The Role of Particle Collisions in Pneumatic Transport

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ABSTRACT

A model of dilute gas-solid flow in vertical risers is developed in which the particle phase is treated as a granular material, the balance equations for rapid granular flow are modified to incorporate the drag force from the gas, and boundary conditions, based on collisional exchanges of momentum and energy at the wall, are employed. In this model, it is assumed that the particle fluctuations are determined by inter-particle collisions only and that the turbulence of the gas is unaffected by the presence of the particles. The model is developed in the context of, but not limited to, steady, fully developed flow. A numerical solution of the resulting governing equations provides concentration profiles generally observed in dilute pneumatic flow, velocity profiles in good agreement with the measurements of Tsuji, et al. (1984), and an explanation for the enhancement of turbulence that they observed.

INTRODUCTION

Gas-solid flows satisfy the principles of mass and momentum conservation. While the Navier–Stokes equations govern the motion of the gas phase, there is not yet unanimous agreement upon the form of the equations for the particle phase. Treating this phase as a continuum provides a framework in which techniques such as volume averaging may be used. This two–fluid approach results in the derivation of partial differential equations of motion. Depending on the situation, the stress tensor for the particle phase can then be modeled in order to close the system.

For small particles, dilute in a turbulent gas, momentum transfer in the particle phase is due to turbulent diffusion of the particles. In this regime, Elghobashi & Abou–Arab (1983) have rigorously derived a two–fluid k–ε model that relates the Reynolds stress of the particles to gradients of the mean particle velocity using an eddy viscosity that is a

fraction of the eddy viscosity of the gas. This model predicts the reduction of turbulent energy observed in the presence of small particles (e.g., Modaress, et al., 1984). Other models for this regime include the works of Pourahmadi & Humphrey (1983), Chen & Wood (1985) and Berker & Tulig (1986). However, these models do not apply to gas-solid flows with large and heavy particles.

For large particles, the experiments of Soo, et al. (1960) have shown that the intensity of particle velocity fluctuations may exceed that of the fluid, an observation that cannot be explained by treating the particle response to turbulence. Min (1967) attributed this high particle "turbulence" to particle–wall collisions. In this context, Lourenco, et al. (1983) have successfully modeled the air flow in a 10cm wide horizontal duct loaded with 500μm glass particles. By analogy with molecular dynamics, these authors treat particle collisions using a particle velocity distribution function that satisfies the Boltzmann transport equation. They assume that the distribution is determined by particle collisions rather than by the gas turbulence. In other words, the gas affects the mean velocity of the particles but not their random motion. This assumption is justified in Lourenco's model, because the ratio of particle relaxation time to a typical large eddy turbulent timescale is large (Hinze, 1972). Clearly, the success of Lourenco, et al. encourages further studies of the regime dominated by particle collisions.

Another shortcoming of the two–fluid approach is the lack of attention given to the formulation of correct boundary conditions for the particle phase. Unlike the fluid phase, particles can slip at a boundary. For dilute suspensions of small particles, Soo (1969) suggested, without derivation, a set of boundary conditions by analogy with rarefied–gas dynamics. Clearly, for regimes where collisions dominate momentum transfer among the particles, the boundary conditions of the particle phase should be carefully considered.

In this study, a two–fluid model of the vertical flow in a gas-solid riser is proposed. For simplicity, the model assumes that the flow is fully-developed and steady. Particles are assumed to be sufficiently large or heavy so that momentum transfer in the particle phase is due to particle collisions alone. The granular theory used to describe the particles is related to the kinetic theory of gases, but it is not limited to dilute situations or to elastic particles (e.g., Jenkins, 1987). The conservation laws for mass and momentum in the particle phase have familiar forms. An additional balance law governs the measure $w = \sqrt{<v'^2>/3}$ of the velocity fluctuations v'. The quantity w is the analog of the molecular temperature. For the particle phase, constitutive relations for the stress tensor and the flux of fluctuation energy are those supplied by the kinetic theory (Chapman & Cowling, 1970) and an existing form

for the collisional dissipation of fluctuation energy is employed (Jenkins & Savage, 1983). Finally, boundary conditions for the mean particle velocity and the granular temperature at a solid surface follow from a detailed description of the particle collision dynamics at the surface (Jenkins, 1988). Such rigorous derivation of the constitutive relations and boundary conditions are a major advantage of the granular flow theory against *ad hoc* continuum models for the particle phase.

In this paper, the granular flow equations given, for example, in Jenkins (1987) are modified to include contributions from the gas. In particular, a drag term is added to the momentum equation and viscous dissipation of the particle rms fluctuating velocity is introduced. The resulting equations, constitutive relations and boundary conditions are presented for a dilute, steady, fully developed, upward flow in a vertical pipe with a circular cross-section. A numerical solution of these equations is obtained and compared with the experimental data of Tsuji, et al. (1984).

THE TWO-FLUID MODEL

Balance Laws

The equations of motion for a steady, fully developed, axi–symmetric upward flow of a dilute gas-solid suspension are:

Gas momentum

$$\frac{1}{r}\frac{d}{dr}(r\,\tau) - \varepsilon\frac{dp}{dz} - \varepsilon\rho g - F = 0 , \tag{1}$$

where ε is the volume fraction of the gas, r and z are radial and vertical coordinates, τ is the mean gas shear stress, p is the gas pressure, and F is the force per unit volume exerted by the gas on the particles;

Particle momentum

$$\frac{1}{r}\frac{d}{dr}(r\,S) - (1-\varepsilon)\rho_s g - (1-\varepsilon)\frac{dp}{dz} + F = 0 \tag{2}$$

and

$$\frac{dN}{dr} = 0 , \tag{3}$$

where S is the particle shear stress, ρ_s is the density of the particle, g is the gravitational acceleration, and N is the particle pressure;

Particle fluctuation energy

$$-\frac{1}{r}\frac{d}{dr}(r\,q) + S\frac{dv}{dr} - D - D' = 0,\qquad(4)$$

where q is the flux of fluctuation energy, v is the mean particle velocity, and D and D' are, respectively, the rates of collisional and viscous dissipation per unit volume.

<u>Constitutive Relations</u>

1) Volume supplies

Equation (1) is the vertical component of the momentum equation for an incompressible gas exerting drag F on the particle phase. The mean drag is equal to the drag force on a single sphere based on the mean velocities, multiplied by the number of particles per unit volume:

$$F = C_d \mid u-v \mid (u-v)\frac{3\rho}{4d}\,(1\text{-}\varepsilon)\,f(\varepsilon),\qquad(5)$$

where u is the mean interstitial gas velocity, d is the particle diameter, and the drag coefficient C_d is given in terms of the particle Reynolds number, Re_p, by

$$C_d = \frac{24}{Re_p}\left[1 + 0.15Re_p^{0.687}\right],\qquad(6)$$

with

$$Re_p = \frac{\mid u\text{-}v\mid \rho d}{\mu},\qquad(7)$$

in which ρ and μ are, respectively, the density and viscosity of the gas. The drag coefficient C_d is taken from Boothroyd (1971) and it is valid for $0 < Re_p < 1000$. The function $f(\varepsilon)$ is an empirical correction to the drag force on a single particle that allows for the presence of other particles. According to Foscolo and Gibilaro (1984), for the same velocity difference, the drag force per particle increases in the presence of other particles according to

$$f(\varepsilon) \approx \varepsilon^2\,\varepsilon^{-3.8},\qquad(8)$$

where the ε^2 term accounts for the ratio of superficial to interstitial velocities.

Equations (2) and (3) are the vertical and radial components of the balance of particle momentum in this simple flow. The shear stress and particle pressure result from transport of momentum between collisions exactly as in a dilute gas. The vertical balance includes forces due to gravity, gas pressure gradient, and drag.

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The first term of equation (4) is the diffusion of particle fluctuation energy. The second term is the energy produced by mean shear, the third is the rate of energy dissipation per unit volume due to inelastic collisions. In the dilute limit of granular flow, the third term is given by

$$D = \frac{24(1-e)\rho_s}{\sqrt{\pi}\ d}\ w^3 (1-\varepsilon)^2 ,\qquad (9)$$

where e is the coefficient of restitution for a particle–particle collision; e=1 for elastic particles. This is obtained by considering the kinetic energy lost in each collision, then averaging over all possible collisions (Jenkins & Savage, 1983).

The fourth term, D', is the rate of energy dissipation per unit volume arising from the drag force on the fluctuating particles. To calculate it, we first write the rate at which energy is gained by a particle as the scalar product of its total velocity and the particle drag force. Ignoring the velocity fluctuations in the gas, this is

$$C_d\ |\ \underline{u} - \underline{v} - \underline{C}\ |\frac{3\rho}{4d}(1-\varepsilon)\ (\underline{u} - \underline{v} - \underline{C})\bullet(\underline{v} + \underline{C}) ,\qquad (10)$$

where $\underline{C}$ is the velocity fluctuation of the particle. Then, upon evaluating the second term in the brackets in equation (6) at the mean velocity difference and averaging over all possible $\underline{C}$, we find that the rate of energy loss per unit volume from the fluctuating motion is

$$D' = \frac{9\rho}{4d}C_d\ |\ u-v\ |\ (1-\varepsilon)\ w^2 .\qquad (11)$$

2) Fluxes

In most situations of practical interest, the flow Reynolds number is high enough for the gas to be turbulent. For simplicity, the mean gas shear stress is modeled using an eddy viscosity μ_t,

$$\tau = \varepsilon\ (\mu + \mu_t)\ \frac{du}{dr} .\qquad (12)$$

In this work, μ_t is approximated using a polynomial fit to the experimental results for turbulent gas flow in a pipe (Hinze, 1975, Sec. 7-13). Thus, we assume that the gas Reynolds stress is unaffected by the presence of particles. We note, however, that several experimental observations have shown that large particles enhance the turbulence, even for conditions as dilute as $(1-\varepsilon) \approx 0.1\%$ (e.g., Tsuji, et al., 1984 and Lee & Durst, 1982). Our view is that any treatment of the gas turbulence should not ignore the details of the particles' motion. In particular, the "pure" turbulent fluctuations in the gas must be distin-

guished from the randomness induced by particle fluctuations and the resulting change of the structure of turbulence must be quantified. Here, for simplicity, we ignore the influence of these induced fluctuations on the eddy viscosity. For a discussion of these effects, see the experiments of Boothroyd (1967), Nouri, et al. (1987), Tsuji, et al. (1984), and Modaress, et al. (1984), and the theoretical discussions of Owen (1969), Elghobashi & Abou–Arab (1983) and Genchev & Karpuzov (1980).

The constitutive relations for the particle shear stress S, the particle pressure N, and the flux of fluctuation energy q for dilute flows of smooth, nearly elastic spheres, are identical to those provided by Chapman & Cowling (1970, Secs. 7.4, 7.41, and 10.21):

$$S = \frac{5\sqrt{\pi}}{96} \rho_s\, d\, w\, \frac{dv}{dr}, \tag{13}$$

$$N = (1\text{-}\epsilon)\, \rho_s\, w^2, \tag{14}$$

and

$$q = -\frac{25\sqrt{\pi}}{64} \rho_s\, d\, w^2 \frac{dw}{dr}. \tag{15}$$

The shear stress is Newtonian and the particle viscosity is proportional to the rms fluctuating velocity and the particle diameter. The energy flux is proportional to the gradient of $3w^2/2$, the fluctuation energy per unit mass, and the corresponding coefficient depends linearly upon d and w. In equations (13) through (15), we assume that the particles do not respond to the turbulent fluctuations, so that the particle velocity distribution function is unaffected by the gas fluctuations. From Hinze (1972), this is true if

$$\frac{\Lambda}{d} \ll \sqrt{\frac{u'\Lambda}{\nu} \frac{\rho_s}{\rho}}, \tag{16}$$

where Λ is the integral lengthscale of turbulence, u' the rms turbulent gas velocity and ν the kinematic viscosity of the gas.

Boundary Conditions

Jenkins (1988) has recently derived boundary conditions for flows of nearly elastic but frictional spheres that interact through collisions with a flat frictional wall. Consideration of the balance of linear momentum, angular momentum, and energy in a single collision and some simple averaging provide the rate per unit area at which momentum and energy are being supplied to the flow by the wall. Boundary conditions are obtained when these are related to the shear stress and energy flux in the flow, evaluated at the wall. When

the mean spin of the particles is assumed to be half the vorticity of the mean particle velocity and fluctuations in the spin are ignored, these conditions are, in the limit of large friction,

$$S = - (2/\pi)^{1/2} (N/7w) [v + (d/4) (dv/dr)] \qquad (17)$$

and

$$q = - (2/\pi)^{1/2} N \{ (1/14w) [v + (d/4) (dv/dr)]^2 - (1-e_w+1/7)w \} , \qquad (18)$$

where e_w is the coefficient of restitution of the particle-wall collision. Consequently, upon adopting the no-slip condition for the gas velocity at the wall and employing the constitutive relations (13) and (15), the values of u, v and w at the wall must satisfy

$$u = 0 , \qquad (19)$$

$$\frac{dv}{dr} = A \frac{v}{d} , \qquad (20)$$

and

$$\frac{dw}{dr} = B \frac{w}{d} , \qquad (21)$$

where

$$A = - 384\sqrt{2} (1-\varepsilon) / \{ 7[20\pi + 96\sqrt{2} (1-\varepsilon)] \} \qquad (22)$$

and

$$B = \frac{32\sqrt{2}}{175\pi} (1-\varepsilon) \{ (\frac{v}{w})^2 [1 - (\frac{A}{4})^2] - 14(1 - e_w + \frac{1}{7}) \} . \qquad (23)$$

The second set of boundary conditions is provided by symmetry. At the centerline,

$$\frac{du}{dr} = \frac{dv}{dr} = \frac{dw}{dr} = 0 . \qquad (24)$$

RESULTS AND DISCUSSION

Equations (1), (2), and (4) are solved numerically using a simple iterative finite difference algorithm. To verify the predictions of the model, gas and particle velocity profiles are now compared with detailed measurements in a vertical pipe. Using a laser-döppler-anemometer, Tsuji, et al. (1984) measured gas and particle velocity profiles in a vertical pipe of 30.5 mm ID. In these experiments, polystyrene particles (ρ_s = 1020 kg/m^3, d = 500μm) were suspended in air with ratios of particle-to-gas mass flow rates (loading) as

high as 3.6. Because the integral lengthscale Λ is of the order of the pipe radius, $\Lambda/d \approx 30$ and $[(u'\Lambda/\nu)\,(\rho_s/\rho)]^{1/2} \approx 600$. Therefore, Hinze's criterion (16) is clearly satisfied. Under these conditions, the particles cannot follow the gas turbulence. In the calculation we assume coefficients of restitution of 0.9 and 0.8 for particle–particle and particle–wall collisions, respectively. We input the pressure gradient in the gas and the particle concentration at the wall and adjust these until we agree with the measured gas velocity at the centerline and the measured loading in the experiments.
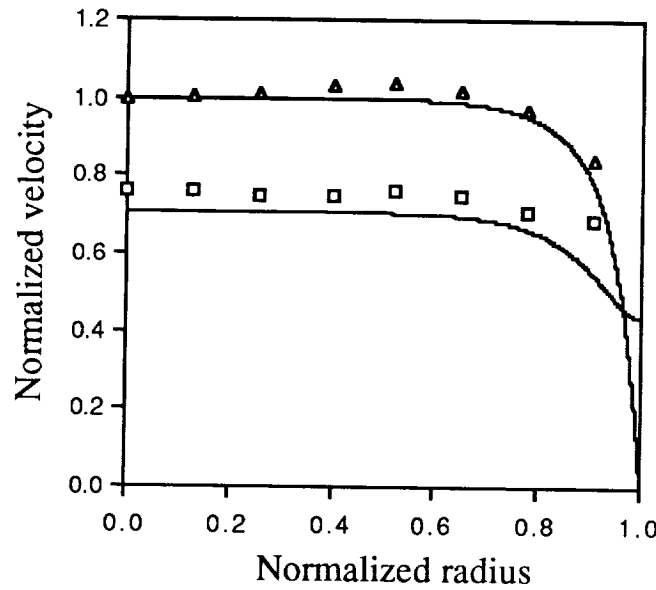
Fig. 1. Calculated gas and particle velocity profiles compared with the data from Tsuji, et al. (1984). The calculated velocities are normalized with the calculated gas velocity at the centerline (7.9 m/s), while the measured velocities (triangles: gas, squares: particles) are normalized with the measured gas velocity at the centerline (8.1 m/s). The ratio of gas–to–particle mass flow rates is 3.6.

Figure 1 shows good agreement between the measured mean velocities and the predictions of the model. At the centerline, the gas velocity is reproduced to within 5%, and the particle slip velocity to within 10%. However, the gas velocity profile near the wall differs from the model predictions. This difference is not altogether surprising, considering the simplicity of the gas equations used. Nevertheless, the overall agreement in the gas phase is good, because the term that dominates the gas momentum equation is not the Reynolds stress, but the difference between the pressure gradient and the particle drag.

The predicted particle velocity at the wall is positive, although not as high as the experimental value. Comparable observations were made by Lee and Durst (1982) in a

similar situation. At first sight, these observation might be surprising; because the gas velocity is zero at the wall, one might expect the particles to fall. In fact, there is a region near the wall where the particles can acquire a velocity higher than that of the gas. This effect results from the shear stress in the particle phase. Particles further from the wall are lifted by the gas and, through the particle shear stress, they lift the particles closer to the wall. Clearly, the details of the momentum exchange in the particle phase are essential for an accurate description of the flow.
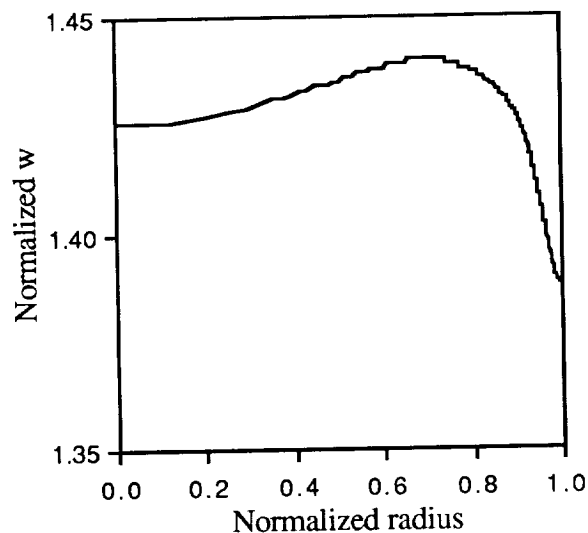

Fig. 2. Particle fluctuation velocity normalized with the calculated gas centerline velocity.

Figure 2 shows the radial distribution of kinetic energy. In this case, w increases away from the wall. There is a slight decrease in energy in the immediate neighborhood of the wall followed by an increase into the interior. In more dense flows the supply or dissipation of energy by the wall is expected to be more important. The particle volume fraction is calculated using equation (14), and it decreases away from the wall (Figure 3). Unfortunately, these predictions cannot be compared with the experiments of Tsuji, et al. (1984), who did not measure the particle velocity fluctuations or the concentration of the particle phase. Nevertheless, the predicted concentration profile agrees qualitatively with other experiments in pneumatic transport (e.g., Boothroyd, 1971 and Kramer & Depew, 1972). Several explanations have been proposed for these larger particle concentrations near the wall. Boothroyd (1971) attributes this to electrostatic forces, while Berker & Tulig (1986) invoke non-gradient turbulent diffusion. In the regime dominated by particle collisions, be-

cause of the constant particle normal stress and the constitutive relation (14), this trend is a direct result of the profile of fluctuation energy.
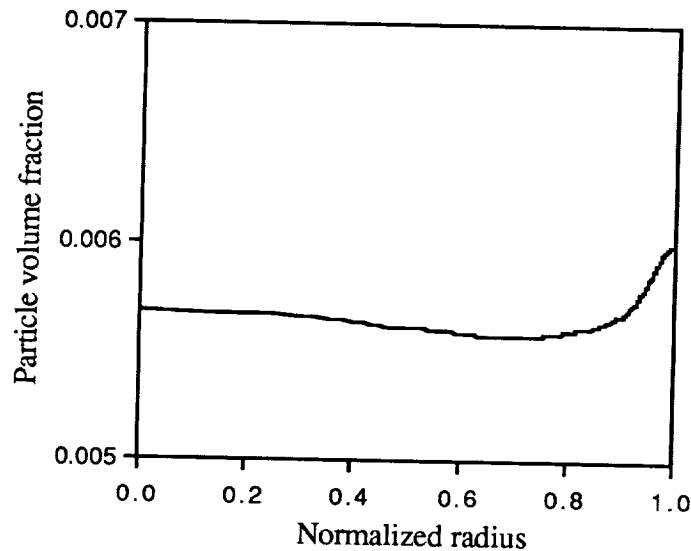
Fig 3. Profile of particle volume fraction.

Finally, we propose that the enhancement of the gas turbulence observed by Tsuji, et al. (1984) in the presence of large particles is due to fluctuations in the gas induced by the random motion of the particles. Turbulent enhancement of the order of our calculated w were measured for the 500μm particles by Tsuji, et al. (1984), and the strength of the turbulence was observed to increase with particle diameter.

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