression

Transverse Deflection of Center of Plate - Smooth Spin-up From 0 - 6 rad/sec in 15 seconds



Results obtained with 3 Assumed Stretch Modes and 3 Assumed Bending Modes

Static Analysis of a Square Plate with Uniform Pressure Distribution Considering only Membrane Stiffness



Maximum inertia force per area in the middle of the plate during the spin-up motion is used as uniform pressure distribution.

Results of Maximum Lateral Deflection

ω \ case	Static	Dynamic	
		Present	Conventional
$\pi/5$	0.18"	0.21"	2.1"
π	0.32"	0.37"	Divergence

Fig. 19 – Static Deflections under High Pressure Loads

CONCLUSIONS

- Existing Flexible Multibody Formalisms Are Limited in Their Ability to Treat Coupled Large Displacement/Small Deformation.
- Alternative Approaches Include Taking Explicit Account of Constraints Geometrically or Within a Nonlinear Strain Measure.
- Consistently-Linearized Models Work Well For Most Problems But Cannot Capture Motion-Induced Membrane Stiffness Variations.
- Second Order Structural Theories Account for Motion-Induced Stiffness Variations But Converge Slowly With Standard Mode Functions.
- Nonlinear Constraint Functions Serve Well as Modal Functions in Order to Improve Convergence in Second Order Structural Theories.