N90-10108

NUMERICALLY EFFICIENT ALGORITHM FOR MODEL DEVELOPMENT OF HIGH-ORDER SYSTEMS

By

L. O. Parada
Calspan Advanced Technology Center
Buffalo, New York

ABSTRACT

Frequency domain parameter identification techniques provide a straightforward approach to transfer function estimation. However, for high-order systems, numerical difficulties may be encountered during the estimation process. Inaccuracies may result because of the large variation of the transfer function polynomial coefficients for high-order systems. The lack of numerical precision to represent this variation may cause the estimation process to break down.

This paper presents a technique for estimating transfer functions in partial fraction expansion form from frequency response data for a high-order system. The problem formulation avoids many of the numerical difficulties associated with high-order polynomials and has the advantage of having the option to fix the damping and frequency of a mode, if known, during the estimation process. The resulting transfer function(s) may be converted to Jordan-Form time domain equations directly.

During the implementation of this technique, a frequency and amplitude normalizing window was developed that maximized the efficiency of the optimization algorithm. The combination of estimating the transfer function in factored form, the ability to fix previously determined parameters and the effectiveness of the normalizing window led to a progressive approach to synthesizing transfer functions from frequency response data for high-order systems.

Abstract

Numerically Efficient Algorithm for Model Development of High Order Systems

L. O. Parada
Calspan Advanced Technology Center
P.O. Box 400
Buffalo, NY 14225
(716) 632-7500

for presentation at the

NASA Langley Research Center Workshop on
Computational Aspects in the Control of Flexible Structures

Frequency domain parameter identification techniques provide a straightforward approach to transfer function estimation. However, for high order systems, numerical difficulties may be encountered during the estimation process. Inaccuracies may result because of the large variation of the transfer function polynomial coefficients for high order systems. The lack of numerical precision to represent this variation may cause the estimation process to break down.

This paper presents a technique for estimating transfer functions in partial fraction expansion form from frequency response data for a high order system. The problem formulation avoids many of the numerical difficulties associated with high order polynomials and has the advantage of having the option to fix the damping and frequency of a mode, if known, during the estimation process. The resulting transfer function(s) may be converted to Jordan-Form time domain equations directly.

During the implementation of this technique, a frequency and amplitude normalizing window was developed that maximized the efficiency of the optimization algorithm. The combination of estimating the transfer function in factored form, the ability to fix previously determined parameters and the effectiveness of the normalizing window led to a progressive approach to synthesizing transfer functions from frequency response data for high order systems.

NUMERICALLY EFFICIENT ALGORITHM FOR MODEL DEVELOPMENT OF HIGH ORDER SYSTEMS

Statement of Problem

Development of Mathematical Models:

Time Domain – Difficult to implement
– Instrumentation Complement
– Input Design
– Noise
– Computational Load

Freq Domain – Simplified Implementation
– Fewer parameters per computation cycle
– Statistical methods applicable

PREVIOUS WORK

Frequency domain parameter identification requires

- Determination of characteristic equation
  (nonlinear or iterative techniques)
- Estimation of numerator polynomials
- Factor characteristic equation
- Estimate zeros or residues

Inaccuracies (for high order systems) due to:

- Variation of transfer function polynomial coefficients
- Transformation errors
- Sensitivity of polynomial roots to variations in polynomial coefficients

SUMMARY OF CONTRIBUTIONS

Development of technique to estimate transfer functions in partial fraction expansion form from frequency response (amplitude and phase) data

Elimination of numerical difficulties associated with high order polynomials

Incorporation of a priori knowledge of system modes (frequency and damping) directly into the estimation process

Development of frequency and amplitude normalizing window that maximizes effectiveness of the optimization algorithm and eliminates the initial guess problem

Stepwise approach for synthesizing transfer functions where order of system is high and unknown

639 C-2

FACTORED FORM ESTIMATION

Classical Nonlinear Regression Problem

Estimate parameters from measured amplitude and phase data

Error Function:

Square of distance between measured and estimated frequency responses summed over all discrete frequency points

$$\epsilon = \sum_{i=1}^{M} \left[F(j\omega_i) - G(j\omega_i) \right]^2$$

where: M = # frequency points

$F(j\omega)$ = measured frequency response

$G(j\omega)$ = estimated frequency response

Estimated Transfer Function – $G(j\omega)$

Sum of 1st and 2nd order terms

$$G(j\omega) = \frac{a_N}{b_N} + \sum_{k=1}^{Q} \frac{n_{1_k}(j\omega) + N_{0_k}}{(j\omega)^2 + d_{1_k}(j\omega) + d_{0_k}} + \sum_{\ell=1}^{N-2Q} \frac{a_\ell}{(j\omega) + b_\ell}$$

where: N = order of system
Q = # of second order terms

Express measured and estimated frequency responses in terms of real and imaginary components

$$F(j\omega) = R(\omega) + j\,I(\omega)$$

$$G(j\omega) = \frac{a_N}{b_N} + \sum_{k=1}^{Q} \frac{N_{0_k}(d_{0_k} - \omega^2) + N_{1_k} d_{1_k}^{\,2}}{(d_{0_k} - \omega^2)^2 + d_{1_k}^{\,2}\,\omega^2} + \sum_{\ell=1}^{N-2Q} \frac{a_\ell b_\ell}{b_\ell^2 + \omega^2}$$

$$+\, j\left[\sum_{k=1}^{Q} \frac{N_{1_k}\,\omega\,(d_{0_k} - \omega^2) - N_{0_k}\, d_{1_k}\,\omega}{(d_{0_k} - \omega^2)^2 + d_{1_k}^{\,2}\,\omega^2} - \sum_{\ell=1}^{N-2Q} \frac{a_\ell \omega}{b_\ell^2 + \omega^2}\right]$$

Substitue for $F(j\omega)$ & $G(j\omega)$ into ϵ

$$\epsilon = \sum_{i=1}^{M} \left[R(\omega_i) - \frac{a_N}{b_N} - \sum_{k=1}^{Q} \frac{N_{0_k}(d_{0_k} - \omega_i^2) + N_{1_k} d_{1_k}\,\omega_i^2}{(d_{0_k} - \omega_i^2)^2 + d_{1_k}^{\,2}\,\omega_i^2} + \sum_{\ell=1}^{N-2Q} \frac{a_\ell b_\ell}{b_\ell^2 + \omega_i^2} \right]$$

$$+ \left[I(\omega_i) + \sum_{k=1}^{Q} \frac{N_{0_k} d_{1_k}\,\omega_i - N_{1_k}\,\omega_i\,(d_{0_k} - \omega_i^2)}{(d_{0_k} - \omega_i^2)^2 + d_{1_k}^{\,2}\,\omega_i^2} + \sum_{\ell=1}^{N-2Q} \frac{a_\ell \omega_i}{b_\ell^2 + \omega_i^2} \right]$$

Solve for unknown parameters
Set partial derivatives equal to zero
Solve using nonlinear optimization technique

641

Fifth Order Single Precision Example

Simulate parameter identification of high order system

Modes distributed over wide frequency range

Single precision: Scale down problem
Reduce number of variables

5th Order Transfer Function:

Cascade form

$$\frac{(s + 5 \times 10^{-3})\ (s + 5 \times 10^{-1})\ (s + 5 \times 10^{+1})\ (s + 5 \times 10^{+3})}{(s^2 + 2 \times 10^{-3} s + 10^{-4})\ (s + 1)\ (s^2 + 1 \times 10^{+4} s + 10^{+8})}$$

Parallel form

$$\frac{1.253955 \times 10^{-3} s + 6.122844 \times 10^{-6}}{s^2 + 2 \times 10^{-3}\ s + 1 \times 10^{-4}} + \frac{1.221073 \times 10^{-3}}{s + 1} + \frac{9.975250 \times 10^{-1} s + 5.024754 \times 10^{+3}}{s^2 + 1 \times 10^{+4}\ s + 10^{+8}}$$

Frequency Range: 1×10^{-4} to $1 \times 10^{+5}$ Hz.

Points/Decade = 30

DENOMINATOR COEFFICIENTS

Term	Exact Coefficient	Additive Components
s^5	$\boxed{1.000000}$	1.0
s^4	$\boxed{10001.00}2$	$1.002 + 1.0 \times 10^{+4}$
s^3	$\boxed{1000100}2.0021$	$2.1 \times 10^{-3} + 1.002 \times 10^{+4} + 10^{+8}$
s^2	$\boxed{1002002}1.0001$	$1 \times 10^{-4} + 2.1 \times 10^{+1} + 1.002 \times 10^{+8}$
s^1	$\boxed{210001.0}$	$1.0 + 2.1 \times 10^{+5}$
s^0	$\boxed{100004.0}$	$1.0 \times 10^{+4}$

$\boxed{}$ = Single Precision Variable Representation

LINEARIZED APPROACH

Initial Error Function:

$$E_k = F(j\omega_k) - \frac{P(j\omega_k)}{Q(j\omega_k)}$$

where: $F(j\omega_k)$ = measured frequency response at ω_k
$P(j\omega_k)$ = estimated numerator polynomial at ω_k
$Q(j\omega_k)$ = estimated denominator polynomial at ω_k

Weighted Error Function:

$$E_k' = E_k Q(j\omega_k) = F(j\omega_k)\, Q(j\omega_k) - P(j\omega_k)$$

Iterative Error Function:

$$E_k'' = \frac{E_k\, Q(j\omega_k)_L}{Q(j\omega_k)_{L-1}} = \frac{F(j\omega_k)\, Q(j\omega_k)_L}{Q(j\omega_k)_{L-1}} - \frac{P(j\omega_k)_L}{Q(j\omega_k)_{L-1}}$$

where: L = iteration #

Minimize E_k'' by taking partial derivatives of E_k'' with respect to each parameter x_i

$$\frac{\partial E_k''}{\partial x_i} = 0$$

Rearrange equations to formulate problem as a set of linear simultaneous algebraic equations:

$$[A]\ [X] = [B]$$

Solve for parameter vector [x]

Iterations converge to minimization of $|E_k|^2$

5th Order example: Polynomial Results

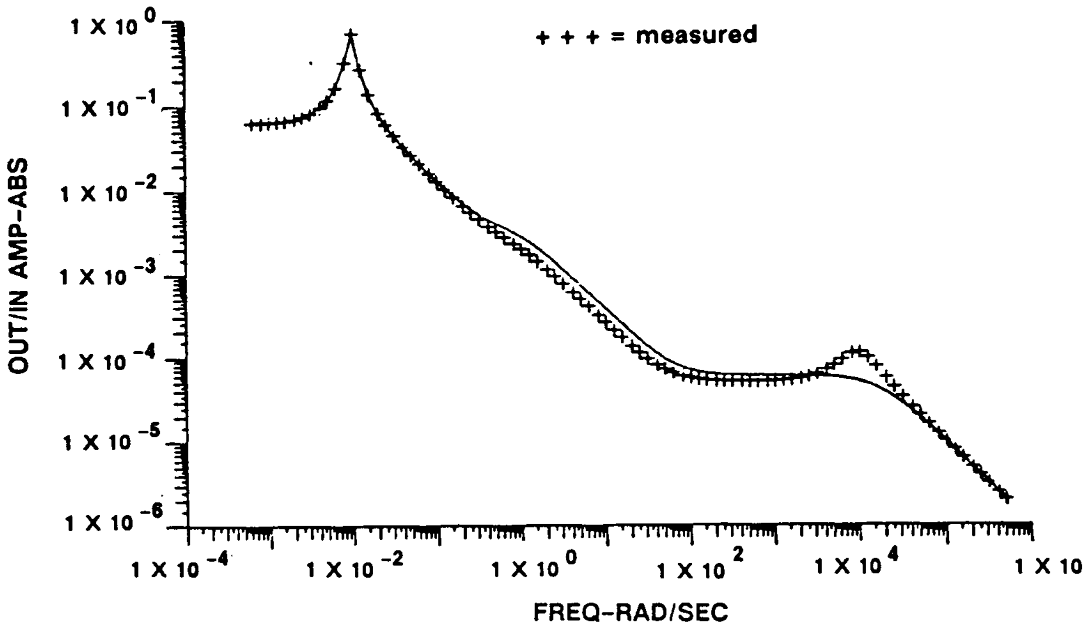
Exact Transfer Function

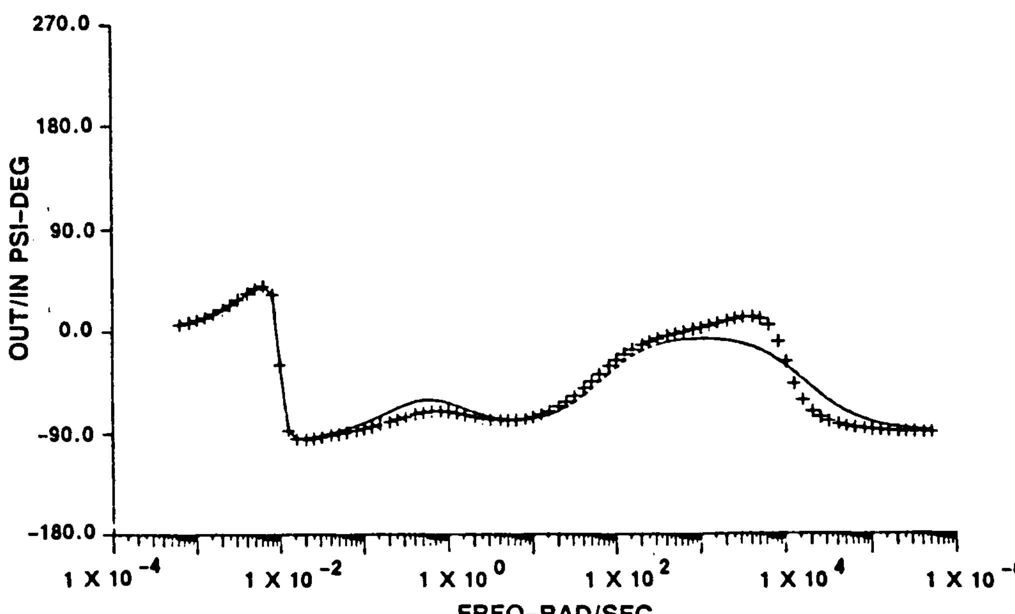
$$\frac{(s + 5 \times 10^{-3}) \quad (s + 5 \times 10^{-1}) \quad (s + 5 \times 10^{+1}) \quad (s + 5 \times 10^{+3})}{(s^2 + 2 \times 10^{-3} s + 10^{-4})(s + 1)\ (s^2 + 1 \times 10^{+4}\ s + 10^{+8})}$$

Linear Results

$$\frac{(s + 5 \times 10^{-3})\ (s + 5 \times 10^{-1})\ (s + 4.24 \times 10^{+1}) \quad (s - 7.17 \times 10^{+3})}{(s^2 + 2 \times 10^{-3} s \ + 10^{-4})\ (s + 9.98 \times 10^{-1})\ (s - 7.17 \times 10^{+3})\ (s + 1.72s\ 10^{+4})}$$

Cost Function 2.37×10^{-10}





FACTORED FORM APPROACH

Nonlinear method of solution – strong initial guess required

Initial Guess – Examine bode plots to approximate frequency and damping of modes

Problem – Error function relatively insensitive to perturbations in parameters of high frequency modes

– Gradient expressions small compared to those of the lower frequency parameters

Solution – Normalizing window

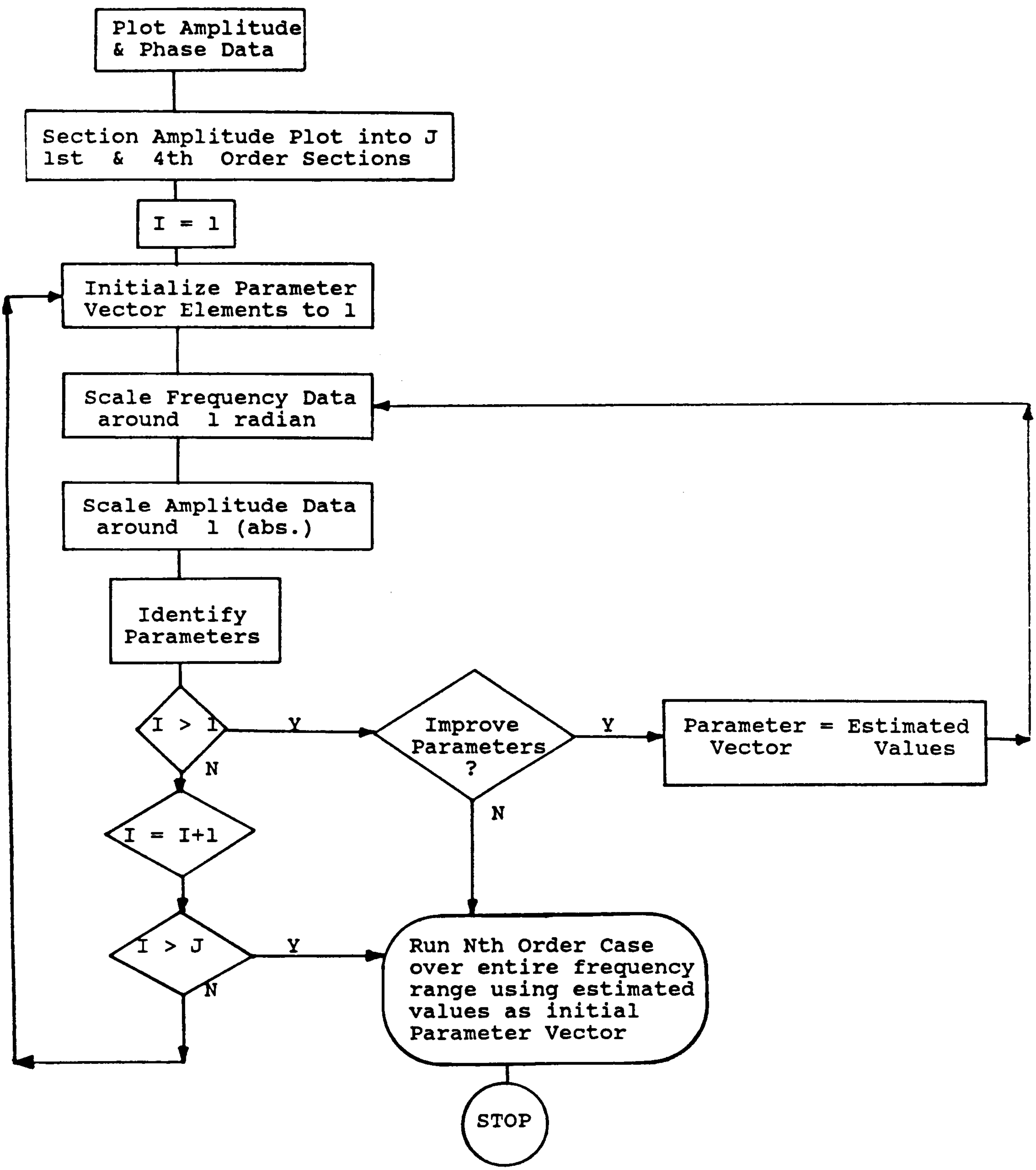
Scale Data Such That:

– Frequency is centered around 1 rad.

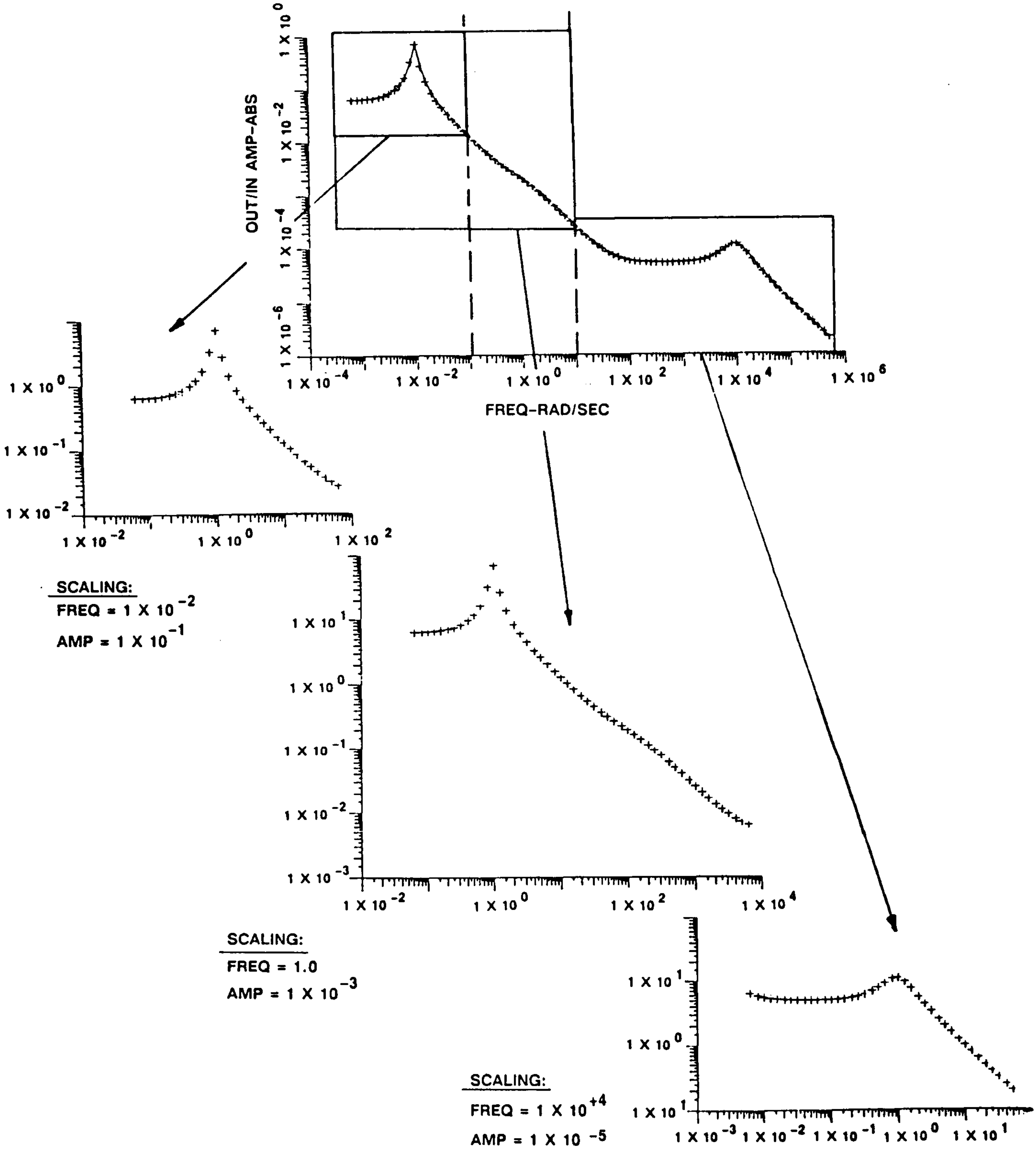
Amplitude is centered around 1 (abs. units)

Benefits – Need for strong initial guess eliminated effectiveness of optimization algorithm maximized

STEPWISE FACTORED FORM TECHNIQUE



5th Order Example: Factored Form Approach



5th Order Example: Factored Form Results

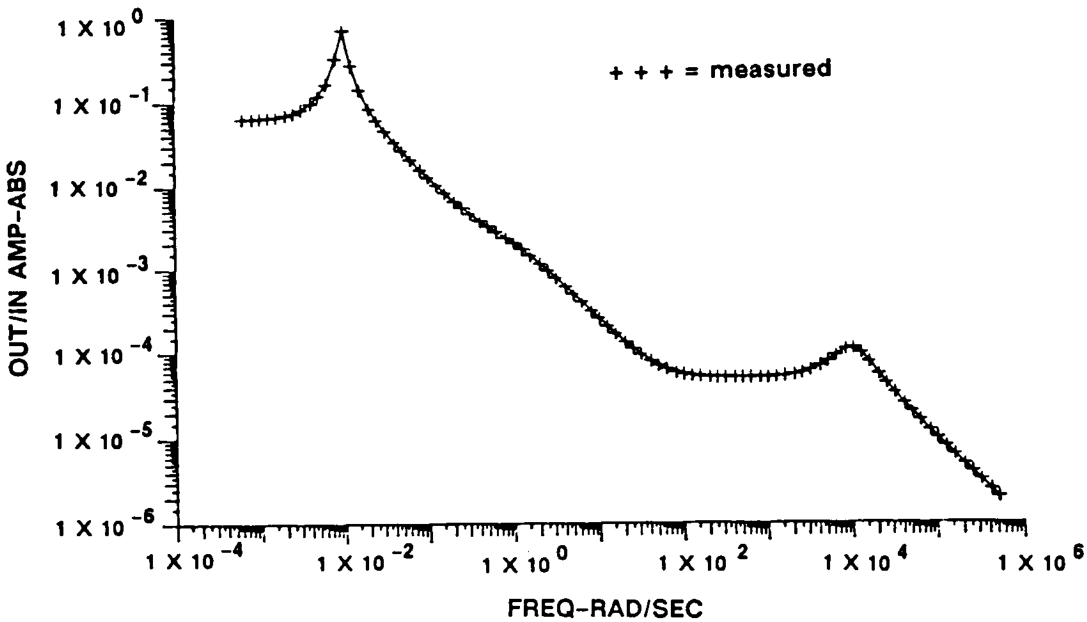
Exact Transfer Function

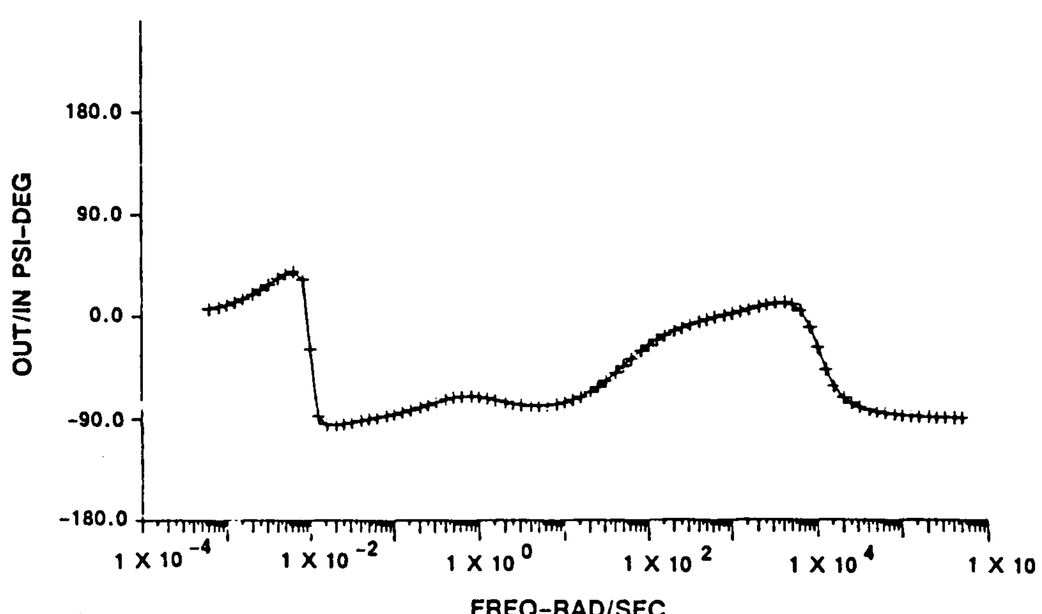
$$\frac{1.25 \times 10^{-3} s + 6.12 \times 10^{-6}}{s^2 + 2 \times 10^{-3} s + 10^{-4}} + \frac{1.22 \times 10^{-3}}{s + 1} + \frac{9.98 \times 10^{-1} s \ + \ 5.02 \times 10^{+3}}{s^2 + 1 \times 10^{+4} s + 10^{+8}}$$

Factored Form Results

$$\frac{1.25 \times 10^{-3} s + 6.12 \times 10^{-6}}{s^2 + 2.00 \times 10^{-3} s + 10^{-4}} + \frac{1.23 \times 10^{-3}}{s + 1.01} + \frac{9.79 \times 10^{-1} s \ + \ 5.21 \times 10^{+3}}{s^2 + 9.95 \times 10^{+3} s + 1.03 \times 10^{+8}}$$

Cost Function 1.90×10^{-12}



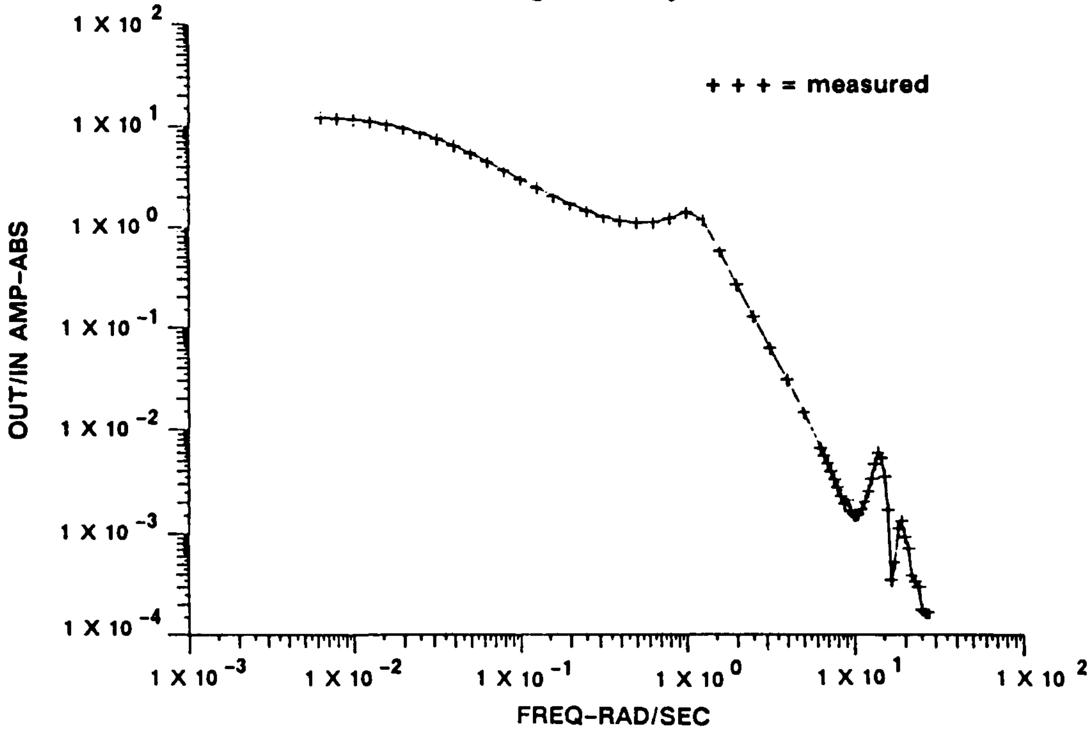


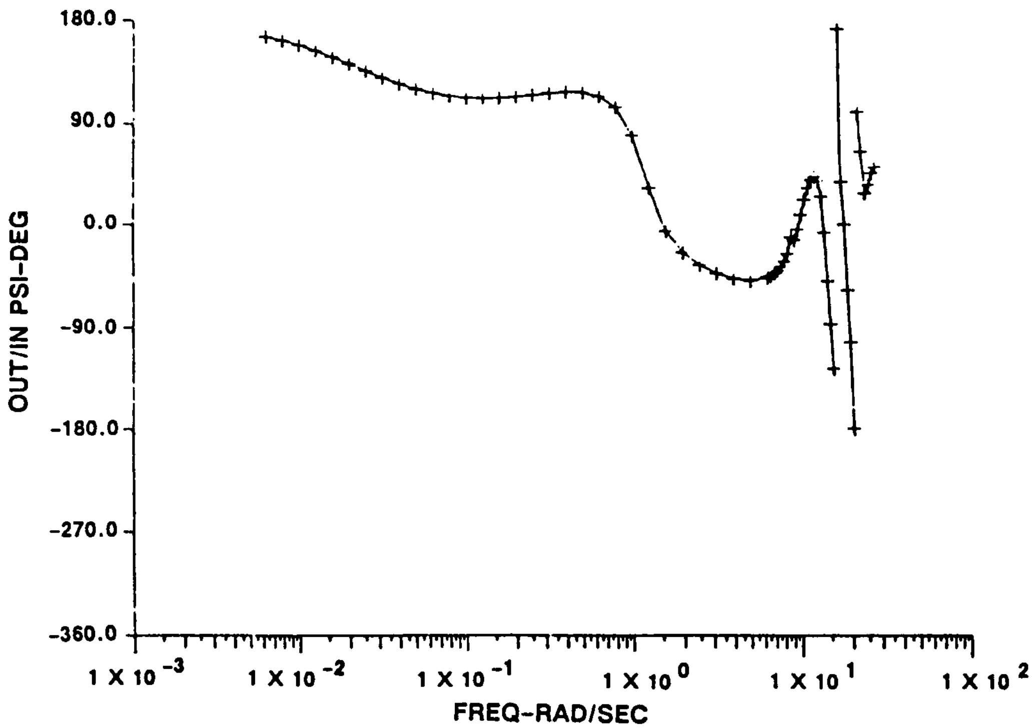
16th Order Transfer Function Estimation

Pcg/Vg: Roll Rate Measured at C.G. of Aircraft

vs.

Unit Gust Along Y-Body Axis





Cost Function: 4.5×10^{-12}

CONCLUSIONS

Development of technique to estimate transfer functions directly in factored form

Advantages:

- Ability to fix damping and frequency of a mode, if known, during the estimation process
- Avoidance of numerical difficulties associated with high order polynomials
- Ability to obtain Jordan–form time domain equations directly

Progressive approach to transfer function estimation through use of a frequency and amplitude normalizing window

Development of frequency and amplitude normalizing window that eliminates the initial guess problem and maximizes the effectiveness of the optimization algorithm