N90-10109

ON MODELLING NONLINEAR DAMPING IN DISTRIBUTED PARAMETER SYSTEMS

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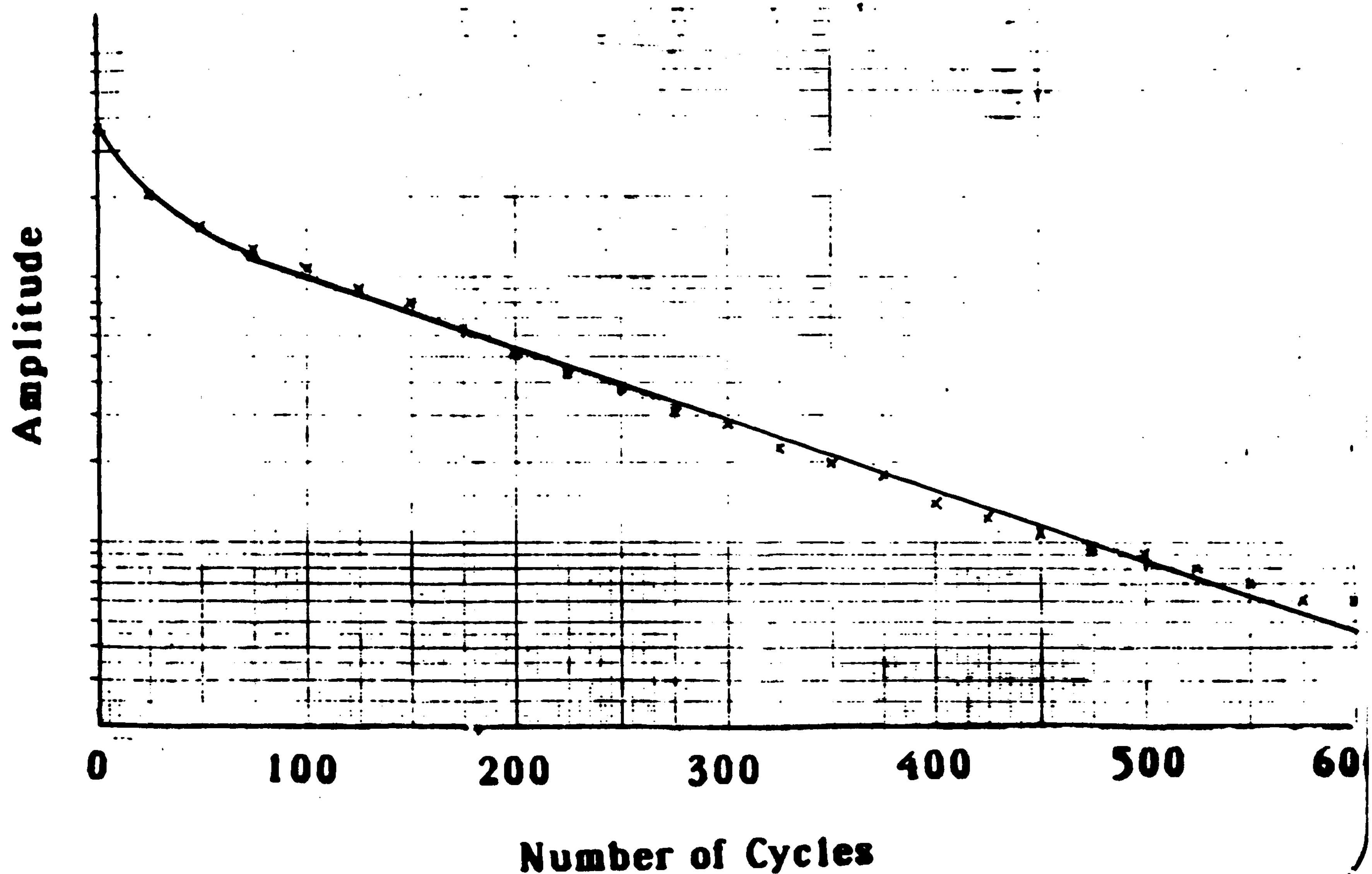
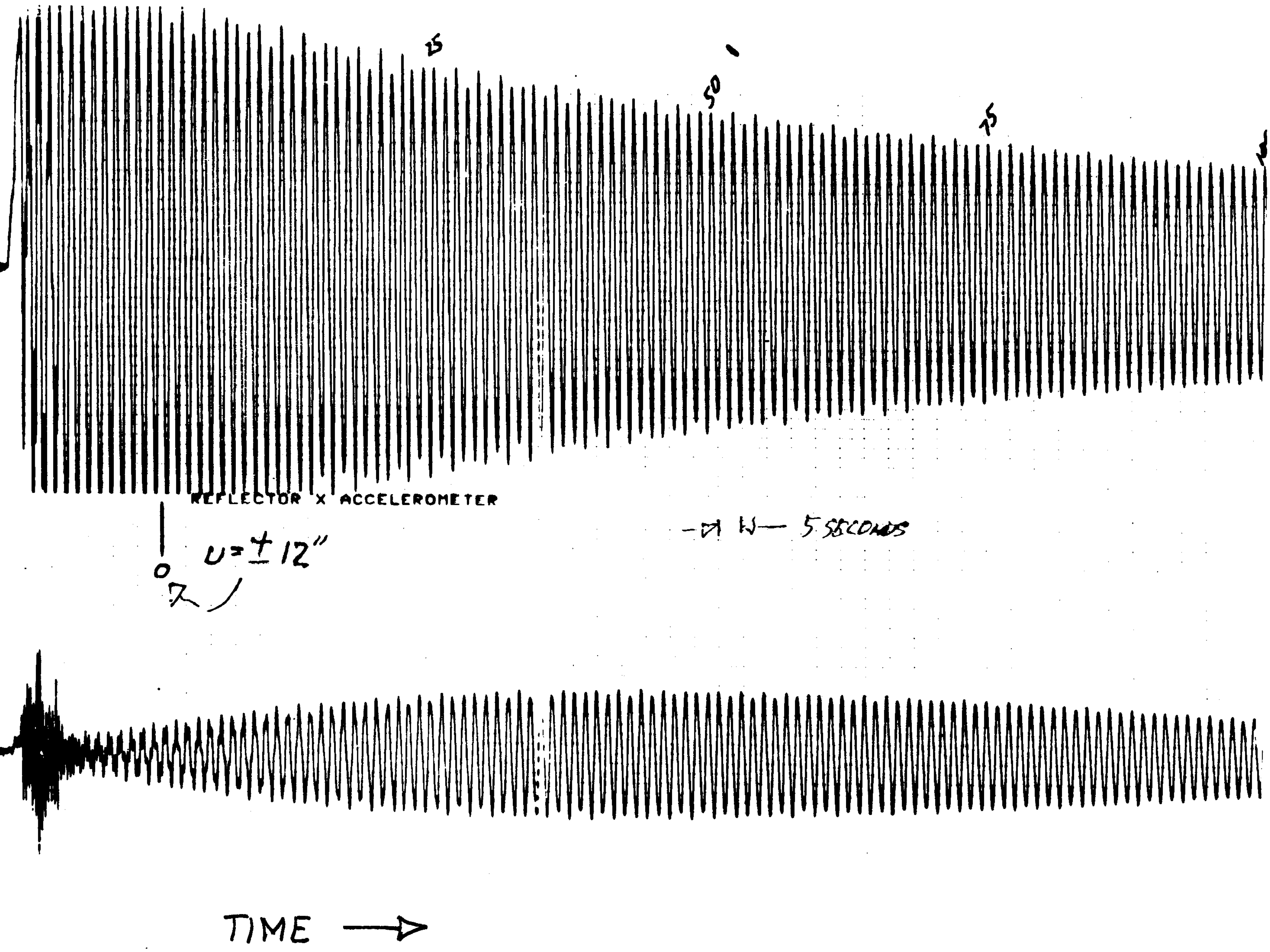
SCOLE DAMPING

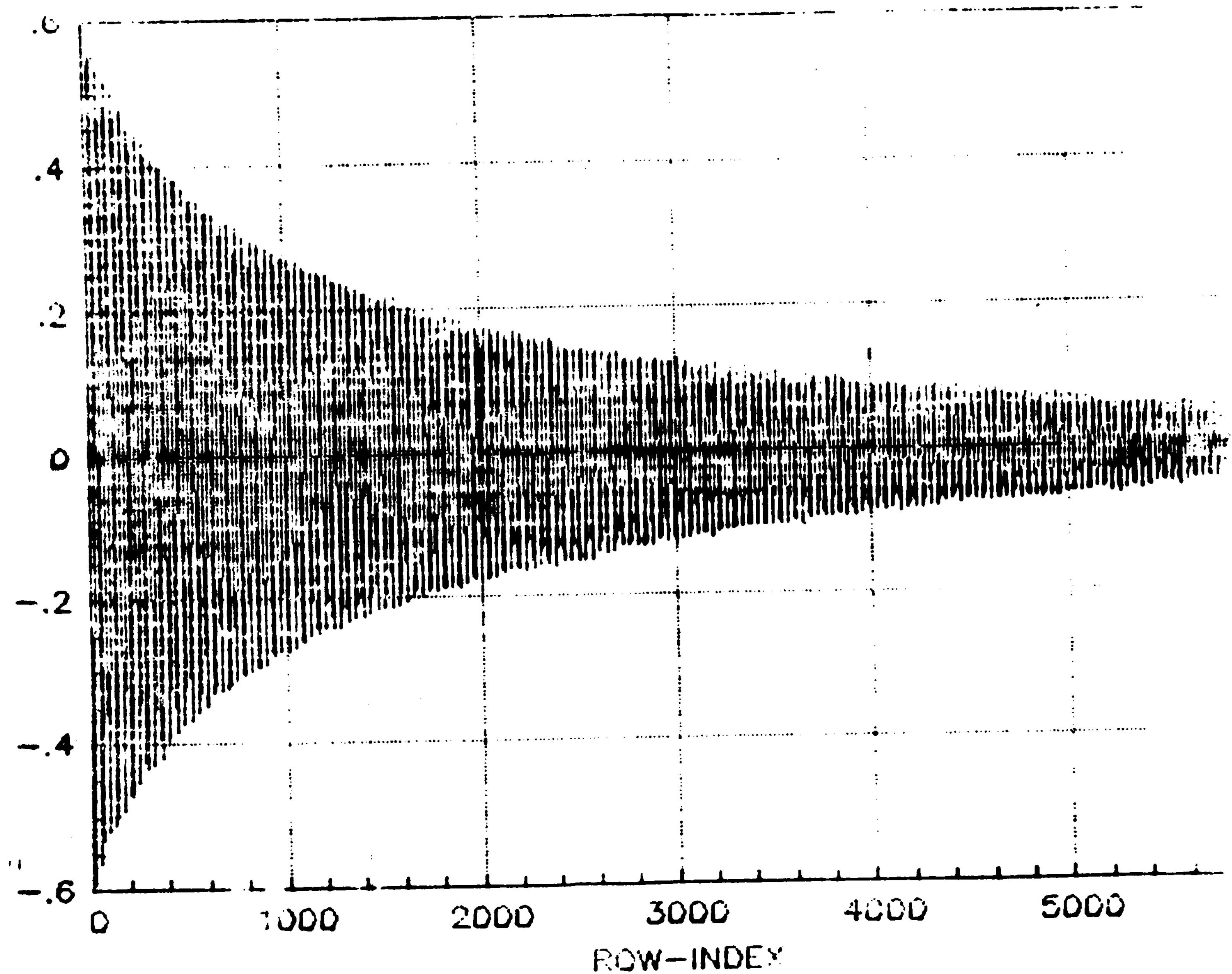


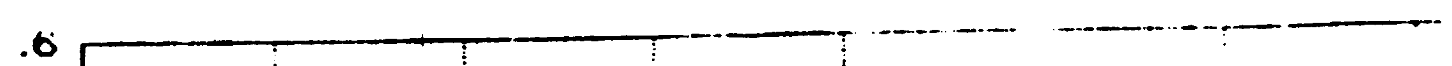
653



Without mass

654



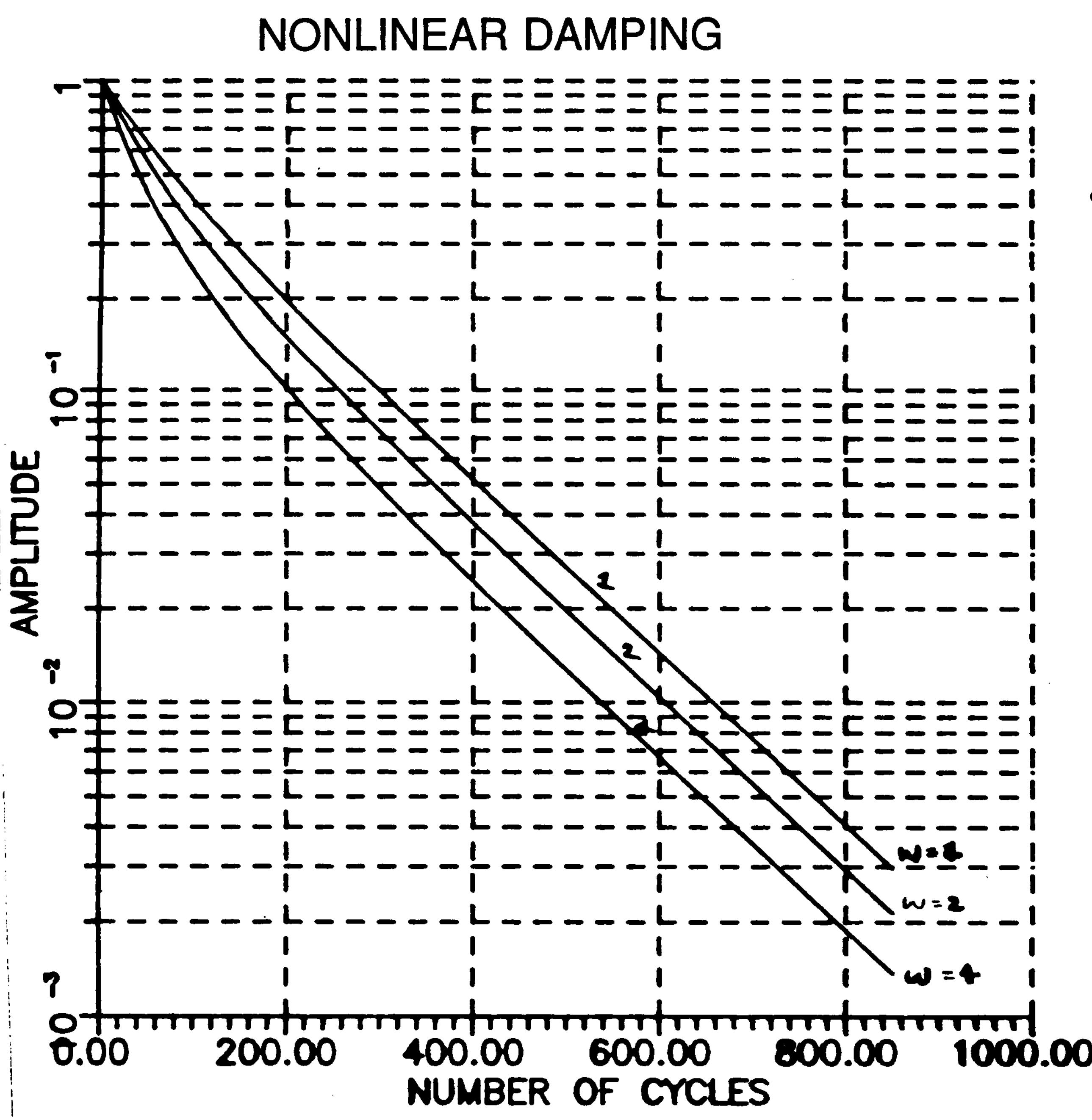


In One Dimension

Without hysteresis

$$
\begin{aligned}
\ddot{x}(t) \;+\; \omega^2 x(t) \;+\; 2\omega\zeta\dot{x}(t) \\
+\; \gamma x(t)^{2m}\,|x(t)|^{\alpha}\,\dot{x}(t)^{2n+1}\,|\dot{x}(t)|^{\beta} \\
+\; Bu(t) \;+\; FN(t) \\
=\; 0
\end{aligned}
$$

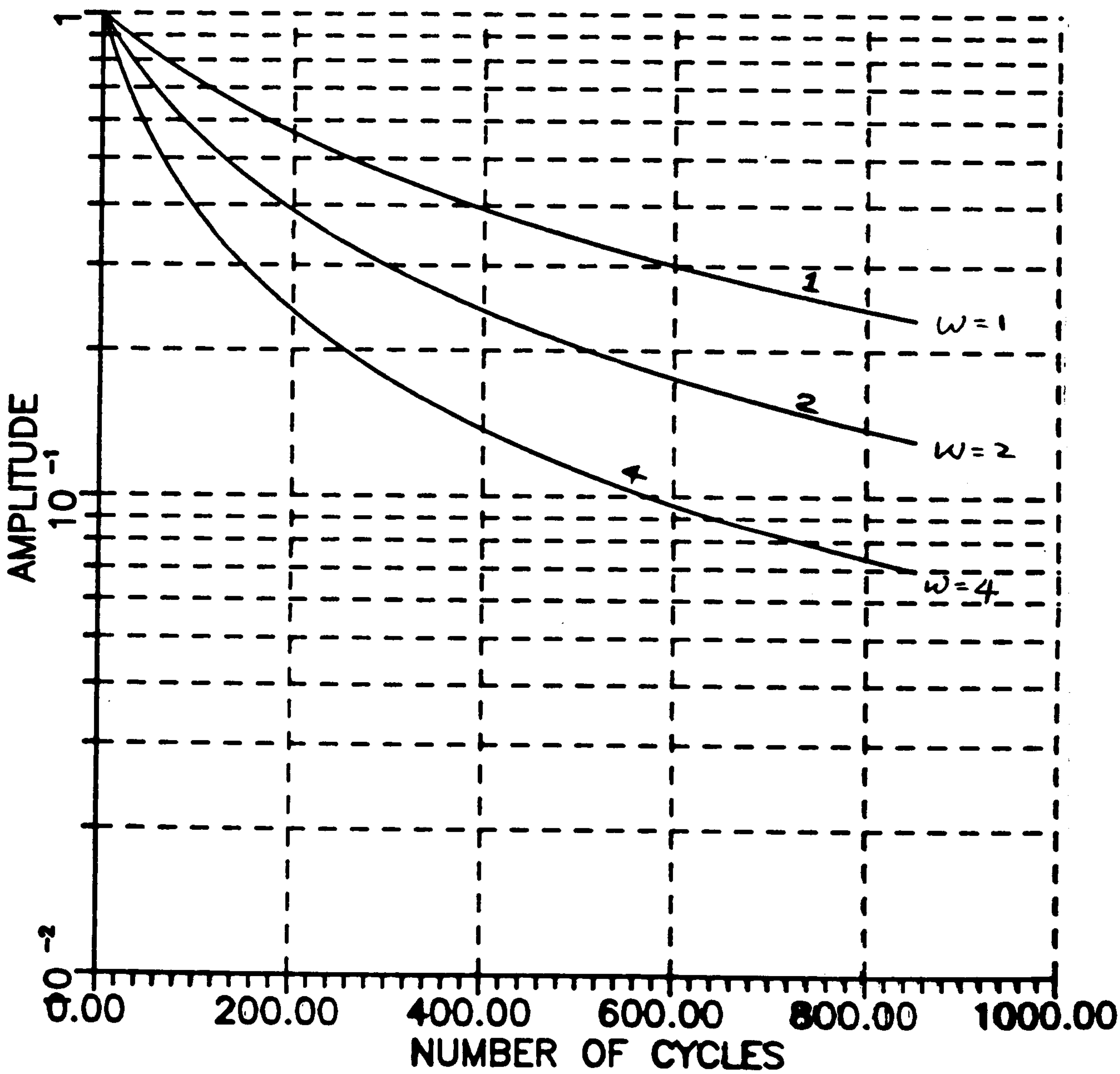
$$0 \le \alpha, \beta \le 1\ .$$



NONLINEAR DAMPING
1
10 -1
10 -2
AMPLITUDE
0.00
200.00
400.00
600.00
800.00
1000.00
NUMBER OF CYCLES
1
2
ω = 2
ω = 4

d_{n+1}

NONLINEAR DAMPING: $\zeta = 0$



Beam Model

$$\ddot{u}(s,\,t) \;+\; \lambda u''''(s,\,t) \;-\; 2\zeta\sqrt{\lambda}\,\dot{u}''(s,\,t)$$

$$-\;\gamma\left(\int_0^L u'(s,t)\,\dot{u}'(s,t)\;ds\right)^{2(n+\beta)+1} u''(s,\,t)$$

$$=\;0\,,$$

$$0 < s < L;\;\; 0 < t$$

n: zero or positive integer

$0 \leq \beta < \frac{1}{2}$

ζ: Linear Damping Ratio

Prime represents space derivative

Dot represents time derivative

$$A \sim \frac{d^4}{ds^4} : \text{ clamped beam}$$

$$A\phi_k = \omega_k^2 \phi_k$$

$$\sqrt{A}\,\phi_k = \omega_k \phi_k$$

$$\sqrt{A} \sim (-1)\,\frac{d^2}{ds^2}$$

$$\int_0^L u'(s, t)\, \dot{u}'(s, t)\, ds$$

$$= {}_0^L[u(s, t)\, \dot{u}'(s, t)]$$

$$- \int_0^L u(s, t)\, \dot{u}''(s, t)\, ds$$

$$x(t) = u(\cdot, t)$$

$$\rightarrow \quad \approx [x(t), \sqrt{A}\, \dot{x}(t)]$$

$$F(x, D\dot{x}) = \gamma([x, \sqrt{A}\, \dot{x}])^{2(n+\beta)+1} \sqrt{A}\, x$$

$$\ddot{x}(t) + \lambda A x(t) + D\dot{x}(t)$$

$$+ F(x(t), D\dot{x}(t)) + Bu(t)$$

$$= 0$$

$$D = 2\zeta\sqrt{\lambda}\,\sqrt{A}$$

$$x(t) = u(\cdot, t)$$

$$M\ddot{x}(t) + \lambda A x(t) + D\dot{x}(t) + F\big(x(t), D\dot{x}(t)\big) + Bu(t) = 0$$

Energy

$$E(t) = \frac{1}{2}\{[\dot{x}(t), \dot{x}(t)] + \lambda[Ax(t), x(t)]\}$$

$$\frac{d}{dt}E(t) = [\ddot{x} + \lambda A x, \ \dot{x}]$$

$$= -[D\dot{x}(t), \dot{x}(t)] - \big[F\big(x(t), D\dot{x}(t)\big), \dot{x}(t)\big]$$

$$[F(x, D\dot{x}), \dot{x}(t)] = ([x, \sqrt{A}\,\dot{x}])^{2(n+\beta)+2}$$

$$\geq 0$$

$$\Rightarrow \frac{dE(t)}{dt} \leq 0$$

$$x = a_k(t)\phi_k$$

$$F(x, D\dot{x})$$

$$= \gamma\big(a_k(t) \quad \omega_k \dot{a}_k(t)\big)^{2(n+\beta)+1} \omega_k a_k(t)\phi_k$$

$$= \gamma a_k(t)^{2(n+\beta)+2} \omega_k^{2(n+\beta)+2} \dot{a}_k(t)^{2(n+\beta)+1} \phi_k$$

$$\alpha = \beta; \qquad m = n$$

$$x(t) \;=\; a_k(t)\phi_k$$

$$\ddot{a}_k(t) \;+\; \lambda\omega_k^2 a_k(t) \;+\; 2\zeta\sqrt{\lambda}\,\omega_k \dot{a}_k(t)$$

$$+\; \gamma(a_k(t)\,\dot{a}_k(t)\,\omega_k)^{2(n+\beta+1)}\;\omega_k a_k(t)$$

$$=\; 0$$

$$\gamma a_k(t)^{2m}\,|a_k(t)|^{\alpha}\,|\dot{a}_k(t)|^{\beta}\,\dot{a}_k(t)^{2n+1}$$

$$\sim\; m \;=\; n\;;$$

$$\alpha \;=\; \beta$$

Alternate Form

$$
\begin{aligned}
u(s,t) &+ \lambda u''''(s,t) - 2\zeta\sqrt{\lambda}\,\dot{u}''(s,t) \\
&+ \gamma\left(\int_0^L u(s,t)\,\dot{u}''(s,t)\,ds\right)^{2(n+\beta)+1} u''(s,t) \\
&= 0\,,
\end{aligned}
$$

$$0 < s < L; \quad 0 < t$$

n: zero or positive integer

$0 \le \beta < \frac{1}{2}$

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Prime represents space derivative

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