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# Development of a Numerical Solution to the Time Dependent Kinetic Equation

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## Abstract

We have developed a numerical solution for the time dependent Fokker-Planck equation for arbitrary distributions of electrons injected into a magnetized plasma. We have calibrated and tested our code which includes energy loss and pitch angle scattering due to Coulomb collisions and changes in pitch angle due to inhomogeneous magnetic fields. The numerical method is versatile so that other scattering or radiation terms can be easily included. Using this code we will investigate many processes associated with the impulsive phase of solar flares.

## Introduction

The evolution of a distribution of electrons in magnetized plasma is given by the equation

$$\frac{\partial f}{\partial t} = -\mu c\beta\frac{\partial f}{\partial s} - \frac{\partial}{\partial \mu}(\dot{\mu} f) - \frac{\partial}{\partial E}(\dot{E} f) + \frac{\partial}{\partial \mu}(D_{\mu\mu}\frac{\partial f}{\partial \mu}) + \frac{\partial}{\partial E}(D_{EE}\frac{\partial f}{\partial E}) + \frac{\partial}{\partial E}(D_{E\mu}\frac{\partial f}{\partial \mu}) + \frac{\partial}{\partial \mu}(D_{E\mu}\frac{\partial f}{\partial E}) + S(E,\mu,s,t), \quad (1)$$

where $E$, $\beta c$, $\mu$, and $s$ are the electron, energy, velocity, pitch angle cosine, and postion respectively. The coefficients $\dot{E}$ and $\dot{\mu}$ are the systematic changes in energy and pitch angle cosine due to external forces, radiation, and scattering, while the diffusion coefficients $D_{ij}$ are due only to scattering processes. The term $S(E,\mu,s,t)$ is the source (or sink) of electrons which accounts for those acceleration (or loss) processes with timescales which are much faster than the processes giving rise to the coefficients $\dot{E}$, $\dot{\mu}$, and $D_{ij}$.

For electrons of energy 10 keV to 1 MeV and an ambient plasma with magnetic field strength $10^2$ to $10^3$ Gauss and plasma density $10^9$ to $10^{14}$ cm$^{-3}$ the dominant processes are Coulomb collisions and magnetic mirroring. Therefore, we need only to determine the coefficients which arise from Coulomb collisions and magnetic mirroring in equation (1) and solve the resulting equation to determine the time evolution of electrons of the above energies for typical solar flare conditions. We obtain

$$\frac{\partial f}{\partial t} = -\mu c\beta\frac{\partial f}{\partial s} + \beta c\frac{d\ln B}{ds}\frac{\partial}{\partial\mu}\left(\frac{(1-\mu^2)}{2}f\right) + \frac{c}{\lambda_0}\frac{\partial}{\partial E}\left(\frac{f}{\beta}\right) + \frac{c}{\lambda_0\beta^3\gamma^2}\frac{\partial}{\partial\mu}\left[(1-\mu^2)\frac{\partial f}{\partial\mu}\right] + S(E,\mu,s,t), \qquad (2)$$

where $\lambda_0 = 10^{24}\mathrm{cm}/n(s)\ln\Lambda$ is the mean free path of an electron with energy equal to its rest mass, $n(s)$ is the background plamsa number density (in cm$^{-3}$), and $\ln\Lambda \approx 20$ is the Coulomb logarithm (see Lu and Petrosian 1988).

### Numerical Solution

Our numerical solution ultilizes a finite difference scheme along with the powerful method of operator splitting. Operator splitting (or time split method) is a method of solving partial differential equations which contain a number of differential operators such as equation (1) or (2). The complete numerical solution is obtained by finding the finite difference solution for each individual operator and then applying these cyclically (see Centrella and Wilson 1984, Hawley, Smarr, and Wilson 1984).

An important aspect of the development of a numerical code for the study of a physical system is code calibration. In order to check the accuracy of our numerical solution, we compare the numerical solution with known analytic solutions for simplified physical conditions. In addition to demonstrating the accuracy of the numercial solution, the calibration also provides us with the limitations of the numerical code as well as the number of grid points necessary to resolve features in the electron distribution.

## Applications

The numerical code that we have developed to solve equation (2) can be used to study the evolution of a distribution of electrons during the impulsive phase of a solar flare under conditions for which an analytic solution is impossible. Therefore, we will be able to compute the expected time evolution of the radiation spectrum and of the spatial variation of the radiation for arbitrary density and magnetic field structures for arbitrary sources of electrons.

We have already begun to utilize the numerical code for study of the impulsive phase of solar flares. In a completed study, we used our solution to investigate the relative timing of microwaves and hard X-rays (Lu and Petrosian 1989). We are also using this code to study the onset of kinetic plasma instabilities which result from the propagation and mirroring of electrons such as the cyclotron instability. In a previous investigation, the correlation of type III bursts and hard X-rays was studied (Hamilton and Petrosian 1989). We will use our numerical solution to analyze the type III burst hard X-ray association. As the ability to resolve temporal and spatial variations in X-rays improves, comparison of the new data with our models will allow us to place more stringent limits on the coronal conditions and specifically the characteristics of the accelerated electrons during the impulsive phase.

Modification of the code to include other processes such as a direct electric field, scattering from Langmuir or Alfvén wave turbulence (Hamilton and Petrosian 1987), or synchrotron radiation losses is straightforward. Therefore, other applications of our code include the study of the acceleration of electrons by parallel electric fields, the scattering and/or the acceleration of electrons by turbulence, and the evolution of continuum gamma-ray emission.

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## References

Centrella, J., and Wilson, J.R. 1984, *Ap. J. Suppl.*, **54**, 229.

Hamilton, R.J., and Petrosian, V. 1987, *Ap. J.*, **321**, 721.

Hamilton, R.J., and Petrosian, V. 1989, *Submitted for publication.*

Hawley, J.F., Smarr, L.L., and Wilson, J.R. 1984, *Ap. J. Suppl.*, **55**, 211.

Lu, E.T., and Petrosian, V. 1988, *Ap. J.*, **327**, 405.

Lu, E.T., and Petrosian, V. 1989, *Submitted for publication.*