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DISCRETE FILTERING TECHNIQUES
APPLIED TO
SEQUENTIAL GPS RANGE MEASUREMENTS

BY

FRANK VAN GRAAS

(NASA-CR-177472) DISCRETE FILTERING
TECHNIQUES APPLIED TO SEQUENTIAL GPS RANGE
MEASUREMENTS (Ohio Univ.) 43 p CSCL 17G

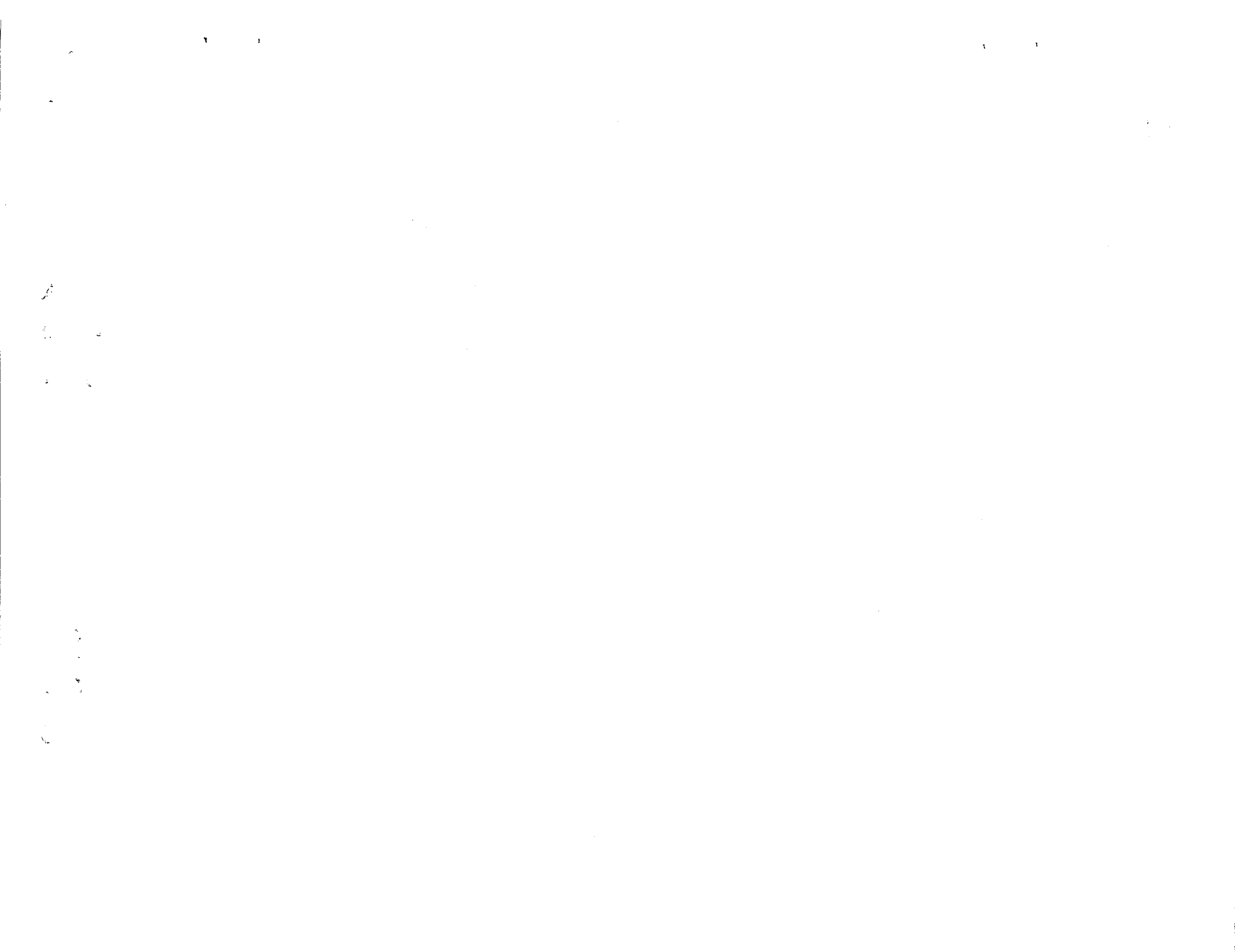
N90-11730

Unclas
G3/04 0243082

CONTRACT NAS2- 11969
DECEMBER 1987

Date for general release December 1989





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PREPARED FOR
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UNDER CONTRACT NAS2-11969
DECEMBER 1987



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ABSTRACT

This paper describes the basic navigation solution for position and velocity based on range and delta range (Doppler) measurements from NAVSTAR Global Positioning System satellites.

The application of discrete filtering techniques is examined to reduce the white noise distortions on the sequential range measurements. A second order (position and velocity states) Kalman filter is implemented to obtain smoothed estimates of range by filtering the dynamics of the signal from each satellite separately.

Test results using a simulated GPS receiver show a steady-state noise reduction, the input noise variance divided by the output noise variance, of a factor of four.

Recommendations for further noise reduction based on higher order Kalman filters or additional delta range measurements are included.

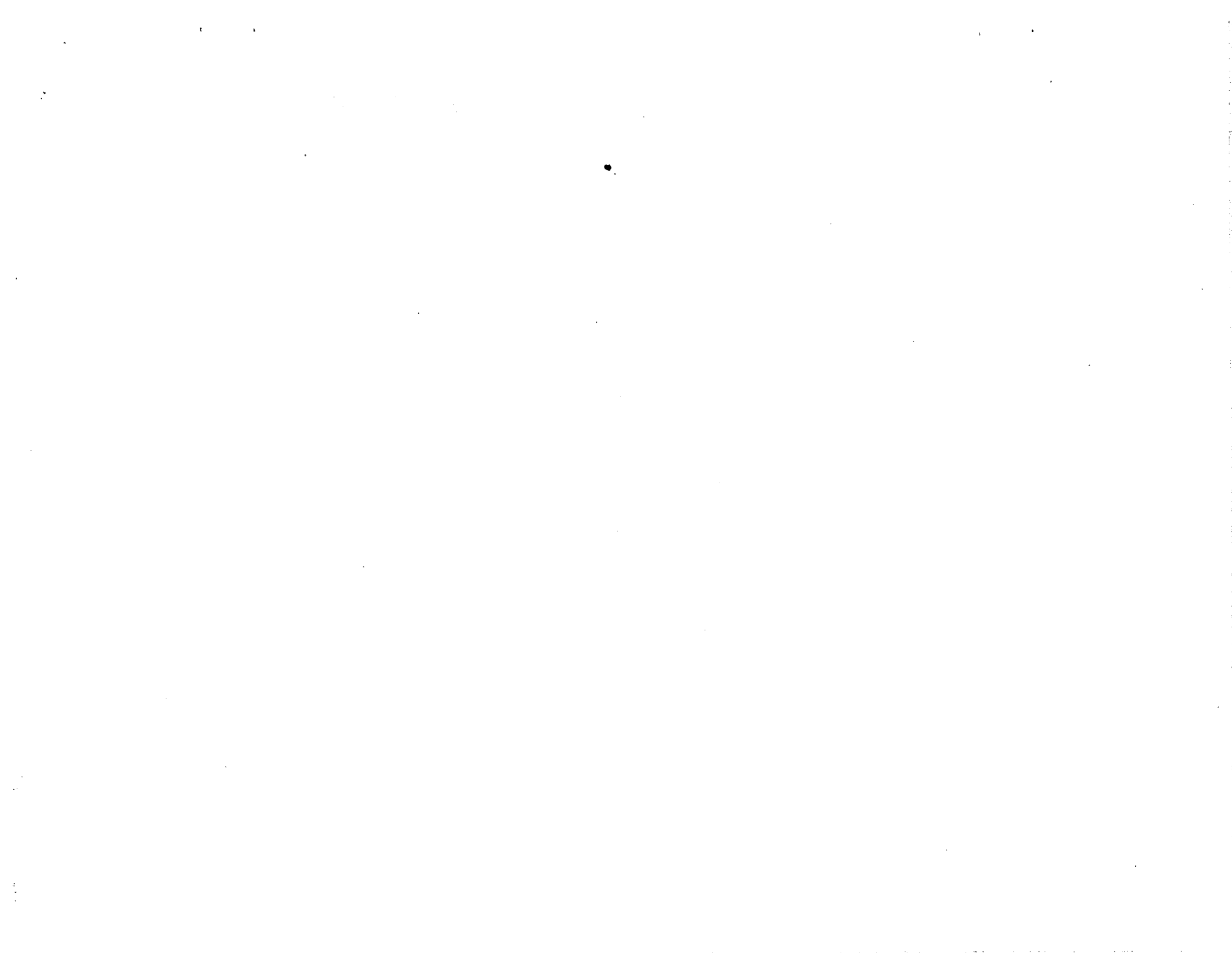
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I. INTRODUCTION TO GPS NAVSTAR

The Global Positioning System (GPS) NAVSTAR is a satellite based navigation system. The system provides the user with highly precise 3D position and velocity information and time worldwide on a continuous, all-weather basis. The system consists of a Space Segment, a Control Segment and a User Segment.

When fully operational in 1990, the Space Segment will consist of 18 satellites in six 12 hour orbits of 3 satellites. Each satellite will continuously broadcast a message containing precise information relative to its own position (ephemeris) and clock accuracy and less precise information relative to the entire constellation position (almanac).

The Control Segment consists of monitor stations and a master control station. The monitor stations transmit satellite tracking data to the master control station, which determines the satellites' orbital parameters and communicates them to the satellites for retransmission to the users.

The User Segment consists of the equipment necessary to derive position, velocity and time from the information received from the satellites.

Four satellites are normally required for navigation purposes, and the four offering the best geometry can be selected manually or automatically by receivers using ephemeris information transmitted by the satellites. Ranges to the four satellites are determined by scaling the signal transmit time by the speed of light. The transmitted message contains ephemeris parameters that enable the user to calculate the position of each satellite at the time of transmission of the signal.

Operation of the system requires precise synchronization of space vehicle (SV) clocks with GPS time, which is accomplished by the use of an atomic frequency standard in each space vehicle and the use of correction parameters that are provided by the Control Segment. The requirement for users to be equipped with precision clocks is eliminated by the use of range measurements from four satellites. If users maintained precision clocks synchronized with GPS time, navigation could be accomplished with only three satellites. In that case, the user could be thought of as being at the intersection of three spheres, with centers located at the satellites. The fourth satellite permits an estimate of the user's clock error. In this case, the user position contains four unknowns consisting of position in three dimensions and the error, or fixed bias, in the user's imprecise clock, which can be solved by simultaneous solution of the four equations.

The range measurements from satellite to user are corrupted by several distortions. Basically, these distortions can be divided into two groups; random and bias distortions. The random distortions can be reduced by use of filtering techniques (see Chapters 4, 6 and 7). It is not possible to make a distinction between the actual range and the bias errors. Therefore, bias distortions can not be reduced by use of filtering techniques.

However, there is a way to obtain a very accurate estimate of the bias distortions, by implementing the so-called differential GPS (DGPS) approach.

The DGPS approach uses a receiver/transmitter at a known, fixed location. This receiver compares its GPS derived position to its actual surveyed location, and transmits the errors to suitably equipped users to allow them to improve their own solutions. Various DGPS concepts can be implemented [1].

In this paper the accent is on the following concept:

- a receiver is placed in a known location and the errors $R(\text{err})$ (bias errors) from user to satellite are measured:

$$R(\text{err}) = (\text{measured range}) - (\text{true range})$$

The correction term, $R(\text{err})$, is then transmitted to the user receiver.

II. GPS SIGNAL STRUCTURE

The GPS signal consists of two components, Link 1 (L1) at a center frequency of 1575.42 MHz and Link 2 (L2) at a center frequency of 1227.6 MHz. Each of these two signals L1 and L2, is modulated by either or both a 10.23 MHz clock rate precision P signal and/or by a 1.023 MHz coarse/acquisition C/A signal. Each of these two binary signals is formed by a P-code or a C/A code which is modulo-2 added to 50 bps data D, to form $P \oplus D$ and $C/A \oplus D$. The $P \oplus D$ and $C/A \oplus D$ signals are modulo-2 added to L1 and L2 in phase quadrature.

The P-code is a pseudo-random sequence with a period of 1 week. The C/A code is a unique Gold code with a period of 1 msec. The user has the capability to duplicate both P and C/A codes and the transmission time is determined by measuring the offset that has to be applied to the locally generated code to synchronize it with the code received from the satellite.

Since the P-code has a wider bandwidth, it will be more difficult to acquire than the C/A code, but it will provide greater accuracy and additional anti-jam protection.

In this report, we are only concerned with the C/A code, as the P-code will only be available for authorized users. However, the C/A code, in combination with differential GPS, will provide an accuracy on the same order of magnitude as the P-code.

III. DEFINITIONS (PSEUDO) RANGE AND DELTA (PSEUDO) RANGE

The principle of GPS navigation is based on the delay between the time the navigation code is transmitted by the satellite and the time the code is received by the user. This time delay can be considered as a range from satellite to user in a vacuum, denoted as the true range:

$$R = c * (T_r - T_t) \quad \text{meters}$$

where c = GPS value for speed of light ($2.99792458 * 10^{**8}$ m/s)

T_r = GPS receive time

T_t = GPS transmit time

However, there are additional time delays, including:

- the user clock bias T_{Bu} with respect to true GPS time;
- atmospheric delay T_{Da} , as the signal is not transmitted in a vacuum;
- receiver delay T_{Dr} between the antenna phase center and the code-correlation point in the receiver;
- the transmit clock bias T_{Bs} with respect to true GPS time

The range measured by the user is called pseudorange and is defined as:

$$PR = R + c * (T_{Da} + T_{Dr} + T_{Bu} - T_{Bs}) \quad \text{meters}$$

The change in true range over a specified time interval is called delta range:

$$DR = R(t + dt) - R(t)$$

where dt = the time interval between the two range measurements

Delta range divided by the specified time interval is the mean relative velocity between the user and satellite for that time interval.

The delta pseudorange is defined as the change in pseudorange over a specified time interval:

$$\begin{aligned} DPR &= PR(t + dt) - PR(t) \\ &= DR - c * (DT_{Bs} - DT_{Bu}) \end{aligned}$$

where DT_{Bs} = the satellite clock bias change over the time interval dt
 DT_{Bu} = the user clock bias change over the time interval dt

Delta range is usually obtained by counting the number of carrier cycles that occur over a finite interval in the receiver phase lock loop. The total phase change (Doppler count) accumulated over the measurement interval is equivalent to the integral of the Doppler phase rate (frequency) due to relative motion between the user and the satellite during the measurement interval.

$$\text{Dopplercount } \Delta N = \int_{t_1}^{t_2} \left[f_g - f_r(t) \right] dt \quad (3.1)$$

where $t_2 = t_1 + \Delta t$

f_g = precise ground station
frequency

Δt = Dopplercount interval

$f_r(t)$ = received frequency

The mean velocity during the measurement interval is given by:

$$\bar{v} = \frac{\Delta N * \lambda}{\Delta t} \quad (3.2)$$

where λ = carrier wavelength

and the delta pseudorange is given by:

$$\text{DPR} = \Delta N * \lambda \quad (3.3)$$

IV. GPS ERRORS

The GPS navigation accuracy is a function of the product of two components:

- Measurement Errors
- Geometric Dilution of Precision GDOP

A. Measurement Errors

Error contributions have been attributed to the various system segment contributors; the Space segment, the Propagation Link and the User segment. The user equivalent range error (UERE) and the user equivalent delta range error (UEDRE) are measures to describe these contributions to pseudorange and delta pseudorange measurements.

B. Geometric Dilution of Precision GDOP

GDOP is the degradation of accuracy that results when the geometry of the satellites is not optimal:

$$\text{GDOP} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_t^2} / \sigma_\rho$$

where σ_x^2 , σ_y^2 , σ_z^2 are the variances of user position
 σ_t^2 is the variance of user time
 σ_ρ is the pseudorange standard deviation

GDOP can be partitioned into separate position and time variances:

$$\text{GDOP}^2 = \text{PDOP}^2 + \text{TDOP}^2$$

$$\text{where PDOP} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} / \sigma_\rho$$

= Position Dilution of Precision

$$\text{TDOP} = \sigma_t / \sigma_\rho$$

= Time Dilution of Precision

In a navigation system PDOP is used for the determination of the position accuracy, knowing time is generally not necessary. PDOP is determined by the satellite constellation, user's geographical location, mask angle, and time of the day [2]. A typical value of PDOP is 3.4 with a probability of 0.9 where the elevation mask is 5 degrees for a 24-satellite constellation.

C. Position Error

The rms position error (x,y,z)

$$\sigma_p \triangleq \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$$

is related to the rms radial range error σ_R by

$$\sigma_p = \text{PDOP} * \sigma_R$$

where σ_p = 1 σ position error in three dimensions
 σ_R = square root of sum of squares of UERE contributions
PDOP = position dilution of precision in three dimensions

D. Velocity Error

The rms velocity error (x,y,z)

$$\sigma_v \triangleq \sqrt{\sigma_{vx}^2 + \sigma_{vy}^2 + \sigma_{vz}^2}$$

is related to the rms radial delta range error σ_{DR} by

$$\sigma_v = \text{PDOP} * \sigma_{DR}$$

where σ_v = 1 σ velocity error in three dimensions
 σ_{DR} = square root of sum of squares of UEDRE contributions

E. Influence of UERE and UEDRE on Range and Delta Range

The influence of the UERE and UEDRE on the range and delta range (Doppler) can be divided accordingly to the summary given in table 1 [3-6].

Using a differential GPS approach, it follows from table 1 that the expected errors in range (position) and delta range (velocity) due to random noise are respectively 14 meters and 1.3 centimeters.

Table 1. Influence of the UERE and UEDRE on the Range and Delta Range (Doppler) for the C/A Code.

ERROR CONTRIBUTORS	RMS RANGE ERROR IN M	RMS DELTA RANGE ERROR IN M	STATISTICS	COMMENTS
satellite ephemeris	1.5		bias	uncorrelated between SV's
satellite group and clock relativistic frequency shifts	1.0		bias	uncorrelated between SV's
satellite clock noise		0.003	white noise	cesium clock
receiver noise and resolution		0.008	white noise	reference [5]
receiver noise	< 10 **		markov	reference [5] and [4]
receiver resolution	0.0005		white noise	reference [5]
ionospheric and tropospheric delays	< 25		bias	
propagation gradient		0.01	markov	
multipath error	10		white noise	

** This error consists mainly of the thermal tracking jitter error, which decreases when C/No increases. In case an accurate measure of C/No in the receiver and the bandwidths is available, a prediction of this error in real time can be obtained [5].

V. THE NAVIGATION SOLUTION

A. Introduction

This Chapter gives an overview of the basic methods to derive user position, velocity, clock bias and clock bias rate from pseudorange and delta pseudorange (Doppler) measurements. Some of these algorithms were implemented in the computer language FORTRAN IV and tested on a IBM System/370 in combination with a GPS simulation program.

In order to simulate the GPS using an 18-satellite constellation, two computer programs were used [7]:

- the NAVSTR program which provides satellite position, satellite velocity, pseudorange and delta pseudorange. The program has the option to introduce several distortions into the pseudorange data.
- the USRPOS program which simulates a helicopter flight pattern containing several maneuvers and provides user position and velocity.

The simulations used in this paper are all based on a flight duration of 270 seconds. The time interval between fixes is 0.3 second. During the first 138 seconds the USRPOS program simulates an unaccelerated straight level flight. During the next 31 seconds a turn and a -2 knots/hour acceleration is simulated. The last part is again a straight level flight.

All the programs are written in double precision (precision of approximately 16.8 digits [8]), instead of single precision (precision of 7.2 digits) because of the large range and delta range values with respect to the desired computation precision. For instance, a delta range value of 9000 nmi requires at the very least a computation accuracy of 8 decimals to obtain 1 meter computation precision.

B. Position

The GPS navigation position fix and time bias can be obtained from the following basic equations (in Earth Centered Earth Fixed (ECEF) coordinates):

$$\begin{aligned}(r_1 - b)^2 &= (u_x - s_{x1})^2 + (u_y - s_{y1})^2 + (u_z - s_{z1})^2 \\(r_2 - b)^2 &= (u_x - s_{x2})^2 + (u_y - s_{y2})^2 + (u_z - s_{z2})^2 \\(r_3 - b)^2 &= (u_x - s_{x3})^2 + (u_y - s_{y3})^2 + (u_z - s_{z3})^2 \\(r_4 - b)^2 &= (u_x - s_{x4})^2 + (u_y - s_{y3})^2 + (u_z - s_{z4})^2\end{aligned}\tag{5.1}$$

where (u_x, u_y, u_z) is the user position; b is the user clock bias with respect to GPS time; r_i is the true range distance between the user and i th satellite; (s_{xi}, s_{yi}, s_{zi}) is the position of the i th satellite.

The user state (u_x, u_y, u_z, b) can be determined by solving eq. 5.1 directly, given the measured pseudoranges $R = (r_1 r_2 r_3 r_4)^T$ and the three satellite position components (s_{xi}, s_{yi}, s_{zi}) , $i = 1, 2, 3, 4$. This approach is computationally unwieldy, especially because the calculations are usually executed in a coordinate system not very close to the problem. A good alternative approach is to linearize (5.1) about an estimate of the user state and solve successively for position corrections based on new measurements.

The basic navigation equations can be linearized by employing incremental relationships as described by Jorgensen [9]. The resulting linearized navigation equations are given by:

$$\frac{u_{nx} - s_{xi}}{r_{ni} - b_n} \Delta x + \frac{u_{ny} - s_{yi}}{r_{ni} - b_n} \Delta y + \frac{u_{nz} - s_{zi}}{r_{ni} - b_n} \Delta z + \Delta b = \Delta r_i \quad (5.2)$$

where $u_{nx}, u_{ny}, u_{nz}, b_n$ are nominal (a priori best estimate) values of u_x, u_y, u_z, b ; $\Delta x, \Delta y, \Delta z, \Delta b$ are the corrections to these nominal values; r_{ni} are the nominal pseudorange measurements from the i th satellite ($i=1,2,3,4$); Δr_i is the difference between the actual and nominal measurements.

The linearized equations can be expressed in matrix notation as follows:

$$H \cdot \Delta U = \Delta R \text{ or } \Delta U = H^{-1} \cdot \Delta R$$

$$\text{where } \Delta U = (\Delta x \Delta y \Delta z \Delta b)^T$$

$$\Delta R = (\Delta r_1 \Delta r_2 \Delta r_3 \Delta r_4)^T$$

$$H = (h_1 h_2 h_3 h_4)^T$$

$$h_i = \left[\begin{array}{cccc} \frac{u_{nx} - s_{xi}}{r_{ni} - b_n} & \frac{u_{ny} - s_{yi}}{r_{ni} - b_n} & \frac{u_{nz} - s_{zi}}{r_{ni} - b_n} & 1 \end{array} \right] \quad (5.4)$$

The known quantities Δr_i are actually incremental pseudorange measurements. They are the differences between the actual measured pseudoranges and the measurements that had been predicted by the user's computer based on the knowledge of satellite position and the user's most current state. The quantities to be computed, $\Delta x, \Delta y, \Delta z$ and Δb are corrections that the user will make to his current state.

C. Position Algorithm

The methodology for calculating the user state based on the linearized equations (5.3) proceeds as follows [10]:

- 1) Guess initial user state $U_n = (u_{nx}, u_{ny}, u_{nz}, b_n)$
- 2) Obtain r_i and s_{xi}, s_{yi}, s_{zi} for $i = 1, 2, 3, 4$
- 3) Compute $r_{ni} = \sqrt{(u_{nx} - s_{xi})^2 + (u_{ny} - s_{yi})^2 + (u_{nz} - s_{zi})^2} + b_n$
- 4) Compute $\Delta r_i = r_i - r_{ni}$ for $i = 1, 2, 3, 4$
- 5) Compute H from (5.4)
- 6) Compute $G = H^{-1}$
- 7) Compute $\Delta U = G * \Delta R$
- 8) Update state estimate $U_n = U_n + \Delta U$
- 9) if $|\Delta U| > \epsilon$, a specified tolerance, go to step 3) for another iteration, else go to step 2) for the next position + clock bias fix

The program has a build-in limit for the maximum number of iterations per fix.

Figure 1 shows the results for the 3-dimensional position error where $\epsilon = 0.001$, and no pseudorange distortions were introduced. The number of iterations per fix to reach the desired accuracy during this simulation was one or two.

Figure 2 shows the magnitude of the 3-dimensional position error when multipath error was introduced. White noise statistics are assumed for the multipath error having a zero mean and a variance of 10 meters.

D. Velocity

The user velocity can be derived from the range information (position) or from the carrier Doppler frequency. The Doppler velocity is expected to have a much greater accuracy (approximately 1.3 centimeters) than the velocity derived from range information. However, to obtain the Doppler velocity, additional hardware is required.

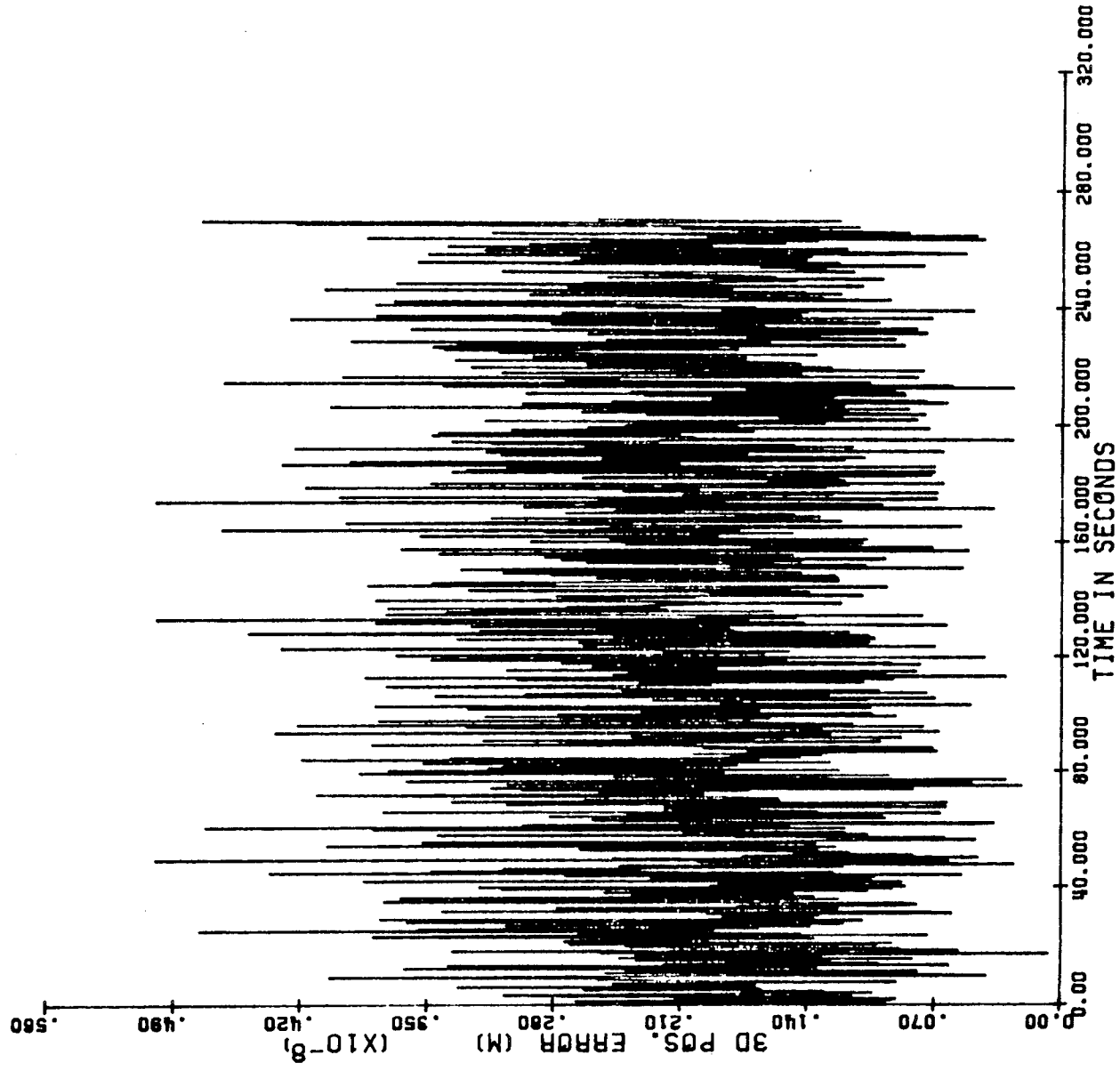


Figure 1. 3D Position Error, No Pseudorange Distortions.

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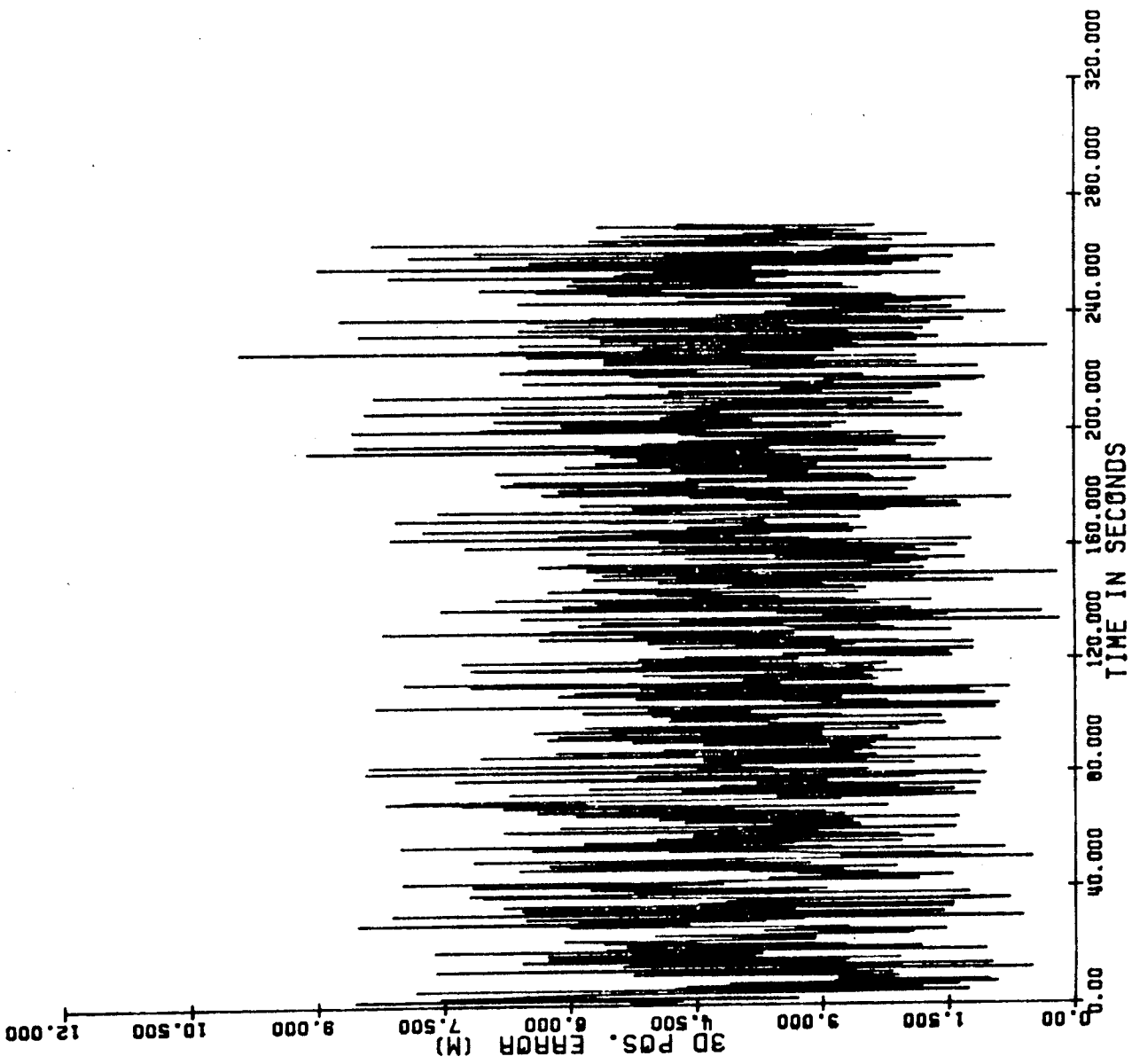


Figure 2. 3D Position Error, Pseudorange Distortion; Multipath.

1. Velocity Derived from Carrier Doppler Frequency

Based on the velocity equation 3.2 we can derive a similar algorithm as was given for the position fix. The user state consists of the user velocity (v_x, v_y, v_z) and the clock bias rate \dot{b} . The velocity measured on the line-of-sight from satellite to user is called r . In the simulation, the satellite velocity is provided by the NAVSTR program. In an actual receiver, the satellite velocity can be easily derived from satellite position within the desired accuracy [11].

The algorithm for the user velocity computation, based on a similar linearization about the user state as used for the position fix, proceeds as follows:

- 1) Guess initial user state $U_n = (v_{nx}, v_{ny}, v_{nz}, \dot{b}_n)$
- 2) Obtain $r_i, sv_{xi}, sv_{yi}, sv_{zi}$ for $i = 1, 2, 3, 4$
- 3) Compute $r_{ni} = (\text{rangeflag} * \sqrt{(sv_{xi} - v_{nx})^2 + (sv_{yi} - v_{ny})^2 + (sv_{zi} - v_{nz})^2}) + \dot{b}_n$

where rangeflag is the sign of the change in range

- 4) Compute $\Delta r_i = r_i - r_{ni}$ for $i = 1, 2, 3, 4$
- 5) Compute $H = (h_1 \ h_2 \ h_3 \ h_4)^T$

$$\text{where } h_i = \begin{bmatrix} \frac{v_{nx} - sv_{xi}}{r_i - \dot{b}_n} & \frac{v_{ny} - sv_{yi}}{r_i - \dot{b}_n} & \frac{v_{nz} - sv_{zi}}{r_i - \dot{b}_n} & 1 \end{bmatrix}$$

- 6) Compute $G = H^{-1}$
- 7) Compute $\Delta U = G * \Delta R$

$$\text{where } \Delta R = (\Delta r_1 \ \Delta r_2 \ \Delta r_3 \ \Delta r_4)^T$$

$$\Delta U = (\Delta v_x \ \Delta v_y \ \Delta v_z \ \Delta \dot{b})^T$$

- 8) Update state estimate $U_n = U_n + \Delta U$
- 9) If $|\Delta U| > \epsilon$, a specified tolerance, go to step 3) for another iteration, else go to step 2) for the next velocity + clock bias rate fix.

The performance of the velocity Doppler algorithm is similar to the performance of the position algorithm as expected (see figure 1).

2. Velocity derived from range information

The basic method to derive the velocity from range information is to fit a n th order function through $n+1$ position measurements based on range information.

2.1. First-order Estimation

The estimated velocity over the time interval between two successive position fixes equals:

$$\bar{v} = \frac{x(T + nT) - x(nT)}{T}$$

The first order estimation provides a reasonable estimate of the user velocity, however the velocity estimates are not available at the same time as the position fixes.

2.2. Second-order Estimation

In fact, this method supposes a constant acceleration for the (vehicle + satellite) dynamics by fitting a second-order function (parabola) through three successive position fixes. The parabola provides an estimation of the position at some intermediate time, t , within the measurements:

$$y(t) = a_1 + a_2 t + a_3 t^2 \quad t \in [nT, nT + 2T]$$

Given the three measurements:

$$y_1(nT) = a_1 + a_2 nT + a_3 (nT)^2$$

$$y_2(nT + T) = a_1 + a_2 (nT + T) + a_3 (nT + T)^2$$

$$y_3(nT + 2T) = a_1 + a_2 (nT + 2T) + a_3 (nT + 2T)^2$$

it can be easily be shown that

$$a_1 = \frac{y_1 (2 + 3n + n^2) + y_2 (-4n - 2n^2) + y_3 (n + n^2)}{2}$$

$$a_2 = \frac{y_1 (-3 - 2n) + y_2 (4 + 4n) + y_3 (-2n - 1)}{2T}$$

$$a_3 = \frac{y_1 - 2y_2 + y_3}{2T^2}$$

The velocity estimate at some intermediate time $t \in [nT, nT + 2T]$ is given by

$$v(t) = a_2 t + 2a_3 t$$

Using a second-order approximation, acceptable velocity estimations are obtained as long as the range data are not corrupted by distortions. When introducing distortions, the velocity errors will be in the same order of magnitude or larger than the position errors for a time interval between fixes smaller than 1 second.

VI. GPS SIGNAL FILTERING

DGPS is capable of removing the bias component in the range data. The random distortion however will still degrade the range data. For example see figure 3 for a multipath distortion of the range data.

There exist two extreme methods [12] for filtering the signals from the satellites and forming the navigation solution for position, velocity, perhaps acceleration, and clock parameters. The most powerful method is to input the raw observables of pseudorange and delta pseudorange from all satellites observed into a total solution filter which solves simultaneously for all the navigation parameters. The filter is typically a Kalman filter, or some modification thereof.

The second extreme method is to filter the dynamics of the signal from each satellite separately, forming smoothed estimates of pseudorange, velocity, and perhaps acceleration, and then form a memoryless flash solution for the navigation parameters, without further smoothing. This method is not as powerful as a general Kalman filter, but often has distinct implementation advantages without significant loss in performance. Many navigation receivers use a combined method, in which the individual signals are tracked and smoothed in phase locked loops, and the loop outputs are further smoothed in the navigation filter. This combined method has the advantage over the first method that the filter update rate does not have to be as high because the raw data are smoothed.

In this paper we will only consider discrete filtering techniques, as the range data are a discrete measurement sequence. The best way to implement discrete filters is by using a digital computer. Some important considerations in using this approach are:

- computer time: for instance the time required for matrix operations in a Kalman filter grows as the cube of the state dimension [13]; also a higher calculation precision requires more computer time.
- computer storage: for instance the storage required for the Kalman covariance matrix grows as the square of the state dimension [13].

For the purpose of establishing feasibility and a performance bound, it is sufficient to consider the second filter type: separate tracking filters for each satellite, followed by a flash navigation solution as presented in Chapter 5.

It is also sufficient to consider the tracking filter only, since the maximum navigation filter position error due to dynamics is equal to the maximum position error in one tracking filter, and since the error in position solution due to random noise is equal to the pseudorange error multiplied by the PDOP which does not depend on filter type [12].

A. One Channel Filtering

Various techniques are available for the separation of the range signal from the random distortions [14] and [15]. In this paper we will

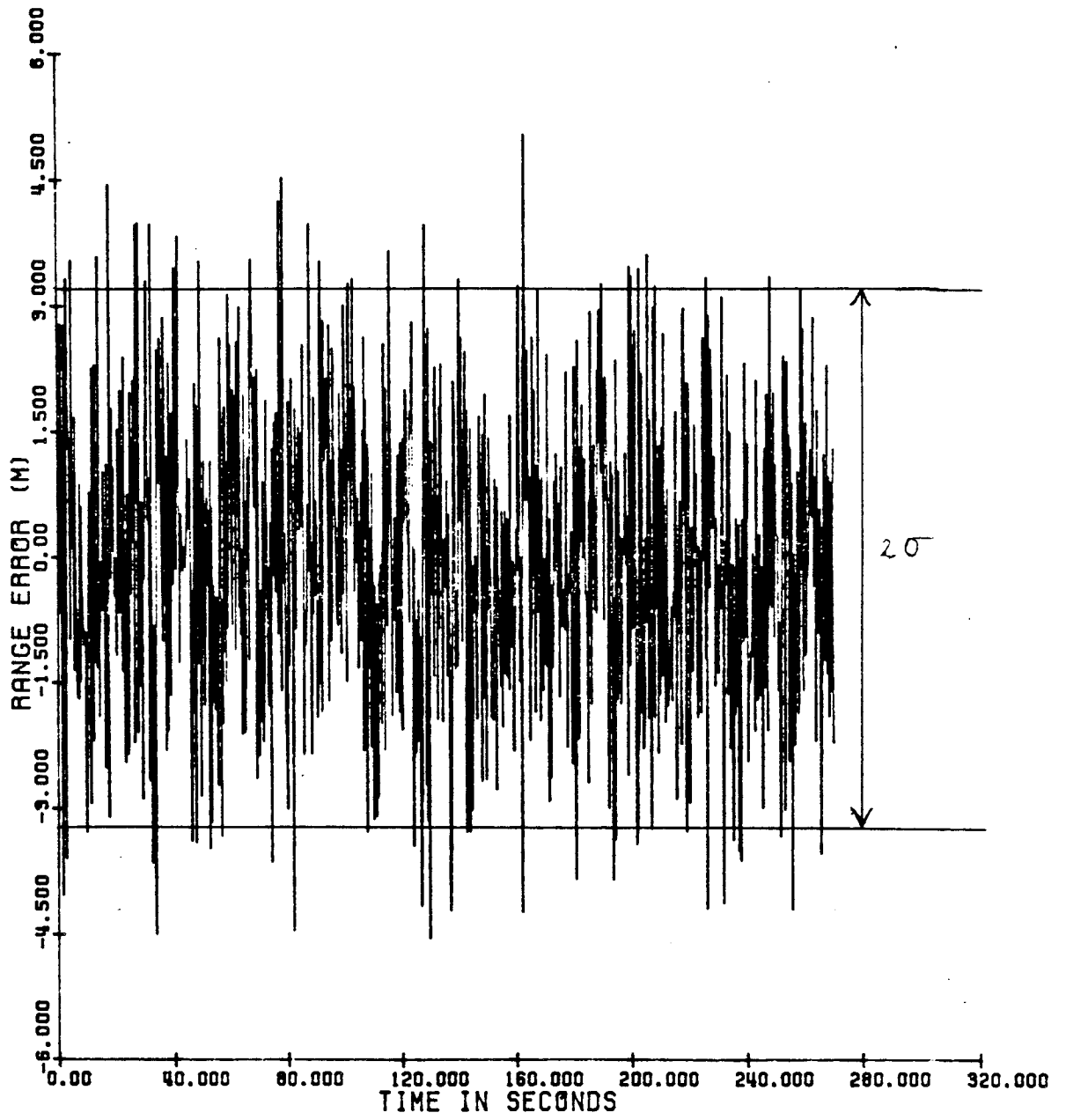


Figure 3. Pseudorange Distortion; Multipath.

mainly consider a particular branch of filters, the so-called least-squares filters. Simply stated, the least-squares filter problem is this: given the spectral characteristics of an additive combination of signal and noise, what linear operation on this input combination will yield the best separation from the noise, where best means minimum mean-square error [14]. Besides the least-squares filtering one other type of filter will be mentioned, the second-order filter as presented by Hurd [12].

1. Second-order Filter

W. J. Hurd [12] describes a second order filter with both poles at the same location, using only range and not delta range as the input. This filter is characterized by one parameter, and when the update rate is sufficiently high this parameter is effectively a time constant T . This is valid when the time constant between updates is small compared to T . The response of the filter to random noise is to reduce the rms error by a factor of 1.12 times the square root of (T divided by the time between inputs).

In case of an update time of 0.3 second and a time constant of 2 seconds, the raw range errors are reduced by a factor of 2.6. However, the position error due to acceleration is equal to acceleration, (a), times T squared. A time constant of 2 seconds will result in a range error due to acceleration of $4a$ meters.

2. Wiener Filter

Reference [14] gives a very good description of the Wiener filtering technique. The main disadvantages using a discrete Wiener filter are:

- the filter is non-recursive, so there is a 'growing memory' problem caused by the need to store all of the past measurement data. This is very unwieldy in on-line applications.
- the filter is difficult to extend to more complicated multiple-input multiple-output problems.

3. Kalman Filter

The main features of the discrete Kalman filter formulation and solution of the filtering problem are [14]:

- vector modeling of the random processes under consideration;
- recursive processing of the noisy measurement (input) data

Beside these features, the Kalman filter approach allows many extensions and variations for specific implementations.

B. One Channel Kalman Filtering

The discrete Kalman filter is for the range filtering problem in this paper superior to both the Wiener and the second order filter. The Wiener

filter is having the 'growing memory' problem and both the second-order filter and the Wiener filter are difficult to extend to a filter having range and delta range as input data.

The approach in which the very accurate delta range derived from Doppler measurements is also fed into the filter is expected to reduce the remaining random noise error in the position compared to the filter with range measurements only.

VII. DEVELOPMENT AND IMPLEMENTATION OF A DISCRETE KALMAN FILTER

A. Description of the Discrete Kalman Filter

The derivation of the discrete Kalman filter is very well documented [14 - 16], therefore in this paper we will only present the final filter equations and the necessary definitions.

If the true state of a Markov process and a measurement are defined by the equations:

$$\underline{x}_{k+1} = \Phi_k \underline{x}_k + \underline{w}_k \quad (7.1)$$

$$\underline{z}_k = H_k \underline{x}_k + \underline{v}_k \quad (7.2)$$

where \underline{x}_k is the state vector at time t_k

Φ_k is the state transition matrix, representing the known system dynamics

\underline{z}_k is the measurement vector (filter input) at time t_k

H_k is the measurement matrix, representing the relationship between the measurement \underline{z}_k and the state vector \underline{x}_k in the absence of noise

\underline{w}_k is the plant noise with covariance matrix Q

\underline{v}_k is the measurement noise with covariance matrix R

$E [\underline{w}_k \underline{v}_i^T] = 0$ for all k and i

then the best estimate \underline{x}_k of the system state vector and its covariance matrix P_k can be updated by the equations:

$$\begin{aligned} \underline{x}_{k+1}^{(-)} &= \Phi_k \underline{x}_k^{(+)} \\ P_{k+1}^{(-)} &= \Phi_k P_k^{(+)} \Phi_k^T + Q_k \\ P_k^{(+)} &= (I - K_k H_k) P_k^{(-)} \end{aligned} \quad (7.3)$$

ENTER PRIOR ESTIMATE $\underline{x}_k^{(-)}$ AND
 ITS ERROR COVARIANCE MATRIX $P_k^{(-)}$
 FOR TIME t_k

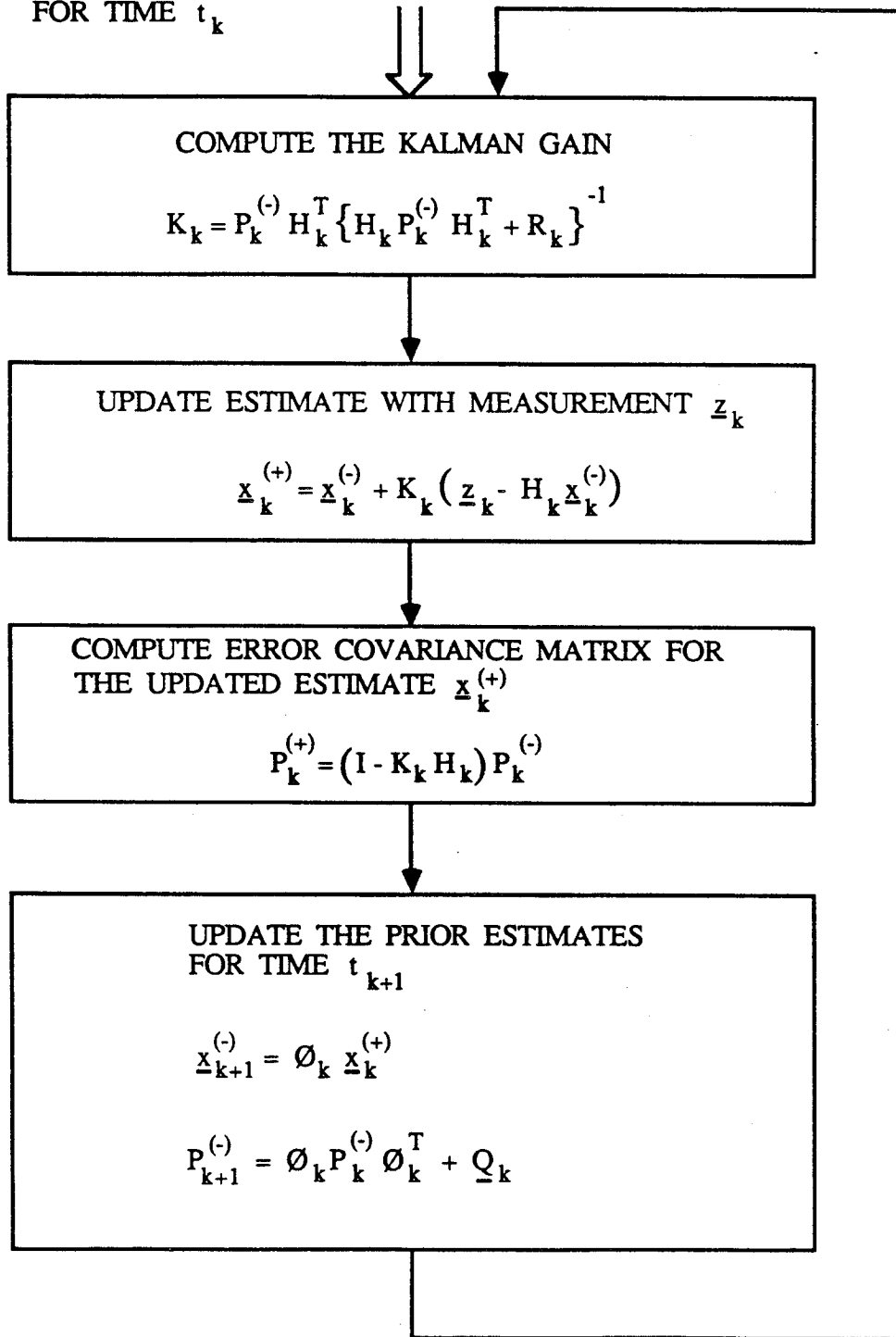


Figure 4. Computational Steps.

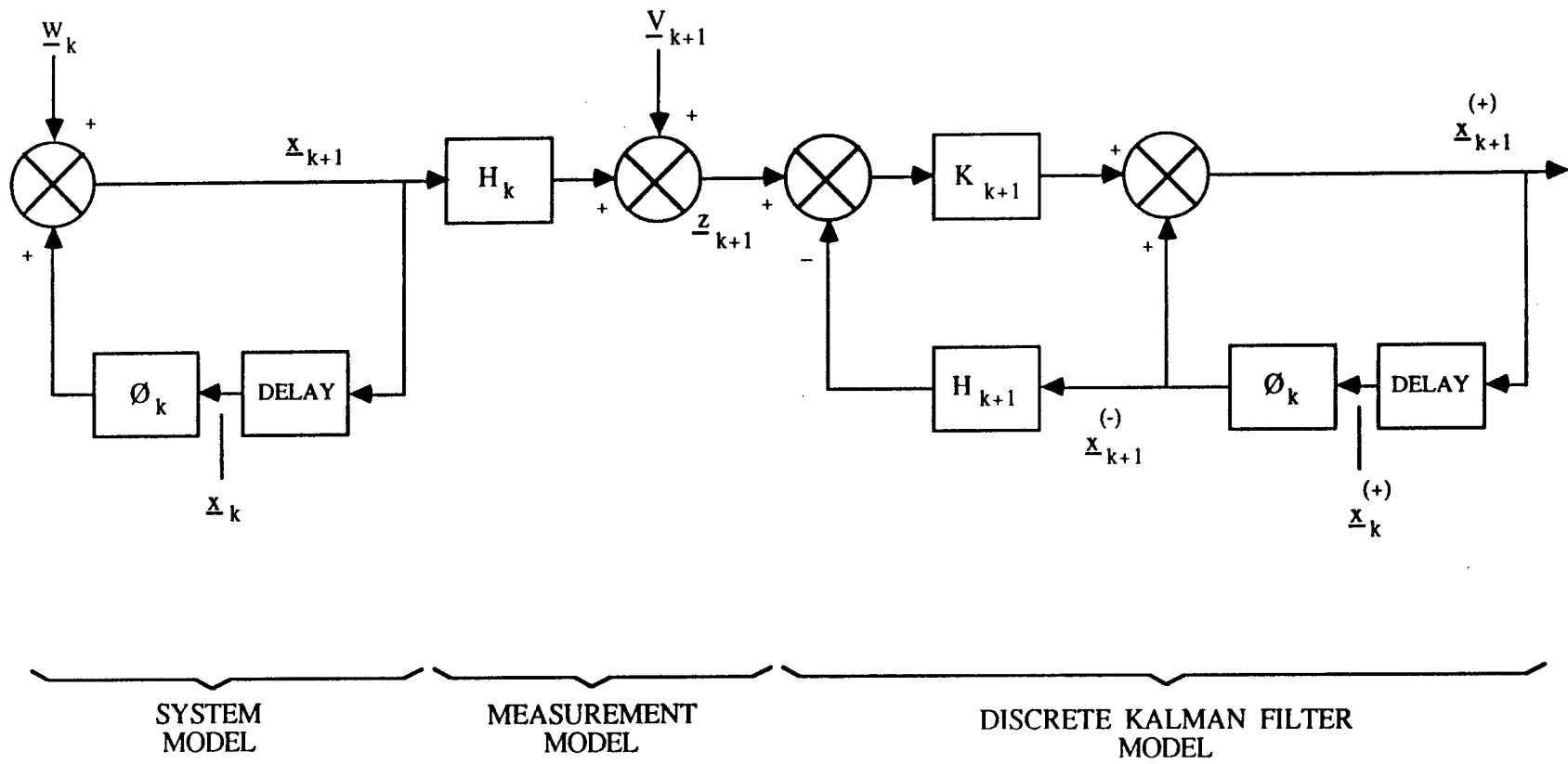


Figure 5. Block Diagram of System Implementation.

$$\underline{x}_k^{(+)} = \underline{x}_k^{(-)} + K_k (\underline{z}_k - H_k \underline{x}_k^{(-)})$$

.4)

$$K_k = P_k^{(-)} H_k^T \{ H_k P_k^{(-)} H_k^T + R_k \}^{-1}$$

where $\underline{x}_k^{(-)}$, $P_k^{(-)}$ and $\underline{x}_k^{(+)}$, $P_k^{(+)}$ are respectively estimates of \underline{x}_k and P_k just before and just after time t_k .

K_k is the Kalman gain (blending factor)

Figure 4 shows the sequence of the computational steps pictorially and figure 5 shows a block diagram of the system model, the measurement and the discrete Kalman filter.

B. Kalman Filter Implementation Considerations

When the discrete Kalman filter equations (7.1 - 7.3) are applied to practical problems, several difficulties quickly become obvious:

- the optimal filter must model all error sources in the system under consideration;
- it is assumed in the filter equations that exact descriptions of the system dynamics, error statistics, and the measurement process are known.

As an unlimited computer capability is not usually available most of the implemented Kalman filters are sub-optimal; less than the optimal number of states are included in the system and/or measurement model and no exact description of the error statistics is available. The performance of a sub-optimal Kalman filter is not yet based on a unified body of theory and practice. Fortunately there are many examples of discrete Kalman filter implementations available [14 - 16] from which concepts for a successful implementation can be derived.

Some of these concepts are:

- derive a system model representing the real system as close as possible;
- use at least double precision in the computation of the filter equations (especially in off-line implementations) to prevent the filter to diverge due to calculation errors (for a description of the performance degradation in digitally implemented Kalman filters [17]);
- implement 'reasonableness checks' in the filter, e.g. remove unlikely input data, when a matrix is expected to be symmetric, force it to be symmetric during or after each operation on that matrix;
- find an acceptable evaluation of the plant covariance matrix Q . In most implementations it is not a trivial task to derive the Q matrix from its definition:

$$Q_k = E [\underline{w}_k \underline{w}_k^T] \quad (7.4)$$

$$= E \left\{ \int_{t_k}^{t_{k+1}} \int_{t_k}^{t_{k+1}} \phi(t_{k+1}, u) G(u) E[\underline{w}(u) \underline{w}^T(v)] * \right. \\ \left. * G^T(v) \phi^T(t_{k+1}, v) du dv \right\}$$

where $E[\underline{w}(u) \underline{w}^T(v)]$ is a matrix of Dirac delta functions that, presumably, is known from the continuous model. G represents the influence of the noise vector \underline{w} on the measurements.

C. System Modeling

In the area of pulse Doppler surveillance radars several discrete Kalman filters were implemented [18]. The signal model of the pulse Doppler radar is very similar to the GPS signal model, therefore, most of the techniques derived for the pulse Doppler radar can be used for the GPS one-channel filtering.

Some of the system models as presented by Fitzgerald [18] are:

- three state exponentially correlated acceleration (ECA) model;
- two state exponentially correlated velocity (ECV) model;
- two or three state integrated white noise or 'random walk' acceleration (RWA) model.

In order to obtain a performance bound and a basis for a feasibility study, a two state RWA model is implemented. The RWA tracking problem is described by the state-space model:

$$\underline{x}_{k+1} = \Phi_k \underline{x}_k + \underline{w}_k \quad (7.5)$$

where $\underline{x}_k = (r \quad \dot{r})^T$ is the state-vector at time t_k

r is the range from satellite to user

\dot{r} is the relative velocity on the line-of-sight between satellite and user

\underline{w}_k is a stationary white noise process representing the random acceleration noise at time t_k

ϕ_k is the state-transition matrix at time t_k

D. Measurement Modeling

Two types of filter input data are available: range and delta range. In a DGPS approach, the range data are degraded by random noise with an rms value of approximately 14 meters (see Chapter 4) and the delta range derived from Doppler is accurate up to 1.3 centimeters. In order to compare the accuracy of the velocity derived from range with Doppler velocity and because of defining a basis for a feasibility study, the filter is fed with range data only. However, the filter design makes allowance for an extension to both range and delta range measurements.

The measurement equation based on range data only becomes:

$$z_k = H_k x_k + v_k$$

where z_k (a scalar) is the range measurement at time t_k
 H_k is the measurement matrix at time t_k
 v_k (a scalar) is a stationary white noise process representing the random measurement noise at time t_k

The measurement matrix H_k is independent of the time. In the absence of measurement noise, the measurement z_k equals the range r , therefore the H matrix becomes (1 0).

E. Computation of the State Transition Matrix ϕ

The dynamical system as described by equation (7.1) has fixed parameters; i.e., the continuous state transition matrix F is a constant. In this case, the state transition matrix ϕ may be written as an exponential series:

$$\phi_k = e^{FT} = I + FT + \frac{(FT)^2}{2!} + \dots$$

where $T = t_{k+1} - t_k$

In the continuous case:

$$\begin{bmatrix} \dot{r} \\ \ddot{r} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \dot{r} \end{bmatrix} + \text{noise}$$

therefore

$$\phi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & T \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \dots = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

F. Computation of the Error Covariance Matrix Q

The Q matrix is defined by equation (7.4):

$$Q = E [\underline{w}_k \underline{w}_k^T]$$

where $\underline{w}_k = [w_1 \ w_2]^T$

$$w_1 = \int_0^T u \cdot a(u) du$$

$$w_2 = \int_0^T a(u) du$$

$a(u)$ is the random white noise acceleration

$$Q_{11} = \int_0^T \int_0^T u \cdot v \cdot E \{a(u)a(v)\} dudv$$

$$E\{a(u)a(v)\} = \sigma_a^2 T \delta(u - v)$$

where σ_a^2 is the variance of the acceleration and $\delta(u-v)$ is the Dirac delta function

$$\begin{cases} \delta(u-v) = 1 & \text{if } u = v \\ \delta(u-v) = 0 & \text{if } u \neq v \end{cases}$$

it follows that

$$Q_{11} = \int_0^T v^2 \sigma_a^2 T \, dv = \frac{1}{3} \sigma_a^2 T^4$$

also

$$Q_{12} = \int_0^T \int_0^T u \cdot E \{ a(u) a(v) \} \, dudv$$

$$= \int_0^T v \cdot \sigma_a^2 \cdot T \, dv = \frac{1}{2} \sigma_a^2 T^3$$

and

$$Q_{22} = \int_0^T \int_0^T E \{ a(u) a(v) \} \, dudv$$

$$= \int_0^T \sigma_a^2 \cdot T \, dv = \sigma_a^2 T^2$$

G. The Measurement Covariance Matrix R

As the measurement vector \underline{z}_k is a scalar, the covariance matrix R is also a scalar, given by the variance σ_x^2 of the measurement noise.

H. Expected Filter Performance

1. Sampling Rate Versus rms Acceleration

In order to keep the accuracy between observations below the measurement (sensor) error, the following equation should be satisfied, from [19]:

$$\sigma_a T^2 \leq 0.24 \sigma_x$$

where σ_x is the measurement accuracy and
 σ_a is the rms acceleration

The maximum error will occur just before a measurement. In the model, the time-constant T is set to 0.3 second, therefore

$$\sqrt{\frac{\sigma_x}{\sigma_a}} > 0.6$$

For instance, if the measurement accuracy $\sigma_x = 1$ foot, then the accuracy between observations will be less than 1 foot if the rms acceleration is below 2.78 ft/s*s.

2. Velocity Accuracy

However, no direct velocity measurements are made; velocity can be derived from position by subtracting two successive position measurements divided by the time interval between them.

The corresponding accuracy in velocity is given by [19]:

$$\sigma_v = \sqrt{P_v} = \sqrt{\frac{1}{2} (\sqrt{1+2r} + 1) \sigma_a^2 T^2}$$

where $r = \frac{4\sigma_x}{\sigma_a T^2}$

For example, if $\sigma_x = 1$ foot; $\sigma_a = 2.78$ ft/s*s; $T = 0.3$ s, then it follows from the above equation that the rms velocity error $\sigma_v = 1.57$ ft/s.

I. Filter Summary

The discrete Kalman filter for one channel range data tracking proceeds as given by the diagram in figure 4, where:

$$\underline{x}_k(-) = \begin{bmatrix} \text{estimated initial range} \\ \text{estimated initial velocity} \end{bmatrix}$$

$$P_k^{(-)} = \begin{bmatrix} p_1^{(-)} & p_2^{(-)} \\ p_2^{(-)} & p_3^{(-)} \end{bmatrix} \quad p_1^{(-)}, p_2^{(-)}, p_3^{(-)} \text{ are initial guesses having approximately the same order of magnitude as the measurement noise variance.}$$

$$H = (1 \quad 0)$$

$$Q = \sigma_a^2 \begin{bmatrix} T^4/3 & T^3/2 \\ T^3/2 & T^2 \end{bmatrix} \quad \phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

$$R = \sigma_x^2$$

J. Simulation Results

The measurement noise is generated by the NAVSTAR simulation program. The simulation was used in the mode where multipath error is added to the range data.

Figure 3 shows the raw range data produced by the NAVSTAR simulation. The variance of the white noise equals 10 meter [7].

Figures 6, 7 and 8 show the filter response for different values of the noise variances σ_x^2 and σ_a^2 , as summarized in table 2.

The calculated parameters for optimal filter response are given by the first row of table 2.

Table 2. Single Channel Discrete Kalman Filter Performance

	σ_x^2 in m	σ_a^2 in m	Steady State Noise Reduction $\sigma_{IN}^2/\sigma_{OUT}^2$	Maximum Position Error in m
FIGURE 6	10	2	4	3
FIGURE 7	10	0.2	3	8
FIGURE 8	1	1	2.5	3

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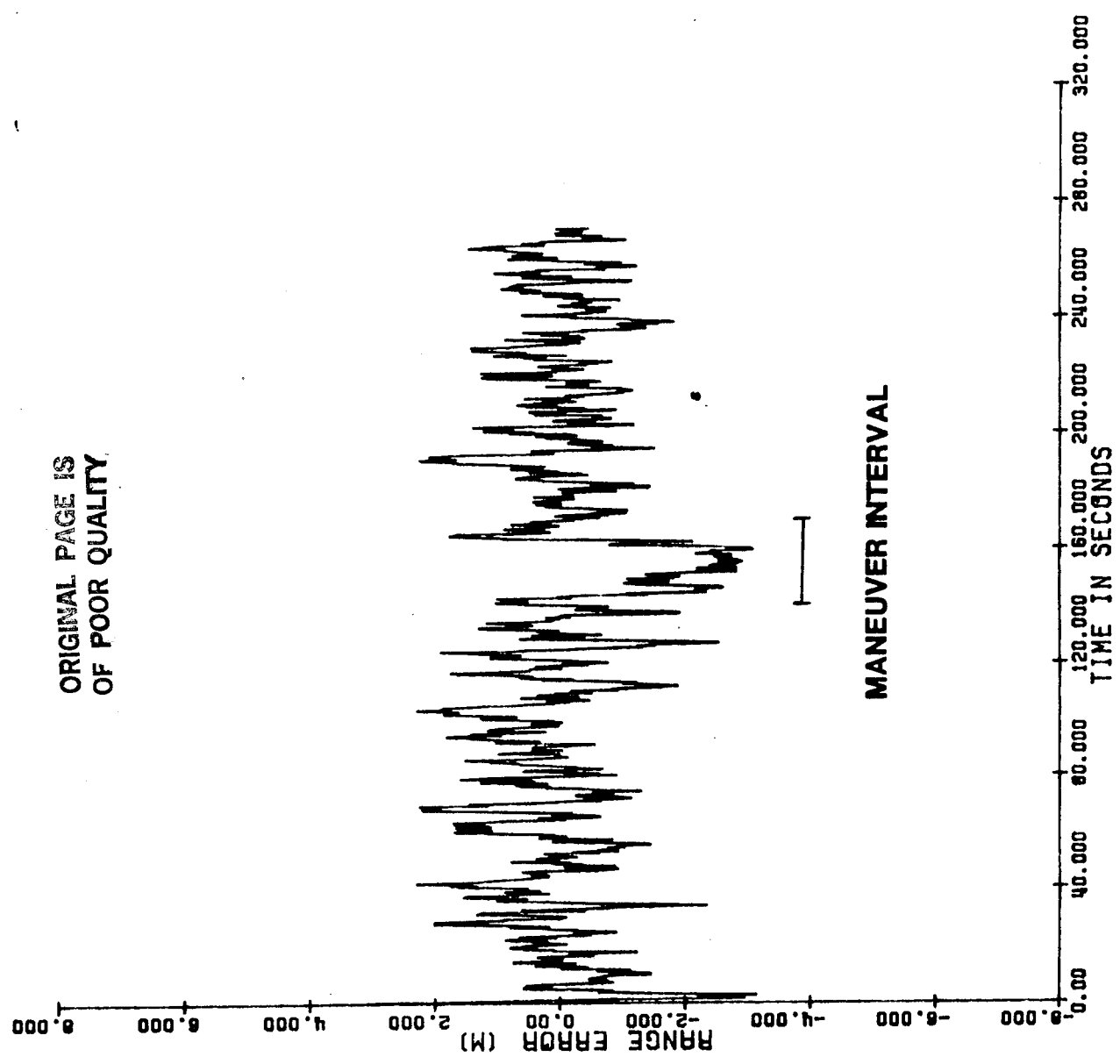


Figure 6. Filter Response; $\sigma_{x^2} = 10$, $\sigma_{a^2} = 2$.

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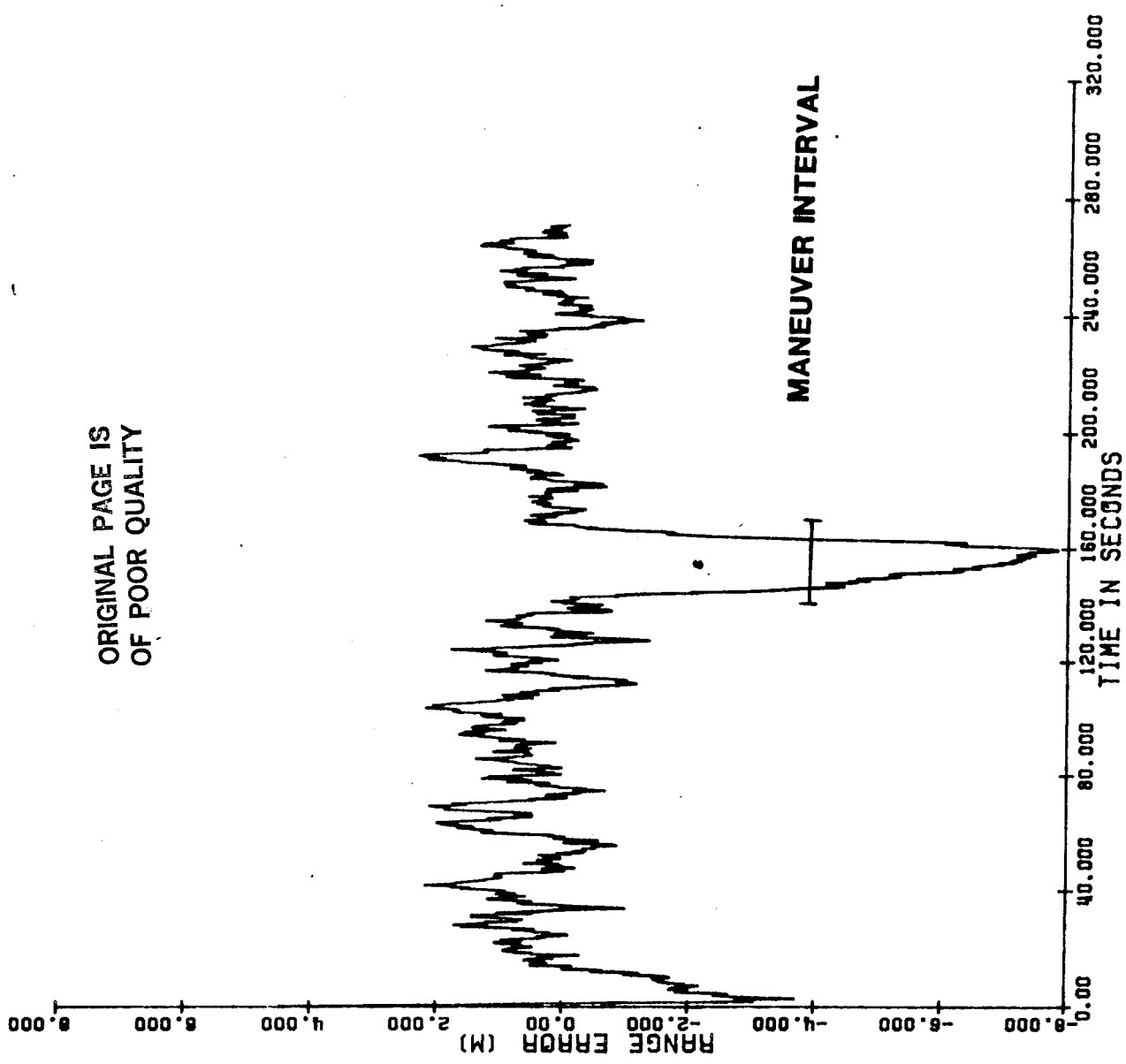


Figure 7. Filter Response; $\sigma_{x^2} = 10$, $\sigma_{a^2} = 0.2$.

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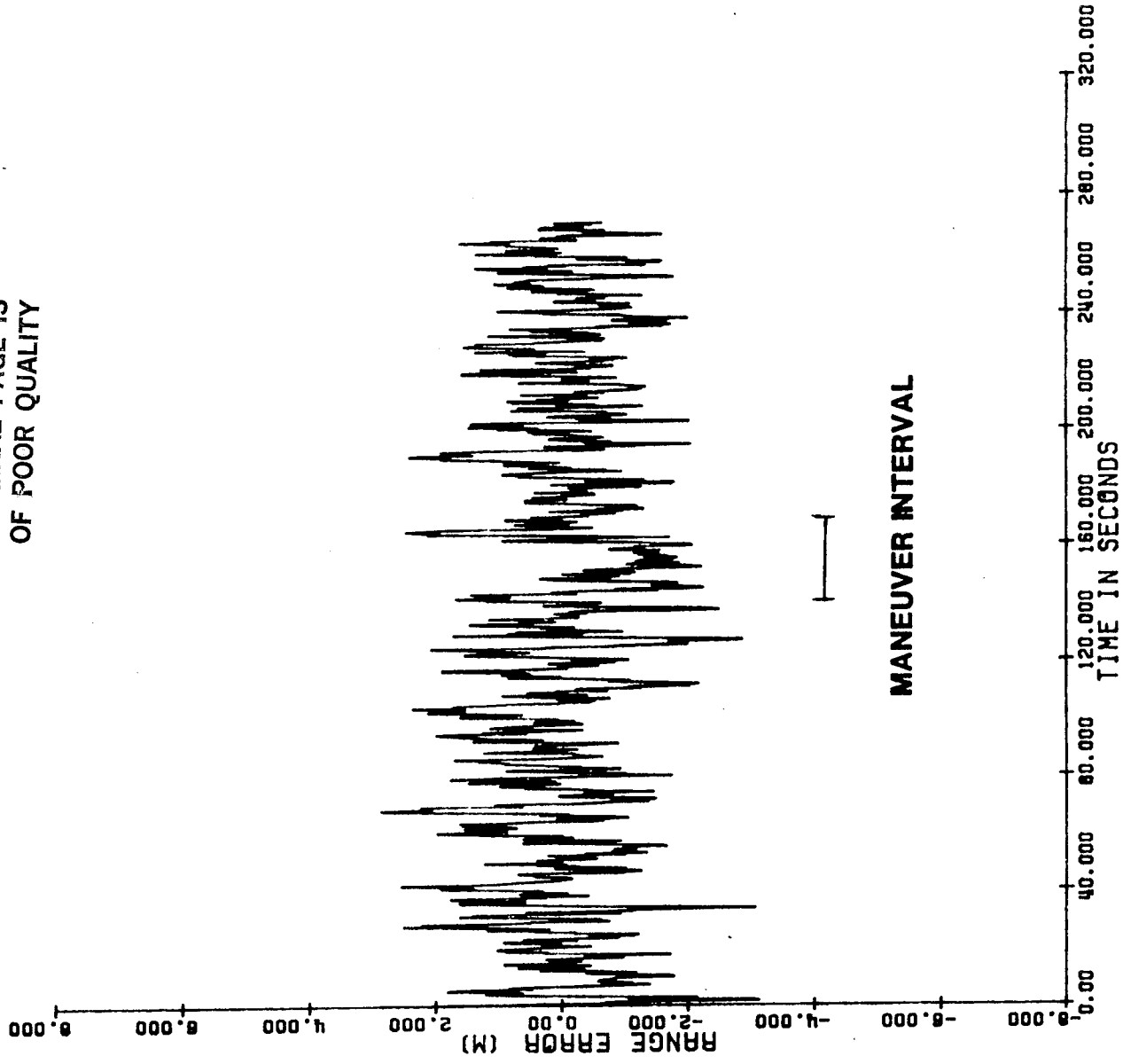


Figure 8. Filter Response; $\sigma_{x^2} = 1$, $\sigma_{a^2} = 1$.

VIII. CONCLUSION AND RECOMMENDATIONS FOR FURTHER RESEARCH

The discrete Kalman filter for tracking GPS signals separately is a very flexible method for an adequate solving of the random noise problem.

Although a very simple filter (range measurements only) was used in this paper, the (steady state) noise variance was reduced with a factor 4 (table 2).

In case the highly accurate delta range (Doppler) measurements are available to the tracking filter, the resulting improvement in position estimation can be an order of magnitude or more [20]. The filter is also relatively easy to extend to a three state filter capable of handling the Markov noise produced by the receiver (see table 1) as well. Another possibility is to add additional states for allowing more complex vehicle dynamics [18, 21, 22].

The results shown in figures 6, 7 and 8 are also representative for the final three dimensional position error; see Chapter 6, also compare figure 2 (magnitude of the 3D position error due to multipath) with figure 3 (single channel multipath error).

The separate channel tracking approach as presented in this paper compared to the simultaneous (sub)optimal Kalman filter may have a distinct implementation advantage in case the GPS receiver has less than four tracking channels. The problem of 'asynchronized filter feeding' can be solved by separate channel filtering, fitting a nth order function through the filter output data (e.g. a second-order function as described in Chapter 5.D.2.2 and finally executing a position flash computation (Chapter 5.C) or even using a simultaneous filter using synchronized data derived from the fitting function.

IX. ACKNOWLEDGEMENTS

This work was supported under NASA contract NAS2-11969 by NASA Ames Research Center, Moffett Field, CA.

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1. Report No. CR 177472	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle Discrete Filtering Techniques Applied to Sequential GPS Range Measurements		5. Report Date December 1987	6. Performing Organization Code
		8. Performing Organization Report No.	
7. Author(s) Frank Van Graas		10. Work Unit No. 505-66-11	11. Contract or Grant No. NAS2-11969
9. Performing Organization Name and Address Avionics Engineering Center Ohio University Athens, OH 45701		13. Type of Report and Period Covered NASA Contractor Report	
		14. Sponsoring Agency Code	
12. Sponsoring Agency Name and Address NASA Ames Research Center Moffett Field, CA 94035		15. Supplementary Notes Point of Contact: Fred Edwards, Ames Research Center, M.S. 210-9 Moffett Field, CA 94035 (415) 694-5437 or FTS 464-5437	
16. Abstract <p>The GPS range measurements from satellite to user are corrupted by several distortions. These distortions can be divided into random and bias components. The random distortions may be reduced by use of filtering techniques, but not the bias component, for it is not possible to distinguish between the bias errors and the actual range. However, there is a way to obtain a very accurate estimate of the bias distortions and this is through the implementing the so-called differential GPS (DGPS) concept. This paper describes the applications of DGPS and discrete range domain filtering techniques to remove the distortions observed in sequential range measurements of a C/A code Global Positioning System receiver. The separate channel tracking approach as presented in the paper has distinct implementation advantage compared to the simultaneous (sub)optimal Kalman filter for the case of a sequential GPS receiver. These advantages are discussed.</p>			
17. Key Words (Suggested by Author(s)) GPS Differential GPS Kalman Filter Ground Station		18. Distribution Statement Unlimited Subject Category: 04	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) U	21. No. of Pages 46	22. Price*

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