NASA Technical Memorandum 101616

APPROXIMATE SIMULATION MODEL FOR ANALYSIS AND OPTIMIZATION IN ENGINEERING SYSTEM DESIGN

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June 1989

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Space Administration

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ABSTRACT

Computational support of the engineering design process routinely requires mathematical models of behavior to inform designers of the system response to external stimuli. However, designers also need to know the effect of the changes in design variable values on the system behavior. For large engineering systems, the conventional way of evaluating these effects by repetitive simulation of behavior for perturbed variables is impractical because of excessive cost and inadequate accuracy. This paper describes an alternative based on recently developed system sensitivity analysis that is combined with extrapolation to form a model of design. This design model is complementary to the model of behavior and capable of direct simulation of the effects of design variable changes.

1. INTRODUCTION

Mathematical models are a well established means for simulation of the behavior of engineering systems to support design decisions. These models usually employ a network of disciplinary behavioral models such as structural analysis, aerodynamic analysis, propulsion analysis, etc. and have one common characteristic: they answer the question "what will be the behavior (response) of the system to a given external stimulus ?" The answers these models provide are typically cast in a numerical form of some behavior variables, for instance, stress and displacement fields induced by a given load acting on a structure.

In design, however, engineers must decide how to change design variables in order to effect a desired change in behavior. To do so they need answers to "what if" questions, e.g. "what will be the change in the behavior if a particular design variable is altered ?" Indeed, one may assert that the design process is not completed until all such questions are answered, at least for the major design variables. The "what if" questions may be answered by repeated use of behavior models combined with design variable perturbations to obtain finite-difference approximations to design derivatives. In large applications, the computational cost of behavior models interconnected in a network and the accuracy problems intrinsic in the finite-difference methods render that approach impractical. As a result, practical applications of systematic, mathematically based optimization in the design of complete engineering systems have been lagging [1], relative to the progress of engineering optimization theory noted in recent times.

In contrast, optimization applications in structural engineering have been growing steadily in number, size, and complexity of successfully solved cases. This growth may be attributed, at least in part, to the concept of decoupling the design space search from the full analysis (the behavior model), and coupling it instead with an approximate analysis based on derivatives of behavior with respect to design variables. That concept, introduced in [2], spurred the development of sensitivity analysis to the point where it became routine for structures [3]. The approximate analysis concept was also a basis for the development of a body of efficient structural optimization procedures, e.g., [4], that have also diversified to applications in other disciplines, e.g., [5] and [6].

The purpose of this paper is to show that the recent development of algorithms for the sensitivity analysis of complex, internally coupled systems that comprise several subsystems and that involve many disciplines made it possible to use the approach initiated in [2] to develop a combination of extrapolation and sensitivity analysis for such systems. That capability will be referred to as a model of design. The purpose of the model of design, complementary to the commonly used model of behavior, is to simulate efficiently the effect of design variable changes on behavior, so it may be used to answer the "what if" questions quickly and inexpensively in order to support formal optimization as well as judgmental decisions in design.

2. PHYSICAL SYSTEM VERSUS SIMULATION SYSTEM

Engineering design of contemporary aircraft, spacecraft, and other complex systems is a prolonged and complicated process that involves human creativity, ingenuity, and judgment, all supported by massive computations. The computations involve a collection of computer programs, each representing a physical subsystem of the engineering system at hand or a particular aspect of that system behavior. By virtue of passing the data to each other the computer programs in that collection become modules in a coupled system that simulates the physical one and is its mathematical model of behavior, just as each module by itself is a mathematical model of behavior for a discipline or a part of the physical system.

For instance in aircraft design (figure 1) we may distinguish the wing and the fuselage as separate subsystems in the aircraft system. We may also consider the structural and aerodynamic analyses as separate modules in a system of programs that supports the design. This example points out that, in general, no one-to-one correspondence exists between the modules and the subsystems. In fact, the structural finite element model processed in the structural analysis may reflect the wing and the fuselage as separate substructures in the airframe, but the aerodynamic analysis may operate on a single digitized model of the aerodynamic surface that extends over the entire wing-fuselage assembly.

In reality, the wing and the fuselage interact aerodynamically and structurally. As shown in figure 2, they modify each other flow fields through the conduit of aerodynamics, and through the conduit of structures they exert forces on each other at the fuselage-wing junction. The corresponding mathematical model may represent

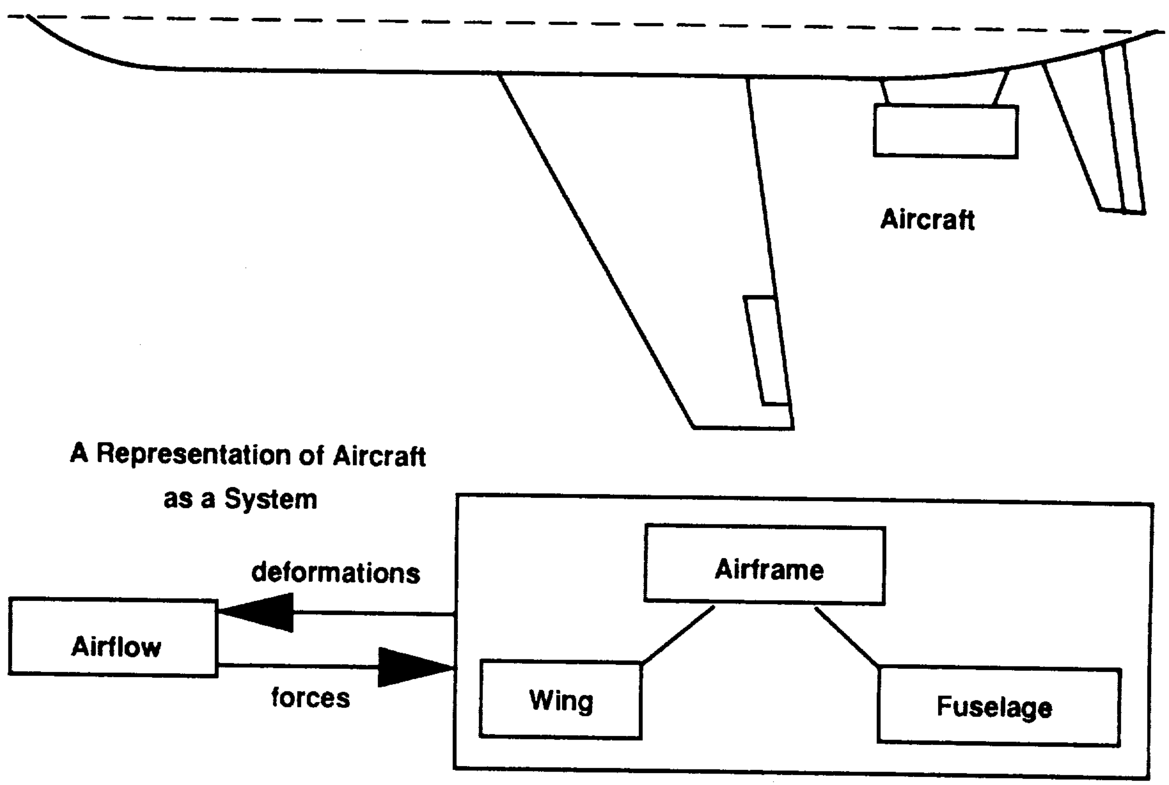

Figure 1 Aircraft system and its aspect and object decompositions.

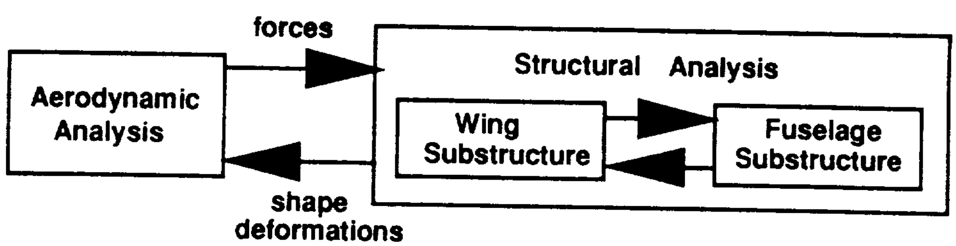

Figure 2 Simulation system.

the interactions by the force-deformation compatibility conditions at the wing-fuselage junction, and by the aerodynamic forces and airframe deformations that couple the aerodynamic and structures analysis modules. The example shows also that for purposes of mathematical simulation, the physical system may be decomposed into smaller parts (the object decomposition) or each aspect of behavior may be assigned a module in the simulation (the aspect decomposition). Both types of decomposition may be used simultaneously, as they are in this example.

Once the real, physical system has been conceptually decomposed and the corresponding mathematical simulation system has been assembled, it is the simulation system that provides computational support for the design process. Basically, that support has two functions: first, to reveal the behavior of the physical system in response to external stimuli; and second, to find out how that behavior may be modified by changing physical attributes (design variables) of the system. The first function calls for a system analysis. The latter answers the "what if" types of questions which is what design is all about, and is performed by sensitivity analysis. In the remainder of this paper, we will examine system analysis only briefly as a prerequisite to sensitivity analysis which will be the discussion focus and will show how the sensitivity analysis can be formalized as a basic ingredient in the mathematical model of design that complements the model of behavior.

3. SYSTEM ANALYSIS AS A MODEL OF BEHAVIOR

Each module in the system may be represented by a function vector notation. For the i-th module in a system of NM modules, we have

$$F^{1}(Z,Y^{1}) = 0 \qquad (1)$$

where F is a vector of functions, NF long, Y is a vector of dependent variables, and Z is a vector of independent variables. The set of NF simultaneous equations in eq. 1 yield NF elements of Y for a given Z and are the governing equations for the physical phenomena simulated by the module. No assumptions are made as to the mathematical nature of eq. 1, they may be nonlinear, transcendental, etc., so that an iterative algorithm may be required to solve for Y. In practice, a module is an entity comprising eq. 1 together with its solution algorithm coded as a computer program, usually including embellishments such as graphics. Then, Z is the input and Y is the output of the program which may be treated as a black box.

Structural finite element analysis and aerodynamic analysis of an aircraft wing are examples of the above. In structural finite element analysis, eq. 1 is

$$KY - P = 0 \qquad (2)$$

and represents an equilibrium of internal forces, the KY term, and the external loads P. Both P and the stiffness matrix K may be functions of input Z that describes the overall wing shape, cross-sectional dimensions, and the loading conditions. These equations and their solution for Y (the displacements and the resulting stress) are typically implemented in large computer programs, yielding tens of thousands of elements of Y and comprising hundreds of thousands of lines of code (e.g., Program NASTRAN, [7]).

Similarly, an example of governing equations for the aerodynamic analysis is

$$(J^{-1}U)_t + F_x + G_y + H_z - (G_v)_y = 0 \qquad (3)$$

where the subscripts indicate differentiation with respect to time t and the coordinates x, y, and z correspond to the streamwise, normal, and chordwise directions. In these equations, the terms are defined in [8], the vector U corresponds to Y in the generic notation used in this paper and the other terms contain input corresponding to Z. The equations yield pressure data for hundred of thousands points over the wing surface (270,000 points were used in [8]). Due to viscosity and compressibility effects, eq. 3 are distinctly nonlinear. Again, implementation of these equations and their solution took form of a large computer code that in an elastic wing system analysis appears as an aerodynamic module.

To simulate an elastic wing behavior, the structural and aerodynamic modules are assembled to make them to interact with each other, simulating the aerodynamics-structure coupling illustrated in figure 2. In reality, the coupling occurs because the aerodynamic forces deform the elastic wing. In turn, the deformation modifies the aerodynamic forces. In the simulating system, the coupling is realized by entering the aerodynamic pressure output Y from the aerodynamic module into the input Z of the structural module as the load data, and by using the deformation output Y from the structural module as the new shape data in the input Z of the aerodynamic module. In the presence of nonlinearities, the simulating system operates by iterating between the modules until the governing equations in each are satisfied to a desired tolerance.

At this point, it is necessary to distinguish three parts in the input Z of any module. The first part of Z consists of the physical quantities X that the designers change to influence the system behavior, the second part includes the constants Q, and the third part comprises the outputs Y from the other modules in the simulating system. Of course, both X and Q remain constant for the duration of analysis and the X elements are changed between the consecutive system analyses. The division between X and Q is not rigid, it is up to the designer to move physical quantities from Q to X and vice versa.

With the above definition of Z, it is now possible to generalize eq. 1 to a set of governing equations for a simulating system consisting of NM modules identified by subscripts:

$$\begin{array}{l} \cdots\cdots\cdots\cdots \\ F^{i-1}((X^{i-1},Q^{i-1},Y^{j}),Y^{i-1}) = 0; \quad j \neq i-1 \\ F^{i}((X^{i},Q^{i},Y^{j}),Y^{i}) = 0; \quad j \neq i \qquad j = 1 \ldots NM \\ F^{i+1},((X^{i+1},Q^{i+1},Y^{j}),Y^{i+1}) = 0; \quad j \neq i+1 \\ \cdots\cdots\cdots\cdots \end{array} \qquad (4)$$

Since each module represents a set of NF^i equations solvable for NF^i elements of Y^i, it follows that the number of equations in eq. 4 is equal to the number of elements in Y concatenated of the Y^i vectors.

In addition to aerodynamic and structural modules discussed in the wing example, support of a complete aircraft design would require modules for propulsion, control, electromagnetics, interior environment control, fuel management, avionics, weaponry, aircraft, performance, and more, all forming a system represented by eq. 4, and coupled internally by the Y cross-feed. To simulate the real system behavior the simulating system has to be iterated to convergence, assuming that nonlinearities exist that preclude solution by a linear algebraic elimination algorithm. The iterations may be nested because some of the modules may require internal iterations for their own solutions. Given the computational size of each module, to converge the solution for one setting of X is a formidable undertaking, even when using present-day supercomputers.

In design, the computational expense of producing the behavior data for one setting of X has to be incurred repeatedly as the designers change that setting in search of one that makes the system behave in an acceptable manner and then again in pursuit of a behavior that is better than merely acceptable.

Thus, the expense of the behavior model operation motivates a proposition that another mathematical model, capable of revealing directly and inexpensively how the behavior will change if a design variable is altered, should be added to the designer's tool box to complement the behavior model already there. That additional model will be referred to as the model of design.

4. MODEL OF DESIGN

One way to create a model of design is to use the model of behavior to obtain the data at several settings of X, each setting interpreted as a point in a hyperspace defined by X. The number of points is limited by the budget available for computational expenses, and their locations are strategically chosen throughout the intervals of interest using methods known as experiment design methods, e.g., [9]. Once the behavior data at these points have been generated, the behavior model is replaced by the hypersurface fitted to the points. The model is invoked as an explicit interpolating functions to obtain data on various aspects of the behavior between the points at essentially no cost, as needed in the course of design. The literature notes a number of applications of that type of design model in support of formal optimization, e.g., [10].

4.1. Extrapolation

One alternative is to obtain the behavior data from a behavior model at a single judiciously chosen setting of X, including among the data their derivatives with respect to the design variables X. The data is used an extrapolation formula, for instance, a Taylor series

$$Y(X) = Y_o + \nabla Y^T(X-X_o) + ½\,(X-X_o)^T [\,\nabla^2 Y\,](X-X_o) + (\textit{higher-order terms}); \qquad (5)$$

where Y quantifies a particular aspect of the behavior of interest, e.g., stress in the wing structure, propulsion thrust, or maximum flight range. Once the expense of using the behavior model to generate the data needed in the series above has been paid, the information about the effect of X on Y is available essentially at no cost, albeit its accuracy deteriorates as one moves away from the reference point where the analysis took place. The advantage of this design model over the one described previously is that one does not need to saturate the entire potentially interesting design space with analysis points. Instead, one starts at a single point and lets the extrapolation formula guide the search for the next point where the behavior data and the derivatives need to be refreshed by new analysis. Structural optimization applications with nearly 100 design variables have been reported, e.g, [11], where good results required only 10 to 20 repetitions of analysis that included calculation of derivatives.

The usefulness of this extrapolation-based design model critically depends on the computational cost of derivatives. Finite differencing as a means to obtain the derivatives may be prohibitively expensive since it adds another outer loop around the iterative analysis loops required for the solution of eq. 4, some of which may be nested already. Moreover, the finite differencing of iteratively obtained solutions may be meaningless for a small difference due to computational noise, while for a larger difference errors due to nonlinearities set in. Therefore, one may assert that the extrapolation-based, design model is incomplete for the purposes of large-scale applications if it does not include a means for efficient and accurate computation of derivatives.

To this end, [12] introduced a direct system sensitivity analysis that bypasses the finite differencing of the system analysis.

4.2. System Sensitivity Analysis

As shown in [12], the derivatives of behavior with respect to a design variable may be obtained from a set of simultaneous linear algebraic equations generated by application of the implicit function theorem to eq. 4. These equations are rewritten to show the Y's as implicit functions of X

$$\begin{array}{ll}
\cdots\cdots & \\
Y^{i-1} = f^{i-1}(X^{i-1}, Q^{i-1}, Y^j);\ \ j \neq i-1 & \qquad (6)\\
Y^i \ \ = f^i\,(X^i \ \ , Q^i \ \ , Y^j\,);\ \ j \neq i & \qquad j = 1 \ldots NM\\
Y^{i+1} = f^{i+1},(X^{i+1}, Q^{i+1}, Y^j);\ \ j \neq i+1 & \\
\cdots\cdots &
\end{array}$$

The derivatives of Y with respect to a particular design variable X k appear as unknowns in a set of equations

$$[A]\,\{\partial Y/\partial X\} = \{RHS\}_k\,;\quad k = 1 \ldots NX \qquad (7)$$

where:

$$[A] = \begin{bmatrix} I & \cdot & \cdot & \cdot & \cdot \\ \cdot & I & \cdot & \cdot A^{ij} & \\ \cdot & \cdot & I & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & I \end{bmatrix};$$

$$Y^T = [Y^{1T}, Y^{2T}, \ldots Y^{iT}, \ldots Y^{NM\,T}];$$
$$[A^{ij}] = [\partial f^i / \partial Y^j]$$
$$RHS^T{}_k = [\{\partial f^1/\partial X_k\}^T, \{\partial f^2/\partial X_k\}^T, \ldots \{\partial f^i/\partial X_k\}^T, \ldots \{\partial f^{NM}/\partial X_k\}^T]$$

The vectors and matrices in the above have the following dimensions

$$\{Y^i\},\ NF^i \times 1;\ [A^{ij}],\ NF^i \times NF^j;\ [A],\ NA \times NA;$$

$$NA = \sum_{i=1}^{NM} NF^i;\ \{RHS\},\ NA \times 1;\ \{X\},\ NX \times 1; \qquad (8)$$

By virtue of the implicit function theorem, eq. 7 are always linear, therefore they may be efficiently solved for many different vectors Y by factoring the A matrix once and storing it. The stored matrix may then be back-substituted over by the RHS vectors, each corresponding to one particular X_k.

The A^{ij} is a Jacobian matrix of the partial derivatives of the output from the i-th module with respect to the output from the j-th module that is received as input in the i-th module. The submatrices on the diagonal of A are identity matrices, and the element of the RHS_k are partial derivatives of the output from the modules with respect to a particular X_k. By definition the partial derivatives in A^{ij} and in RHS_k may be computed for each i-th module independently of each other. This enables one to use specialized methods for sensitivity analysis that have been developed for many engineering disciplines, e.g., [3], and [13], or even use finite differencing but on one module at a time, thus avoiding the cost of repetitive solution of the system equations, eq. 4. On the other hand, the computational cost of generating and solving eq. 7 grows superlinearly with the volume of the coupling data passed from one module to another as measured by NF^i in eq. 8. Indeed, in the limiting case of all A_{ij} being null matrices there is no coupling and the trivial solution of eq. 7 is Y = RHS because A = I. A later will show how one may keep NF^j from growing inordinately by using physical insight in selecting a minimal number of the pieces of data to be transmitted among the modules. Numerical conditioning of A was examined in [12] which concluded that singularity is not a danger if eq. 4 represent a well-posed problem.

Once the system sensitivity analysis has been reduced to solving the linear equations (eq. 7) it is possible to calculate the higher-order derivatives of Y with respect to X as derivatives of the first derivatives obtained from these equations. This approach was implemented in [14] by applying the same implicit function theorem to eq. 7 that was used to derive eq. 7 from eq. 4. Repetitive use of that theorem generates a recursive chain of formulas for the higher-order derivatives shown below in a compact notation which is defined first

$$\begin{aligned} &(\ )^r{}_{klm\ldots} = \partial^r(\ )/\partial X_k \partial X_l \partial X_m \ldots \\ &Z^o = Y^1{}_k \\ &Z^1{}_l = Y^2{}_{kl} \\ &Z^2{}_{lm} = Y^3{}_{klm} \qquad (9) \\ &\quad \cdot \qquad \cdot \\ &Z^N{}_{lm\ldots} = Y^{N+1}{}_{klm\ldots} \end{aligned}$$

where any subscript may be repeated as required to form a mixed derivative with respect to any combination of variables X.

In the above notation, the second- and higher-order derivatives are

$$\begin{aligned} A Z^1{}_l &= R^1{}_l - A^1{}_l Z^o; \qquad (10) \\ A Z^2{}_{lm} &= R^2{}_{lm} - A^1{}_m Z^1{}_l - D^1{}_m(A^1{}_l Z^o); \\ A Z^3{}_{lmn} &= R^3{}_{lmn} - A^1{}_n Z^2{}_{lm} - D^1{}_n(A^1{}_m Z^1{}_l) \\ &\quad - D^2{}_{mn}(A^1{}_l Z^o); \\ A Z^4{}_{lmnp} &= R^4{}_{lmnp} - A^1{}_p Z^3{}_{lmn} - D^1{}_p(A^l{}_n Z^2{}_{lm}) \\ &\quad - D^2{}_{np}(A^1{}_m Z^1{}_l) - D^3{}_{mnp}(A^1{}_l Z^o); \\ &\cdot \end{aligned}$$

etc

where $D^q{}_{lmn}(\ )$ is a shorthand for the q-th mixed derivative of the product of the pair of functions in the parentheses, obtained by the usual rules of product differentiation. Once the derivatives of Z are obtained, the derivatives of Y are available from eq. 9 . The above regular pattern can extended easily beyond the first four derivatives shown above.

It is apparent from eq. 7 and 10, that the computational cost may be reduced by factoring A only once, since A is the matrix of coefficients in equations for derivatives of every order. On the other hand, that cost escalates super-linearly with the derivative order because of the increase of the number of the derivatives to be computed and the accumulation of the prerequisite data required by the recursivity of eq. 10. By weighing that computational cost against the accuracy improvement attained by the use of the higher-order derivatives in the extrapolation (eq. 5) one decides to which order the sensitivity analysis should be extended. The current practice tends to include only the first, and occasionally, the second derivatives in large scale applications, but these practical limits are likely to go up as the progress in computer technology continues to lower the computational cost.

The extrapolation in eq. 5 together with the sensitivity analysis defined by eq. 7 through 10 define a model of design complementary to the model of behavior represented by eq. 4. Once the derivatives have been calculated and substituted in eq. 5, one may compute the effect of any X_k on the behavior practically instantaneously and at relatively negligible cost, provided that the increment of X_k is kept within extrapolation bounds (move limits) consistent with the problem nonlinearity and the order of extrapolation.

4.3. Enhancing the Design Model by Replacement Variables

The extrapolation bounds (move limits) may be widened by introducing artificially a degree of nonlinearity into the design model by a judicious replacement of the design variables. One such replacement is described in [15]. The Y^1_j behavior variable is tested for the sign of its first derivative with respect to X_k. If that derivative is positive, the extrapolation of Y^1_j continues to be done with respect to X_k, but for a negative derivative the extrapolation is done with respect to a replacement variable $R_k = 1/X_k$.

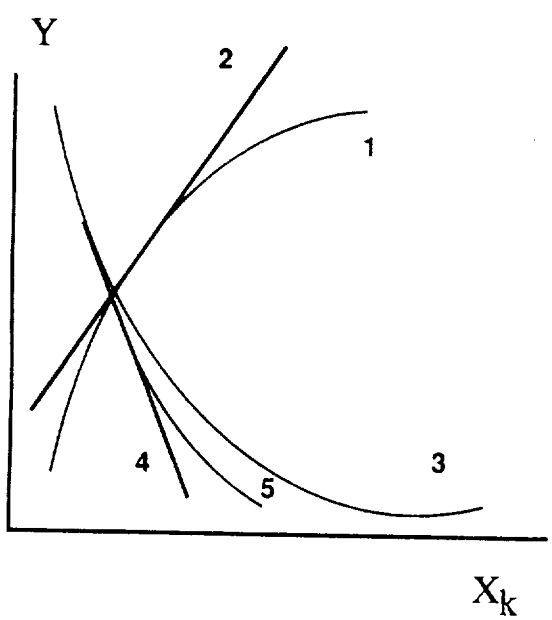

Figure 3 Replacement variables in extrapolation.

Justification for such selective replacement may be explained graphically as shown in figure 3. A positive derivative characterizes the function as ascending. An ascending function in a physical system is likely to obey the law of diminishing returns, hence it should look like curve 1. Extrapolation with respect to X_k by means of tangent 2 is likely to overestimate, and thus be a conservative approximation - a desirable feature in engineering design. Conversely, a negative derivative identifies a descending function portrayed by curve 3. Again, the law of diminishing returns is likely to render that curve asymptotic to the X-axis, so that tangent 4 would be an undesirable nonconservative approximation that underpredicts the value of the function. To reduce the error, one may extrapolate with respect to the reciprocal X_k, in effect following curve 5, and thus preserving the asymptotic nature of the true function 3 and at least some of its nonlinearity.

5. EXAMPLES

The first system optimization using a design model of the type described above was reported in [16]. It was a simple test case of a cantilever beam (structural analysis module) whose dynamic response to a ramp-shaped load impulse was controlled by exerting forces on the beam with actuators commanded by a control system (control module). Successful optimization for the minimum weight of the entire system, including the weights of the beam and of the actuators, employed a linear design model.

Four examples described in this section were selected to represent information accumulated since the above case was completed. Each of these examples illustrates different aspects of the foregoing discussion. The first example shows the role of physical insight in setting up the system sensitivity analysis. The second example demonstrates how greatly the system derivatives, with respect to design variables, may differ from the partial derivatives. The third example addresses the issue of the accuracy of the extrapolation. Finally, the last example shows the effect of including the second derivatives in the model of design used in a formal optimization.

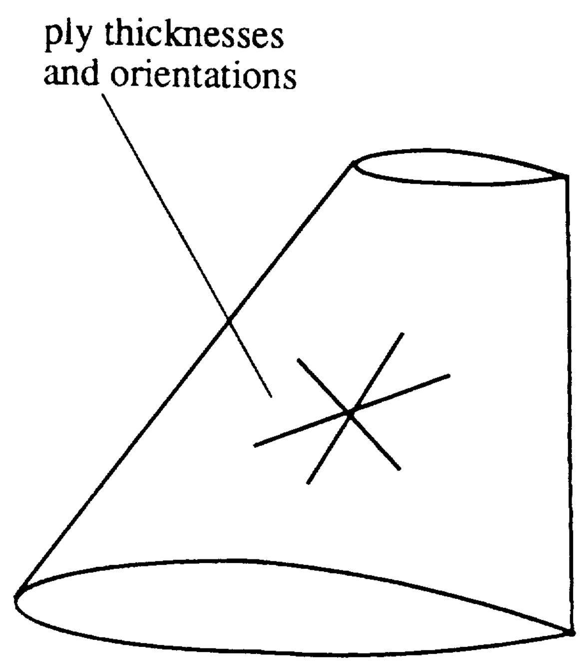

Figure 4 A transport aircraft wing.

5.1. Elastic, Slender, Composite Wing

As the first example, consider a slender wing structure of a transport aircraft (figure 4) with skin made of a composite material. Suppose that optimization of the composite skin involves 30 design variables comprising the ply thicknesses and orientation angles. The behavior model of the wing comprises the aerodynamic and structural modules that exchange the aerodynamic force and structural deformation data. The aerodynamic module uses a nonlinear method of analysis for a transonic flow and outputs an aerodynamic pressure value for each of 1000 discrete points over the wing surface. The structural analysis employs a finite-element method and outputs 1500 discrete displacement values for the finite-element model grid. A gradient-guided optimization program uses derivatives of aerodynamic pressure and structural displacement to guide the search for a constrained optimum in the design space.

If the derivatives needed in optimization were to be obtained by finite differencing performed on the model of behavior, the optimization would be organized as shown by the flowchart in figure 5. First, for a trial setting of X, one would iterate the aerodynamics and structures, loop 1, until convergence of the aerodynamic loads and structural deformations is obtained to a satisfactory tolerance. This iteration constitutes the system analysis for this case.

Next, the finite differencing would proceed in loop 2. Under the simplest finite-difference, one-step-forward scheme, each pass through loop 2 involves reanalysis of the system for the design variables perturbed one at a time. In that reanalysis, the aerodynamics-structure iteration in loop 1 must be reconverged. Assuming 5 passes to converge in loop 1, the total number of executions of each of the modules to carry out the system analysis once to obtain the reference solution and once for each design variable to obtain the solution first derivatives would be, assuming 30 design variables: 5 x (1 + 30) = 155. The solution and its derivatives are used in the extrapolation employing eq. 5 curtailed to the linear part of the series. The extrapolation is coupled to the optimization program in loop 3. The optimization yields a new, presumably improved, design for which the system solution and its derivatives have to be refreshed so loop 4 has to be traversed several times until overall convergence. Estimating that loop 4 would have to be repeated 15 times, one gets 4 x 155 = 620 executions of each module, if optimization is coupled directly to the model of behavior. The aggregate cost of these executions accounts for nearly the total cost of the entire optimization because the cost of executing loop 3 is trivial.

Introduction of a model of design that comprises the system sensitivity analysis by eq. 7 and the extrapolation by eq. 5 enables one to change the optimization organization from the one illustrated in figure 5 to the one shown

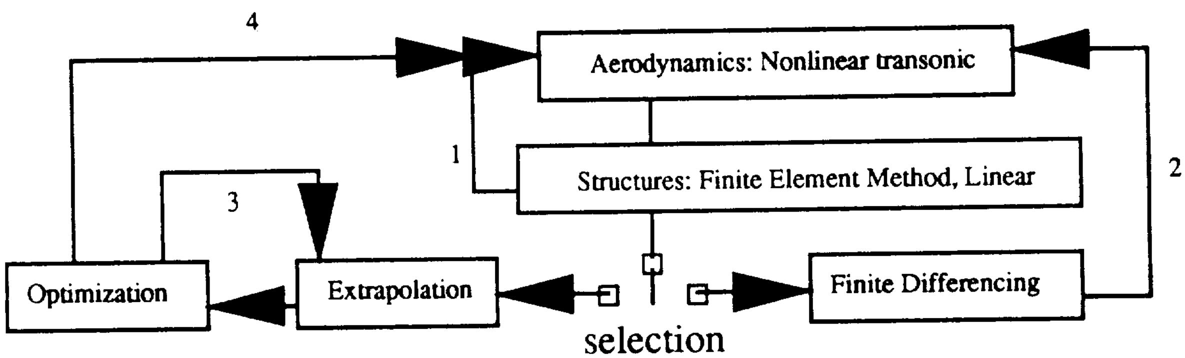

Figure 5 Optimization with dervatives obtained by finite differencing of the behavior model.

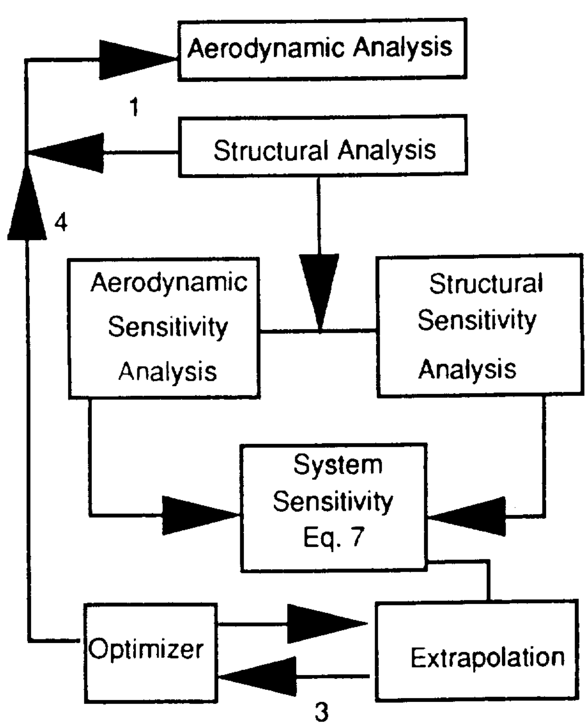

Figure 6 Optimization based on derivatives obtained by system sensitivity analysis.

in figure 6. Loop 1 is the same as in figure 5, but the finite differencing that engages the system analysis, loop 2 in figure 5, is replaced by two disciplinary sensitivity analyses, one for aerodynamics and one for structures, that are executed independently of each other (an opportunity for parallel processing). As mentioned before, algorithms for disciplinary sensitivity analyses become routine in structures [3], begin to be available in aerodynamic analysis [17], and are generally much less computationally costly than finite differencing.

The volume of data to be exchanged between the modules governs the computational cost of the disciplinary sensitivity analyses. As assumed in this example, there are 1000 pressure data output from the aerodynamic module and 1500 displacement data output from the structures module. Labeling the aerodynamic and structures modules as 1 and 2 respectively, the Jacobian matrices in eq. 7 would have the dimensions 1000 x 1500 for A^{12} and 1500 x 1000 for A^{21}. In other words, one would have to take the partial derivatives of each pressure datum with respect to each displacement datum and vice versa. Despite the efficiency of disciplinary sensitivity analysis, computation of that many partial derivatives would still be economically prohibitive. However, the number of derivatives needed may be radically reduced by physical insight.

Since the wing is slender (high aspect ratio), its chordwise, plate-like bending is negligible relative to the spanwise, beam-like bending. Furthermore, the wing aerodynamic forces are affected by the changes of the streamwise airfoil angle of attack caused by the structural twist (and also bending in the case of a swept wing. Since the wing twist angle and bending deflections are known to be distributed quite smoothly spanwise, it is a reasonable assumption to transmit only, say, 5 values of the angle-of-attack changes at 5 spanwise wing locations as a description of structural deformations. Conversely, the 1000 aerodynamic pressure data may be collected into a vector of, say, 10 concentrated forces at 10 spanwise wing locations.

This condensation may easily be implemented as postprocessing so it becomes a part of each module. Then, the dimensions of the Jacobians in eq. 7 reduce to 10 x 5 and 5 x 10 for A^{12} and A^{21}, respectively. By the same token, RHS for this example would be reduced to only 15 elements. In this RHS, all elements are null except the bottom partition of 5 elements that contains the partial derivatives of displacement with respect to the a structural design variable.

Once the partial derivatives from the two disciplinary sensitivity analyses are obtained, the system derivatives are calculated from eq. 7 and optimization using eq. 5 executes in loop 3 followed by loop 4, the same as in figure 5.

In summary, the use of a model of design that embeds a system sensitivity analysis has reduced the number of executions of each modules in this example from 620 to merely 5 x 15 = 75 (loop 1 nested in loop 4), at the price of adding the cost of the disciplinary sensitivity analyses and the cost of solving eq. 7 (nested in loop 4 hence incurred 15 times under the assumptions used in the example). Even with that added cost, the overall optimization cost reduction is likely to be more than an order of magnitude. An additional potential benefit is time saved due to the parallel processing of the independently executed disciplinary sensitivity analyses.

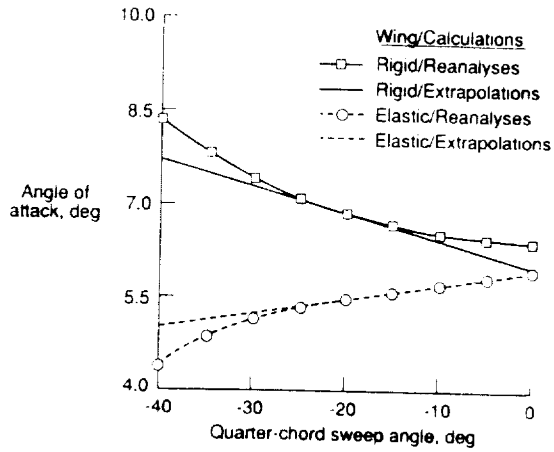

Figure 7 Trimmed angle of attack as a function of sweep angle.

5.2. Forward-Swept, Elastic Wing

A result reported for a forward-swept, elastic wing in [18], is an example of a drastic difference between the values of the system derivative and the partial derivative of a behavior variable with respect to a design variable. The system again comprises the aerodynamics and structures modules, the behavior variable is the trimmed angle of attack (the angle of attack at the reference chord required to maintain a prescribed lift), and the design variable is the sweep angle.

The sweep angle is the horizontal coordinate in figure 7 (negative degrees indicate forward sweep) and the trimmed angle of attack is the vertical coordinate. The trimmed angle of attack as a function of the sweep angle is shown for the rigid and flexible wing. The slopes of the tangents represent the derivatives at an arbitrarily chosen sweep angle of 20 degrees. The partial derivative corresponds to the tangent slope of the rigid wing curve because it was obtained from the aerodynamics alone. The system derivative is represented by the tangent slope of the flexible wing curve since it reflects the interaction of two modules in the wing system. It is apparent that the interaction was so strong in this case that the derivative and the partial derivative have opposite signs.

The lesson from this example is that a trend predicted on the basis of only one module in a coupled system may completely misguide the design decisions.

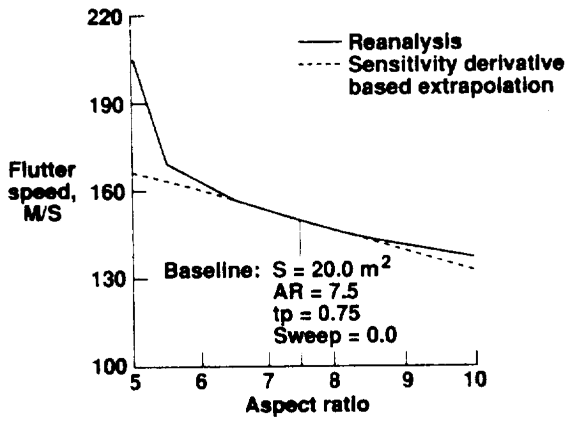

Figure 8 Flutter speed as a function of wing aspect ratio.

5.3. Wing Flutter

Usefulness of the design model as a predictor of the effects of the design variable changes depends on the degree of nonlinearity of these effects. Results that shed light on that issue were reported in [18] and [19]. The previous example showed good accuracy of the linear extrapolation over a broad numerical range of the design variable for a static type of the behavior. Reference [19] included results for a dynamic type of aeroelastic behavior. An example of such a result is given in figure 8 that shows the flutter speed as a function of the wing aspect ratio (slenderness).

The function exhibits a low degree of nonlinearity for the aspect ratio greater than 6.5 which makes the linear extrapolation in that range a very good predictor of the aspect ratio influence on the flutter speed, as illustrated by the dotted line tangent. On the other hand, the function is strongly nonlinear for the aspect ratio smaller than 6.5, and the linear extrapolation in that interval would have to be subject to fairly narrow move limits to be reliable. Slope discontinuities in the function caused by the flutter mode switching are also detrimental to the extrapolation accuracy. This example indicates that care is needed in setting the extrapolation move limits and that the use of higher derivatives in the model of design should be considered as means to avoid excessively narrow move limits.

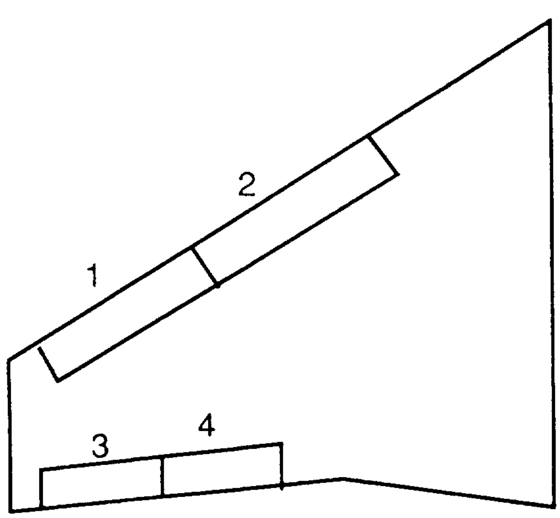

Figure 9 A fight wing configuration with control surfaces.

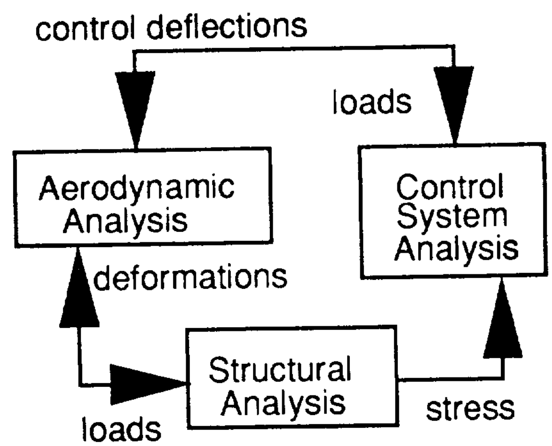

Figure 10 A simulation system for the fighter wing.

5.4. Optimization Using a Quadratic Design Model

An example of the use of the second-order derivatives in the extrapolation coupled with an optimization program was given in [20]. The object of the study was a wing shown in figure 9, that was equipped with two control surfaces on the leading edge and two on the trailing edge. A control system was programmed to deflect these surfaces to reduce the wing-root bending moment while maintaining a constant lift. The model of behavior included the modules representing aerodynamic, structural, and control analyses coupled as depicted in figure 10. Optimization used the wing-root bending moment as the objective function to be minimized by manipulating the control surface deflection as design variables under the constraint of the constant lift.

The volume of data communicated between the aerodynamics and structures modules was judiciously limited by physical insight in a manner similar to that discussed in the first example. For instance, the airfoil lift coefficients were computed at only four spanwise locations.

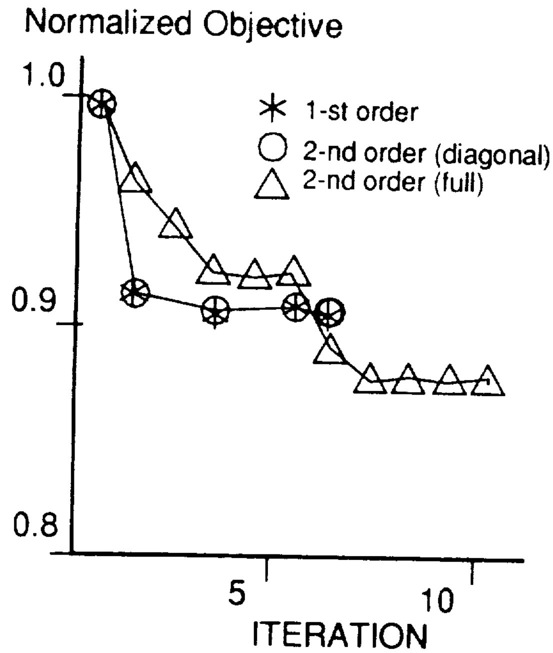

Figure 11 Histogram of optimization of the fighter wing.

Optimization results obtained with the use of a model of design based on the first derivatives, the diagonal terms of the second derivative matrix, and the full second derivative matrix are illustrated in figure 11. They show that in this particular case a meaningful reduction of the minimum objective function was brought about by including a full set of the second derivatives.

6. CONCLUDING REMARKS

Mathematical simulation of complex engineering systems is commonly used in the design of these systems to obtain information about the system response to external stimuli, in effect it is used as a mathematical model of behavior. This paper shows that means exist for such simulation to be extended to answering the "what if" questions concerning effects of the design variables on behavior - the questions that must be answered in the quest for an optimal design. System sensitivity analysis quantifies answers to such questions by computing derivatives of behavior with respect to design variables without the costly finite differencing of system analysis. The algorithm for system analysis offers accuracy and an opportunity for parallel processing. The algorithm also allows the use of specialty methods for partial-sensitivity analysis in the disciplines involved in the system at hand.

Sensitivity analysis yields derivatives of the first- and higher-orders that may be coupled with an extrapolation based on these derivatives to form a model of design. That model is capable of answering the designer's queries about the effect of design variables practically instantaneously, and at a negligible cost comparing to the use of finite differencing on the model of behavior.

Complementing the model of behavior with the model of design extends the array of tools that assist an engineer in the design of a physical system. As shown in figure 12, that array affords the designer the option of getting answers to three basic questions that occur in the design process. The "what now" question about the system response will be answered by the model of behavior. The "what if" question about the effects of a design variable will be answered by the model of design. Finally, the question "what is the best" in search for an optimal setting of many design variables, under complex and possibly competing considerations, may be answered by a formal optimization that calls on both models.

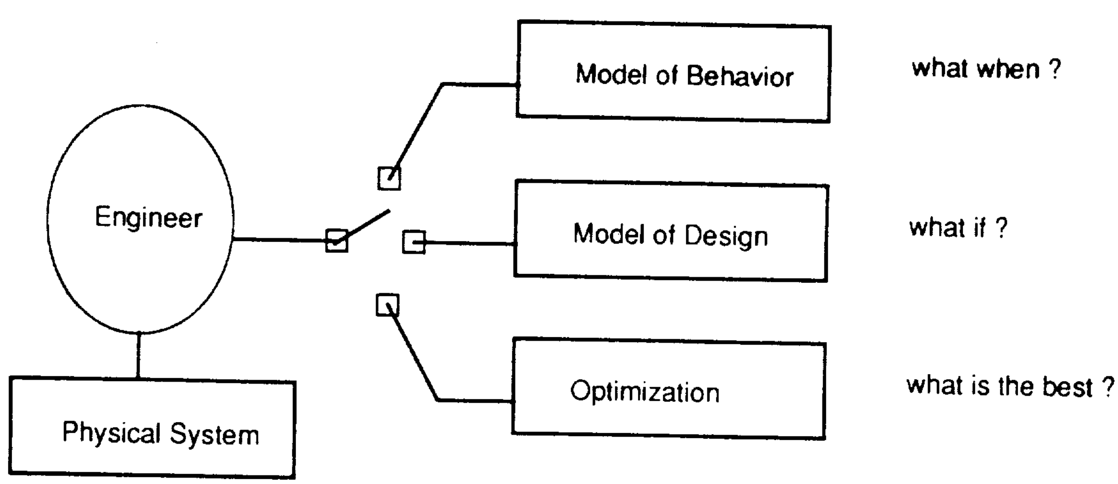

Figure 12 Array of tools for system design.

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NASA National Aeronautics and Space Administration

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-101616		

4. Title and Subtitle	5. Report Date
Approximate Simulation Model for Analysis and Optimization in Engineering System Design	June 1989
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Jaroslaw Sobieszczanski-Sobieski	
	10. Work Unit No. 506-43-41-01
9. Performing Organization Name and Address: NASA Langley Research Center, Hampton, VA 23665-5225	11. Contract or Grant No.
	13. Type of Report and Period Covered: Technical Memorandum
12. Sponsoring Agency Name and Address: National Aeronautics and Space Administration, Washington, DC 20546-0001	14. Sponsoring Agency Code

15. Supplementary Notes

To be published in proceedings of ISAOC '89 - International Symposium on Approximation, Optimization and Computing held July 3-7, 1989, Dalian, China.

16. Abstract

Computational support of the engineering design process routinely requires mathematical models of behavior to inform designers of the system response to external stimuli. However, designers also need to know the effect of the changes in design variable values on the system behavior. For large engineering systems, the conventional way of evaluating these effects by repetitive simulation of behavior for perturbed variables is impractical because of excessive cost and inadequate accuracy. This paper describes an alternative based on recently developed system sensitivity analysis that is combined with extrapolation to form a model of design. This design model is complementary to the model of behavior and capable of direct simulation of the effects of design variable changes.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Optimization Simulation Design	Unclassified - Unlimited Star Category - 31

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
UNCLASSIFIED	UNCLASSIFIED	12	A03

NASA FORM 1626 OCT 86