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ACTIVE VIBRATION MITIGATION OF DISTRIBUTED PARAMETER, SMART-TYPE STRUCTURES USING PSEUDO-FEEDBACK OPTIMAL CONTROL (PFOC)

By

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ABSTRACT

A new, near-optimal feedback control technique is introduced that is shown to provide excellent vibration attenuation for those distributed parameter systems that are often encountered in the areas of aeroservoelasticity and large space systems. The technique relies on a novel solution methodology for the classical optimal control problem. Specifically, the quadratic regulator control problem for a flexible vibrating structure is first cast in a weak functional form that admits an approximate solution. The necessary conditions (first-order) are then solved via a time finite-element method. The procedure produces a low dimensional, algebraic parameterization of the optimal control problem that provides a rigorous basis for a discrete controller with a first-order "like" hold output.

Simulation has shown that the algorithm can successfully control a wide variety of plant forms including multi-input/multi-output systems and systems exhibiting significant nonlinearities. In order to firmly establish the efficacy of the algorithm, a laboratory control experiment was implemented to provide planar (bending) vibration attenuation of a highly flexible beam (with a first clamped-free mode of approximately 0.5 Hz). Base actuation for the cantilever was accomplished using a three degree-of-freedom active bay (variable geometry truss) actuator. On-line processing was accomplished with a 14 mhz "AT" type microcomputer with data acquisition capability. The results of the tests corroborate the utility of the method.

ACTIVE VIBRATION MITIGATION OF DISTRIBUTED PARAMETER SYSTEMS USING PSEUDO-FEEDBACK OPTIMAL CONTROL

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VA TECH

ORGANIZATION OF PAPER

- OVERVIEW OF METHOD (PFCA)
- EXPERIMENTAL RESULTS
- DISCUSSION

VARIATIONAL FORMULATION

OPTIMAL CONTROL PROBLEM

$$\text{Min } J_0 [\bar{u}] \equiv C(\bar{x}(T)) + \int_{\Upsilon} g(\bar{x}, \bar{u}, t)\, dt$$

$$\dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, t) + \bar{\bar{B}}(\bar{x}, t)\, \bar{u} \qquad \bar{x}(O) = \bar{x}_0$$

$$H = g + \langle \bar{y}, (\bar{f} + \bar{\bar{B}}\, \bar{u}) \rangle$$

$$J_1 [\bar{u}] = \int_{\Upsilon} \{ H - \langle \bar{y}, \dot{\bar{x}} \rangle \}\, dt$$

NECESSARY CONDITIONS

$$0 = \int_{\Upsilon} \{ \langle \bar{e}_1, \delta \bar{x} \rangle + \langle \bar{e}_2, \delta \bar{y} \rangle + \langle \bar{e}_3, \delta \bar{u} \rangle \} \, dt$$

$$0 = \langle (c_{,\bar{x}} - \bar{y}), \delta \bar{x} \rangle \Big|_{T}$$

$$\bar{0} = \bar{e}_1 = \bar{H}_{,x} + \dot{\bar{y}}$$

$$\bar{0} = \bar{e}_3 = \bar{H}_{,u}$$

$$\bar{0} = \bar{e}_2 = \bar{H}_{,y} - \dot{\bar{x}}$$

CLASSICAL SOLUTION TECHNIQUES

- LINEAR SYSTEMS
  - CLOSED FORM SOLUTIONS
  - RICCATI METHODS
- NONLINEAR SYSTEMS
  - GRADIENT METHODS
  - SHOOTING METHODS
  - QUASILINEARIZATION

WEAK (VARIATIONAL) FORMULATION

WHY?

- TECHNIQUE FAMILIAR TO COMMUNITY
- EASE OF IMPLEMENTATION

  USING FINITE ELEMENT PARADIGMS AND CODES
- PROVIDES AUTOMATED BASIS FOR FEEDBACK CONTROL LAWS

  FOR LINEAR AND NONLINEAR SYSTEMS

WHAT IS

PSEUDO

FEEDBACK CONTROL

- DRIVER -IN-THE-LOOP CONTROL

  (OHIO STATE --LATE 60'S)

OVERVIEW OF ANALYTICAL DEVELOPMENT

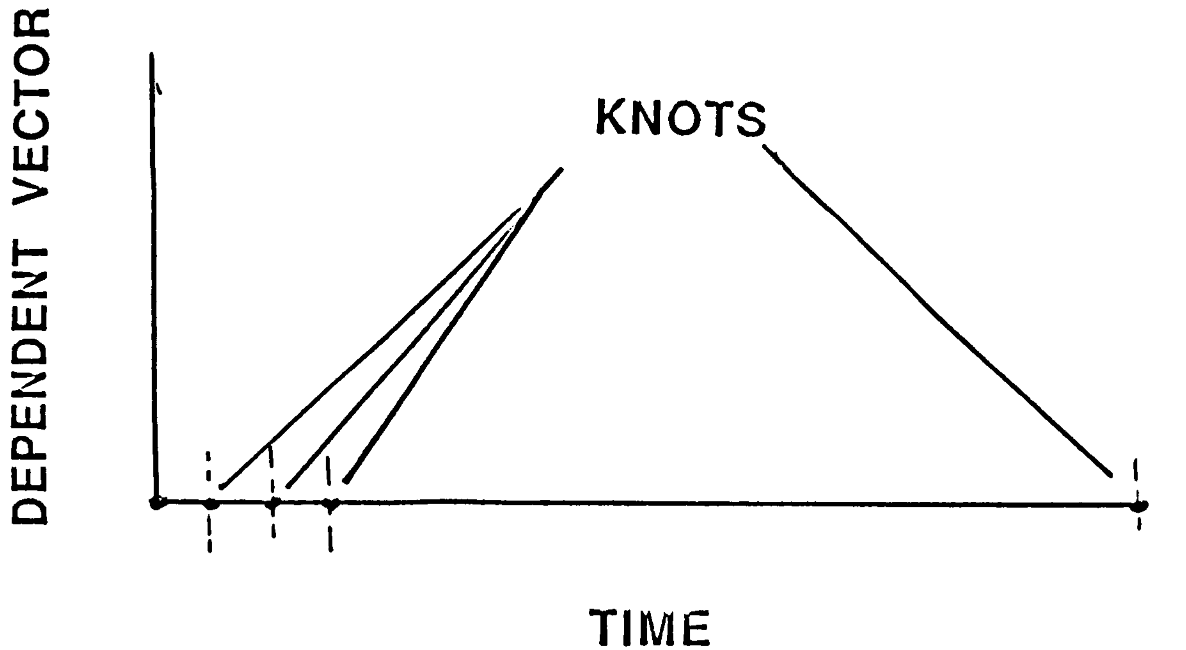
TWO-POINT BOUNDARY VALUE PROBLEM

WEAK FORM OF NECESSARY CONDITIONS

$$\bar{u} = -\bar{\bar{R}}^{-} \cdot \bar{\bar{B}}^{t} \cdot \bar{\gamma}$$

$$0 = \int_{\Upsilon} \langle \bar{e}_1, \delta \bar{x} \rangle \, dt \qquad 0 = \int_{\Upsilon} \langle \bar{e}_2, \delta \bar{\gamma} \rangle \, dt$$

DISCRETIZE DOMAIN OF OPERATOR (TIME)



SLIDING INTERVAL CONTROL

CONSTRUCT INTERPOLENTS TO APPROXIMATE STATE, COSTATE AND CONTROL VECTORS ON EACH OF THE SUBDOMAINS

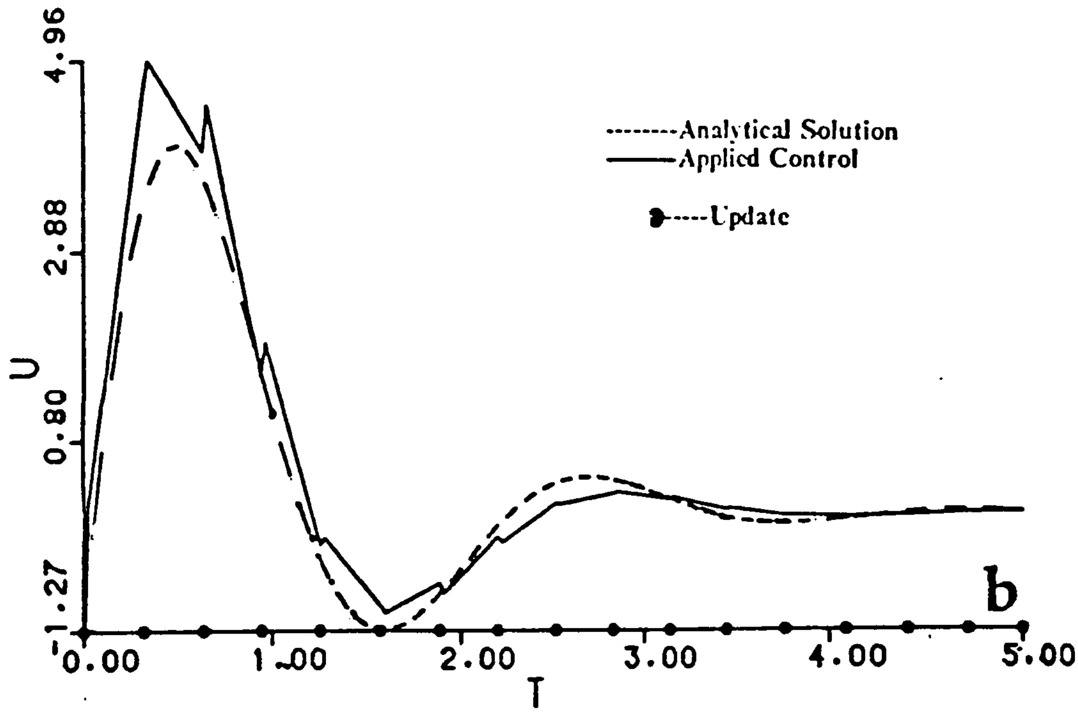
$$||\bar{z}||^2_{W_2^1} \equiv \int_{\Upsilon} [\, \dot{\bar{z}}^t \cdot \dot{\bar{z}} + \bar{z}^t \cdot \bar{z} \,] \, dt$$

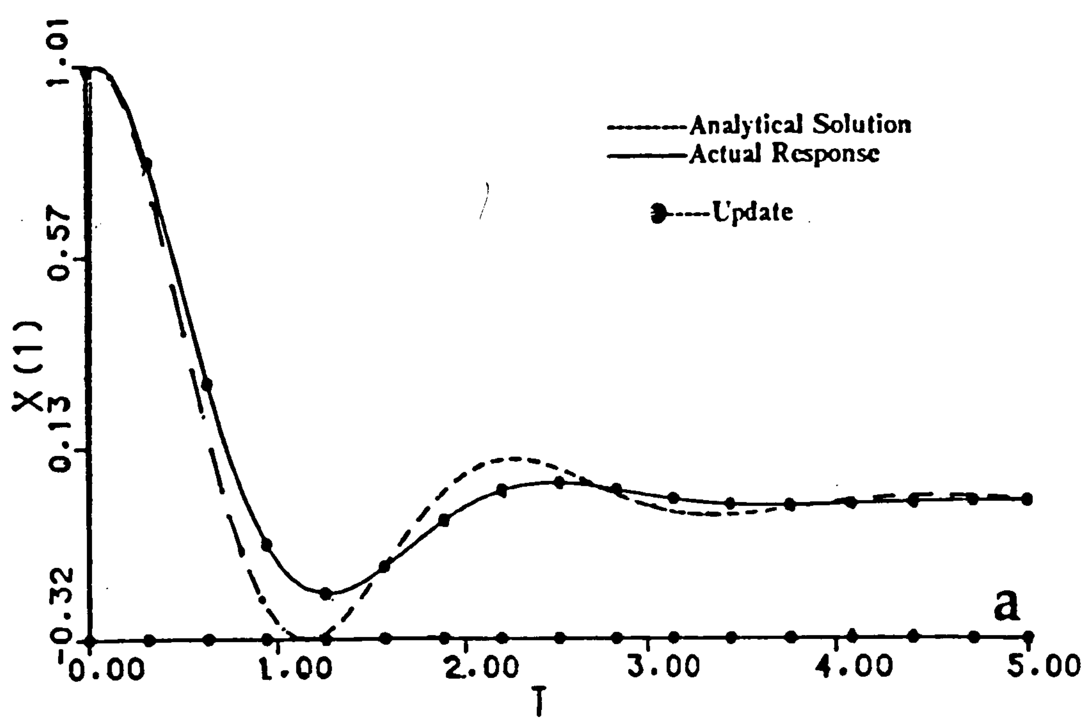
$$\bar{x} \cong \sum_{e=1}^{n} \bar{x}^e = \sum_{e=1}^{n} \bar{\bar{N}}^e \cdot \bar{q}^e = \sum_{e=1}^{n} \bar{\bar{N}}^e \cdot \bar{\bar{T}}^e \cdot g_{\bar{q}}$$

$$\bar{\gamma} \cong \sum_{e=1}^{n} \bar{\gamma}^e = \sum_{e=1}^{n} \bar{\bar{N}}^e \cdot \bar{p}^e = \sum_{e=1}^{n} \bar{\bar{N}}^e \cdot \bar{\bar{T}}^e \cdot g_{\bar{p}}$$

EXAMPLE

LINEAR SECOND ORDER OSCILLATOR (UNDAMPED)





EXAMPLE

NONLINEAR SECOND ORDER MODEL OF

WINGROCK EXHIBITED BY

FREE-TO-ROLL MODEL OF

X-29

NO WR2

EQUATION OF MOTION

$$\ddot{\phi} + \omega^2\phi = \mu_1\dot{\phi} + b_1\phi^3 + \mu_2\phi^2\dot{\phi} + b_2\phi\dot{\phi}^2$$

where

$$\omega^2 = -c_1 a_1$$

$$\mu_1 = c_1 a_2 - c_2$$

$$b_1 = c_1 a_3$$

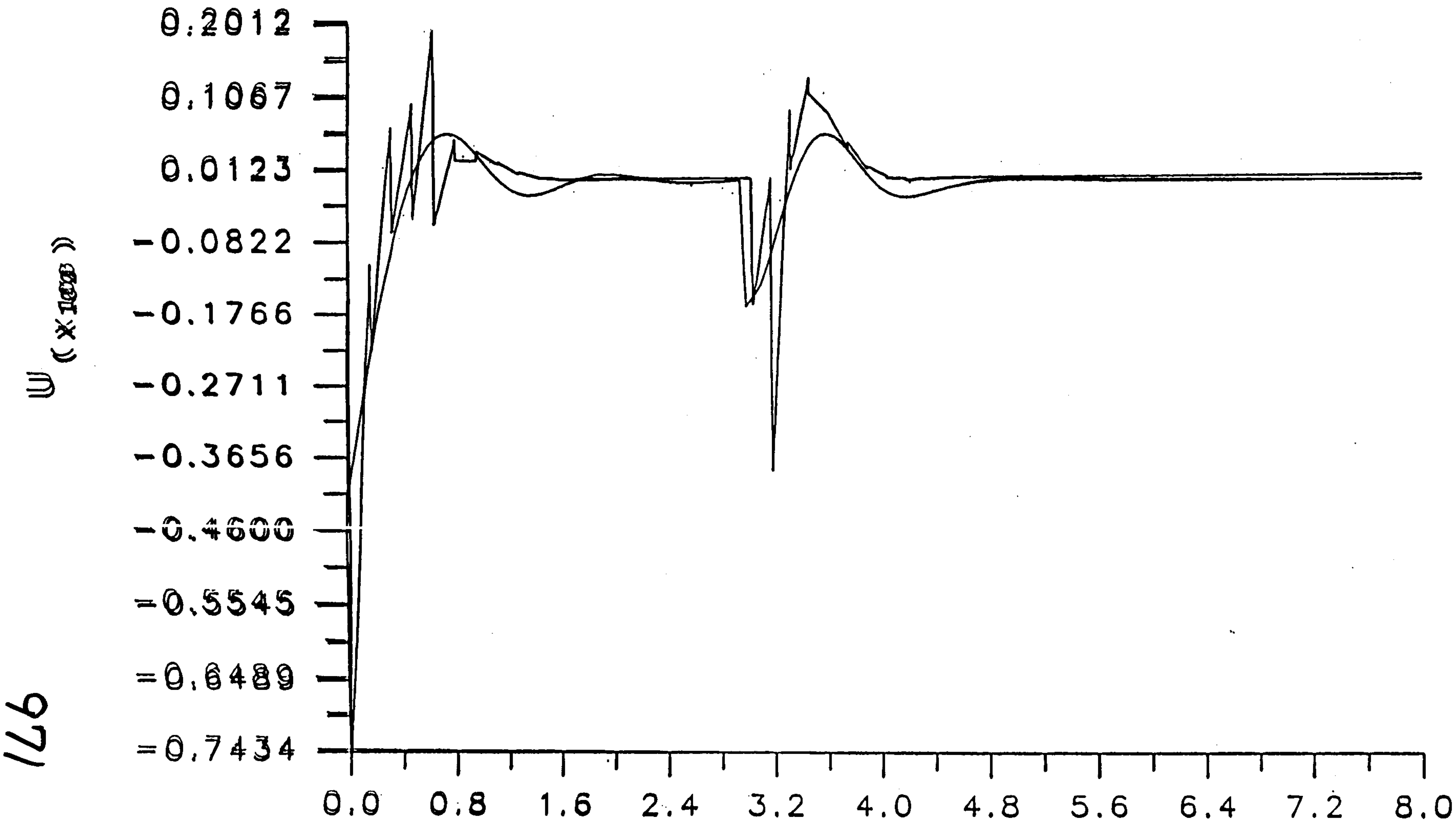
$$\mu_2 = c_1 a_4$$

$$b_2 = c_1 a_5$$

C_1 - CONSTANT
C_2 - CONSTANT
a_i - $f(\alpha)$ $\quad i = 1,5$

CONTROL-U .VS. TIME

Figure 6



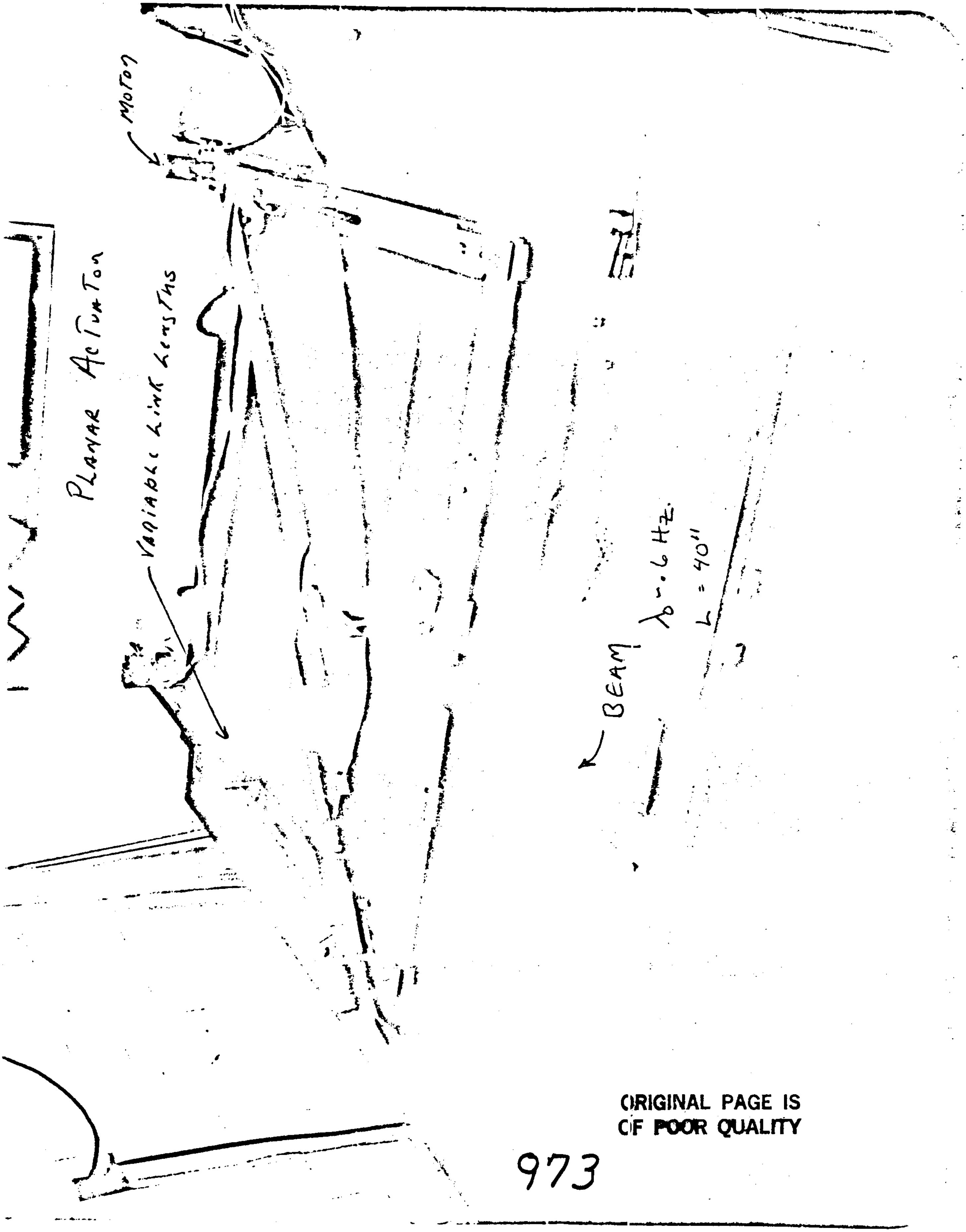
EXPERIMENTAL VALIDATION

OF ALGORITHIM

THE CONTROL OF

A FLEXIBLE BEAM USING

A PLANAR TRUSS ACTUATOR


PLANAR ACTUATOR
MOTOR
VARIABLE LINK LENGTHS
BEAM
$\lambda_0 \sim .6$ Hz
L = 40"

PROCEDURE

$$\dot{X} = AX + BU$$

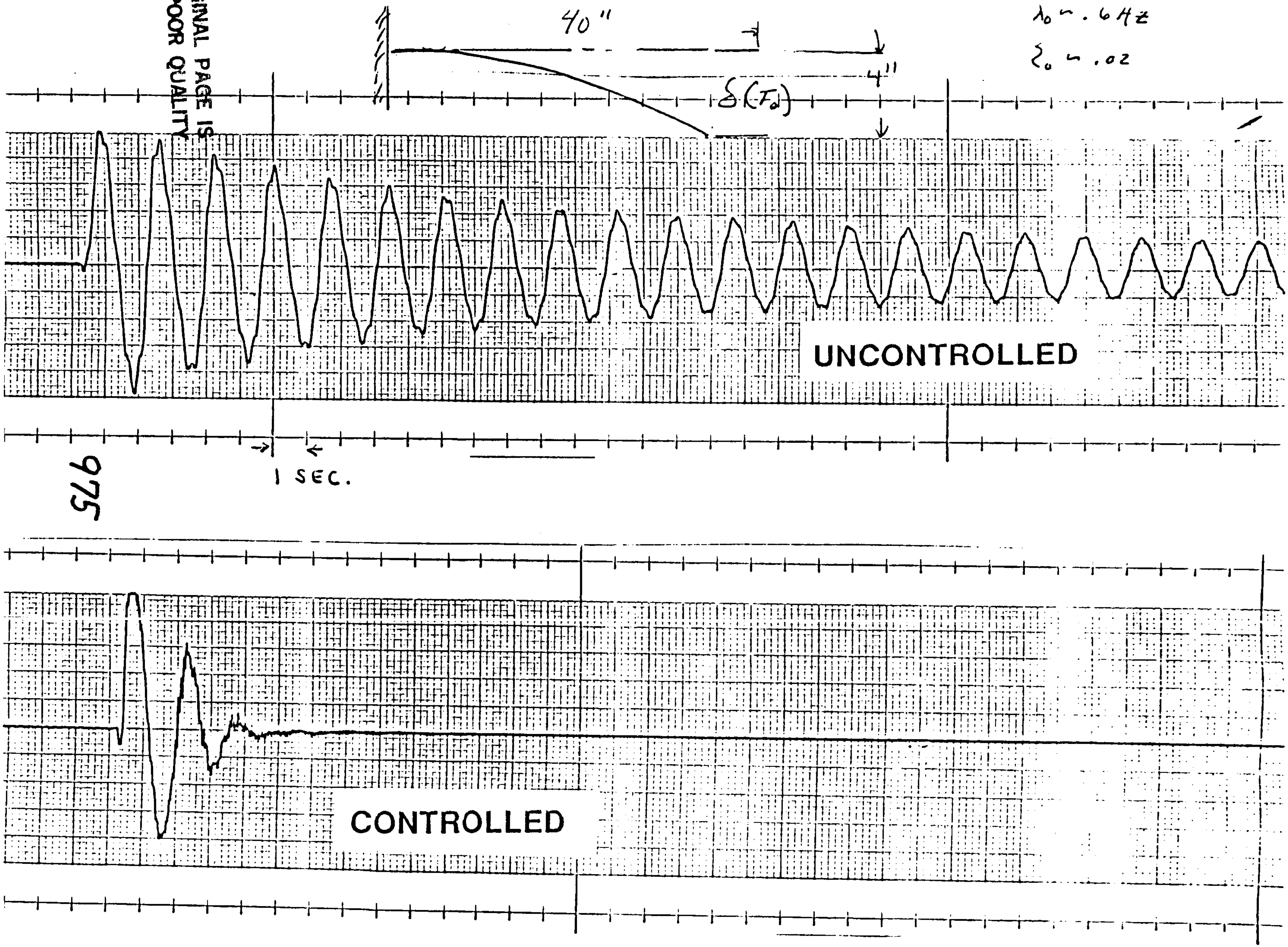
X = [LINK STATES, BEAM STATES]

APPLY FEM APPROXIMATION

ALGEBRAIC EVOLUTION OPERATOR

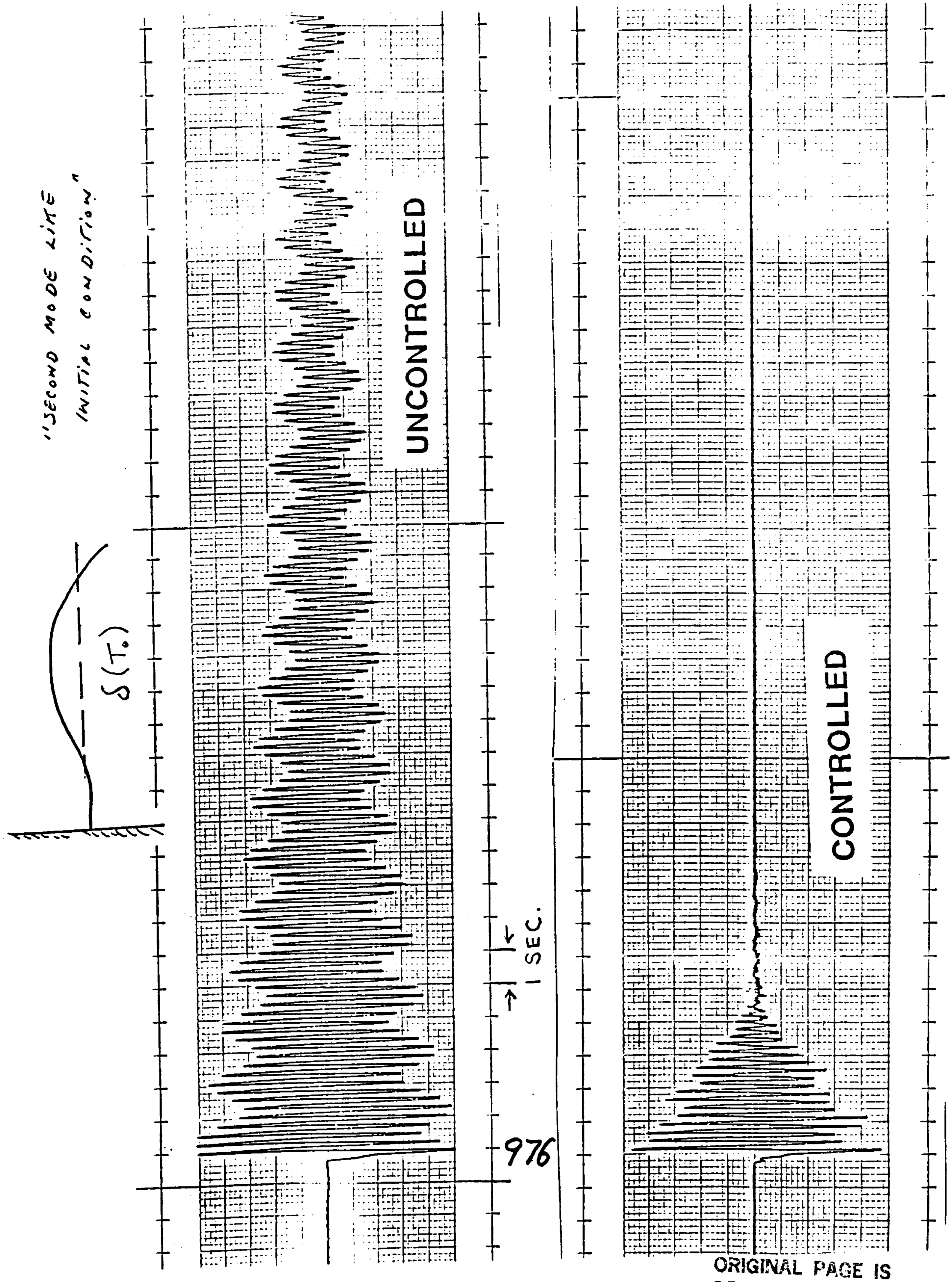
$$Z = K Z_0$$

Z = [STATE VECTORS, COSTATE VECTORS]

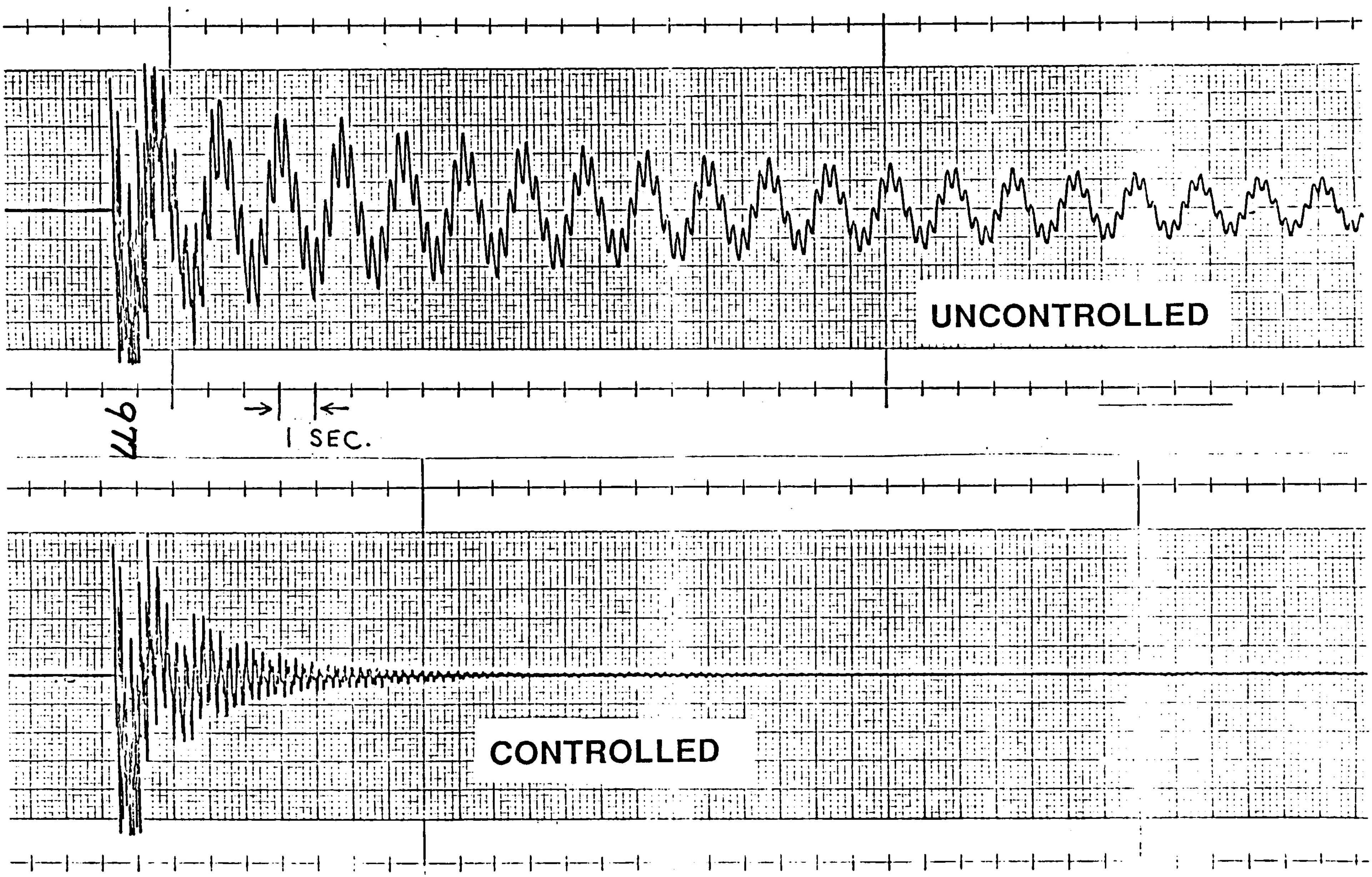


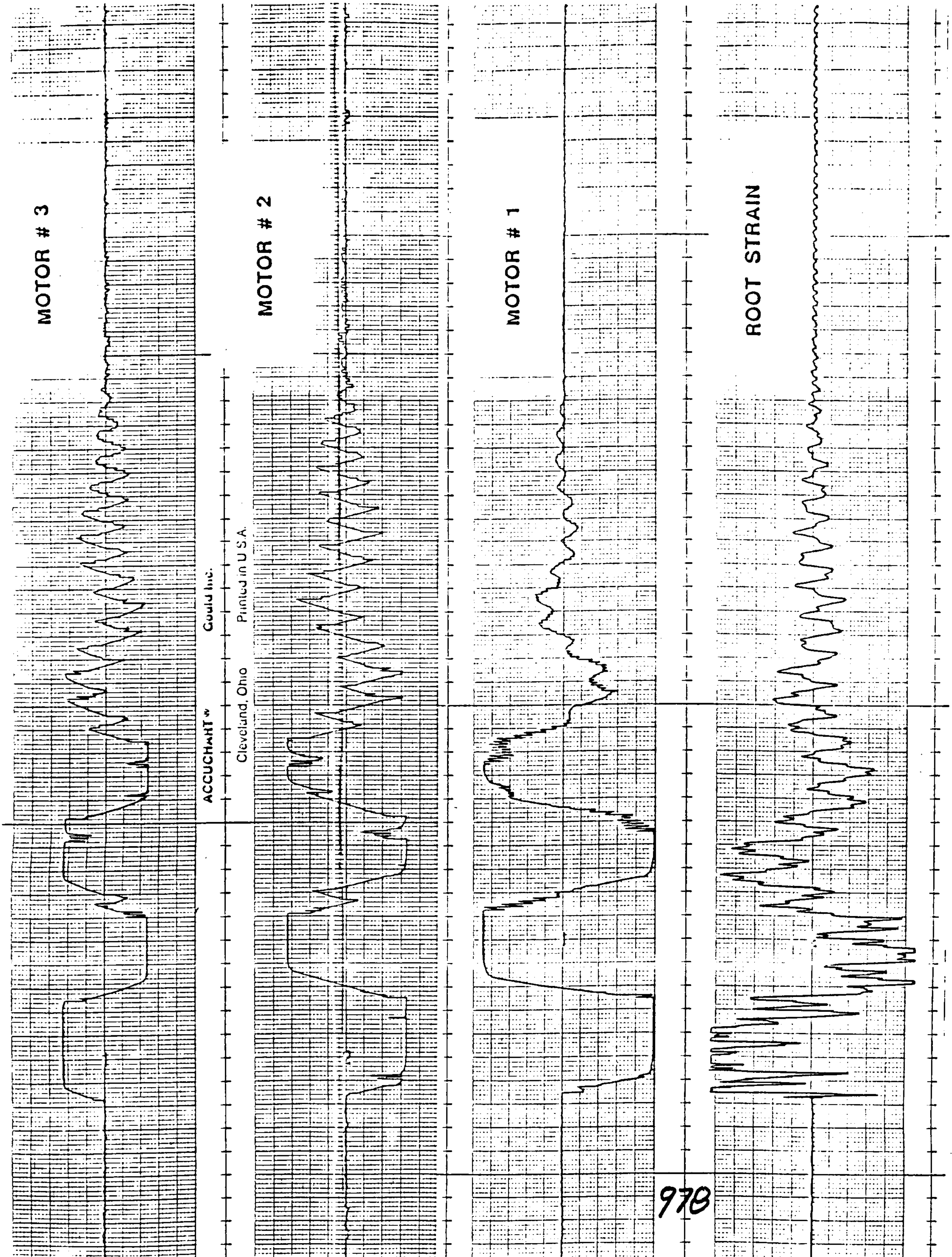
40"
4"
$\delta(F_d)$
$\lambda_0 \sim .6 Hz$
$\zeta_0 \sim .02$
UNCONTROLLED
1 SEC.
CONTROLLED

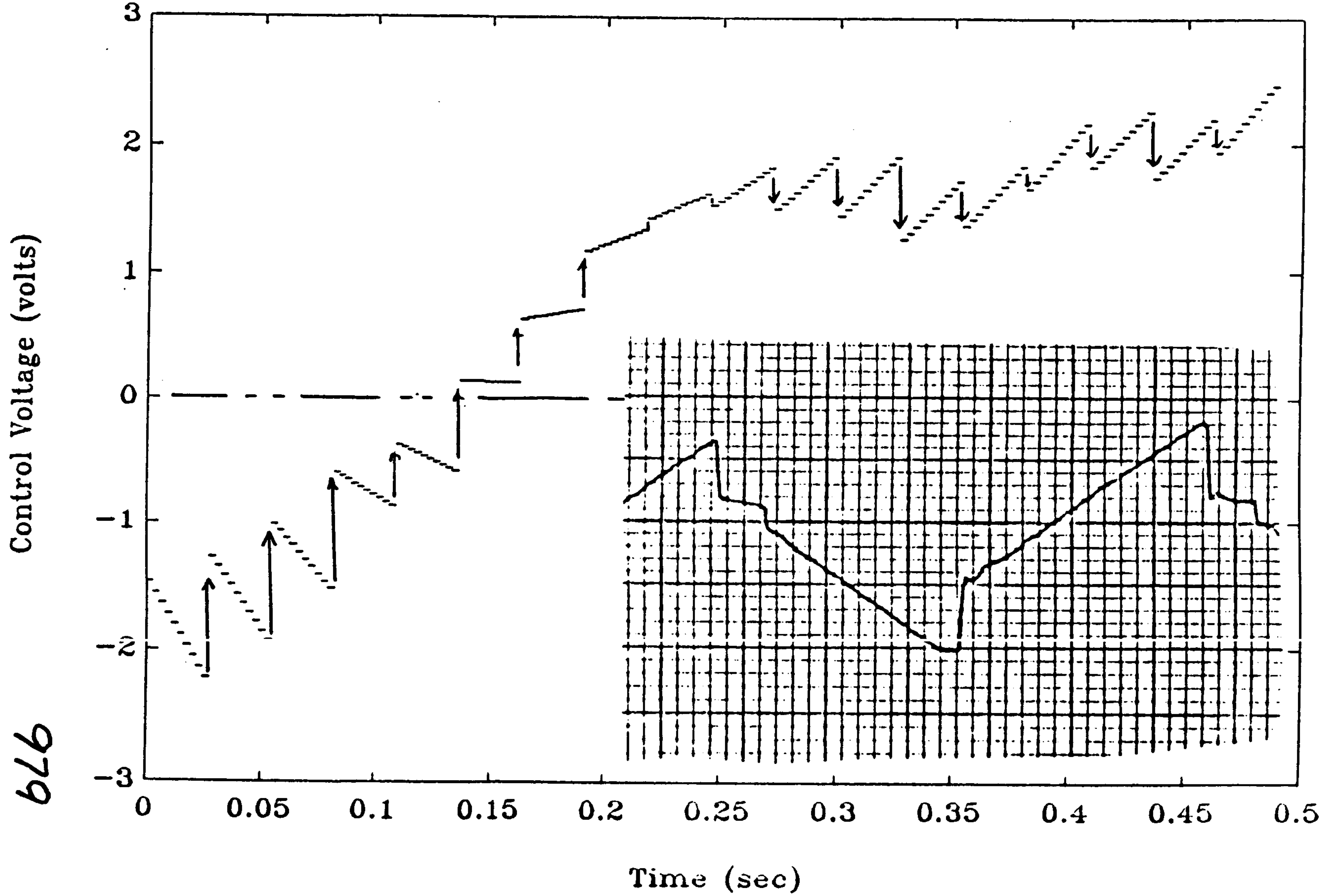
975


"SECOND MODE LIKE
INITIAL CONDITION"
δ(T₀)
UNCONTROLLED
CONTROLLED
1 SEC.
976

IMPULSE RESPONSE




MOTOR # 3
MOTOR # 2
MOTOR # 1
ROOT STRAIN
ACCUCHART
Gould Inc.
Cleveland, Ohio
Printed in U.S.A.
978



Control Voltage (volts)
3
2
1
0
−1
−2
−3
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Time (sec)

626

CONCLUSIONS

- ALGORITHIM VALIDATED

- REAL TIME IMPLEMENTATION POSSIBLE ON LOW-LEVEL PC ARCHITECTURE
  (80286 WITH 27.5 KHZ D/A)

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