KSC-DM-3265

COMPUTATION OF GENERALIZED MODAL LOADS IN AN ACOUSTIC FIELD DEFINED BY A DISTRIBUTION OF CORRELATED PRESSURES

AUGUST 1989

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JOHN F. KENNEDY SPACE CENTER, NASA

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FOREWORD

This report is an aid to designers of structures with large area-to-mass ratios that are subject to high acoustic pressures during rocket launches. It provides a means of determining generalized modal loads using AJ-coefficients defined by the design procedure documented in KSC-DM-3147. AJ-coefficients are a measure of a vibroacoustic coupling between the structure and the acoustic field.

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ABBREVIATIONS, ACRONYMS, AND SYMBOLS

a	distance between the CPD and the center of a span
Ai	Area overlap between Ap and As
AJ	product of reference area and joint acceptance
Ap	pressure correlation area
Ap	area under $\varphi_1(x)$
As	structural area
Ax	reference area in the x direction
Ay	reference area in the y direction
b	beam width
cg	center of gravity
cos	cosine
CPD	correlated pressure distribution
d	cantilever span
DM	Director Mechanical Engineering
e	distance from the CPD to the support
E	material modulus of elasticity
EI	constant stiffness
f	frequency
FEM	finite element model
ft	foot
G	generalized modal load
h	horizontal
Hz	hertz
I	moment of inertia
in	inch
in-lb	inch-pound
in^3	cubic inch
in^4	inch to the fourth power
Jx	joint acceptance in the x direction
Jy	joint acceptance in the y direction
K	loaded span number
KSC	John F. Kennedy Space Center
l	span
L	total length of a cantilevered beam
$lb\text{-}in^2$	pound-inch squared
$lb\text{-}sec^2/in^2$	pound-second squared per inch squared
lb/in^2	pound per square inch
LC	launch complex
lim	limit
M	generalized modal mass
m	uniformaly distributed mass
max	maximum
MAX	maximum value of arguments
Meq	equivalent uniformaly distributed mass
MIN	minimum value of arguments
M_{rb}	response bending moment

ABBREVIATIONS, ACRONYMS, AND SYMBOLS (cont)

n	total number of equal spans; mode number
NASTRAN	NASA structural analysis
no.	number
P(x,y)	distribution of correlated pressures $0<x<1$
P_1	linear function in the interval
PCL	pressure correlated length
PSD	power spectral density
psi	pound per square inch
q	modal coordinate
r	radius vector measured from the center of the CPD
rad/sec	radian per second
$(rad/sec)^2$	radian per second squared
S	secton modulus
S(a,b)	center point of the CPD in the x,y coordinates
sin	sine
u	coordinate
v	coordinate
v	vertical
Y	ordinate of response spectra
Z	response amplitude
φ	normal modal displacement
α	position of the CPD within a loaded span
β	nondimensional horizontal axis of plots
λ	pressure correlation length
℄	centerline
$\Phi(x,y)$	normal mode displacement
ω_n	circular natural frequency of the n^{th} mode
φ_o	maximum normal modal displacement
δ	maximum unit stress of the response bending moment

MATHEMATICAL SIGNS AND SYMBOLS

$=$	equals
$-$	negative
$-$	minus (sign of subtraction)
$+$	positive
$+$	plus (sign of addition)
$\equiv$	identical with
$\neq$	not equal to
$\|x\|$	absolute value of x
$\geqslant$	equal to or greater than
$\leqslant$	equal to or less than
$\rightarrow$	approaches
π	pi, 3.14159+
$\sqrt{}$	square root of
) [ ] { }	parentheses, brackets, and braces; quantities enclosed by them to be taken together in multiplying, dividing, etc.
$\approx$	approximately equals
$\partial u/\partial x$	partial derivative of u with respect to x
$\int$	integral of
$\int_b^a$	integral of, between limits a and b
$\gg$	much greater than
dx	differential of x
$/$	divided by
$\cong$	approximately equal to; congruent

SECTION I

INTRODUCTION

During a Shuttle launch, structures in the proximity of the launch pad are subjected to acoustic pressures generated by rocket exhausts. The design of some structures, particularly those having a large area-to-mass ratio, is governed by the launch-generated acoustic environment, a relatively short but very intense pressure transient.

A procedure documented in KSC-DM-3147, Procedure and Criteria for Conducting a Dynamic Response Analysis of Orbiter Weather Protection System on LC-39B Fixed Service Structure, was developed to calculate the peak dynamic response of a normal vibration mode of a structure to an input transient. The procedure is based on modal parameters of the structure, response spectra to acoustic pressures, and a definition of acoustic field by means of pressure correlation lengths. Modal parameters and the area of correlated pressures bounded by pressure correlation lengths are a prerequisite to calculations of generalized modal loads required as input to response spectra for a consequent estimate of peak responses.

Generalized modal loads are defined by means of AJ-coefficients, which are a measure of a vibroacoustic coupling between the structure and the acoustic field. The stronger the vibroacoustic coupling is, the higher the vibration of the structure in the coupled mode, and vice versa. Thus, an accurate estimate of AJ for each vibration mode is essential to calculations of dynamic response. Even for a structure of an average complexity, calculations of AJ-coefficients may become a cumbersome task because existing dynamic analysis codes do not have the capability to automatically account for all parameters leading to the computation of the required extreme values of AJ's. These coefficients depend on a surface integral of the product between modal displacements and the correlated pressure distribution, a function of pressure correlation lengths that depend upon a particular acoustic field and the resonance frequency of the mode. The relative position between the mode and the center of distribution affects variable integral limits and the values of AJ-coefficients.

A position resulting in the extreme value of AJ and in the highest dynamic response of a mode may not be obvious from an examination of integrand constituents and variable integral limits. Particularly in vibration modes where modal displacements of the surface under an acoustic load change signs in relation to the outward surface normal, a proper optimization of the load position becomes critical and, if not performed, may lead to significant differences (by factors exceeding 100 percent) in the estimate of a peak dynamic response. Generally, a different position of correlated pressure distribution corresponds to each vibration mode, resulting in the strongest vibroacoustic coupling and the highest response.

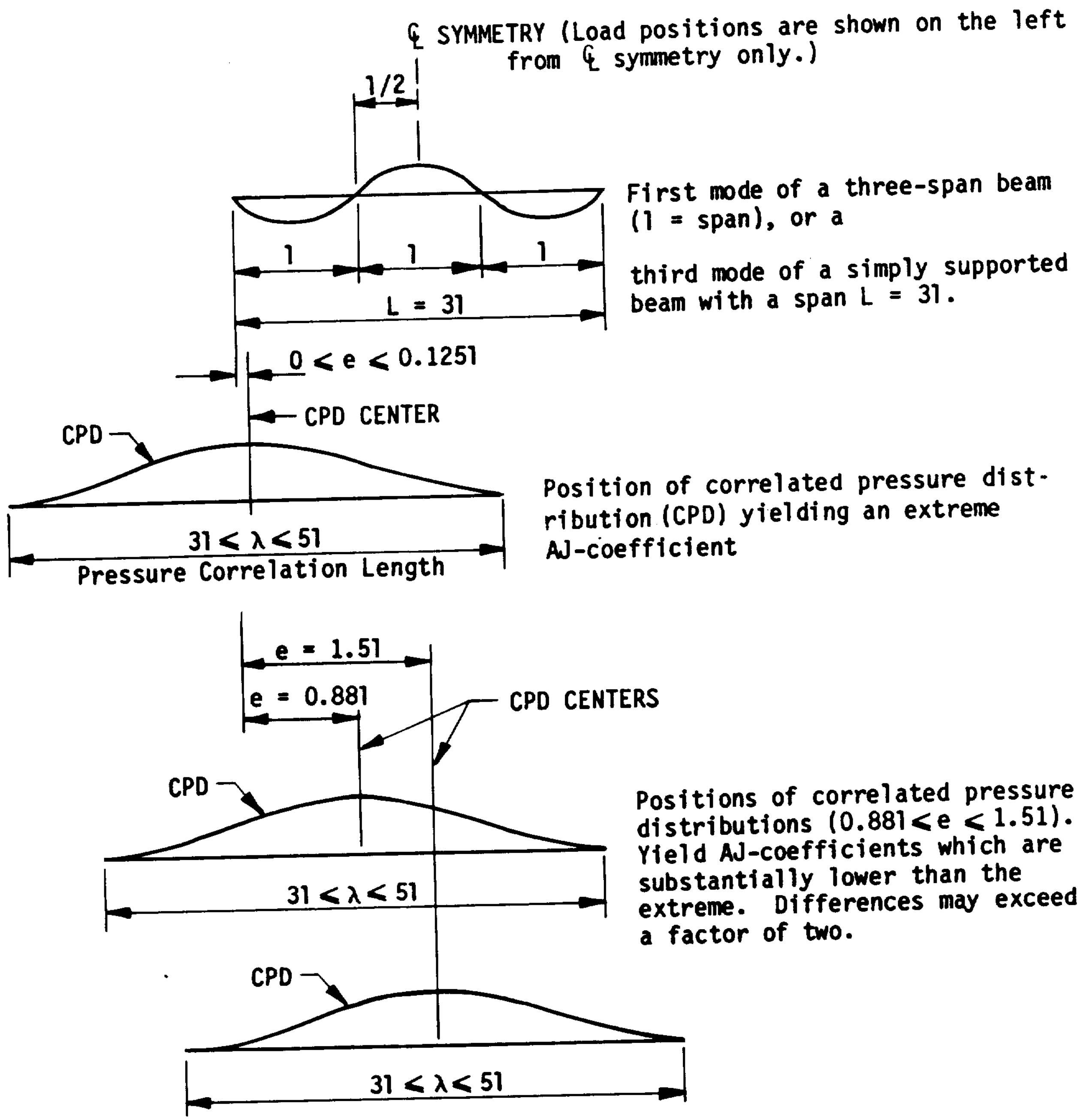
An example of the required optimization of the load position is a case of the fundamental mode of a three-span continuous beam and/or a case of the third mode of a simply supported beam, illustrated in figure 1-1. When the pressure correlation length in the direction of the beam span is equal to three to five times the distance between nodal points, then the position of the distribution center relative to the mode becomes critical to the estimate of AJ-coefficients. The extreme value of AJ occurs when the center of correlated pressure distribution (load center) is located near the end nodal point (the end support of a three-span beam), a load position that is certainly not apparent. If the load position is not optimized and the load center is located anywhere near and between the second and the third nodal points (near and within the middle span of a three-span beam), the value of AJ becomes substantially lower than the extreme value, generally below 50 percent of the extreme. An underestimate of response for fundamental modes of structural components may result in failure of the components in a launch environment because design safety factors are usually near or below a factor of two.

This example of how sensitive computations of AJ-coefficients to load position may become is typical in multispan structures. An exact solution for AJ-coefficients of a three-equal-span continuous beam is provided in this report. It allows a user to obtain a numerical comparison between the exact solution and the output of a dynamic analysis code.

This report presents diagrams for simple and multispan continuous beams from which corresponding AJ-coefficients can be calculated. The diagrams are applicable to fundamental and higher modes. Conditions and requirements for a valid application are stated in the presented derivation, either explicitly or by means of cited assumptions required to perform numerical computations. While many vibration modes are excited in a wide frequency band acoustic field, stress-strain extremes governing a design occur mainly in a single fundamental mode of each structural component.

Applications may be extended to a grid of beam-type structures and to plates. The use of continuous versus simple beams in a grid results in a significant reduction of dynamic response because of two effects. The first effect is a decrease of vibroacoustic coupling reflected in lower AJ-coefficients. A comparison between diagrams for simple and continuous beams proves the advantage offered by the continuity. The second effect is an increase of generalized modal mass (M in AJ/M factor) in direct proportion to the number of continuous spans. Thus, a multifold reduction of response can be achieved by implementing a structural continuity.

The beam is assumed to have a uniformly distributed mass (m) and constant stiffness (EI).



Horizontal scale represents a case λ = 4l.

Figure 1-1. Effect of Load Position (CPD Center) on AJ-Coefficients, a Measure of Vibroacoustic Coupling

SECTION II

PROBLEM DEFINITION

Utilization of response spectra, $Y=q/(AJ/M/\omega_n^2)$, to acoustic pressures, p(t), in an application to the analysis of peak structural responses, requires a computation of a generalized modal load, G(t)=AJp(t), for each normal mode of a vibrating structure. The time history of acoustic pressures, p(t), is assumed to be known from measurements taken in the acoustic field where the structure is located. Generally, a multitude of measurements is required for a proper definition of all basic parameters of an acoustic field. In the following equations, it is assumed that measurements are available and required parameters are defined, so that the mathematical formulation can reference them as if they were obtained from a single set of measurements.

Response spectra and pressure correlation lengths (PCL's) are assumed to be available for the frequency range containing resonance frequencies of all normal modes considered in the analysis. Computations of generalized modal loads are then reduced to the problem of estimating AJ-coefficients for each normal mode, and peak response modal coordinates, q's, are calculated from response spectra:

$$q = Y(AJ/M/\omega_n^2)$$

The equation defining AJ for a mode of a planar structure in x,y plane is:

$$AJ = \int_{Ai} \Phi(x,y)P(x,y)dA \tag{1}$$

where:

$\Phi(x,y)$ is normal mode displacements perpendicular to the x,y plane.

$P(x,y)$ is the general form of the distribution of correlated pressures in the x,y plane and within the correlation area, Ap. Implicitly, distribution is also a parametric function of the resonance frequency, fn, of the mode $\Phi(x,y)$. Condition $0 \leqslant P(x,y) \leqslant 1$ limits the values of the distribution.

Ai is an area defining the extent of the integral. It is equal to the overlapping area between the pressure correlation area, Ap, and the structural area, As, exposed to acoustic pressures. By defining an overlap, Ai strongly depends on the position of the center of correlated pressure distribution relative to the structure and, thus, relative to modal displacements.

The coordinates of the distribution center, a and b, satisfy condition P(a,b)=1. However, a and b are also independent parameters that must be varied to yield an extreme value of AJ. Therefore, P(x,y) must be defined relative to its center while, in a homogeneous acoustic field, the center may be positioned anywhere on the surface of the structure. In the local coordinates u=x-a and v=y-b, the definition of distribution relative to its center becomes:

$$P(x,y) = P(a+u,b+v) = P_{uv}(u,v) \tag{2}$$

with limits on variation of u and v:

$$-\lambda_x/2 \leqslant u \leqslant \lambda_x/2 \text{ and } -\lambda_y/2 \leqslant v \leqslant \lambda_y/2 \tag{3}$$

where λ_x and λ_y are pressure correlation lengths in the direction of the x and y axes.

The equation:

$$P_{uv}(u,v) = 0 \tag{4}$$

defines a contour of the pressure correlation area, Ap, in the x,y plane. Distribution $P_{uv}(u,v)$ is zero everywhere on and outside of this contour. Because pressure correlation lengths, λ_x and λ_y, are obtained experimentally from measurements of coherence and phase between available sensor pairs (see KSC-DM-3147, Appendix B) and because the number of measurements is limited, a verification of the contour shape where $P_{uv}(u,v)=0$ was not possible. The contour is assumed to be an ellipse with axes λ_x and λ_y. The elliptic contour introduces a constraint on the allowable variation of local coordinates u and v that is more stringent than (3):

$$4\lambda_y^2 u^2 + 4\lambda_x^2 v^2 \leqslant \lambda_x^2 \lambda_y^2 \tag{5}$$

Consequently for any point with coordinates u,v, new variables (η, λ, r) are defined:

$$\eta = \frac{v}{u} \ ; \ \lambda = \lambda_x\lambda_y\left[\frac{1+\eta^2}{\lambda_y^2 + \eta^2\lambda_x^2}\right]^{1/2} \ ; \ r = \pm(u^2 + v^2)^{1/2} \tag{6}$$

Note that: for $v = 0$, $\lambda = \lambda_x$

for $u = 0$, $\lim\limits_{u \to 0} \lambda = \lambda_y$

Then, correlated pressure distribution (CPD) is explicitly defined:

$$P_{uv}(u,v) = \frac{1}{2}\left(1 + \cos\frac{2\pi r}{\lambda}\right) \geqslant 0 \tag{7}$$

with a constraint

$$-\frac{\lambda}{2} \leqslant r \leqslant \frac{\lambda}{2} \tag{8}$$

Which ensures that point u,v remains inside the contour of the correlated pressure area. Constraint (8) is equivalent to and replaces (5). The sign of r in (6) is irrelevant to the definition of $P_{uv}(u,v)$.

The contour of the correlation area was assumed to be an ellipse resulting in the smallest value of the correlation area (min Ap), but not necessarily in a lowering of AJ in (1). An assumption of a rectangular contour with sides λ_x and λ_y results in the largest value of the correlation area (max Ap). The corresponding distribution becomes:

$$P_{uv}(u,v) = \frac{1}{4}\left(1 + \cos\frac{2\pi u}{\lambda_x}\right)\left(1 + \cos\frac{2\pi v}{\lambda_y}\right) \tag{9}$$

with the constraint on the variation of u and v provided by (3). The form of (9) is equivalent to a definition of the CPD by means of two separate distributions (separation of variables) along each coordinate axis:

$$P_{uv}(u,v) = P_u(u)P_v(v) \tag{10}$$

Bracketing of the extents of CPD's by (7) and (9) is not expected to have a significant effect on AJ.

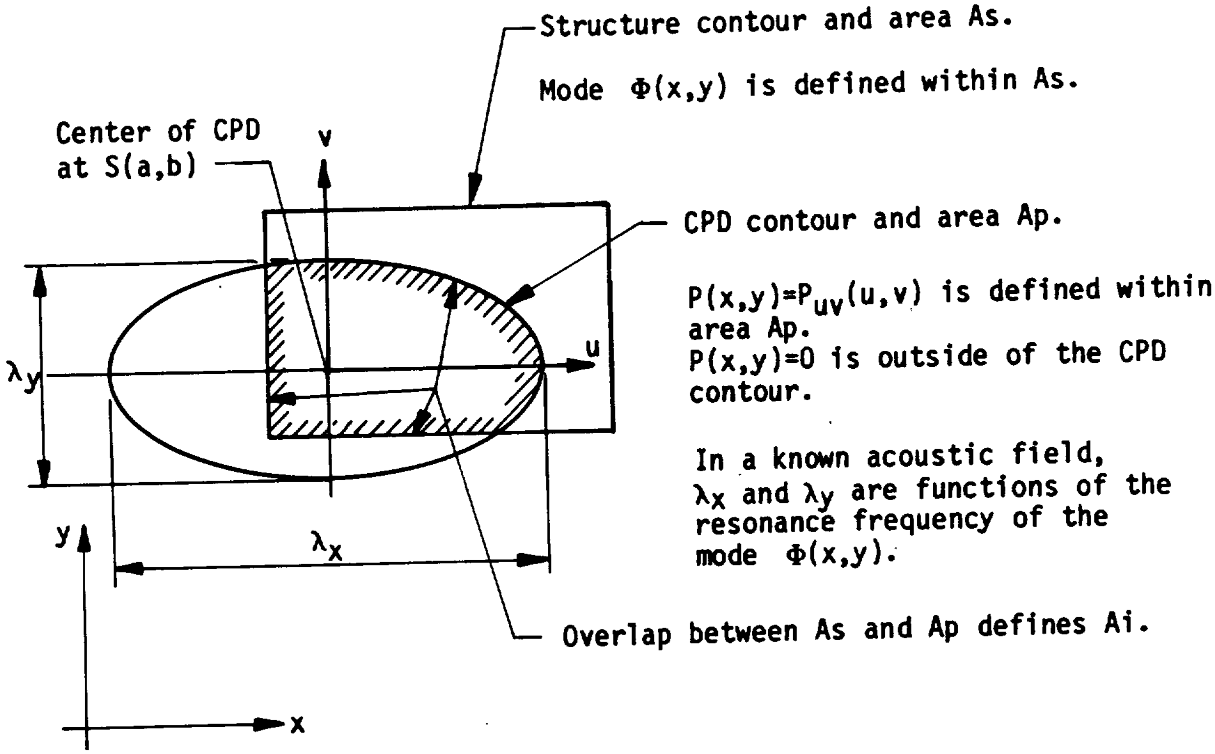
At this point, all constituents of AJ in (1) are formally defined. $\Phi(x,y)$ is known from either the finite element model (FEM) or from experimental modal analysis. $P_{uv}(u,v)$ is defined by either (7) or (9) and by corresponding constraints in (8) or (3). By locating the center of CPD at a point S(a,b) and by applying a coordinate transformation

$$u = x-a \ , \ v = y-b \tag{11}$$

to either $\Phi(x,y) \rightarrow \Phi_{uv}(u,v)$ or to $P_{uv}(u,v) \rightarrow P(x,y)$ so that both functions and their constraints are expressed in the same coordinates, the extent of the integral, the area Ai, can be defined. An additional constraint is imposed on the allowable variation of integrand coordinates by the choice of the point S(a,b). This constraint contains previously defined (8) or (3) by requiring that integrand coordinates be contained within the contour of the overlap area Ai. An illustration of this problem is shown in figure 2-1.

The integration of (1) can be performed in either the x,y or u,v coordinates and a value of AJ can be calculated. This value becomes a function of coordinates of the point S(a,b) that was selected by an essentially arbitrary choice. In order for AJ to become a required extreme, it must satisfy two conditions:

$$\frac{\partial AJ}{\partial a} = \frac{\partial AJ}{\partial b} = 0 \tag{12}$$



Problem: $AJ = \int_{Ai} \Phi(x,y)\ P(x,y)\ dA$

Find a position of S(a,b) relative to the mode $\Phi(x,y)$ such that AJ becomes an extreme [maximum or minimum depending on the sign of $\Phi(x,y)$].

AJ is a measure of vibroacoustic coupling.

Figure 2-1. Illustration of a Vibroacoustic Coupling for a Planar Structure

For a general case of a mode, conditions (12) present a computational problem that, sofar, remains unresolved. With few exceptions (a case of very simple modes where a mode has the same sign within the contour of As or where Ap>>As) the choice of S(a,b) that satisfies (12) is far from being obvious.

A strategy to find S(a,b) satisfying (12) may use a lengthy technique of a single parameter variation; for example, parameter a is varied until an extreme where $\partial AJ/\partial a \approx 0$ is found at $a=a_1$. Then the second parameter b is varied and another extreme where $\partial AJ/\partial b \approx 0$ is found at $b=b_1$. The entire process can be repeated as many times as necessary to obtain an acceptable convergence. The strategy may appear faultless and, perhaps, acceptable if performed by a computer code.

Aside from the fact that such an optimization procedure does not exist in dynamic analysis codes, there will be a serious caveat if such strategy were developed. The point $S(a_1,b_1)$, where conditions (12) are satisfied, may define a local extreme of AJ rather than an absolute extreme. Particularly in asymmetric structures where a mode asymmetry may be caused by the configuration, placement of supports, mass, or stiffness distributions, the integrand $\Phi(x,y)P(x,y)$ can have local peaks that may occur within almost any region containing the center of CPD.

The problem of multiple extremes may occur in the fundamental modes of structural components and is certain to occur in modes higher than fundamental modes. Examples can be found even in simple fundamental modes of multispan continuous beams. Fortunately, the absolute extremes are discernable in these cases. An examination of the diagrams in appendixes A and B provides a position of the CPD center along a single axis, either the coordinate $S(a_1,0)$ along the x-axis or the coordinate $S(0,b_1)$ along the y-axis, which yields an absolute extreme of AJ. If a mode of a structure resembles modes of continuous beams, then the presented diagrams may be helpful to analysts and designers in their task to establish a load position resulting in the absolute extreme of the AJ-coefficient for such a mode.

An important application case occurs whenever the mode $\Phi(x,y)$ may be approximated in the form of separated variables. In such a case, the estimate of AJ may be obtained as a product of two separate estimates and, perhaps, using the diagrams presented in appendixes A and B.

If

$$\Phi(x,y) = \Phi_{uv}(u,v) = \Phi_u(u)\Phi_v(v) \tag{1.1}$$

Then using form (10) for $P_{uv}(u,v)$ and considering dA=dudv:

$$AJ = \int_{A_u} \Phi_u(u)P_u(u)du \int_{A_v} \Phi_v(v)P_v(v)dv \tag{1.2}$$

A piece-wise approximation of the Ai contour is always possible to ensure that $A_u \equiv A_u(u)$ and $A_v \equiv A_v(v)$. Then each integral in (1.2) can be evaluated separately:

$$A_x J_x = \int_{A_u} \Phi_u(u) P_u(u) du \text{ and } A_y J_y = \int_{A_v} \Phi_v(v) P_v(v) dv \tag{1.3}$$

so that:

$$AJ = A_x A_y J_x J_y \tag{1.4}$$

If the approximation (1.1) is possible, then the coordinates of the point S(a,b) where AJ becomes an extreme can be established from a separate examination of each component function, $\Phi_u(u)$ and $\Phi_v(v)$. This conclusion directly follows from (1.4) because (12) now becomes:

$$\frac{\partial AJ}{\partial a} = A_y J_y \frac{\partial (A_x J_x)}{\partial a} = 0 \quad \text{when} \quad \frac{\partial (A_x J_x)}{\partial a} = 0 \tag{12a}$$

$$\frac{\partial AJ}{\partial b} = A_x J_x \frac{\partial (A_y J_y)}{\partial b} = 0 \quad \text{when} \quad \frac{\partial (A_y J_y)}{\partial b} = 0 \tag{12b}$$

When the mode shapes become sufficiently complex, the approximation by (1.1) should be attempted, at least for the purpose of establishing the position of the CPD center where AJ becomes an extreme.

SECTION III

SOLUTION FOR A UNIFORM CONTINUOUS BEAM WITH EQUAL SPANS

The example presented in this section is, perhaps, the simplest case in which one can obtain an explicit solution for AJ-coefficients by a direct integration of (1) and without recourse to FEM's and a numerical integration. The derivation of the explicit solution is documented in every detail and only a few cumbersome algebraic operations are omitted.

Variables defining a one-dimension problem are shown in figure 3-1. A listing of indefinite integrals used in the derivation is shown in figure 3-2 for reference.

The origin of the x,y coordinate system is located at midspan of the "loaded" span containing the center of CPD. The general solution is valid for any location of the CPD center - it is a function of integral limits defined by the overlap area Ai. The concept of the "loaded" span is introduced as a convenience for future reference and presentation of results.

In the x,y system:

$$\Phi(x,y) = \varphi(x) = \cos \frac{\pi x}{l} \tag{13}$$

Thus the solution will be valid for a strip of unit width in the y-direction. The mode $\varphi(x)$ is normalized to have a maximum displacement of one unit. This defines a corresponding generalized modal mass of a uniform (m and EI are constants) continuous beam, M=1/2 nml, with n equal spans, each of length l. The solution will also be valid for higher than the fundamental mode, in which case l is the distance between nodal points and n is the total number of half cycles of cosine function representing the mode (n = mode number for a simply supported beam).

A definite normalization of the mode $\varphi(x)$ is necessary in order to calculate uniquely defined AJ-coefficients. If the mode were normalized to an amplitude φ_o, then:

a. $\varphi(x) = \varphi_o \cos \frac{\pi x}{l}$; $M_o = \frac{1}{2}\varphi_o^2 \text{ nml} = \varphi_o^2 M$

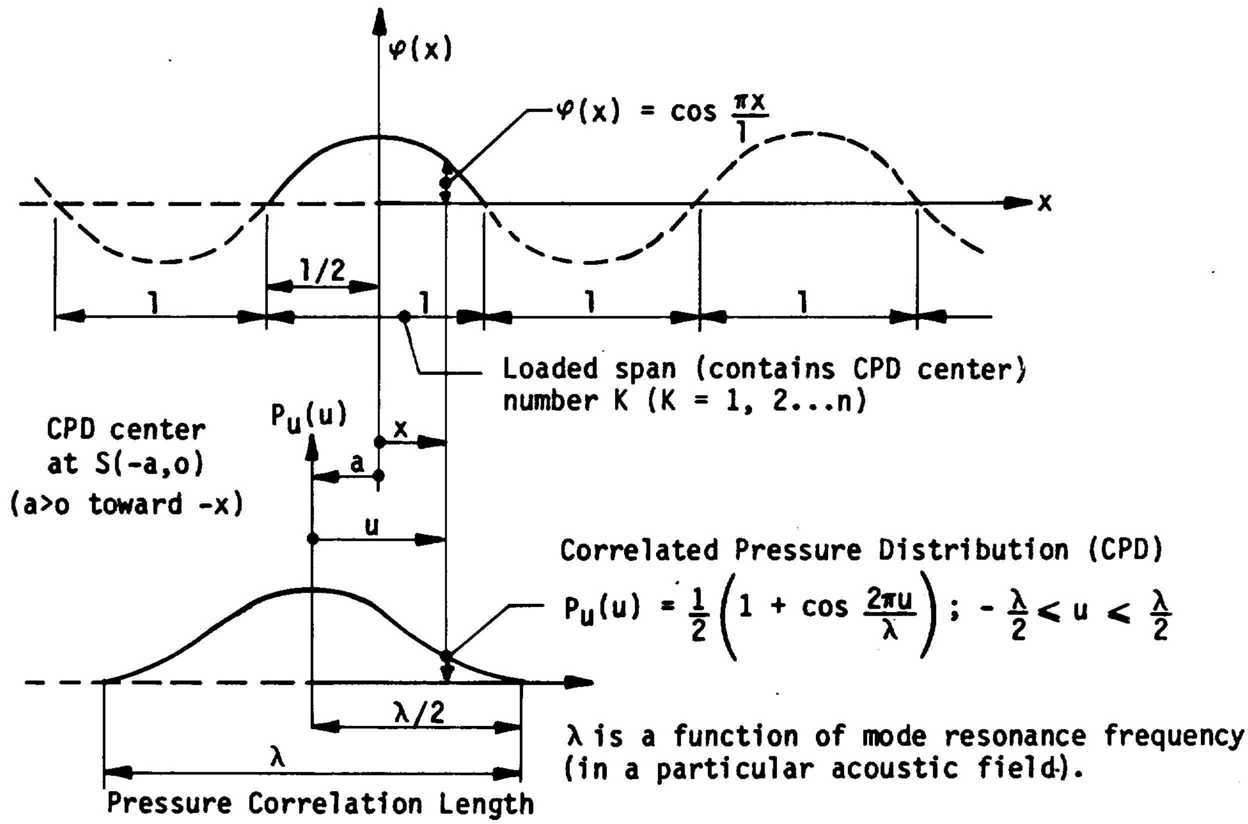
b. Presented AJ-coefficients would require a multiplier φ_o.

c. Corresponding AJ/M and the response modal coordinate $q=Y(AJ/M/\omega_n^2)$ would contain a multiplier $1/\varphi_o$. However, the resulting peak response modal displacement (response amplitude) $Z=q\cdot\varphi(x)$ is independent from φ_o, and so are all other response quantities (stresses, reactions, etc.).

The above relations should be kept in mind whenever a mode is not normalized to the form assumed in (13).

A continuous beam with pinned end supports. The beam is assumed to have a uniformly distributed mass m ($lb\text{-}sec^2/in^2$) and a constant stiffness EI ($lb\text{-}in^2$).

Total number of equal spans is n.



First mode resonsance frequency (undamped): $f = \frac{\pi}{2l^2}\sqrt{\frac{EI}{M}}$ (Hz)

Origin of coordinate system φ,x is located at midspan of the loaded span no. K.

First bending mode: $\varphi(x) = \cos \frac{\pi x}{l}$

Limits on x : $-\left(K - \frac{1}{2}\right)l < x < \left(n - K + \frac{1}{2}\right)l$

When the mode is normalized to have a maximum displacement of one unit (1 inch), then the generalized modal mass, M, is equal to one-half of the total mass: $M = \frac{1}{2}\,nml$

Define: $\alpha = \frac{a}{l}$ and $\beta = \frac{\lambda}{l}$ parameters of an explicit solution.

Figure 3-1. Parameters for Calculation of AJ-Coefficients for a Continuous Beam

Indefinite integrals used in derivation:

$$\int \sin axdx = -\frac{1}{a}\cos ax$$

$$\int \cos axdx = \frac{1}{a}\sin ax$$

$$\int \sin^2 axdx = \frac{1}{2}x - \frac{1}{4a}\sin 2ax$$

$$\int \cos^2 axdx = \frac{1}{2}x + \frac{1}{4a}\sin 2ax$$

$$\int \sin ax \cos ax\, dx = \frac{1}{2a}\sin^2 ax$$

For $|a| \neq |b|$:

$$\int \sin ax \cos bxdx = -\frac{\cos (a+b)x}{2(a+b)} - \frac{\cos (a-b)x}{2(a-b)}$$

$$\int \cos ax \cos bxdx = \frac{\sin (a-b)x}{2(a-b)} + \frac{\sin (a+b)x}{2(a+b)}$$

$$\int \sin ax \sin bxdx = \frac{\sin (a-b)x}{2(a-b)} - \frac{\sin (a+b)x}{2(a+b)}$$

Figure 3-2. Listing of Indefinite Integrals Used in the Derivation

If the center of CPD is selected at point S(-a,0), then:

$$u = a+x \; ; \; x = u-a \; ; \; dx = du \tag{14}$$

If the integration of (1) is performed in the local u coordinate and variables are transformed accordingly by (14):

$$\Phi(x,y) \rightarrow \Phi_{uv}(u,v) = \varphi_u(u) = \cos \frac{\pi(u-a)}{l}$$

$$\varphi_u(u) = \cos \frac{\pi a}{l} \cos \frac{\pi u}{l} + \sin \frac{\pi a}{l} \sin \frac{\pi u}{l} \tag{15}$$

$$P(x,y) \rightarrow P_{uv}(u,v) = P_u(u) = \frac{1}{2}\left(1 + \cos \frac{2\pi u}{\lambda}\right) \tag{16}$$

The integrand in (1) becomes:

$$\varphi_u(u)P_u(u) = \frac{1}{2} \cos \frac{\pi a}{l}\left[\cos \frac{\pi u}{l} + \cos \frac{\pi u}{l} \cos \frac{2\pi u}{\lambda}\right]$$

$$+ \frac{1}{2} \sin \frac{\pi a}{l}\left[\sin \frac{\pi u}{l} + \sin \frac{\pi u}{l} \cos \frac{2\pi u}{\lambda}\right] \tag{17}$$

There are two constraints on the allowable variation of u. The first is imposed by the extent of the structure (same as the constraint by As):

$$\frac{a}{l} - K + \frac{1}{2} \leqslant \frac{u}{l} \leqslant \frac{a}{l} + n - K + \frac{1}{2} \tag{18}$$

The second is imposed by the extent of the pressure correlation length λ (same as the constraint by Ap):

$$-\frac{\lambda}{2l} < \frac{u}{l} < \frac{\lambda}{2l} \tag{19}$$

Where K is the number of the span containing the CPD center (counting from left to right, in the direction of the +x-axis).

The overlap between constraints (18) and (19) defines lower and upper limits of the integral (1), the same as the constraint provide by Ai.

$$\int_{Ai} \Phi(x,y)P(x,y)dA = \int_{u_l}^{u_u} \varphi_u(u)P_u(u)du = AJ \tag{1a}$$

Where u_l is the larger (considering signs) value between:

$$l\left(\frac{a}{l} - K + \frac{1}{2}\right) \text{ and } -\frac{\lambda}{2} \tag{20}$$

u_u is the smaller value between:

$$l\left(\frac{a}{l} + n - K + \frac{1}{2}\right) \text{ and } +\frac{\lambda}{2} \tag{21}$$

At this point, all constituents of (1) and (1a) are defined by (17), (20), and (21) and AJ can be calculated explicitly by using pertinent integrals from figure 3-2. Before this is done, there remain a few aspects that require clarification. These are presented next.

Only the total value of an AJ coefficient that is both required and sufficient for intended application in response spectra can be calculated. Neither A nor J from AJ can be separated without introducing a convention of how to do it. In the analysis of a steady-state response to random acoustic pressures by means of input power spectra, a concept of Joint Acceptance, usually designated by the symbol J^2, is introduced. There are tables and diagrams depicting J^2 for fundamental and higher modes of single-span structures such as beams, plates, etc. Calculated quantity is actually $(AJ)^2$ where A is assumed to be the total area of the structure (a single span only) so that J^2 can be uniquely defined. In order to remain compatible with the concept of Joint Acceptance and in order to eliminate the dependence of J upon the span length l, it is assumed that A=1. This convention, actually a normalization, allows a presentation of J in the form of a nondimensional coefficient that has the same physical meaning as the square root of Joint Acceptance. In the following equations, AJ and lJ are used interchangeably to denote a product of two defined quantities.

A general formula for the integral in (1a) contains indeterminate terms of the type 0/0, when λ=2l and when λ=0. The last one is a trival case of zero load when a corresponding AJ=0. The case λ=2l requires a different formula for J which does not contain indeterminate terms.

In order to simplify an already cumbersome explicit formula for J, the following definitions are used:

$$\alpha = \frac{a}{l}, \quad \beta = \frac{\lambda}{l}, \quad X = \frac{u_l}{l}, \quad Y = \frac{u_u}{l}$$

$$C = \frac{\pi(\beta - 2)}{\beta}, \quad D = \frac{\pi(\beta + 2)}{\beta} \tag{22}$$

Then, constraints on u_l and u_u, (20) and (21), become constraints on X and Y:

$$X = MAX\left(-\frac{\beta}{2}, \alpha - K + \frac{1}{2}\right); \; Y = MIN\left(\frac{\beta}{2}, \alpha + n - K + \frac{1}{2}\right) \tag{23}$$

where K=1,2...n

For $\beta \neq 0$ and $\beta \neq 2$

$$J = \frac{1}{l}\int_{u_l}^{u_u} \varphi_u(u)P_u(u)du =$$

$$\frac{\cos \pi\alpha}{2}\left[\frac{\sin \pi Y - \sin \pi X}{\pi} + \frac{\sin CY - \sin CX}{2C} + \frac{\sin DY - \sin DX}{2D}\right]$$

$$- \frac{\sin \pi\alpha}{2}\left[\frac{\cos \pi Y - \cos \pi X}{\pi} + \frac{\cos CY - \cos CX}{2C} + \frac{\cos DY - \cos DX}{2D}\right] \tag{24}$$

For $\beta = 2$

$$J = \frac{\cos \pi\alpha}{2\pi}\left[\sin \pi Y - \sin \pi X + \frac{\sin 2\pi Y - \sin 2\pi X}{4} + \frac{\pi(Y-X)}{2}\right]$$

$$- \frac{\sin \pi\alpha}{2\pi}\left[\cos \pi Y - \cos \pi X + \frac{\cos 2\pi Y - \cos 2\pi X}{4}\right] \tag{25}$$

(26)

For β=0, J=0

Explicit equations (24) through (26), definitions (22) and constraints (23) complete the solution of equation (1) for the case of a uniform continuous beam with equal spans.

The examples in appendix A contain calculated J-coefficients ranging from the case of a single span beam (a trivial case included mainly for a comparison) to the case of a five-span continuous beam which appears to represent the largest number of continuous spans encountered in practice. The accompanying text on the plots provides an explanation of the basic plot parameters. The user is also referred to figure 3-1, which depicts plot parameters.

SECTION IV

AN EXTENSION TO SPECIAL CASES

The solution obtained in Section III can be extended to a few special cases:

a. A simply supported beam with two equal cantilevers

b. A free-free beam

c. A single cantilever

Extension of the existing solution requires only a few simple modifications in the formulation of constraints on the integral limits and an addition of a constant term to the equation defining the fundamental mode of case structures. It should be noted that new solutions are approximate, although quite accurate, because they are based on an assumed mode shape rather than on an exact solution for the mode. While exact solutions exist, their formulation contains both trigonometric and hyperbolic functions so that the roots of characteristic equation which define resonance frequencies cannot be obtained explicitly but only by numerical methods. A check of the accuracy of an assumed mode solution may be based on a comparison between fundamental resonance frequencies obtained from the exact solution and an approximate solution. Such a check shows that for a free-free beam (a particular case of a simply supported beam with equal cantilevers) the error is approximately 1.3 percent. When cantilevers increase relative to the midspan, in the limiting case of the midspan length approaching zero (a case of two equal cantilevers with a common fixed end), the error increases to a maximum of 4.1 percent. Thus, within the entire range of the approximate solution, the error from the approximation remains within acceptable limits.

Figure 4-1 shows a new configuration and the variables defining the extension of the previous solution to special cases considered in this report. The assumed fundamental bending mode is:

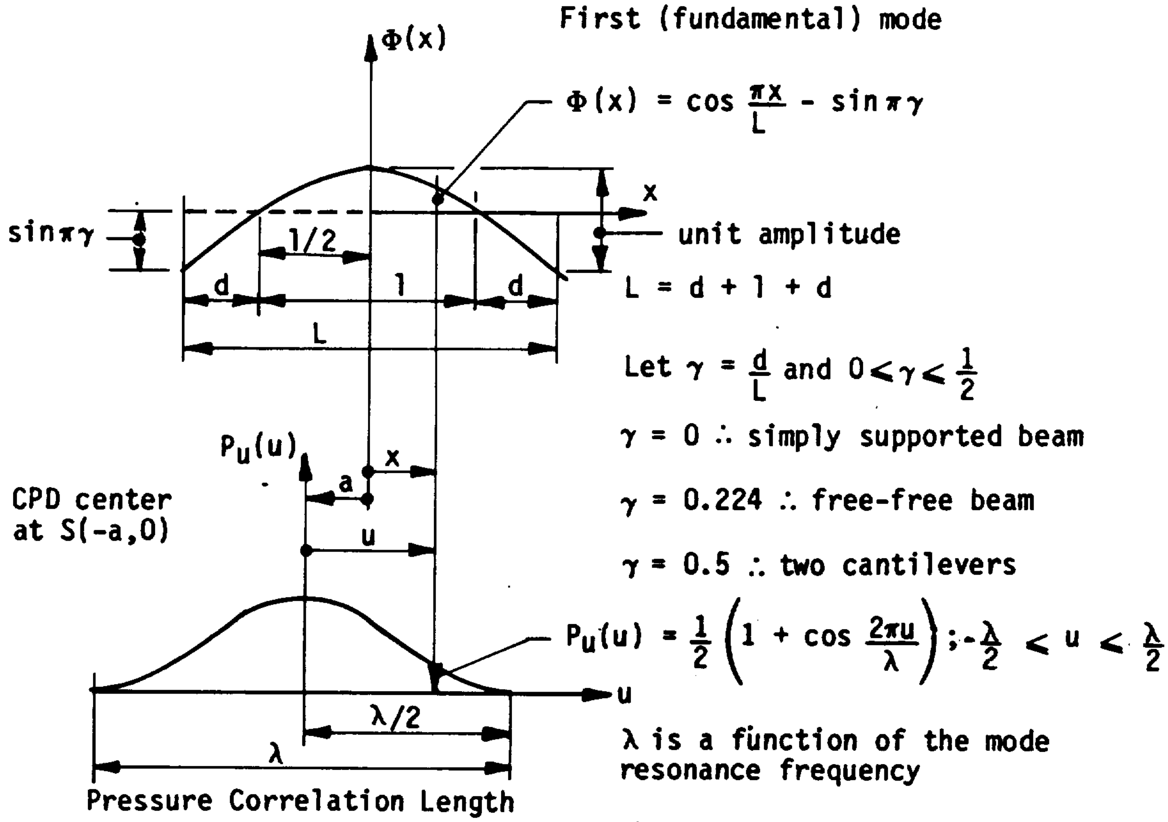
$$\Phi(x) = \cos\frac{\pi x}{L} - \sin\pi\gamma = \Phi_{SB}(x) - F \tag{27}$$

where

$$\Phi_{SB}(x) = \cos\frac{\pi x}{L}$$

is the first bending mode of a simply supported beam with the span L. The form of $\Phi_{SB}(x)$ is identical to that of $\varphi(x)$ in (13) when l is substituted by the new variable $L=l+2d$ = the total length of the beam (center span and two cantilevers). The solution for $J=J_{SB}$, when the mode is $\Phi_{SB}(x)$, is already provided by equations (22) through (26) after the substitution of l by L is made.

A simply supported beam with two equal cantilevers, beam is assumed to have a uniformly distributed mass M ($lb\text{-}sec^2/in^2$) and a constant stiffness EI ($lb\text{-}in^2$)



First mode undamped resonance frequency (by Rayleigh energy method) is:

$$f = \frac{\pi}{2L^2}\sqrt{\frac{EI}{m}}\left[1 - \frac{8}{\pi}\sin\pi\gamma + 2\sin^2\pi\gamma\right]^{-1/2} = \frac{\pi}{2L^2}\sqrt{\frac{EIL}{2M}} \quad (Hz)$$

Generalized modal mass: $M = \frac{mL}{2}\left[1 - \frac{8}{\pi}\sin\pi\gamma + 2\sin^2\pi\gamma\right]$

Define: $\alpha = \frac{a}{L}$ and $\beta = \frac{\lambda}{L}$

Figure 4-1. Configuration and Parameters for Calculation of AJ-Coefficients in a Case of a Simply Supported Beam With Two Equal Cantilevers

$F = \sin\pi\gamma$ is a constant in the case of a particular structure. The new variable, γ, defines a ratio between the cantilever length, d, and the total length, L. As noted in figure 4-1, each value of γ defines a different structure. Consequently, J-coefficients for different structures are not directly comparable.

$\Phi(x)$ in (27) represents a superposition of two modes: $\Phi_{SB}(x)$ (a bending mode) and F (a translation mode). Both modes have the same resonance frequency. Consequently, the J-coefficient for $\Phi(x)$ becomes a sum (a superposition) of two coefficients, J_{SB} and J_F, that correspond to $\Phi_{SB}(x)$ and F modes. Such a summation is valid, provided that J_F is referenced to the same A=L as J_{SB} is referenced to in equations (24) and (25):

$$J = J_{SB} - J_F \tag{28}$$

The same result as (28) can be obtained by using $\Phi(x)$ in (27) and by repeating the same procedure outlined in equations (14) through (26). The chosen route is simplier.

This paragraph provides an explicit definition for the generalized modal mass, M, for the mode $\Phi(x)$. The component mode $\Phi_{SB}(x)$ is normalized to the unit maximum displacement at x=0. The amplitude of the second component mode, F, is a function of parameter γ. Unlike the case of a continuous beam with equal spans where M is equal to one-half the total mass, the generalized modal mass and the resonance frequency of a cantilevered beam become functions of the parameter γ:

$$M = \int_{-L/2}^{L/2} \Phi 2(x) m dx = \frac{mL}{2}\left[1 - \frac{8}{\pi}\sin\pi\gamma + 2\sin^2\pi\gamma\right] \tag{29}$$

If an "equivalent" uniformly distrubuted mass

$$M_{eq} = \frac{2M}{L}$$

is introduced, then the fundamental resonance frequency of a cantilevered beam attains the same form as the fundamental resonance of a simply supported beam:

$$f = \frac{\pi}{2L^2}\sqrt{\frac{EI}{M_{eq}}} = \frac{\pi}{2L^2}\sqrt{\frac{EIL}{2M}} \tag{30}$$

New parameters in (22) are now defined in terms of L:

$$\alpha = \frac{a}{L}, \quad \beta = \frac{\lambda}{L}, \quad X = \frac{u_l}{L}, \quad Y = \frac{u_u}{L} \tag{22a}$$

The form of parameters C and D in (22) remains unchanged. The constraints on X and Y become:

$$X = \mathrm{MAX}\left(-\frac{\beta}{2}, \alpha - \frac{1}{2}\right), \quad Y = \mathrm{MIN}\left(\frac{\beta}{2}, \alpha + \frac{1}{2}\right) \tag{23a}$$

In terms of the parameters of (22a) and constraints of (23a), equations (24) through (26) now define J_{SB}. The equation for the remaining term J_F in (28) is:

$$J_F = \frac{F}{L}\int_{u_1}^{u_u} P_u(u)du = \sin\pi\gamma\left[\frac{Y-X}{2} + \frac{\beta}{4\pi}\left(\sin\frac{2\pi Y}{\beta} - \sin\frac{2\pi X}{\beta}\right)\right] \tag{31}$$

Equations (28), (22a), (23a), (24) through (26), and (31) complete the explicit solution for J-coefficients of a cantilevered beam and a free-free (γ=0.224) beam. A comparison between J-coefficients for cantilevered beams with the same total length L is not meaningful because the values of corresponding M's in (29) and f's in (30) are variable functions of the parameter γ.

The solution for a single cantilever with the span d, for example the left cantilever in figure 4-1, is obtained from the proceeding solution by setting γ=1/2 and by constraining the Y-extent of the CPD load to:

$$Y = \mathrm{MIN}\left(\frac{\beta}{2}, \alpha\right) \tag{23b}$$

The corresponding generalized modal mass and the fundamental resonance frequency are (approximate error 4.1 percent in f) from (29):

$$M = \frac{md}{2}\left[3 - \frac{8}{\pi}\right] \approx 0.227md$$

The exact solution yields M=0.25md (error ≈ 9.3 percent) from (30):

$$f = \frac{0.583}{d^2}\sqrt{\frac{EI}{M}}$$

(The exact coefficient is 0.56 instead of 0.583.) Then, using (28) and L=2d:

$$AJ = LJ = L(J_{SB} - J_F) = 2d(J_{SB} - J_F) = dJ_C \tag{32}$$

$$\text{and } J_C = 2(J_{SB} - J_F) \tag{28a}$$

Here, J_C is the J-coefficient for a single cantilever, referenced to the cantilever span d. All parameters in (22a) entering computations of J_{SB} and J_F) remain defined by L=2d. Thus, horizontal plot axis, β, defines a ratio between the pressure correlation length and two cantilever lengths. These special cases are shown in appendix B.

SECTION V

SUMMARY AND RECOMMENDATIONS

The existing dynamic analysis codes do not have the capability to utilize recently developed methods involving response spectra and the distributions of correlated pressures in the analysis of structural responses to random acoustic loads. The implementation of such a capability within an existing code would require a substantial modification of the code that may be far beyond what the code structure allows to be accomplished by a user. Perhaps, only the originator of the code may have the capability to implement such modifications.

A different route should be considered. This route would use either an existing FEM code or a modal analysis test to establish the modal parameters of a structure. The capability to have a dual source of modal parameters is essential for a test/analysis correlation and for a dynamic analysis of structures where only the test can establish modal parameters. A separate code should be developed for the analysis of structural response to random acoustic loads occurring during a launch. The code would input a structure geometry, modal parameters, pressure correlation lengths, and response spectra. The input of geometry and modal parameters should be made by the transfer, via a magnetic tape or other media, from the modal analysis code to the new response analysis code. For a general case of a three-dimensional structure, a corresponding three-dimensional definition of the contours of correlated pressure distributions over the structure surface must be obtained from the ellipsoid of pressure correlation lengths. The original grid of nodal points, which was generated by the requirements of modal analysis, may have to be either replaced or appended by a high-density grid where the density is governed by the nonuniformity of the correlated pressure distribution and by the requirement to have a reasonably small delta-A associated with each grid point to ensure the accuracy of a numerical integration. Modal vectors will also require a transformation from the modal analysis grid to the high-density pressure distribution grid. Then, the estimate of AJ extremes and peak response modal coordinates can be performed in the high-density grid coordinates. Computed modal coordinates would be either input back into the modal analysis code for the calculation of peak responses (stresses, displacements, etc.) or the same task may be accomplished within the response analysis code, in which case additional inputs, such as matrices of internal stresses due to normal modes, must be obtained from the modal analysis code.

In the above outline, the optimization of the load position leading to an absolute extreme of AJ-coefficients and modal coordinates may never become entirely automated, and the problem is not expected to be solved for a general case. Because each mode requires a different load position for the extreme response in that mode, the initial positioning of the CPD center may have to be left to the judgement of the user, followed by some limited programmed optimization. A graphic display of the mode and an interactive input of the initial load position would certainly facilitate the choice of input.

In this scheme, a separate response analysis code provides a definite advantage before a general modal analysis code. However, the communication problems between the two codes involving data transfers, stops and restarts, are not expected to be resolved without the involvement of personnel who originated and presently maintain the general modal analysis code.

A development of the capability to perform a response analysis for a general case of a structural configuration would require substantial effort and time. Perhaps, it should be accomplished in steps of a gradually increasing complexity by limiting the initial development to only certain types of structures, such as planar structures, etc.

Meanwhile, in order to satisfy the immediate needs of design groups, designers must resort to approximations, such as breaking down a structure into simpler components for which adequate solutions either already exist or can be obtained by means of simple programs and diagrams like those presented in this report.

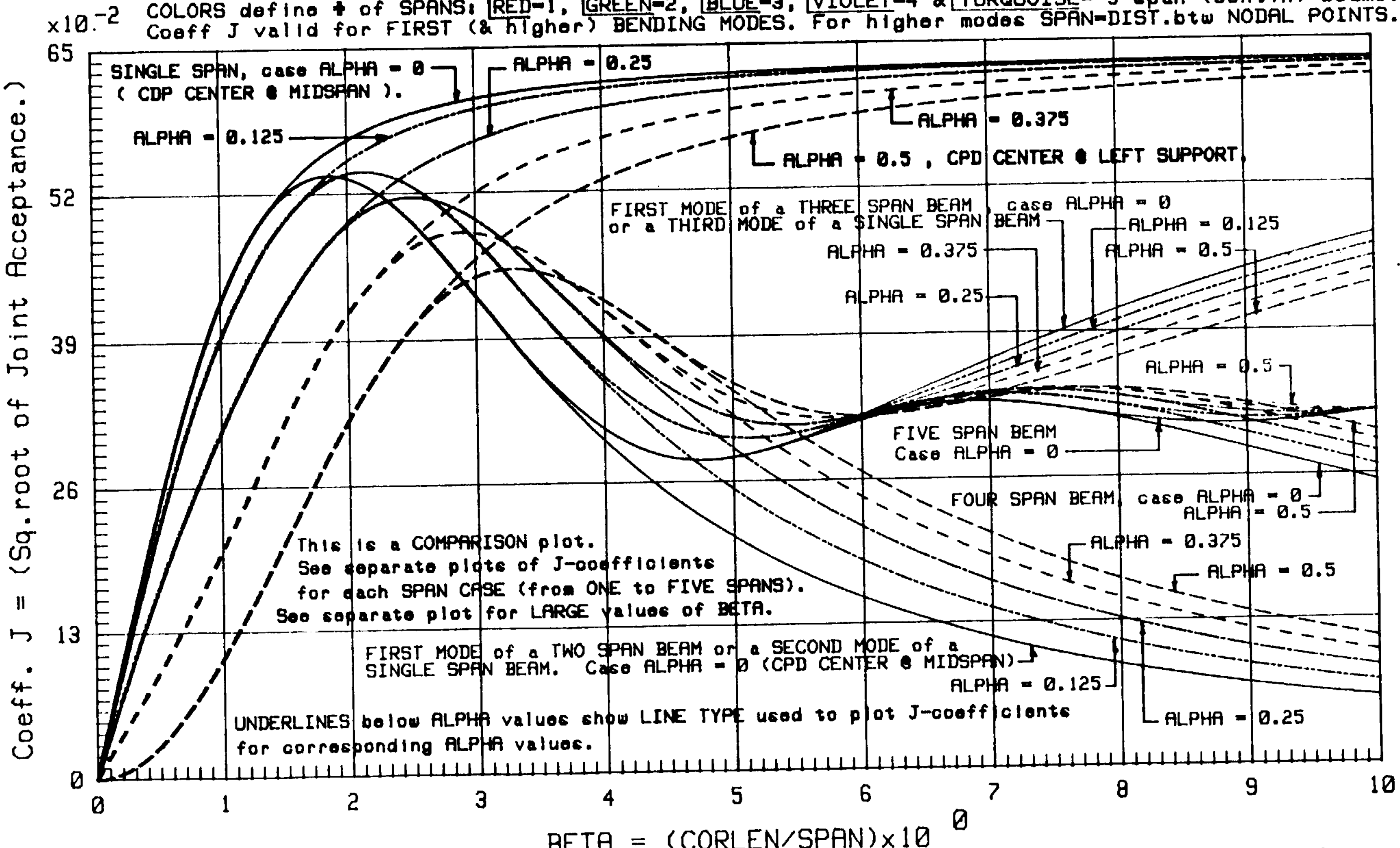
APPENDIX A

J-COEFFICIENTS FOR ONE, TWO, THREE, FOUR, AND FIVE SPAN BEAMS

Coefficients J for ONE, TWO, THREE, FOUR & FIVE span beams. CPD CENTER in span # 1.

ALPHA: A=0 A=.125 A=.25 A=.375 A=.5 position of CPD CENTER within loaded span.

Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly, of SPAN, & A=Lx*Ly.
Param. ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN.
ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT of LEFT-most (loaded) SPAN # 1.
COLORS define # of SPANS: RED=1, GREEN=2, BLUE=3, VIOLET=4 & TURQUOISE= 5 span (contin) beams.
Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST.btw NODAL POINTS.

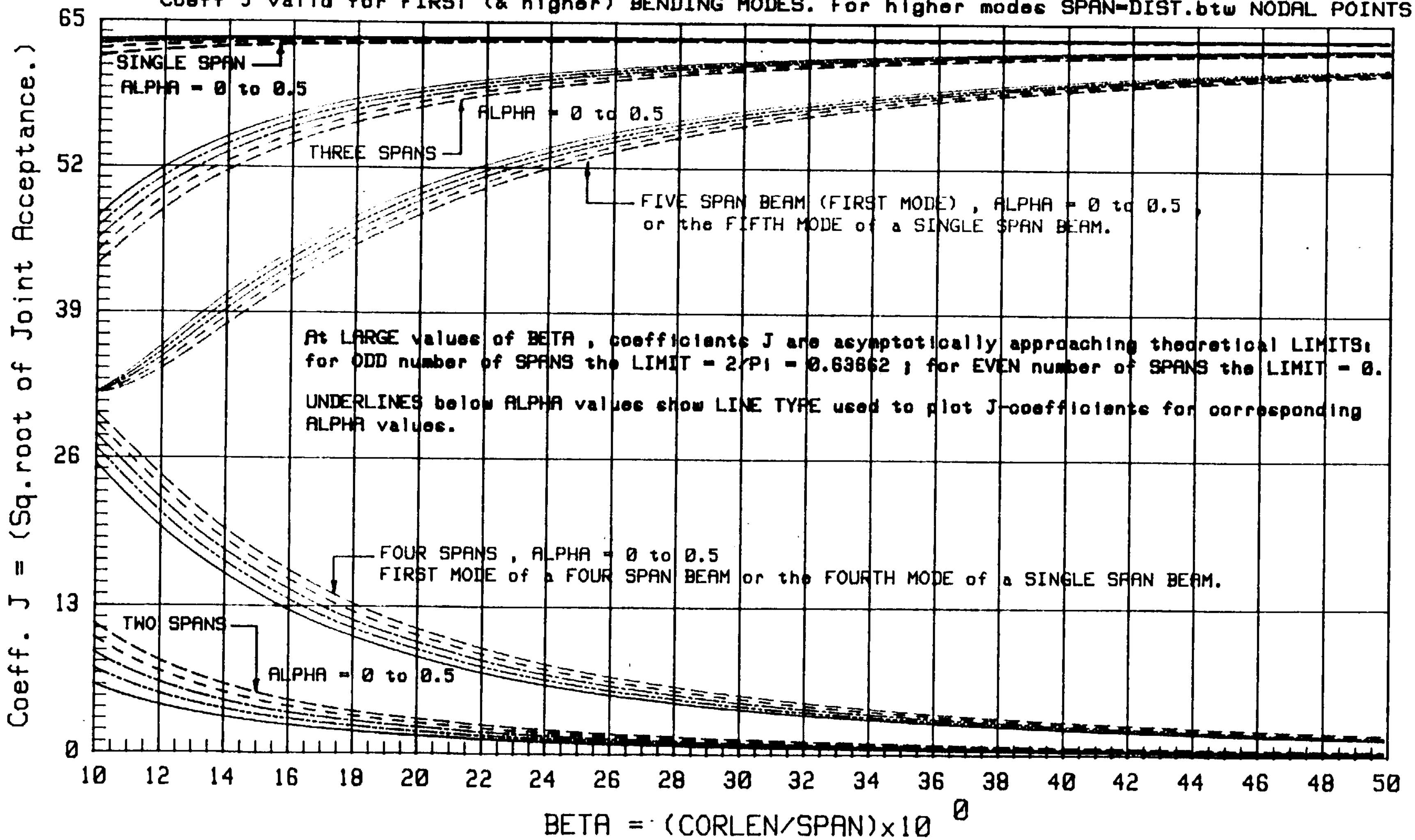


Horizontal Axis, BETA, defines a RATIO between PRESSURE CORRELATION LENGTH and either the BEAM SPAN (case of the FIRST MODE) or the DISTANCE between NODAL (zero modal disp) POINTS for higher modes.

* aouload. NASA KSC DM-MED-33 *

Coefficients J for ONE, TWO, THREE, FOUR & FIVE span beams. CPD CENTER in span # 1.
ALPHA: A=0 A=.125 A=.25 A=.375 A=.5 position of CPD CENTER within loaded span.

Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly, of SPAN, & A=Lx*Ly.
Param. ALPHA defines POSITION of the CENTER of CORRELATED PRESS. DISTRIBUTION (CPD) within LOADED SPAN.
ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT of LEFT-most (loaded) SPAN # 1.
COLORS define # of SPANS: RED=1, GREEN=2, BLUE=3, VIOLET=4 & TURQUOISE= 5 span (contin) beams.
Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST. btw NODAL POINTS.



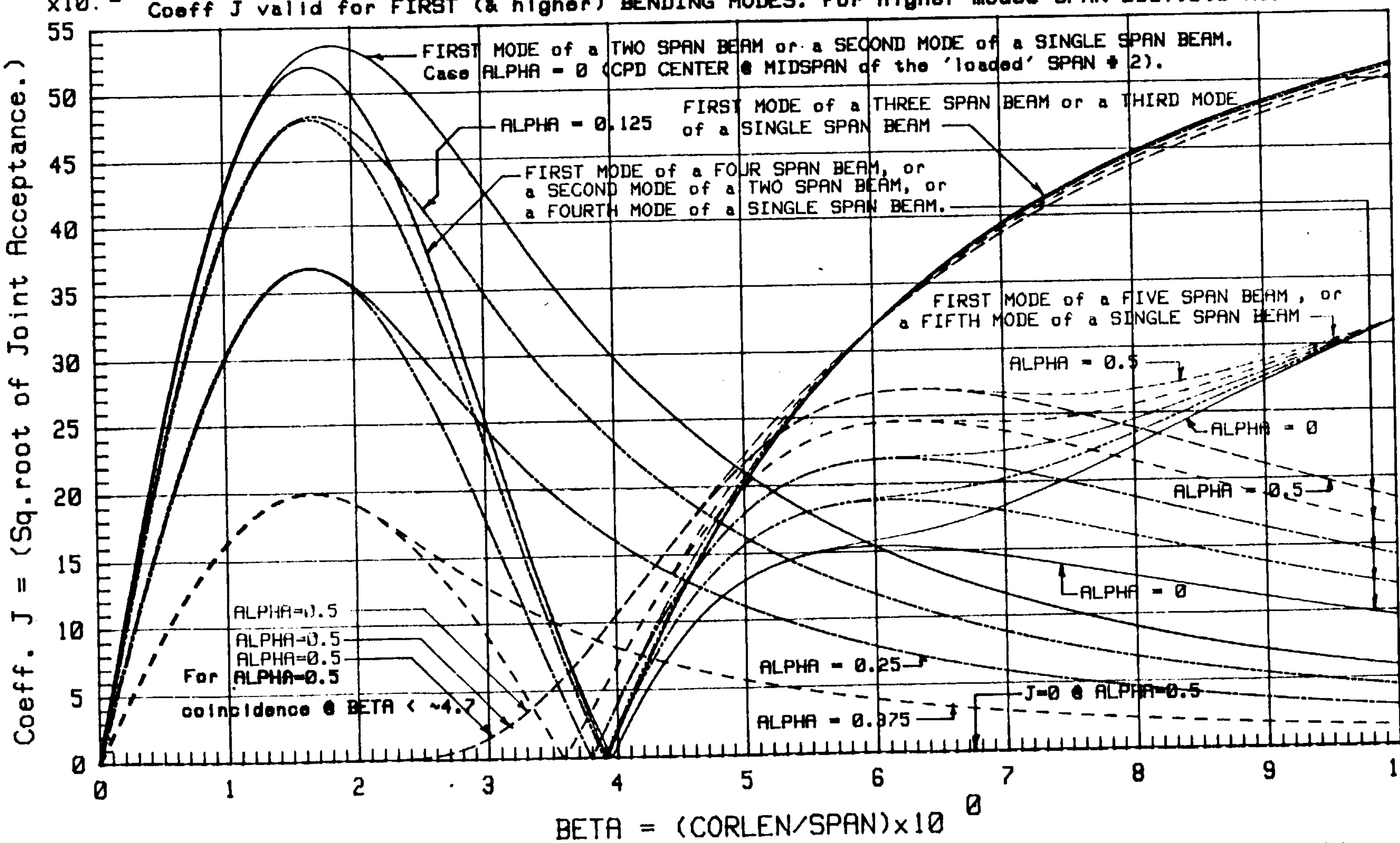
Horizontal Axis, BETA, defines a RATIO between PRESSURE CORRELATION LENGTH and either the BEAM SPAN (case of the FIRST MODE) or the DISTANCE between NODAL (zero modal displ) POINTS for higher modes.

* aouload. NASA KSC DM-MED-33 *

Coefficients J for TWO, THREE, FOUR and FIVE SPAN BEAMS. CPD CENTER in span # 2.

ALPHA: A=0 A=.125 A=.25 A=.375 A=.5 position of CPD CENTER within loaded span.

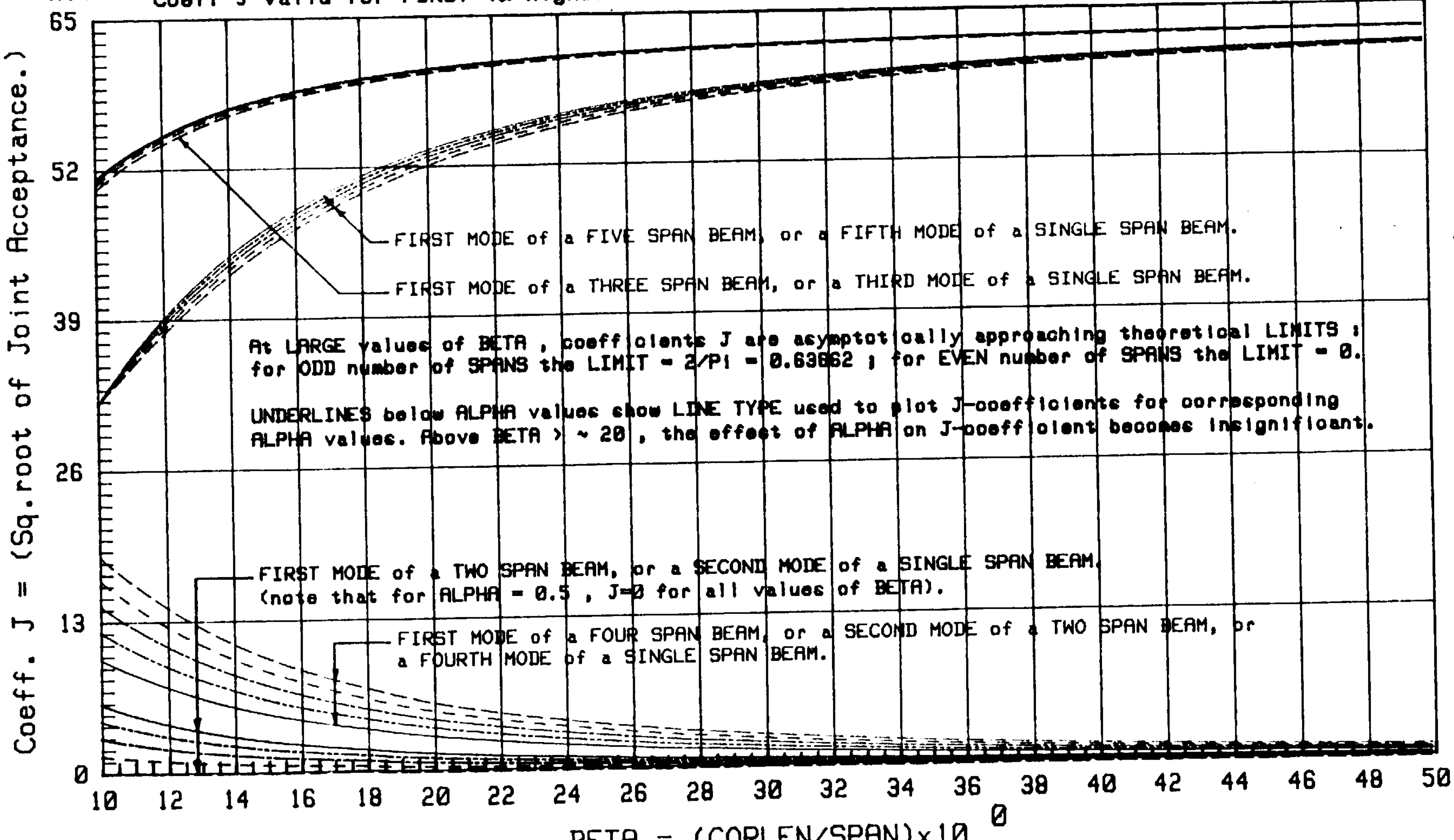
Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly ,of SPAN ,& A=Lx*Ly.
Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN.
ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA =-0.5 for CPD @ RIGHT SUPPORT.
COLORS define # of SPANS : GREEN=2, BLUE=3, VIOLET=4 & TURQUOISE= 5 span (contin) beams.
Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST.btw NODAL POINTS.



Horizontal Axis , BETA , defines a RATIO between PRESSURE CORRELATION LENGTH and either the BEAM SPAN (case of the FIRST MODE) or the DISTANCE between NODAL (zero modal disp) POINTS for higher modes.

* aouload. NASA KSC DM-MED-aa *

Coefficients J for TWO, THREE, FOUR and FIVE SPAN BEAMS. CPD CENTER in span # 2.
ALPHA: A=0 A=.125 A=.25 A=.375 A=.5 position of CPD CENTER within loaded span.
Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly ,of SPAN ,& A=Lx*Ly.
Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN.
ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA =-0.5 for CPD @ RIGHT SUPPORT.
COLORS define # of SPANS : GREEN=2, BLUE=3, VIOLET=4 & TURQUOISE= 5 span (contin) beams.
Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN-DIST.btw NODAL POINTS.



Horizontal Axis , BETA , defines a RATIO between PRESSURE CORRELATION LENGTH and either the BEAM SPAN
(case of the FIRST MODE) or the DISTANCE between NODAL (zero modal disp) POINTS for higher modes.

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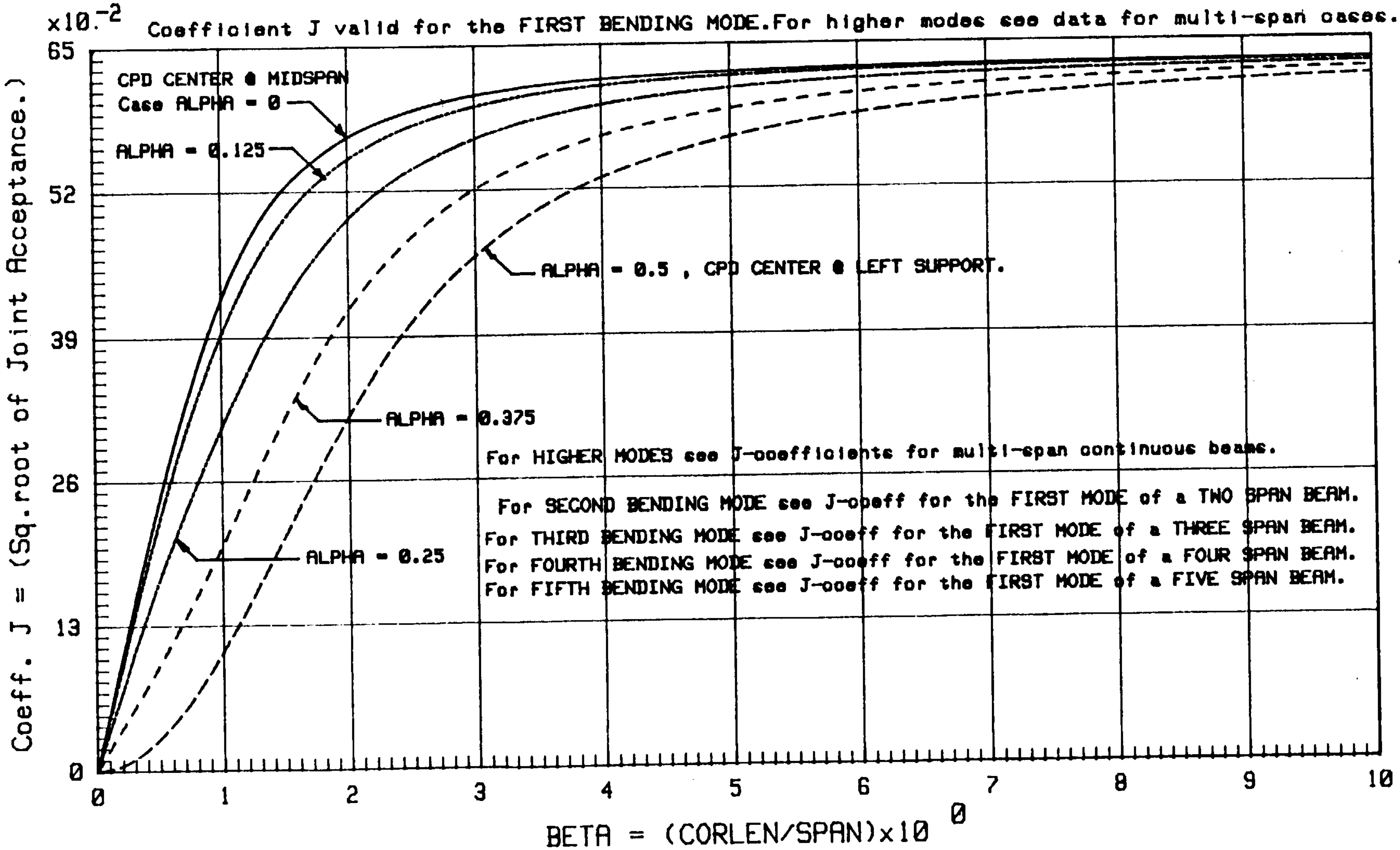
Coefficients J for a SINGLE SPAN SIMPLY SUPPORTED BEAM.   CPD CENTER within SPAN.

ALPHA: A=0 A=.125 A=.25 A=.375 A=.5 position of CPD CENTER within loaded span.

Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly, of SPAN, & A=Lx*Ly.
Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN.
ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA = -0.5 for CPD @ RIGHT SUPPORT.

Coefficient J valid for the FIRST BENDING MODE. For higher modes see data for multi-span cases.



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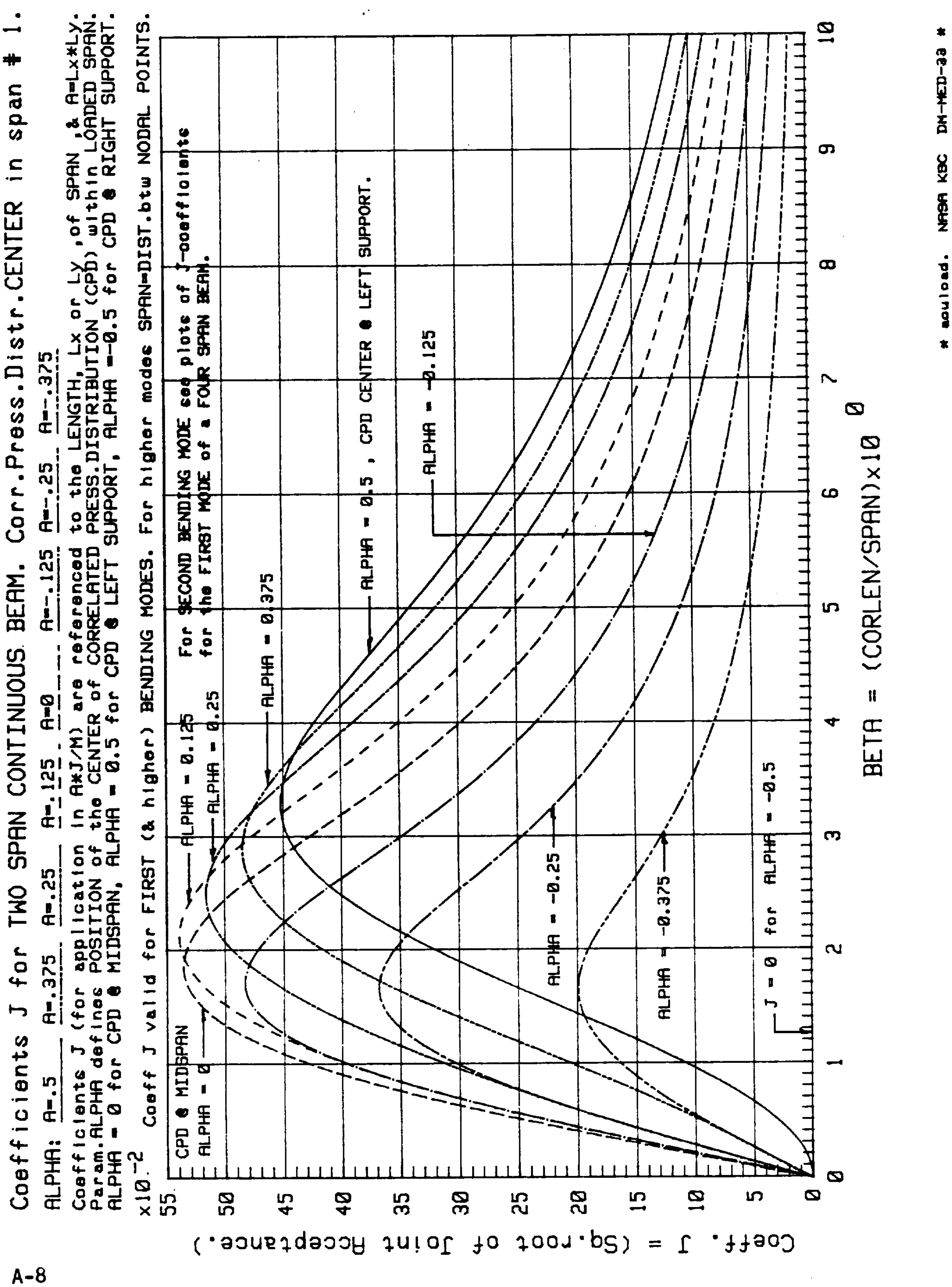
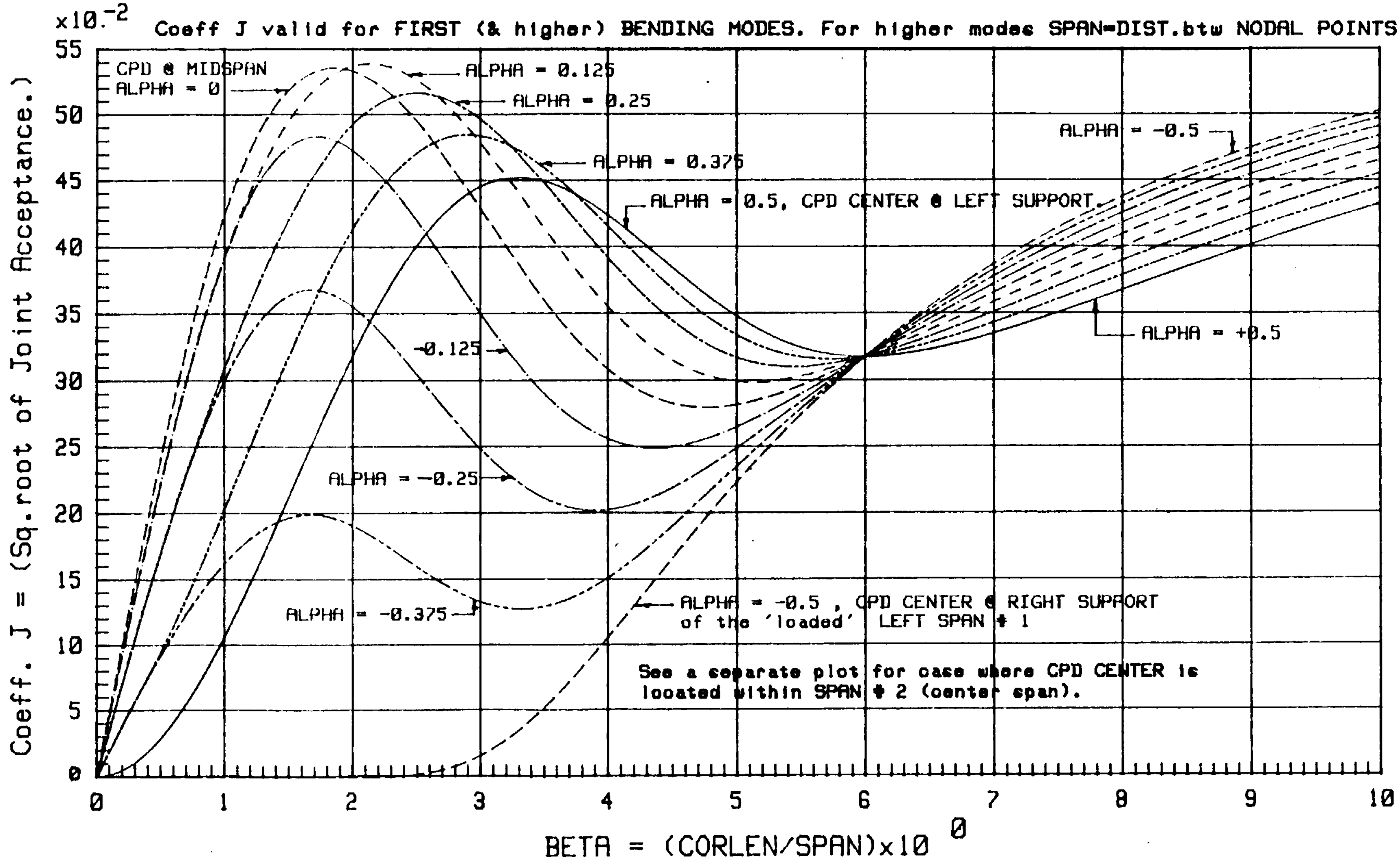

Coefficients J for TWO SPAN CONTINUOUS BEAM. Corr.Press.Distr.CENTER in span # 1.
ALPHA: A=.5 A=.375 A=.25 A=.125 A=0 A=-.125 A=-.25 A=-.375
Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly, of SPAN, & A=Lx*Ly.
Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN.
ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA =-0.5 for CPD @ RIGHT SUPPORT.
x10^-2
Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST.btw NODAL POINTS.
For SECOND BENDING MODE see plots of J-coefficients for the FIRST MODE of a FOUR SPAN BEAM.
CPD @ MIDSPAN
ALPHA = 0
ALPHA = 0.125
ALPHA = 0.25
ALPHA = 0.375
ALPHA = 0.5, CPD CENTER @ LEFT SUPPORT.
ALPHA = -0.125
ALPHA = -0.25
ALPHA = -0.375
J = 0 for ALPHA = -0.5
Coeff. J = (Sq.root of Joint Acceptance.)
BETA = (CORLEN/SPAN)x10
0 1 2 3 4 5 6 7 8 9 10
55 50 45 40 35 30 25 20 15 10 5 0

Coefficients J for THREE SPAN CONTINUOUS BEAM. Corr.Press.Dis.CENTER in span # 1.

ALPHA: A=.5 A=.375 A=.25 A=.125 A=0 A=-.125 A=-.25 A=-.375 A=-.5

Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly ,of SPAN ,& A=Lx*Ly. Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN. ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA =-0.5 for CPD @ RIGHT SUPPORT.

Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST.btw NODAL POINTS.



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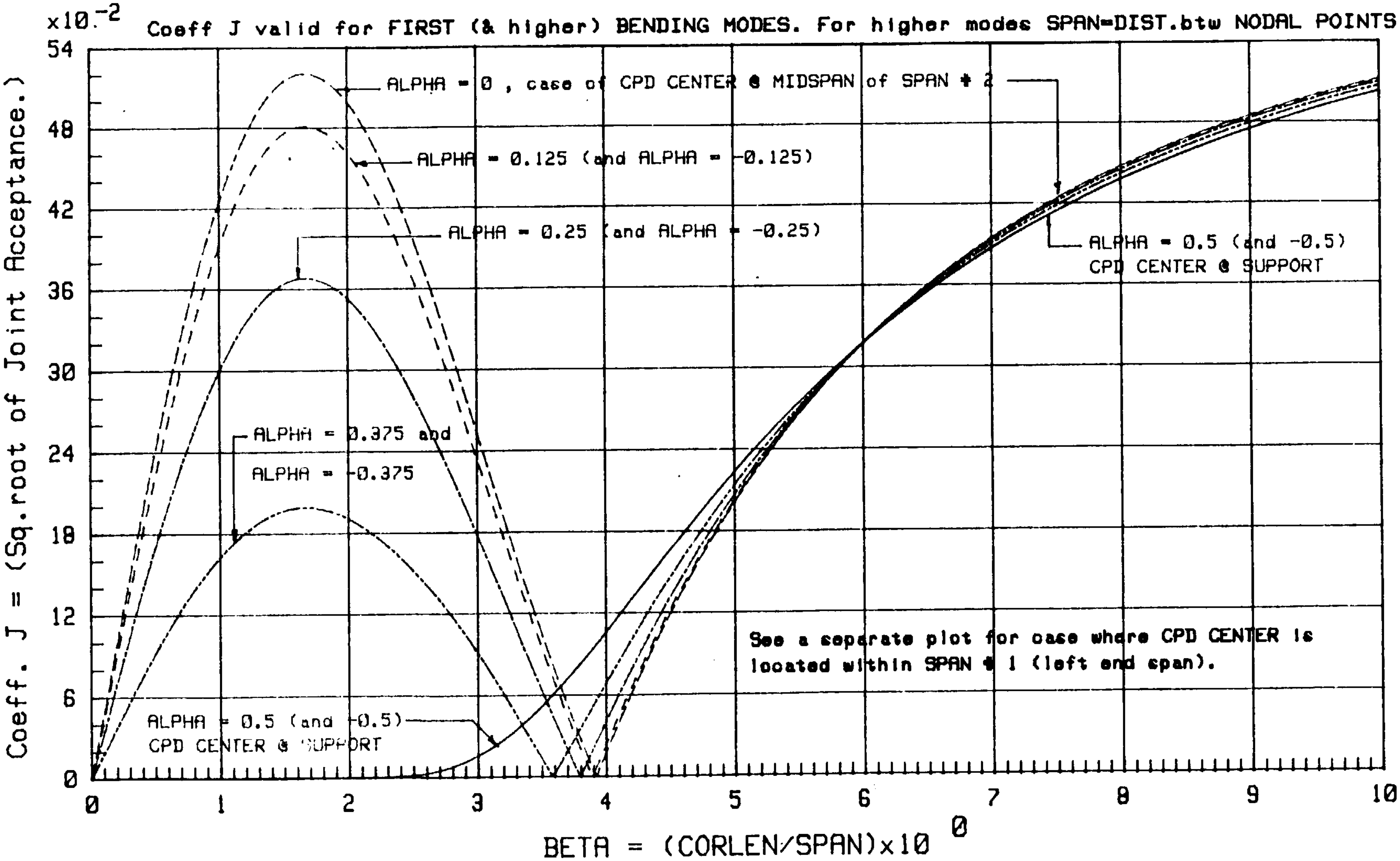
Coefficients J for THREE SPAN CONTINUOUS BEAM. Corr.Press.Dis.CENTER in span # 2.
ALPHA: A=.5 A=.375 A=.25 A=.125 A=0 position of CPD CENTER within loaded span.
Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly ,of SPAN ,& A=Lx*Ly.
Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN.
ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA =-0.5 for CPD @ RIGHT SUPPORT.
Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST.btw NODAL POINTS.



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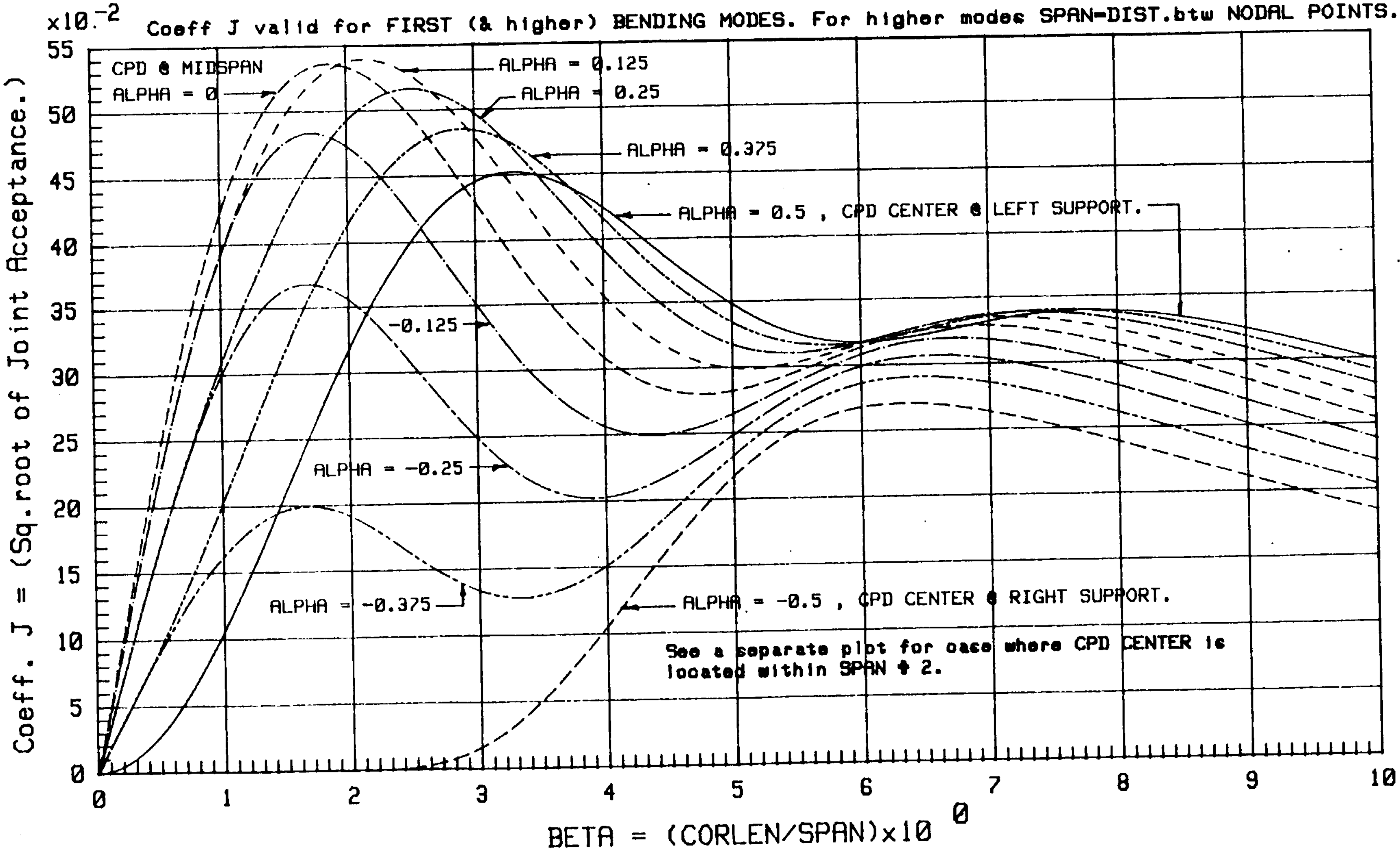
Coefficients J for FOUR SPAN CONTINUOUS BEAM. Corr.Press.Dist.CENTER in span # 1.

ALPHA: A=.5 A=.375 A=.25 A=.125 A=0 A=-.125 A=-.25 A=-.375 A=-.5

Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly ,of SPAN ,& A=Lx*Ly.
Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN.
ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA =-0.5 for CPD @ RIGHT SUPPORT.

Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST.btw NODAL POINTS.



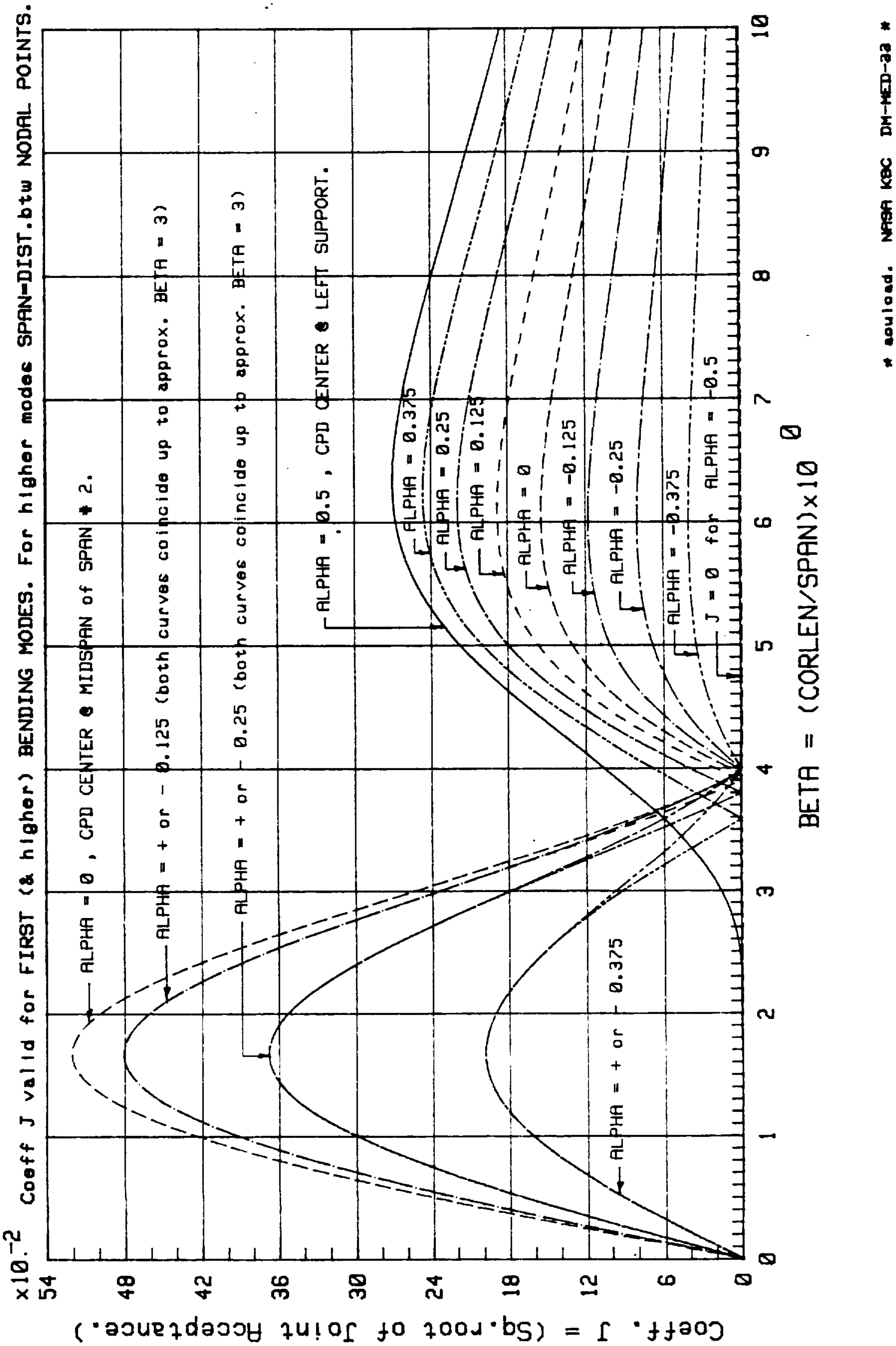
* aouload. NASA KSC DM-MED-33 *

Coefficients J for FOUR SPAN CONTINUOUS BEAM. Corr.Press.Dist.CENTER in span # 2.

ALPHA: A=.5 A=.375 A=.25 A=.125 A=0 A=-.125 A=-.25 A=-.375 A=-.5

Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly, of SPAN, & A=Lx*Ly. Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN. ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA = -0.5 for CPD @ RIGHT SUPPORT.

Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST.btw NODAL POINTS.

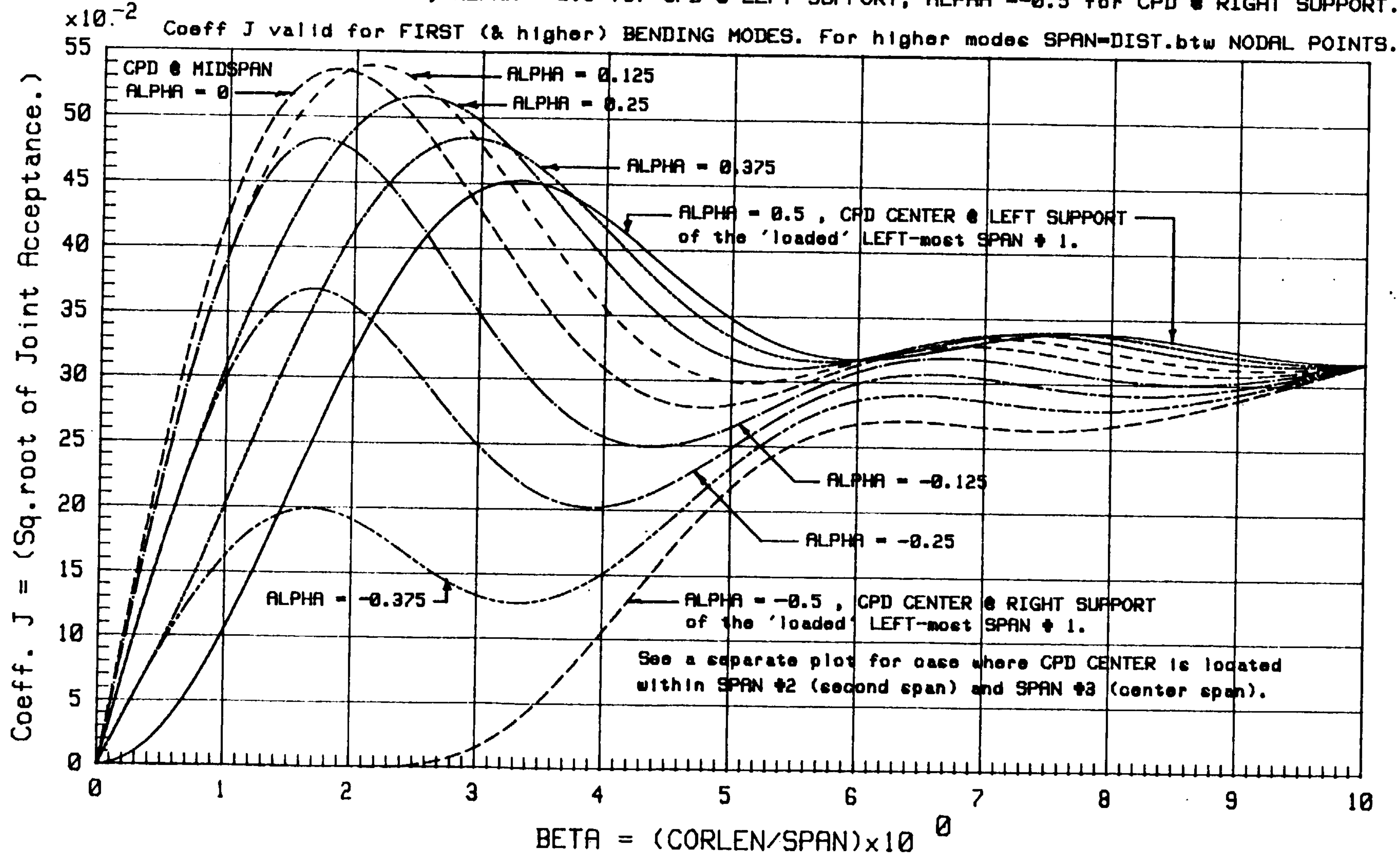


Coefficients J for FIVE SPAN CONTINUOUS BEAM. Corr.Press.Dist.CENTER in span # 1.

ALPHA: A=.5 A=.375 A=.25 A=.125 A=0 A=-.125 A=-.25 A=-.375 A=-.5

Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly ,of SPAN ,& A=Lx*Ly. Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN. ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA =-0.5 for CPD @ RIGHT SUPPORT.

Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST.btw NODAL POINTS.



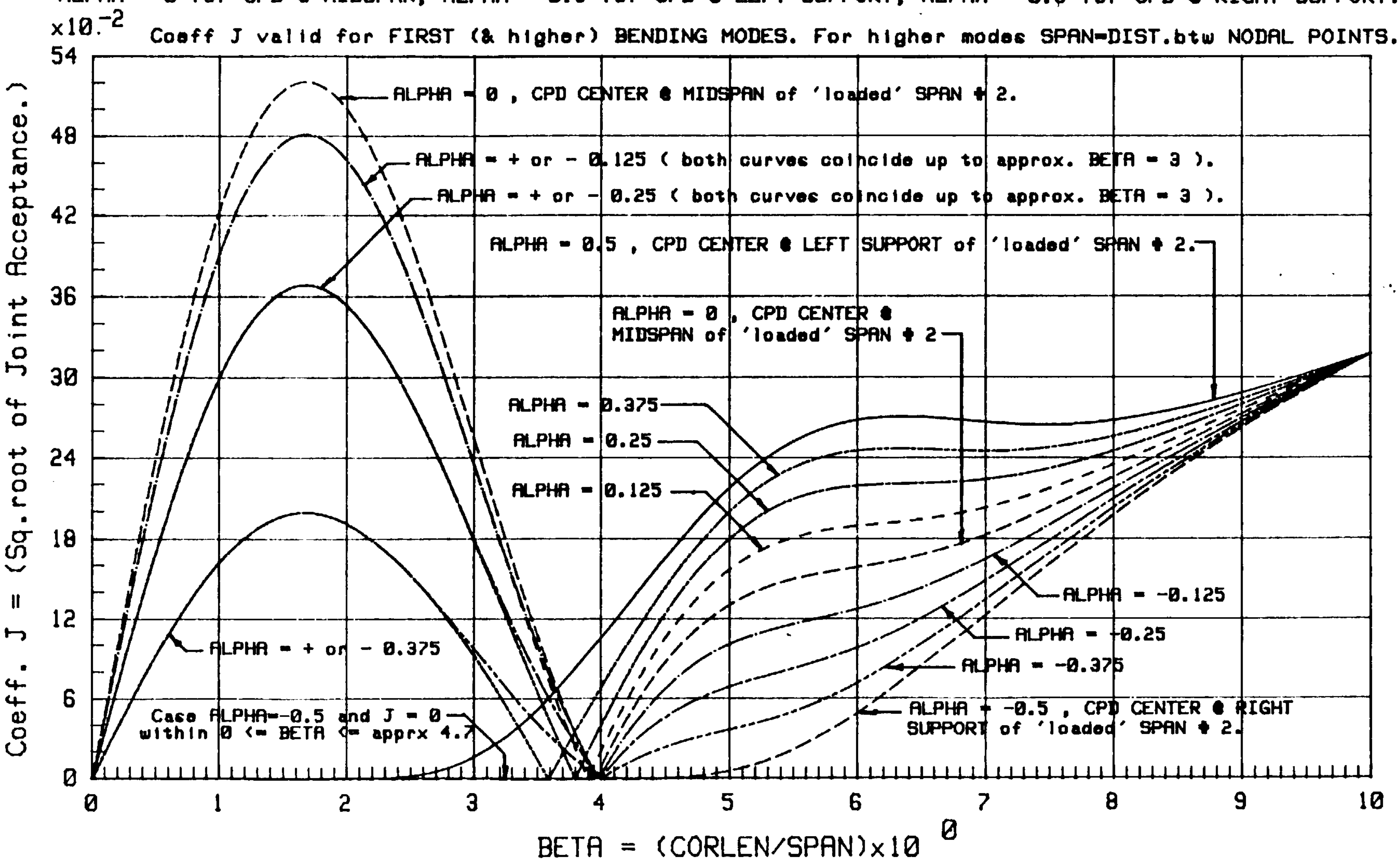
* aouload. NASA KSC DM-MED-32 *

Coefficients J for FIVE SPAN CONTINUOUS BEAM. Corr.Press.Dist.CENTER in span # 2.

ALPHA: A=.5 A=.375 A=.25 A=.125 A=0 A=-.125 A=-.25 A=-.375 A=-.5

Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly ,of SPAN ,& A=Lx*Ly.
Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN.
ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA =-0.5 for CPD @ RIGHT SUPPORT.

Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST.btw NODAL POINTS.



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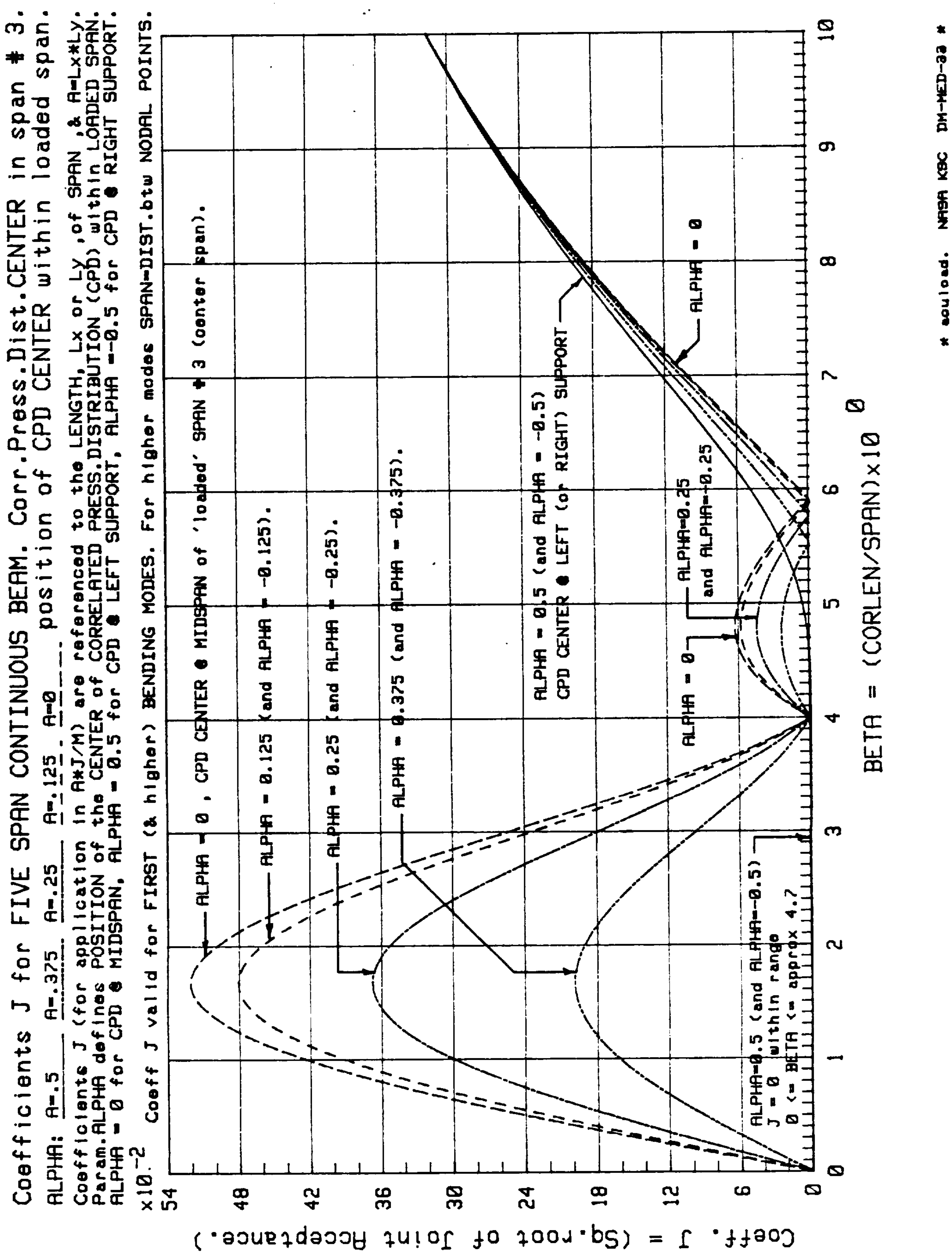


Coefficients J for FIVE SPAN CONTINUOUS BEAM. Corr.Press.Dist.CENTER in span # 3.
ALPHA: A=.5 A=.375 A=.25 A=.125 A=0 position of CPD CENTER within loaded span.
Coefficients J (for application in A*J/M) are referenced to the LENGTH, Lx or Ly ,of SPAN ,& A=Lx*Ly.
Param.ALPHA defines POSITION of the CENTER of CORRELATED PRESS.DISTRIBUTION (CPD) within LOADED SPAN.
ALPHA = 0 for CPD @ MIDSPAN, ALPHA = 0.5 for CPD @ LEFT SUPPORT, ALPHA =-0.5 for CPD @ RIGHT SUPPORT.
Coeff J valid for FIRST (& higher) BENDING MODES. For higher modes SPAN=DIST.btw NODAL POINTS.
x10^-2
ALPHA = 0 , CPD CENTER @ MIDSPAN of 'loaded' SPAN # 3 (center span).
ALPHA = 0.125 (and ALPHA = -0.125).
ALPHA = 0.25 (and ALPHA = -0.25).
ALPHA = 0.375 (and ALPHA = -0.375).
ALPHA = 0.5 (and ALPHA = -0.5)
CPD CENTER @ LEFT (or RIGHT) SUPPORT
ALPHA = 0
ALPHA=0.25
and ALPHA=-0.25
ALPHA = 0
ALPHA=0.5 (and ALPHA=-0.5)
J = 0 within range
0 <= BETA <= approx 4.7
Coeff. J = (Sq.root of Joint Acceptance.)
BETA = (CORLEN/SPAN)x10^0
0 6 12 18 24 30 36 42 48 54
0 1 2 3 4 5 6 7 8 9 10
* aouload. NASA KSC DM-MED-33 *

APPENDIX B

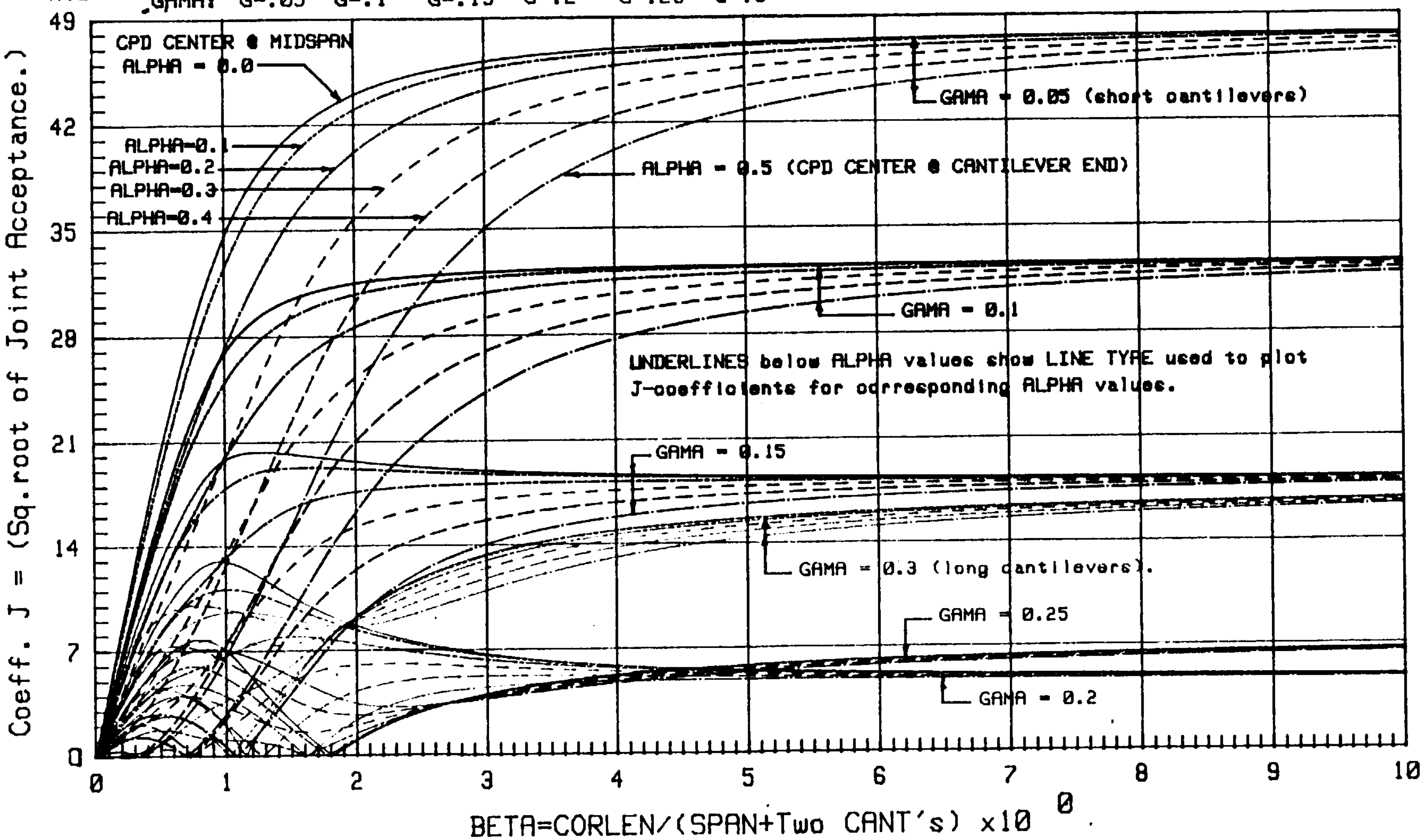
J-COEFFICIENTS FOR A CANTILEVERED BEAM, A FREE-FREE BEAM, AND A SINGLE CANTILEVER

Coeff. J for the FIRST MODE of a SIMPLY SUPPORTED beam with TWO EQUAL CANTILEVERS.

ALPHA: A=0 A=.1 A=.2 A=.3 A=.4 A=.5 posit.of CPD CENTER ref.to midspan.

Coefficients J (for A*J/M) are referenced to the LENGTH Lx=d+l+d, l=center span, d=cantilever length. ALPHA defines POSITION of CPD CENTER: ALPHA=0 for CENTER @ midspan, ALPHA=0.5 for CENTER @ cantil.end. GAMA = d/Lx. To each value of GAMA corresponds a series of six (color) J-coefficients for each ALPHA. Gen.Mod.MASS (approx): M=(Total Mass)*(0.5-1.27*SIN(PI*GAMA)+(SIN(PI*GAMA))^2). A=Lx*Ly (Ly=1).

GAMA: G=.05 G=.1 G=.15 G=.2 G=.25 G=.3



Horizontal axis, BETA, defines a RATIO between PRESSURE CORRELATION LENGTH and the TOTAL LENGTH Lx.

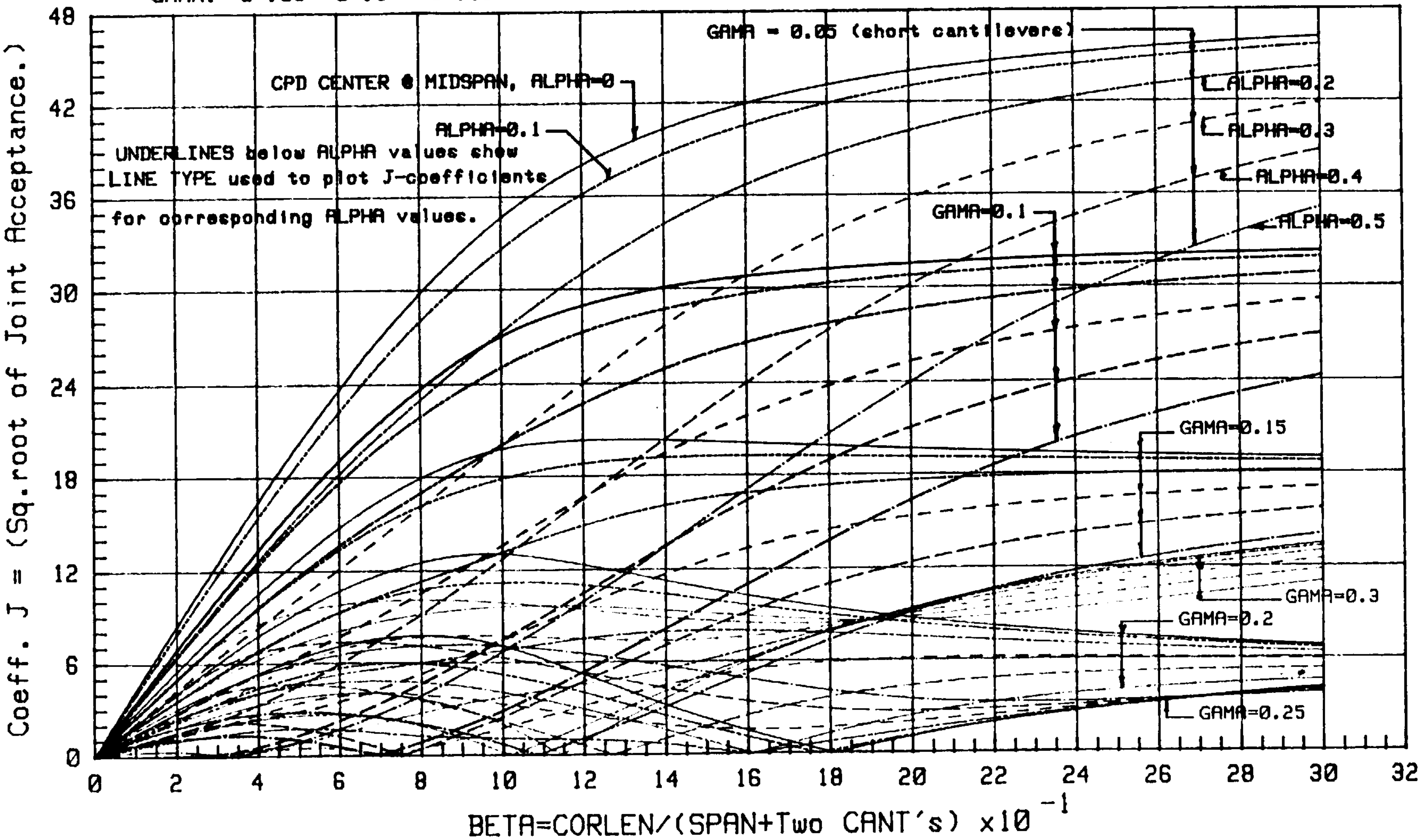
* aouload. NASA KSC DM-MED-32 *

Coeff. J for the FIRST MODE of a SIMPLY SUPPORTED beam with TWO EQUAL CANTILEVERS.

ALPHA: A=0 A=.1 A=.2 A=.3 A=.4 A=.5 posit.of CPD CENTER ref.to midspan.

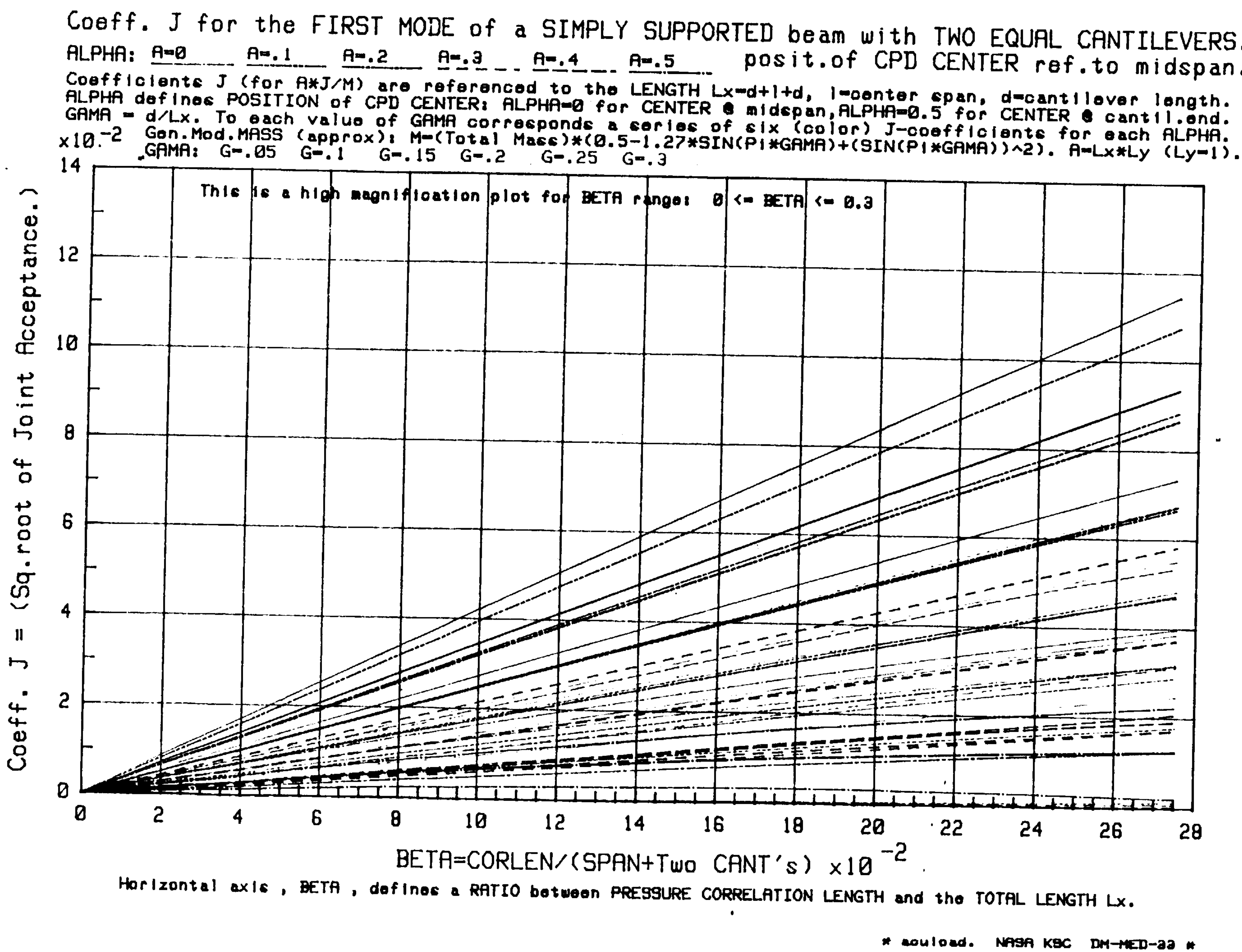
Coefficients J (for A*J/M) are referenced to the LENGTH Lx=d+l+d, l=center span, d=cantilever length. ALPHA defines POSITION of CPD CENTER: ALPHA=0 for CENTER @ midspan,ALPHA=0.5 for CENTER @ cantil.end. GAMA = d/Lx. To each value of GAMA corresponds a series of six (color) J-coefficients for each ALPHA. Gen.Mod.MASS (approx): M=(Total Mass)*(0.5-1.27*SIN(PI*GAMA)+(SIN(PI*GAMA))^2). A=Lx*Ly (Ly=1).

GAMA: G=.05 G=.1 G=.15 G=.2 G=.25 G=.3



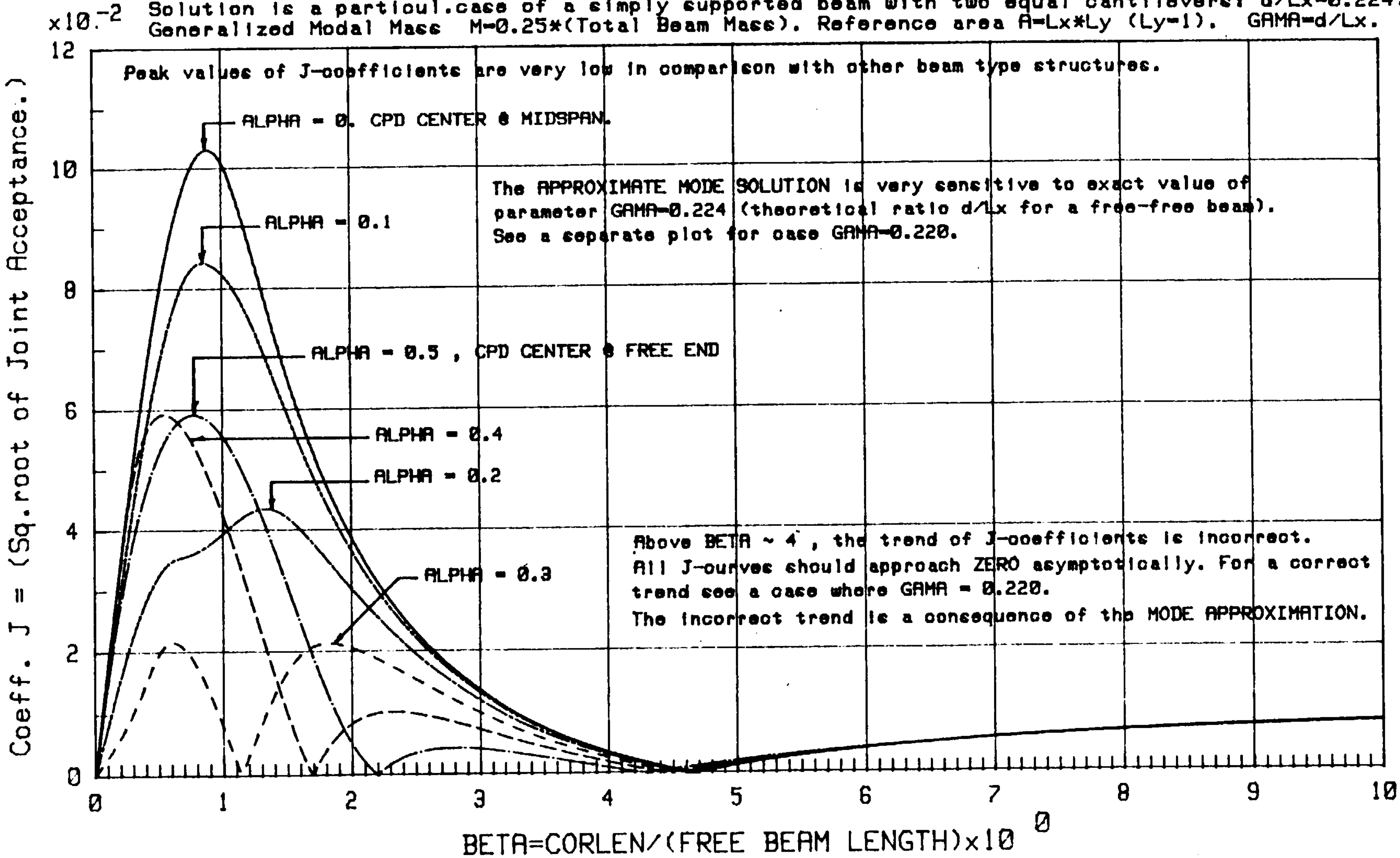
Horizontal axis , BETA , defines a RATIO between PRESSURE CORRELATION LENGTH and the TOTAL LENGTH Lx.

* aouload. NASA KSC DM-MED-33 *


Coeff. J for the FIRST MODE of a SIMPLY SUPPORTED beam with TWO EQUAL CANTILEVERS.
ALPHA: A=0 A=.1 A=.2 A=.3 A=.4 A=.5 posit.of CPD CENTER ref.to midspan.
Coefficients J (for A*J/M) are referenced to the LENGTH Lx=d+l+d, l=center span, d=cantilever length.
ALPHA defines POSITION of CPD CENTER: ALPHA=0 for CENTER @ midspan,ALPHA=0.5 for CENTER @ cantil.end.
GAMA = d/Lx. To each value of GAMA corresponds a series of six (color) J-coefficients for each ALPHA.
Gen.Mod.MASS (approx): M=(Total Mass)*(0.5-1.27*SIN(Pi*GAMA)+(SIN(Pi*GAMA))^2). A=Lx*Ly (Ly=1).
GAMA: G=.05 G=.1 G=.15 G=.2 G=.25 G=.3
This is a high magnification plot for BETA range: 0 <= BETA <= 0.3
x10^-2
Coeff. J = (Sq.root of Joint Acceptance.)
0
2
4
6
8
10
12
14
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
BETA=CORLEN/(SPAN+Two CANT's) x10^-2
Horizontal axis , BETA , defines a RATIO between PRESSURE CORRELATION LENGTH and the TOTAL LENGTH Lx.
* acuload. NASA KBC DM-MED-33 *

Coefficients J for the FIRST BENDING MODE of a uniform FREE-FREE BEAM. GAMA=0.224.
ALPHA: A=0 A=.1 A=.2 A=.3 A=.4 A=.5 posit.of CPD CENTER ref.to midspan.

Coefficients J (for A*J/M) are referenced to the FREE-FREE LENGTH : Lx=d+l+d, l=0.552*Lx, d=0.224*Lx.
ALPHA defines POSITION of CPD CENTER: ALPHA=0 for CENTER @ midspan, ALPHA=0.5 for CENTER @ a free end.
Lx=total beam lengths. l=distance between zero nodal points. d=dist. from zero nodal point to free end.
Solution is a particul. case of a simply supported beam with two equal cantilevers: d/Lx=0.224.
Generalized Modal Mass M=0.25*(Total Beam Mass). Reference area A=Lx*Ly (Ly=1). GAMA=d/Lx.



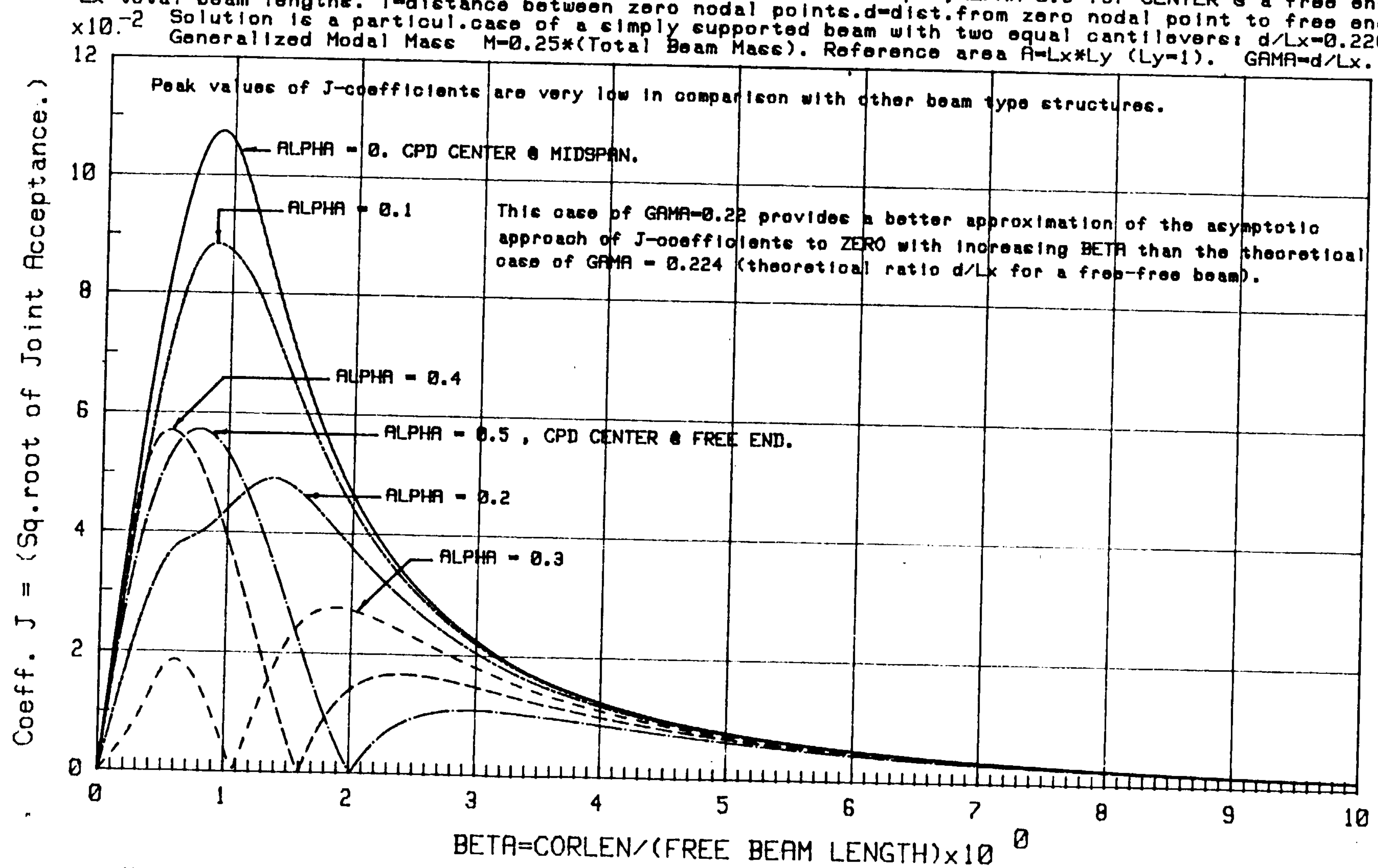
Horizontal axis , BETA , defines a RATIO between PRESSURE CORRELATION LENGTH and the TOTAL BEAM LENGTH.

* aouload. NASA KSC DM-MED-33 *

Coefficients J for the FIRST BENDING MODE of a uniform FREE-FREE BEAM. GAMA=0.220.

ALPHA: A=0 A=.1 A=.2 A=.3 A=.4 A=.5 posit.of CPD CENTER ref.to midspan.

Coefficients J (for A*J/M) are referenced to the FREE-FREE LENGTH : Lx=d+l+d, l=0.560*Lx, d=0.220*Lx.
ALPHA defines POSITION of CPD CENTER: ALPHA=0 for CENTER @ midspan,ALPHA=0.5 for CENTER @ a free end.
Lx=total beam lengths. l=distance between zero nodal points.d=dist.from zero nodal point to free end.
Solution is a particul.case of a simply supported beam with two equal cantilevers: d/Lx=0.220.
Generalized Modal Mass M=0.25*(Total Beam Mass). Reference area A=Lx*Ly (Ly=1). GAMA=d/Lx.

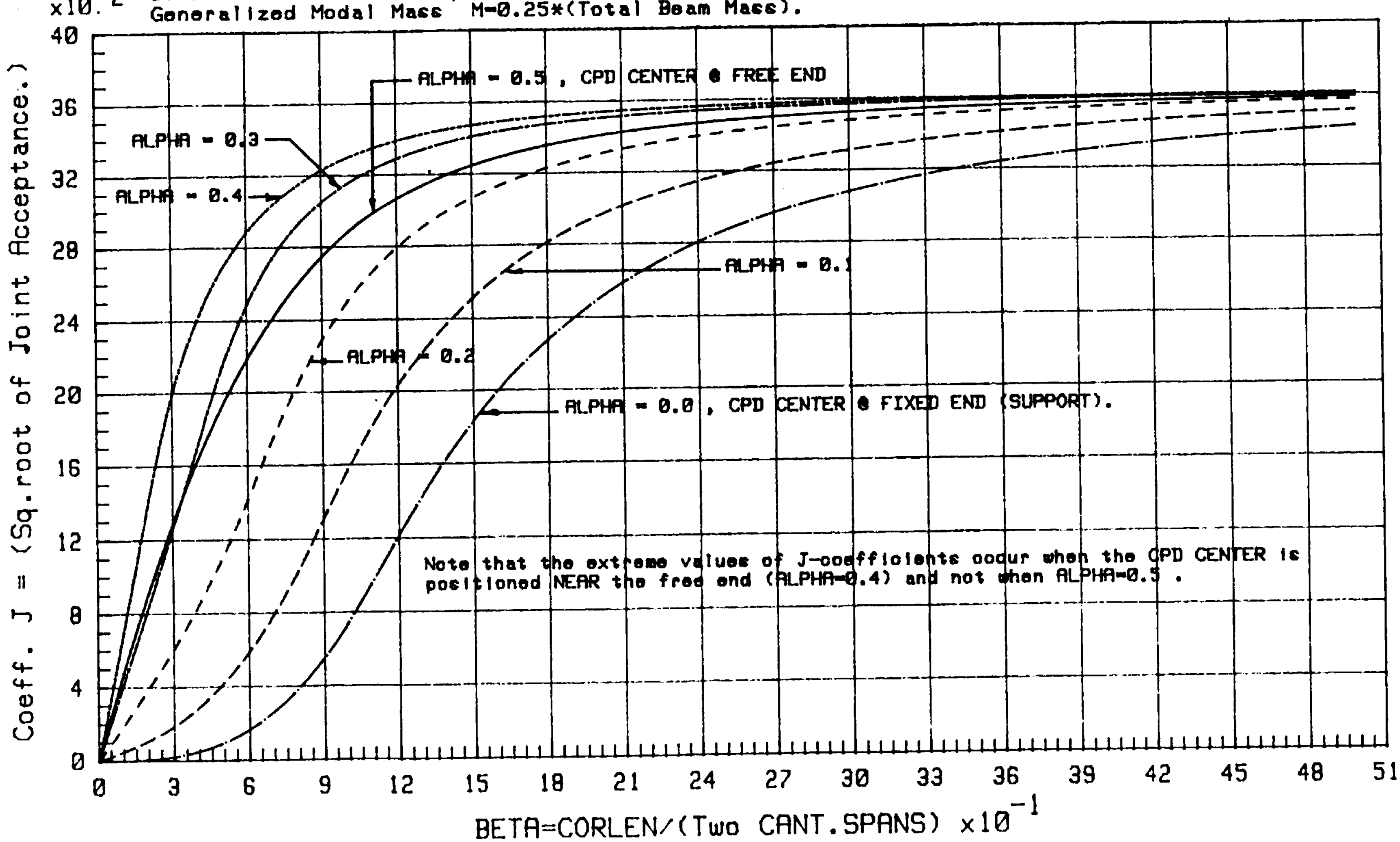


Horizontal axis , BETA , defines a RATIO between PRESSURE CORRELATION LENGTH and the TOTAL BEAM LENGTH.

* aouload. NASA KSC DM-MED-aa *

Coefficients J for the FIRST BENDING MODE of a unif. CANTILEVER (FIXED-FREE) BEAM.
ALPHA: A=.5 A=.4 A=.3 A=.2 A=.1 A=0 posit.of CPD CENTER ref.to fix.end.

Coefficients J (for A*J/M) are referenced to the CANTILEVER LENGTH , Lx , so that A=Lx*Ly (Ly=1).
ALPHA defines POSITION of CPD CENTER: ALPHA=0 for CENTER @ fixed end,ALPHA=0.5 for CENT.@ a free end.
Solution is a particular case of a simply supported beam with two equal cantilevers when the length of one cantilever is equal to one-half of the total beam length (parameter GAMA=0.5).
Generalized Modal Mass M=0.25*(Total Beam Mass).



Horizontal axis , BETA , defines a RATIO between PRESSURE CORRELATION LENGTH and TWO CANTILEVER LENGTHS.

* aouload. NASA KSC DM-MED-33 *

APPENDIX C

EXAMPLE PROBLEM FOR A THREE-SPAN CONTINUOUS BEAM

This appendix contains numerical examples illustrating the application of the presented theory to a computation of response of a three-span continuous beam.

In the presented examples, the response spectra, pressure correlation lengths, and modal parameters are assumed to be known. Under this assumption, the response computations are shown. Most of the text is devoted to step-by-step explanations of the employed procedure.

The example of a three-span continuous beam is shown in figure C-1.

Beam parameters are:

Number of spans	$n = 3$
Span length	$l = 58.0$ in (4.833 ft)
Beam width	$b = 7.2$ in
Uniform mass	$m = 1.6814 \times 10^{-3}$ lb-sec^2/in^2
Material modulus of elasticity	$E = 29.0 \times 10^6$ lb/in^2
Moment of inertia	$I = 0.2393$ in^4
Section modulus	$S = 0.282$ in^3

C.1 FIRST SINUSODIAL MODE

The resonance frequency (undamped) is:

$$f_1 = \frac{\pi}{2l^2} \sqrt{\frac{EI}{m}} = 30 \text{ Hz}$$

$$\omega_1 = 2\pi f_1 = 188.5 \text{ rad/sec}$$

$$\omega_1^2 = 3.553 \times 10^4 \text{ (rad/sec)}^2$$

The generalized modal mass (mode normalized to the maximum unit displacement at the centerline span) is:

$$M = \frac{1}{2}\, nml = 0.1463 \text{ lb-sec}^2\text{/in}$$

The area exposed to acoustic pressures (for application in the diagrams of J-coefficients) is:

$$A = bl = 417.6 \text{ in}^2$$

The response spectrum ordinate for an assumed 2-percent damping (see figure C-2) at f_1=30 Hz is:

$$Y = q/(AJ/M/\omega_1^2) = 6.5$$

This is a maximum value from 12 launches.

Note that the median value of Y is around 4. The assumed value, Y=6.5, is the maximum design value.

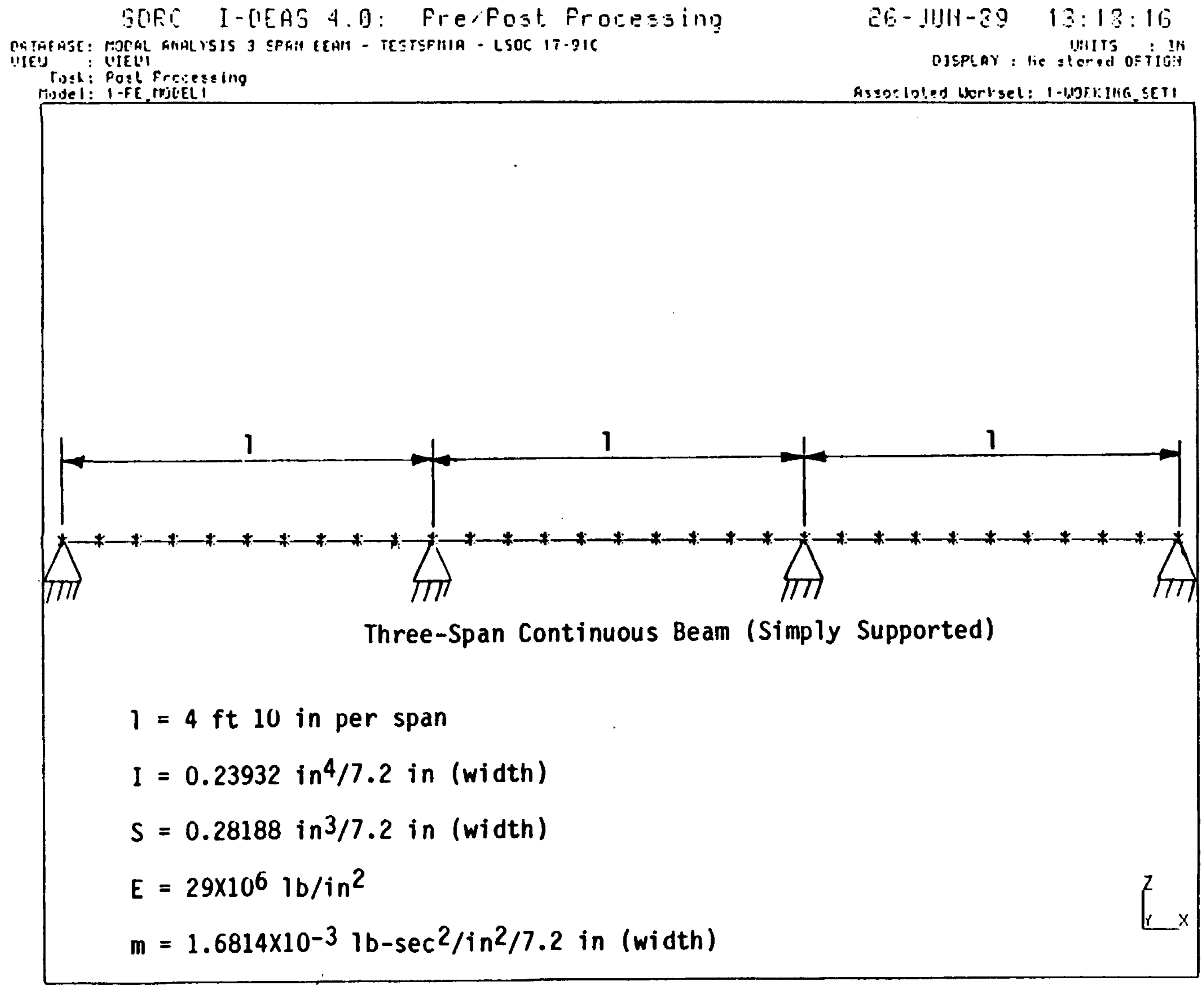


Figure C-1. Test Model Data

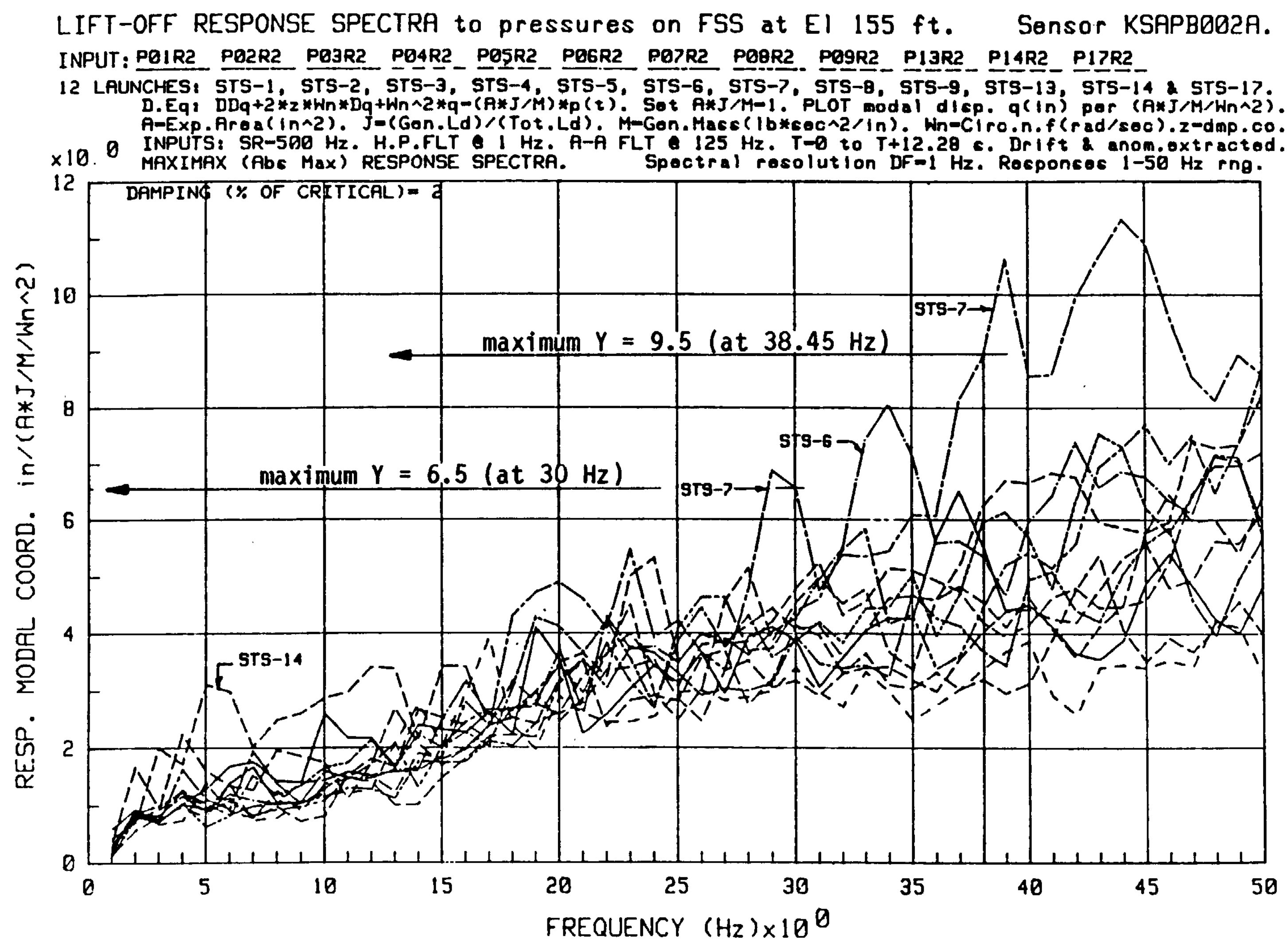


Figure C-2. Lift-Off Response Spectra to Pressure on the Fixed Service Structure (FSS) at 155-Foot Elevation (Sensor KSAPB002A)

The pressure correlation length, λ, depends on the direction of the beam (see figure C-3), whether the beam is vertical or horizontal.

C.1.1 CASE OF A VERTICAL BEAM.

At f_1 = 30 Hz, $\lambda_v \approx 19$ ft

$$\beta_v = \frac{\lambda_v}{l} = 3.93$$

From the diagrams of J-coefficients for a three-span continuous beam, maximum J_v at β_v occurs when the CPD center is locted in span no. 1:

maximum J_v = 0.43 (see figure C-4)

Note that if the CPD center were located in span no. 2, the error in the response estimate would exceed a factor of 4 (see figure C-5).

$$AJ_v/M/\omega_1^2 = 417.6 \times 0.43/0.1463/3.553 \times 10^4 = 3.455 \times 10^{-2}$$

The response modal coordinate (modal participation factor) is:

$$q_{1v} = Y\left[AJ_v/M/\omega_1^2\right] = 6.5 \times 3.455 \times 10^{-2} = 0.2245$$

Because the mode was normalized to the maximum unit displacement, q_{1v}=0.2245 inches is the maximum vibration amplitude at the centerline span.

C.1.2 CASE OF A HORIZONTAL BEAM.

At f_1 = 30 Hz, λ_h = 75 ft

$$\beta_h = \frac{\lambda_h}{l} = 15.5$$

Because of a large β_h, the location of the CPD center yielding a maximum J_h is not critical, whether it is in span no. 1 or no. 2.

For the CPD center in span no. 1, J_h = 0.563 (see figure C-6).

For the CPD center in span no. 2, J_h = 0.58 (maximum) (see figure C-7).

$$AJ_h/M/\omega_1^2 = 417.6 \times 0.58/0.1463/3.553 \times 10^4 = 4.66 \times 10^{-2}$$

$$q_{1h} = Y\left[AJ_h/M/\omega_1^2\right] = 6.5 \times 4.66 \times 10^{-2} = 0.3029$$

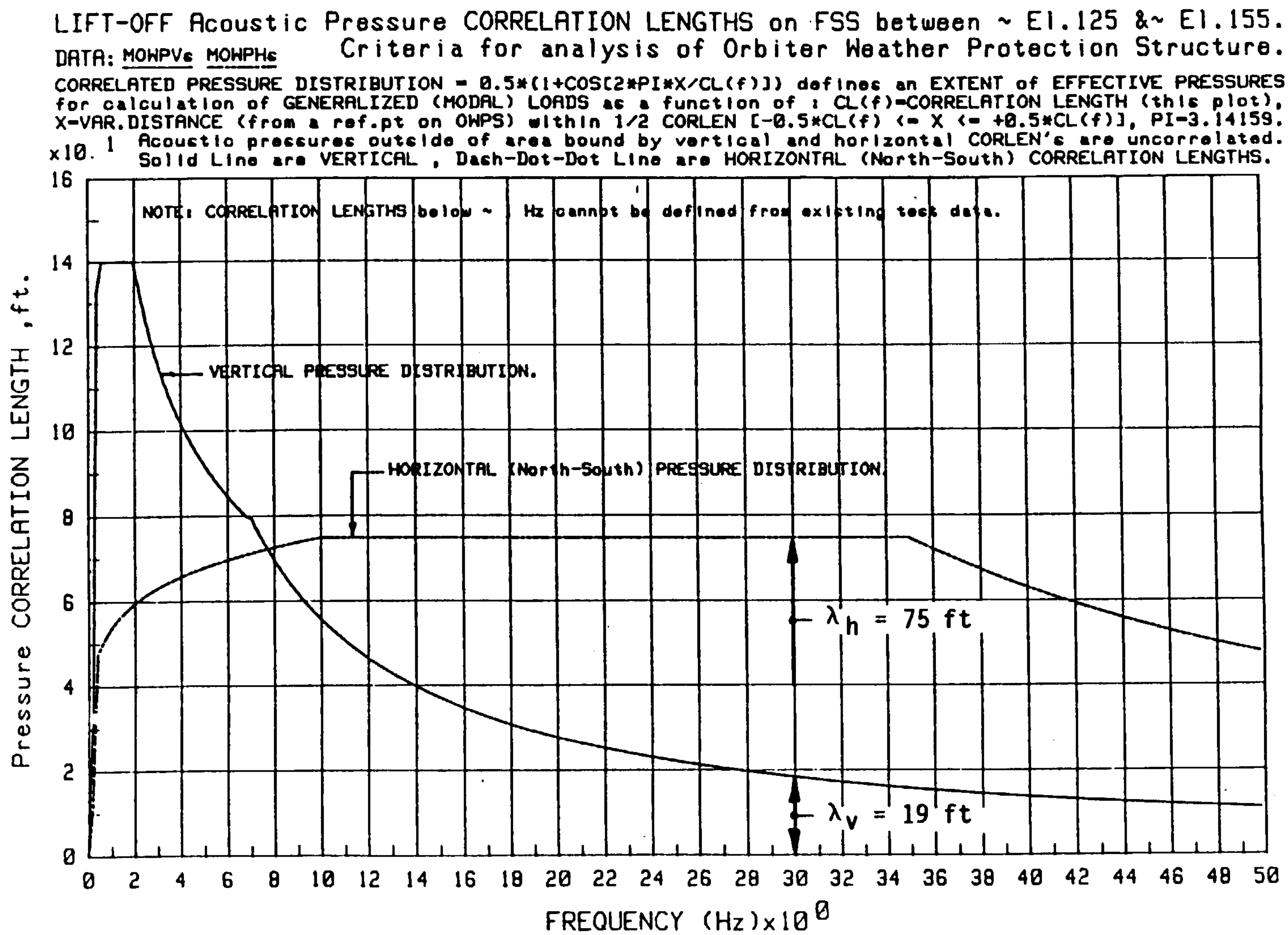


Figure C-3. Lift-Off Acoustic Pressure Correlation Lengths on FSS Between 125- and 155-Foot Elevation

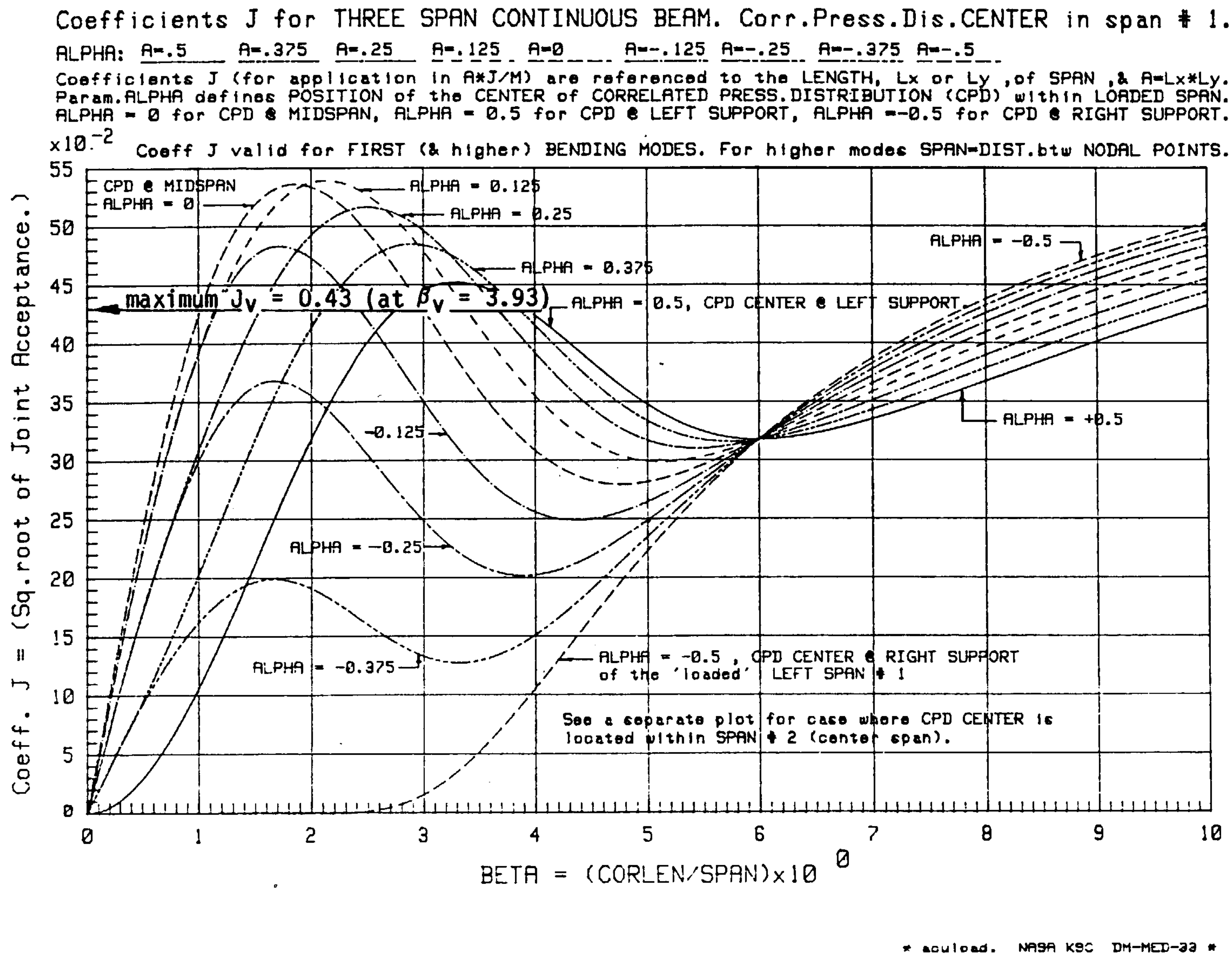
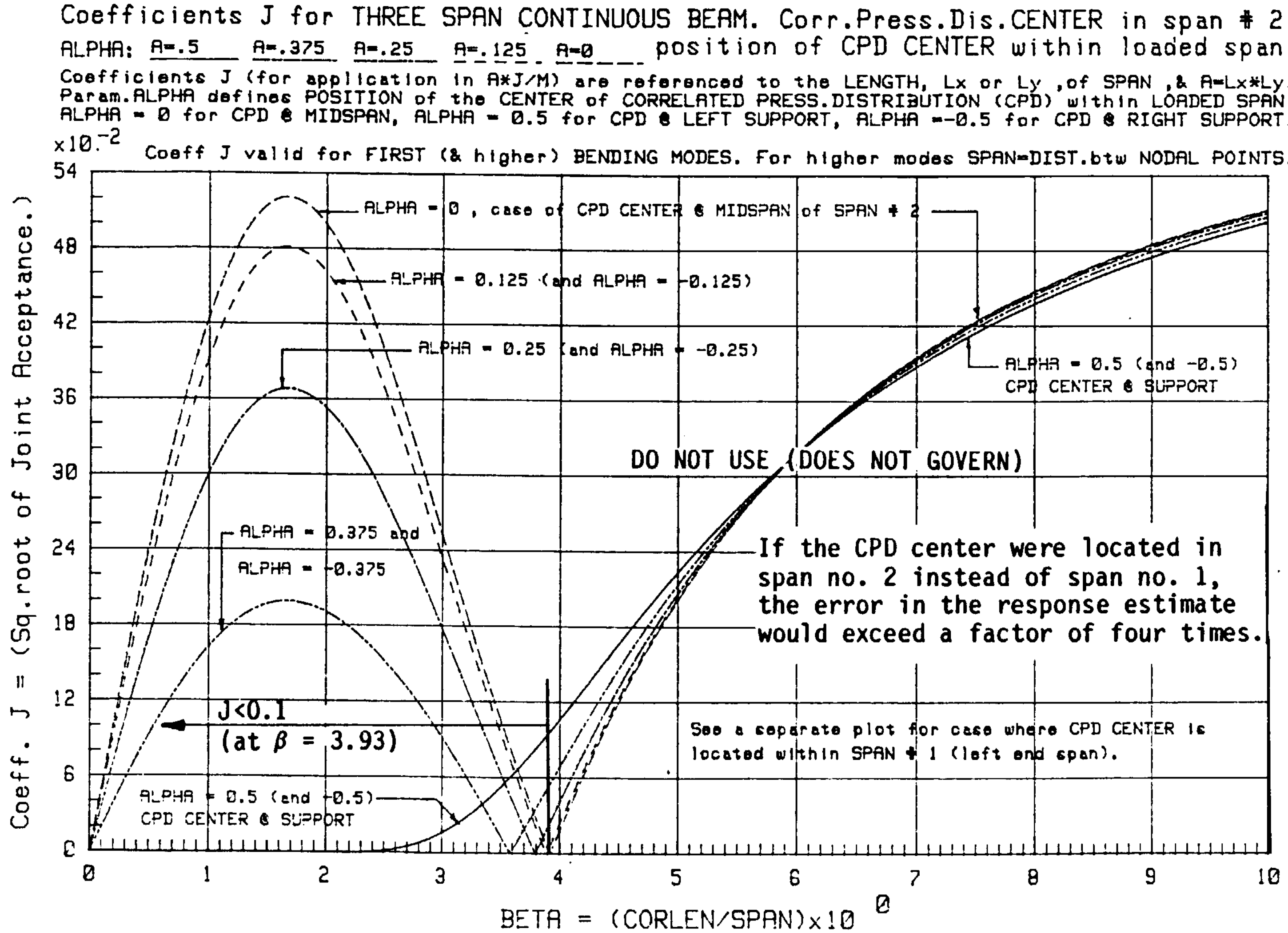


Figure C-4. J-Coefficients for a Three-Span Continuous Beam, Correlated Pressure Distribution Center in Span No. 1



* acuload. NASA KSC DM-MED-22 *

Figure C-5. J-Coefficients for a Three-Span Continuous Beam, Correlated Pressure Distribution Center in Span No. 2

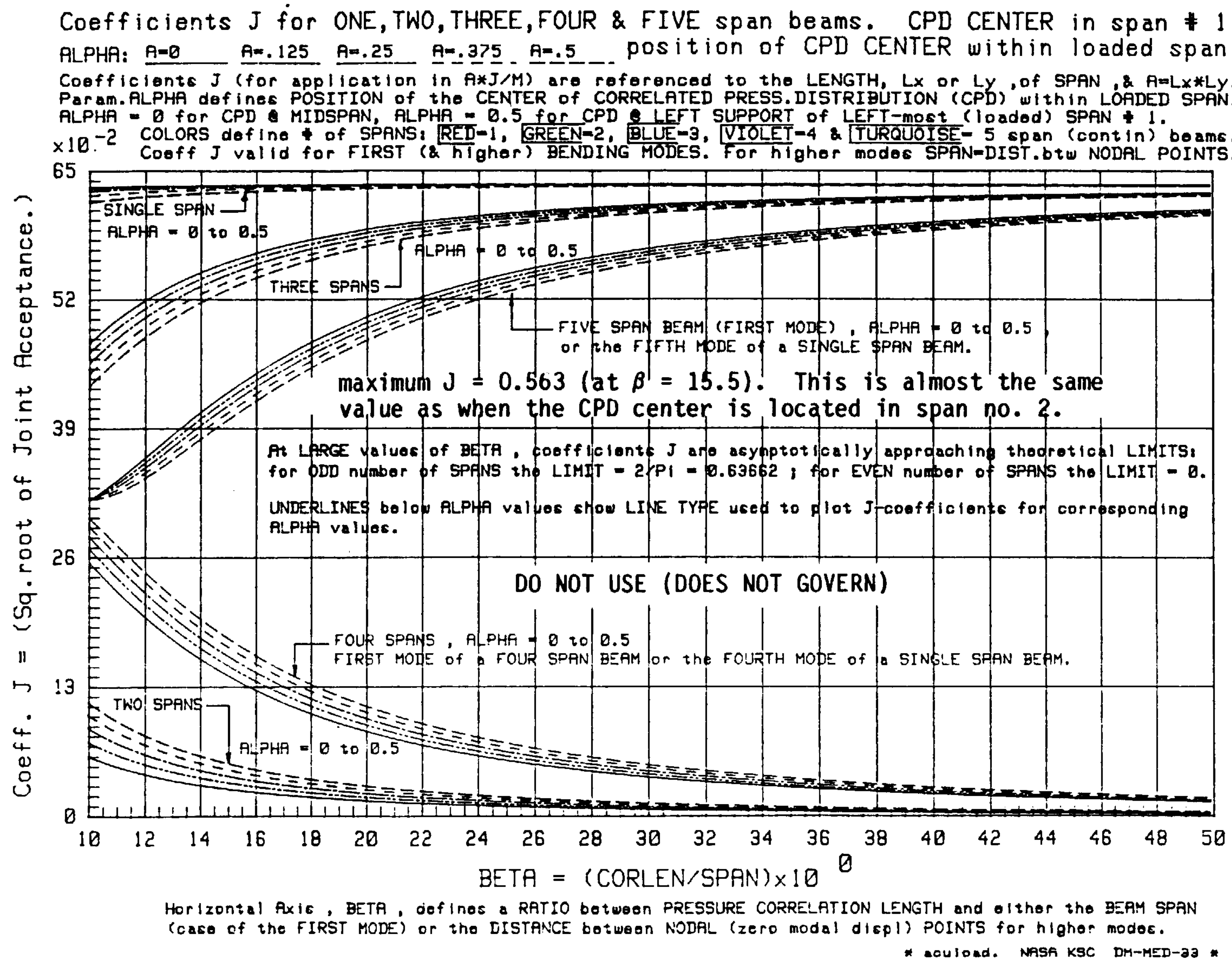


Figure C-6. J-Coefficients for One-, Two-, Three-, Four-, and Five-Span Beams, Correlated Pressure Distribution Center in Span No. 1

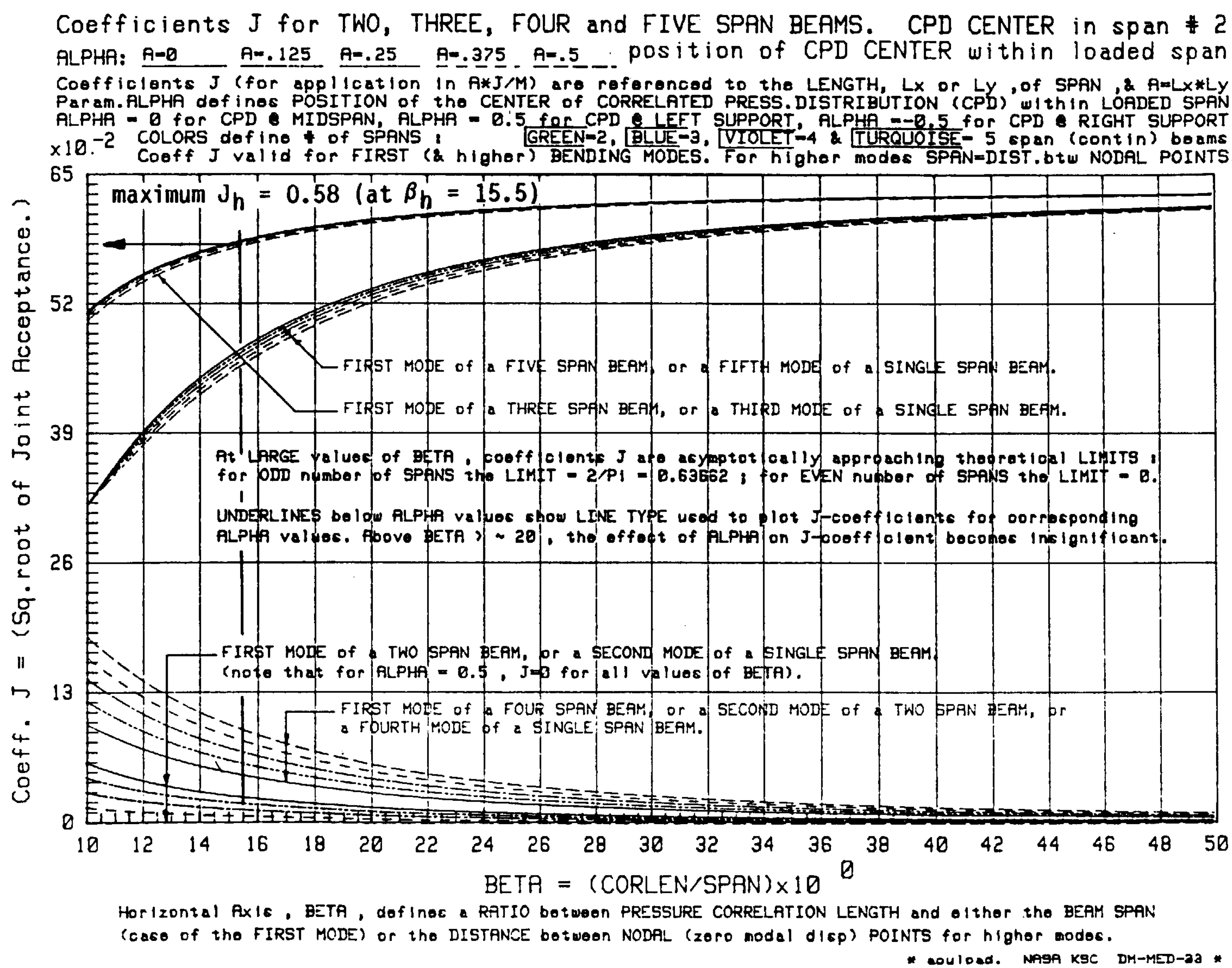


Figure C-7. J-Coefficients for Two-, Three-, Four-, and Five-Span Beams, Correlated Pressure Distribution Center in Span No. 1

C.1.3 FIRST BENDING MODE SUMMARY. Modal coordinates, either q_{1v} or q_{1h}, are the multipliers of the normal mode no. 1 stress matrix. For the first mode, $\varphi=\sin(\pi x/l)$, stresses can be computed explicitly without resorting to the FEM. Thus, the stress matrix bending moment is:

$$M_{b1} = -EI \frac{\partial^2 \varphi_1}{\partial x^2} = EI \left(\frac{\pi}{l}\right)^2 \sin(\pi x/l)$$

$$M_{b1} \text{ (at } x = l/2) = EI \left(\frac{\pi}{l}\right)^2 = 2.036 \times 10^4 \text{ lb-in/(normal mode amplitude)}$$

For a vertical beam, the maximum response bending moment at midspan ($x=l/2$) is:

$$\delta M_{rb1v} = q_{1v} M_{b1} = 0.2245 \times 2.036 \times 10^4 = 4.571 \times 10^3 \text{ in-lb}$$

and the maximum unit stress is:

$$\delta_{rb1v} = M_{rb1v}/S = 4.571 \times 10^3/0.282 = 16{,}209 \text{ psi}$$

For a horizontal beam:

$$\delta_{rb1h} = q_{1h} M_{b1}/S = 21{,}870 \text{ psi}$$

C.2 SECOND BENDING MODE (ANTISYMMETRIC)

An approximate estimate of beam response in this mode is possible using J-coefficient diagrams presented in this report. Note that in the following example, J-coefficients are not those of a three-span beam.

Modal parameters are taken from the FEM [NASA Structural Analysis (NASTRAN)] output:

Resonance frequency $f_2 = 38.45$ Hz

Generalized modal mass $M_2 = 9.599 \times 10^{-2}$ lb-sec^2/in

(mode normalized to maximum unit displacement in spans no. 1 and no. 3)

$$\omega_2^2 = (2\pi f_2)^2 = 5.836 \times 10^4$$

C.2.1 CASE OF A VERTICAL BEAM.

At f_2 = 38.45 Hz, $\lambda_{v2} \approx 15$ ft

$$\beta_{v2} = \frac{\lambda_{v2}}{l} = \frac{15}{4.833} = 3.10$$

Locate the CPD center at midspan of span no. 1 and draw (in scale) the correlated pressure distribution,

$$P(x) = \frac{1}{2}\left[1 + \cos\frac{2\pi x}{\lambda_{v2}}\right], \text{ considering that}$$

$$\frac{\lambda_{v2}}{2} = \frac{\beta_{v2}}{2}\,l = 1.551, \text{ as shown in figure C-8.}$$

Write the expression for AJ, paying attention to integral limits:

$$AJ = \int_{-l/2}^{\lambda_{v2}/2} \varphi_2(x)P(x)dx = \int_{-l/2}^{l/2} \varphi_2(x)P(x)dx + \int_{l/2}^{\lambda_{v2}/2} \varphi_2(x)P(x)dx$$

Note that:

a. For $l/2 \leqslant x \leqslant \lambda_{v2}/2$, $\varphi_2(x)$ is small compared to $\varphi_2(x)$ in span no. 1.

b. $\displaystyle\int_{l/2}^{\lambda_{v2}/2} \varphi_2(x)P(x)dx$ will have the sign opposite to

$$\int_{-l/2}^{l/2} \varphi_2(x)P(x)dx$$

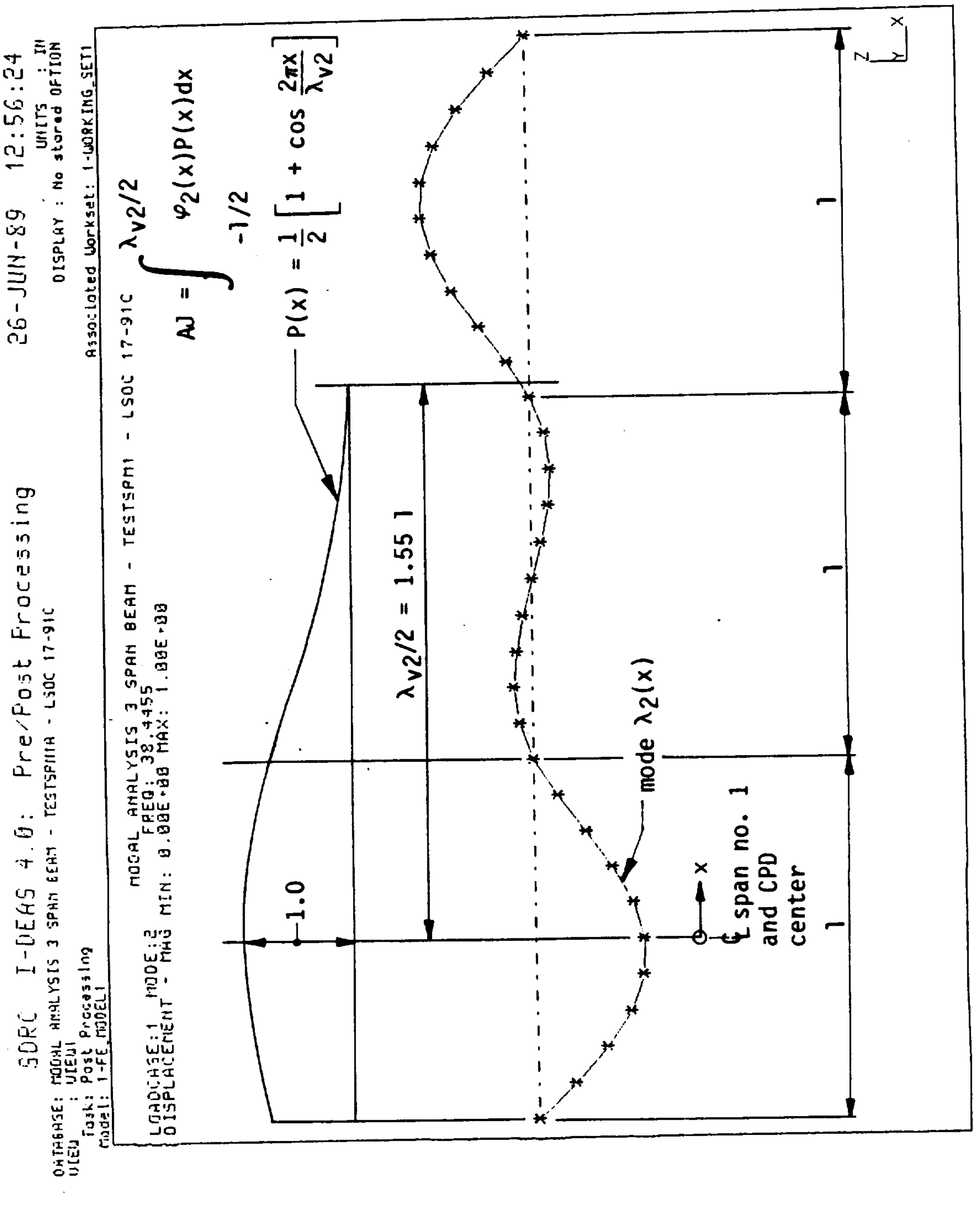


Figure C-8. Second Bending Mode in the Case of a Vertical Beam

Therefore, a conservative estimate of AJ can be obtained by calculating only the first of the two integrals:

$$AJ \approx \int_{-1/2}^{1/2} \varphi_2(x)P(x)dx$$

Note that in span no. 1, $\varphi_2(x)$ is similar to the first mode of a simply supported beam with span 1. Therefore, one can compute AJ and J from the diagram (see figure C-9) for a simply supported beam at β_{v2}=3.10 , J_{v2}=0.607.

Corresponding $A=A_{v2}$ is the same as for the first mode.

$$AJ_{v2}/M_2/\omega_2^2 = 417.6 \times 0.607/9.599 \times 10^{-2}/5.836 \times 10^4 = 0.0452$$

The response spectrum ordinate for an assumed 2-percent damping (same plot as for the first mode) at 38.45 Hz is Y_2=9.5. Y_2=9.5 is a 12-launch maximum. The next highest Y is around 6.5, approximately 32 percent lower.

$$q_{2v} = Y_2 \left[AJ_{v2}/M_2/\omega_2^2 \right] = 9.5 \times 0.0452 = 0.430$$

Response in the second mode exceeds that in the first mode. This rather unusual case is a consequence of small β_{v2} and high Y_2 at launch STS-7.

From the modal stress matrix (NASTRAN output) the bending moment at the midspan of span no. 1 is:

$$M_{b2} = 2.385 \times 10^4 \text{ lb-in/(normal mode amplitude)}$$

The response bending moment is:

$$M_{rb2v} = q_{2v}\, M_{b2} = 0.430 \times 2.385 \times 10^4 = 1.025 \times 10^4 \text{ in-lb}$$

The maximum unit stress is:

$$\delta_{rb2v} = \frac{M_{rb2v}}{S} = 36{,}356 \text{ psi}$$

C.2.2 CASE OF A HORIZONTAL BEAM.

At f_2 = 38.45 Hz, $\lambda_{h2} \approx 67$ ft

$$\beta_{h2} = \frac{\lambda_{h2}}{1} = 13.87$$

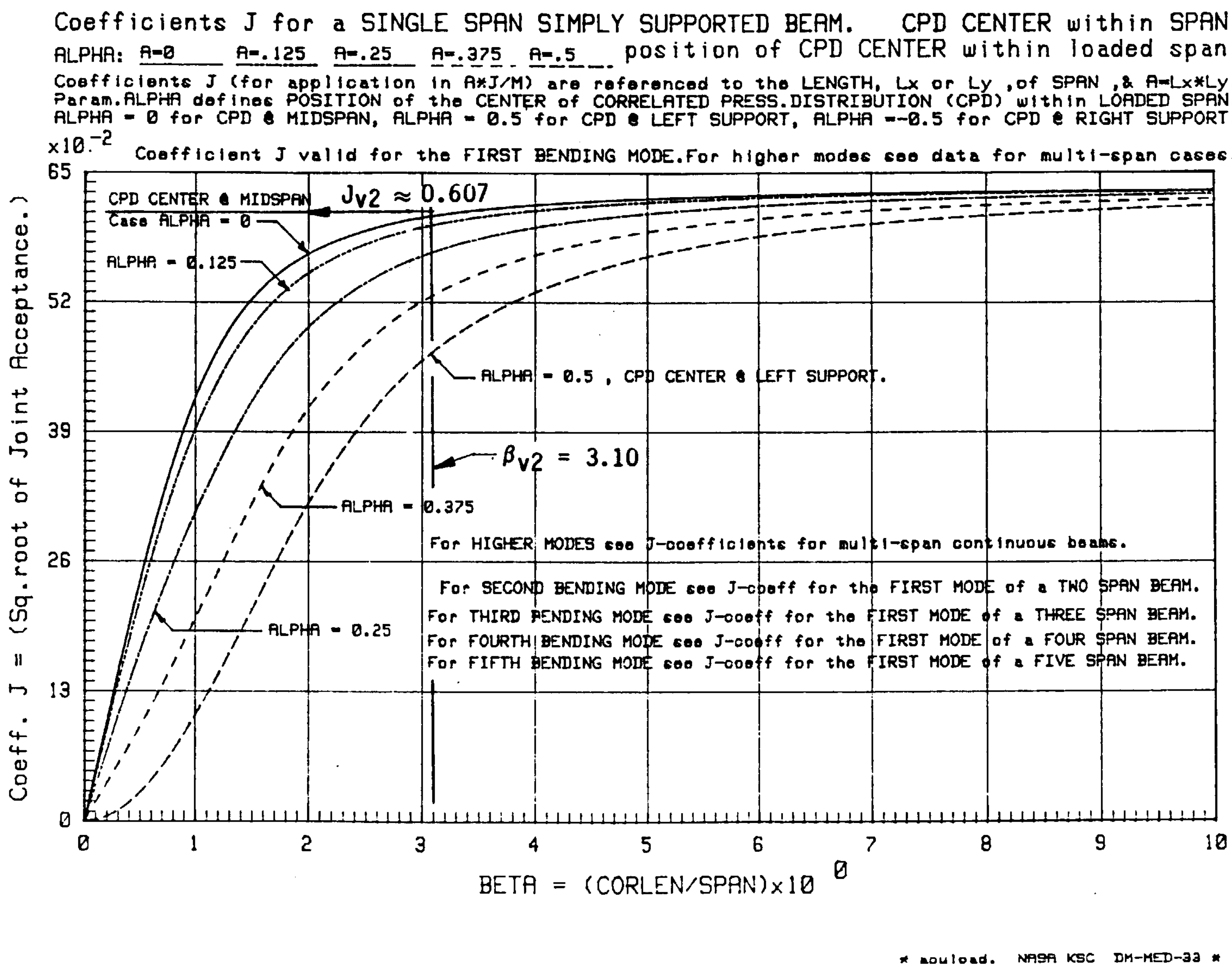


Figure C-9. J-Coefficients for a Single-Span Simply Supported Beam, Correlated Pressure Distribution Center Within the Loaded Span

If the same steps are followed as for the vertical beam and the CPD is drawn relative to the mode, considering that:

$$\frac{\lambda_{h2}}{2} = 6.941,$$

one finds that the entire asymmetric mode is "loaded" by a nearly uniform distribution. If the distribution were exactly uniform, the mode could not be excited since

$$\int_{o}^{3l} \varphi(x)dx = 0$$

An approximate estimate of AJ can be made by a direct computation. The CPD center is positioned at the left support of span no. 1 because this position results in the most nonuniform load and maximum AJ. This is shown in figure C-10.

At $x = l/2$, $\frac{2\pi x}{\lambda_{h2}} = \frac{\pi l}{\lambda_{h2}} = \frac{\pi}{\beta_{h2}} = 0.2265$

$$P(l/2) = \frac{1}{2}[1 + \cos(0.2265)] = 0.9872 = P_1$$

At $x = 5l/2$, $\frac{2\pi x}{\lambda_{h2}} = \frac{5\pi}{\lambda_{h2}} = 1.1325$

$$P\left(\frac{5l}{2}\right) = \frac{1}{2}[1 + \cos(1.1325)] = 0.7122 = P_2$$

It is assumed that:

$$\int_{l}^{2l} \varphi(x)P(x)dx = 0$$

The assumption is conservative since the actual value of this integral is negative and it would, if computed, decrease the total estimate of AJ.

$$(AJ)_h = \int_{o}^{3l} b\varphi(x)P(x)dx \approx b\,\frac{2}{3}\,l(P_1-P_2)$$

$$= 7.2 \times \frac{2}{3} \times 58 \times (0.9872 - 0.7122) = 76.6 \text{ in}^2$$

$$(AJ)_h/M2/\omega_2^2 = 76.6/9.599 \times 10^{-2}/5.836 \times 10^4 = 1.367 \times 10^{-2}$$

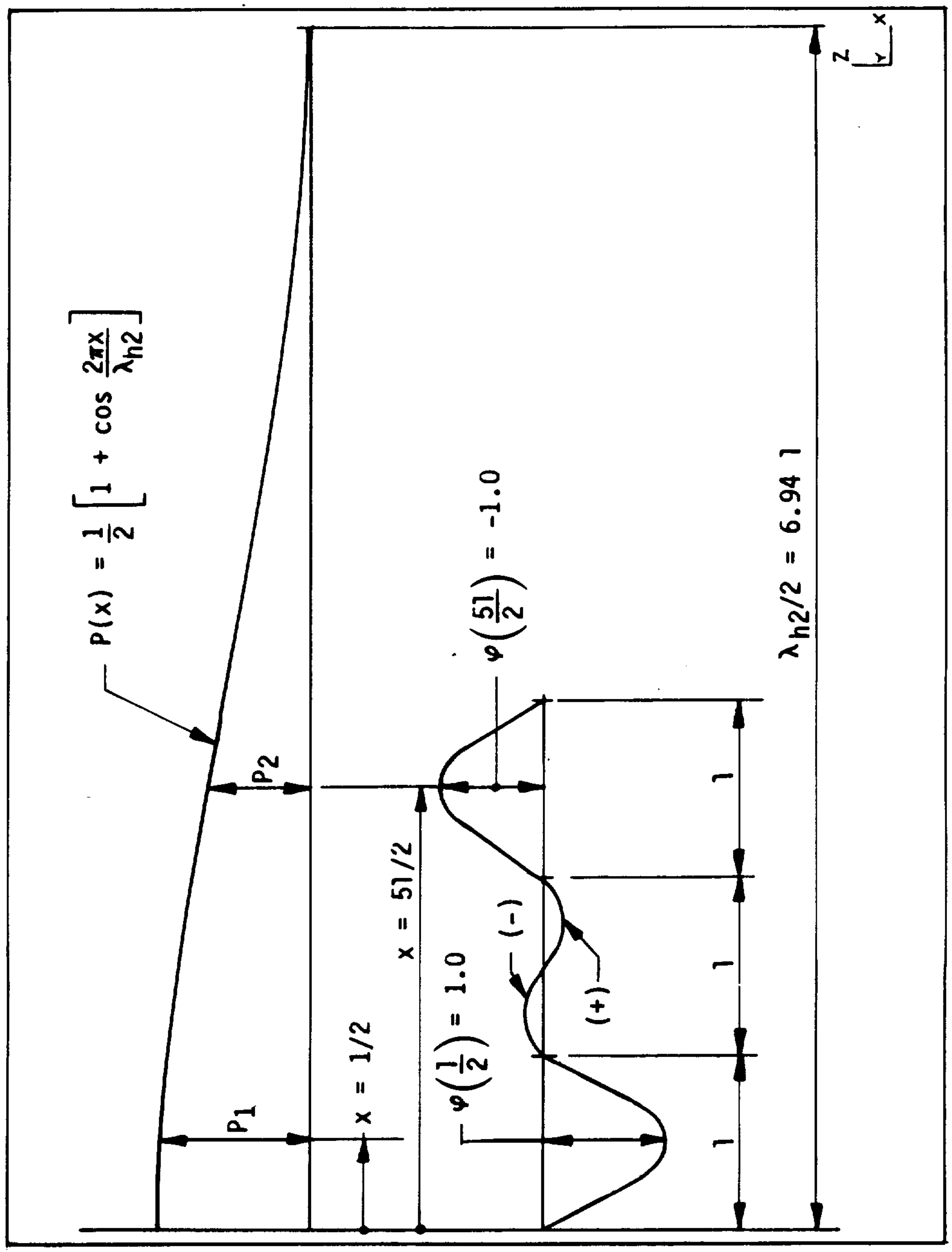


Figure C-10. Second Bending Mode in the Case of a Horizontal Beam

$$q_{2h} = Y_2 \left[(AJ)_h / M_2 / \omega_2^2 \right] = 9.5 \times 1.367 \times 10^{-2} = 0.1298$$

The response bending moment is:

$$M_{rb2h} = q_{2h}\, M_{b2} = 0.1298 \times 2.385 \times 10^4 = 3.097 \times 10^3 \text{ in-lb}$$

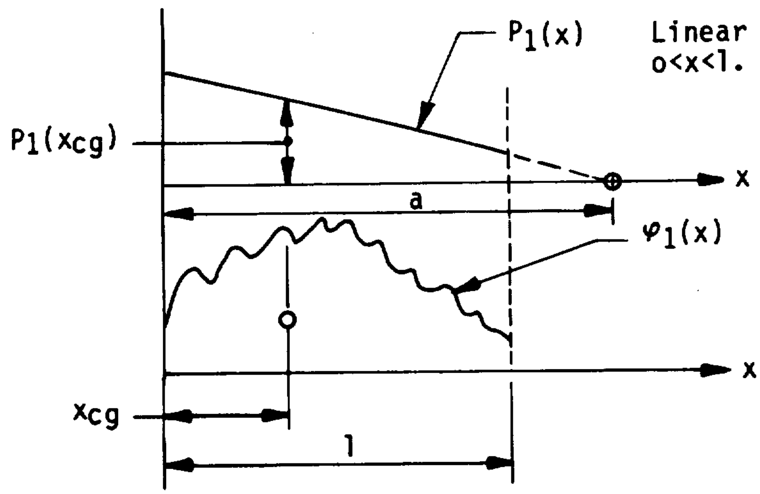
The maximum unit stress is:

$$\delta_{rb2h} = \frac{M_{rb2h}}{S} = 10{,}981 \text{ psi}$$

C.2.3 SECOND BENDING MODE SUMMARY. Estimates of modal coordinates are approximate and conservative. For a vertical beam, J was computed from the diagram (see figure C-9) for a single span beam. Such approximation was possible because β_{y2} is small and the contribution to AJ from the center span is negligible [small $\varphi_2(x)$]. For a horizontal beam, AJ was estimated directly by calculating the integral definition of AJ. In this computation, the following general theorem was used (see the following illustration). The theorem states that:

$$\int_0^1 \varphi_1(x) P_1(x) dx = A_\varphi P_1(x_{cg})$$

The integral is equal to the area, A_φ, under $\varphi_1(x)$ times the ordinate of the linear function, $P_1(x_{cg})$, taken at the abscissa, x_{cg}, corresponding to the location of the center of gravity of A_φ.



Area under $\varphi_1(x)$ $\qquad A_\varphi = \int_o^1 \varphi_1(x)dx$

Position of the "area center" $\qquad x_{cg} = \frac{1}{A_\varphi}\int_o^1 \varphi_1(x)xdx$

Proof of theorem:

$$P_1(x) = P_1(o)\left(1 - \frac{x}{a}\right)$$

$$\int_o^1 P_1(x)\varphi_1(x)dx = P(o)\int_o^1 \left(1- \frac{x}{a}\right)\varphi_1(x)dx$$

$$= P_1(o)\left[\int_o^1 \varphi_1(x)dx - \frac{1}{a}\int_o^1 \varphi_1(x)xdx\right]$$

$$= P_1(o)\left[A_\varphi - A_\varphi \frac{x_{cg}}{a}\right] = A_\varphi P_1(o)\left[1 - \frac{x_{cg}}{a}\right]$$

$$= A_\varphi P_1(x_{cg})$$

C.3 THIRD BENDING MODE (SYMMETRIC)

This mode is a result of a mathematical solution of the eigenvalue problem. This mode cannot be excited in the "real world." The mode is shown in figure C-11. However, a formal computation of J-coefficients (in order to perhaps check the program) can be computed (approximately) using the following procedure:

a. Locate the CPD center at the center of span no. 2.

b. Use a procedure similar to that in C.2.2, applying the theorem in C.2.3.

c. When calculating the area of modal displacements in spans no. 1 and no. 3, take into consideration modal amplitudes in these spans (they are $\neq 1.0$).

d. Within each span, the correlated pressure distribution function

$$P(x) = \frac{1}{2}\left[1 + \cos\frac{2\pi x}{\lambda}\right]$$

shall be substituted by a linear approximation so that the theorem in C.2.3 will apply.

C.4 FOURTH BENDING MODE (ANTISYMMETRIC)

This mode shape is the same as the first bending mode of a six-span beam with the span l_4=1/2=29 in=2.42 ft. The mode is shown in figure C-12. Because the mode is antisymmetric, the critical position of the CPD center is always in the first span. As long as $\lambda/2$ (λ = either vertical or horizontal PCL) does not exceed $5xl_4$=12.08 ft, existing diagrams of J-coefficients derived for a five-span beam can be used.

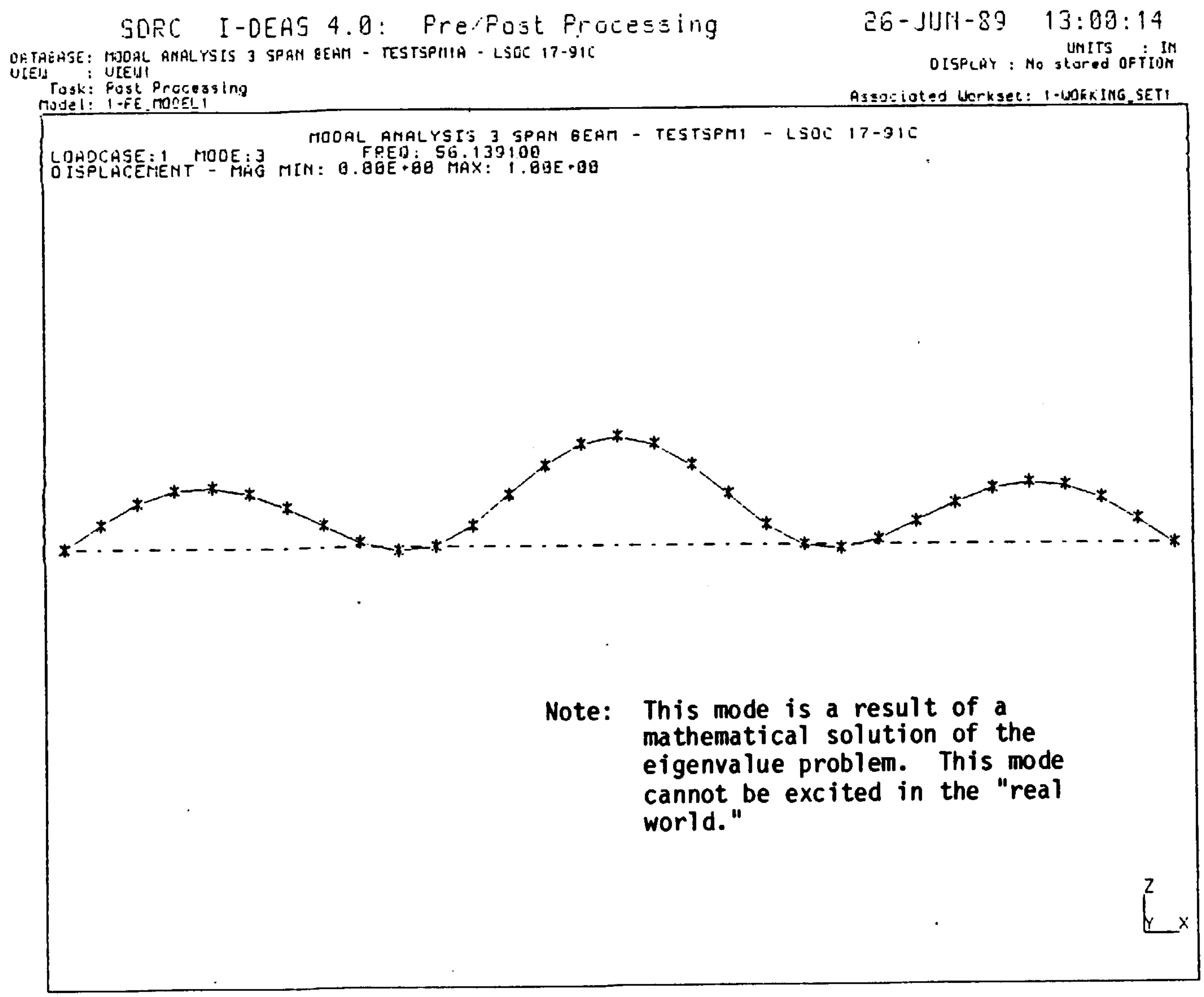


Figure C-11. Third Bending Mode

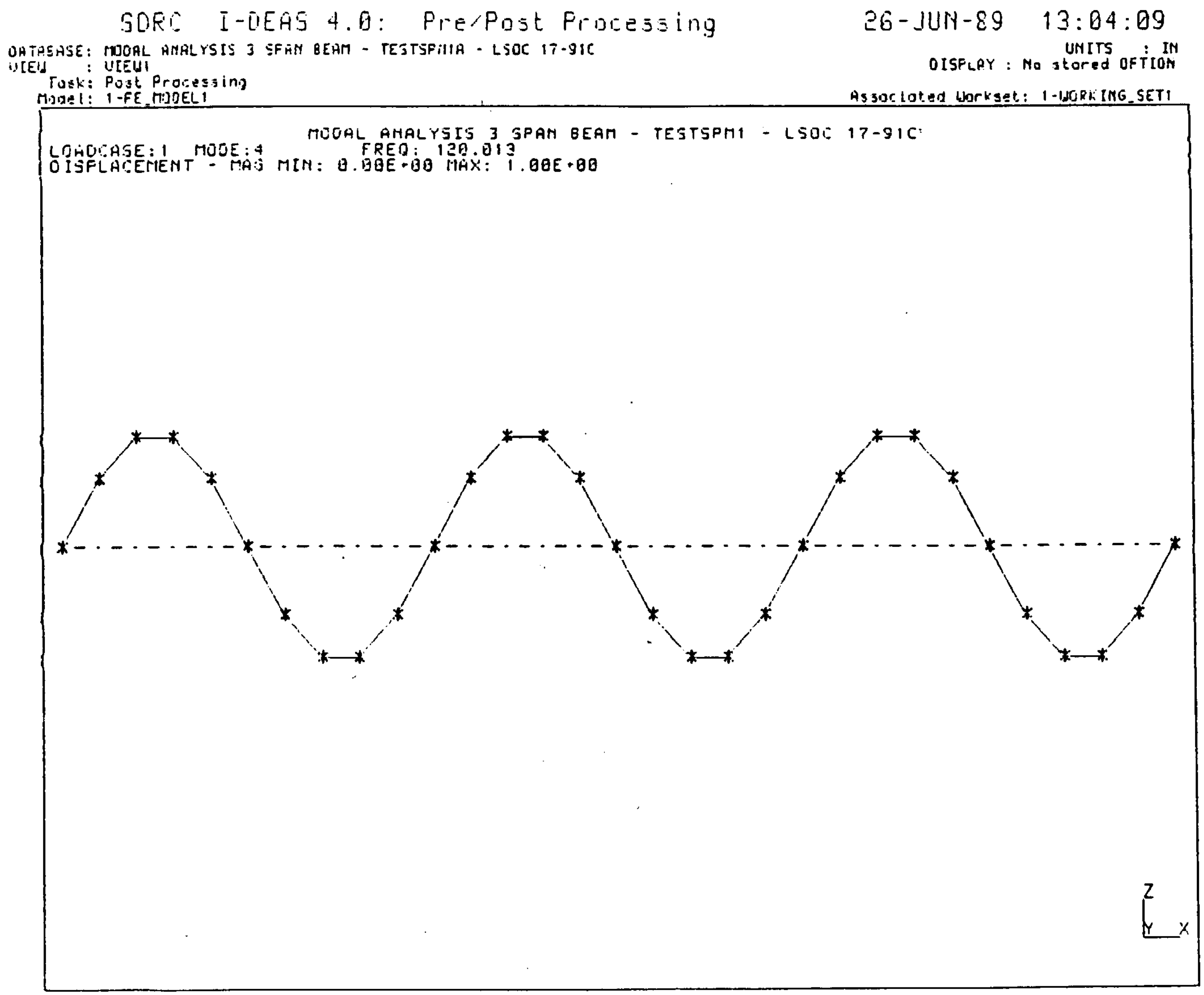


Figure C-12. Fourth Bending Mode

In this case,

$$A_4 = bl_4 = bl/2 = A/2 = 208.8 \text{ in}^2$$

Mode no. 4 parameters (from NASTRAN output) are:

Resonance frequency $f_4 = 120$ Hz

Generalized modal mass $M_4 = 0.1617$

$$\omega_4^2 = (2\pi f_4)^2 = 5.685 \times 10^5$$

Because the mode resonance, f_4, exceeds the frequency range of available PCL's and response spectra, the required values of λ's and Y_4 will be extrapolated/ assumed. From a consideration of available power spectral densities (PSD's), the value of the response spectra ordinate at 120 Hz is Y_4=12 (±10 percent).

C.4.1 CASE OF A VERTICAL BEAM.

Extrapolated $\lambda_{v4} = 4.8$ ft

$$\beta_{v4} = \frac{\lambda_{v4}}{l_4} \cong 2$$

From the diagram of J-coefficient for a five-span beam (see figure C-13), at $\beta_{v4}=2$, $J_{v4}=0.538$.

$$A_4 J_{v4}/M_4/\omega_4^2 = 208.8 \times 0.538/0.1617/5.685 \times 10^5 = 1.222 \times 10^{-3}$$

$$q_{4v} = Y_4 \left[A_4 J_{v4}/M_4/\omega_4^2\right] = 12 \times 1.222 \times 10^{-3} = 0.0147$$

From NASTRAN output, $M_{b4}=-8.416\times10^4$ at approximately one-quarter of the span ("element 4" in NASTRAN ID). The bending moment at the midspan in this mode is zero. The response bending moment at l/4 is:

$$M_{rb4v} = 8.416 \times 10^4 \times 0.0147 = 1{,}234 \text{ in-lb}$$

The maximum unit stress is:

$$\delta_{rb4v} = \frac{M_{rb4v}}{S} = 4{,}376 \text{ psi}$$

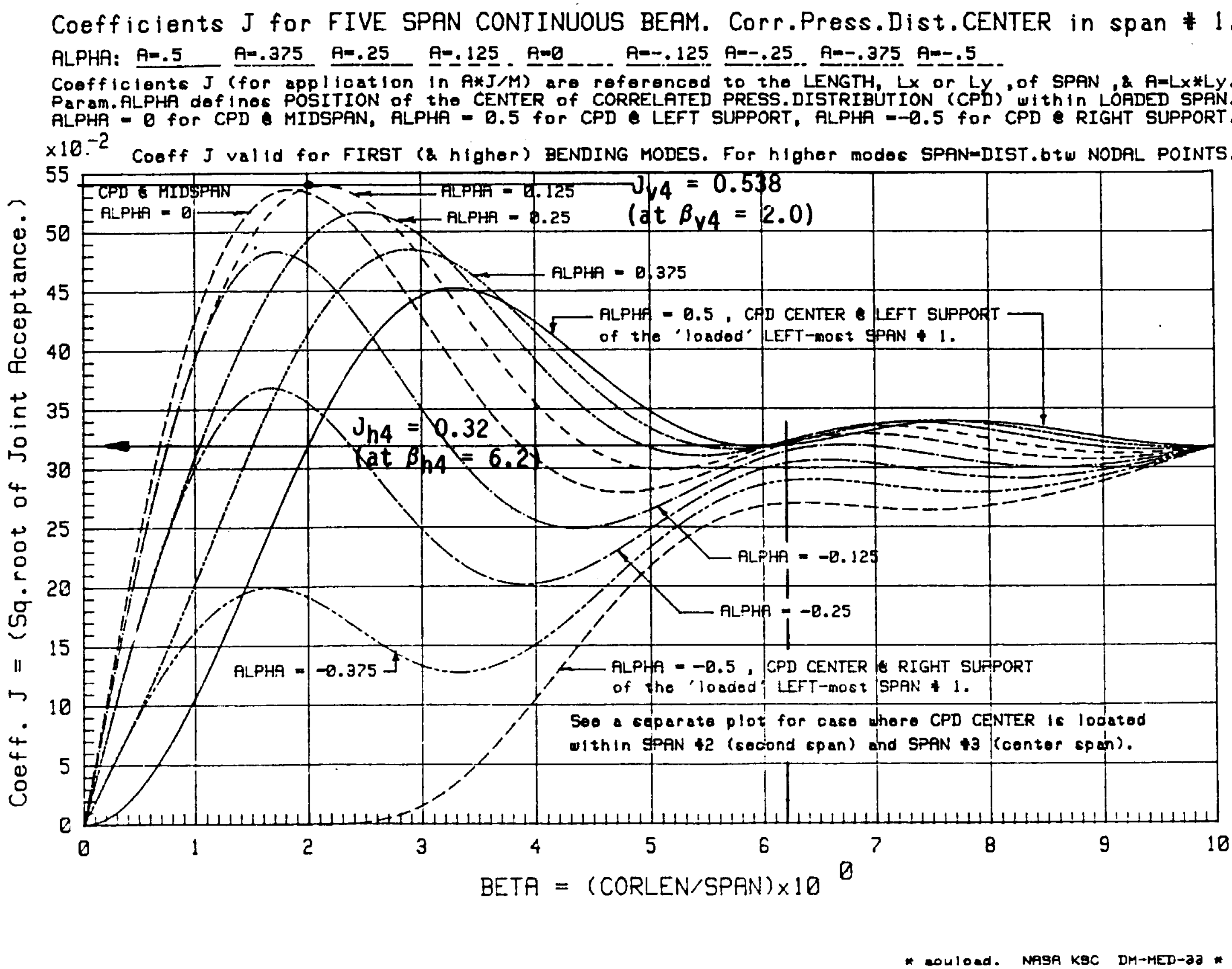


Figure C-13. J-Coefficients for a Five-Span Continuous Beam, Correlated Pressure Distribution Center in Span No. 1

C.4.2 CASE OF A HORIZONTAL BEAM.

Extrapolated λ_{h4} = 15 ft

$$\beta_{h4} = \frac{\lambda_{h4}}{l_4} = 6.2$$

From the diagram of a five-span beam (see figure C-13) with the CPD center in span no. 1, $J_{h4} = 0.32$

$$A_4 J_{h4}/M_4/\omega_4^2 = 7.27 \times 10^{-4}$$

$$q_{4h} = Y_4 \left[A_4 J_{h4}/M_4/\omega_4^2\right] = 8.72 \times 10^{-3}$$

The response bending moment at 1/4 is:

$$M_{rb4h} = q_{4h}\, M_{b4} = 734 \text{ in-lb}$$

The maximum unit stress is:

$$\delta_{rb4h} = 2603 \text{ psi}$$

C.4.3 FOURTH BENDING MODE SUMMARY. Response was computed using J-coefficients for a five-span beam. The response is diminishing. No higher modes need to be considered.