Cosmic Strings and Ultra-high Energy Cosmic Rays

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Abstract

We calculate the flux of ultra-high energy protons due to the process of "cusp evaporation" from cosmic string loops. For the 'standard' value of the dimensionless cosmic string parameter $\epsilon \equiv G\mu \approx 10^{-6}$, the flux is several orders of magnitude below the observed cosmic-ray flux of ultra-high energy protons. However, the flux at any energy initially increases as the value of ϵ is decreased. This at first suggests that there may be a lower limit on the value of ϵ, which would imply a lower limit on the temperature of a cosmic-string-forming phase transition in the early universe. However, our calculation shows that this is not the case— the particle flux at any energy reaches its highest value at $\epsilon \approx 10^{-15}$ and it then *decreases* for further decrease of the value of ϵ. This is due to the fact that for too small values of $\epsilon (< 10^{-15})$, the energy-loss of the loops through the cusp evaporation process itself(rather than gravitational energy-loss of the loops) becomes the dominant factor that controls the behavior of the number-density of the loops at the relevant times of emission of the particles. The highest flux at any energy remains at least four orders of magnitude below the observed flux. There is thus no lower limit on ϵ.

value of ϵ —an assumption, which, as we have mentioned above and shall discuss below, is not valid. We will report the explicit calculations for the case of neutrinos elsewhere, but from the results of Ref. 4 and the discussions given below, it already appears that the use of the correct formulas for the loop number densities would also eliminate the lower bounds on ϵ found in Ref. 4.

In Sec. II, we briefly describe the process of cusp evaporation from CS loops and estimate the number of primary particles emitted from the string per unit time. The UHE proton injection spectrum, resulting from the decay of the primary particles and the subsequent hadronization of the decay products, is estimated in Sec. III by using a suitable hadronic jet fragmentation distribution function. A general expression for the predicted flux in the present epoch is written down in Sec. IV. In Sec. V, we briefly discuss the main processes by which UHE protons lose energy during their propagation through the cosmic medium, and discuss how the effective maximum possible redshift of injection is determined for a given value of the energy of the proton in the present epoch. The CS loop length distribution function required for our calculation is obtained in Sec. VI. The main calculation of the flux is described in Sec. VII, and the results, discussions and conclusions are presented in Sec. VIII.

Except where otherwise stated, we use natural units, $\hbar = c = 1$, so that $\sqrt{G} = M_{Pl}^{-1} = t_{Pl}$, where M_{Pl} is the Planck mass and t_{Pl} is the Planck time. The Hubble constant is $H_0 = 100.h \ Km.s^{-1}.Mpc^{-1}$, and we use $h = 0.75$. Also t_{eq} is the time of equal matter- and radiation energy density, z_{eq} is the corresponding redshift and t_0 is the present age of the universe. We assume a $\Omega_0 = 1$ universe.

II. CUSP EVAPORATION

A non-self-intersecting,[5,6] freely oscillating CS loop has one or more points which momentarily achieve the speed of light once during every oscillation period. These points called "cusps" appear[6] if the motion of the loop is described by the Nambu action, which is valid for infinitely thin strings. In reality, CS have a finite width, and so the Nambu action is, strictly speaking, not valid for CS and true cusps may not form. Nevertheless, "near cusp" points are likely to occur where the string moves with very high Lorentz factor. At a cusp, two string segments overlap, and it has been pointed out[3] that interactions of the underlying fields lead to 'evaporation'of the overlapped region whereby the energy contained in the overlapped region of the loop is released in the form of particles, thus smoothing out the cusp. New cusps continue to form and evaporate during each period of oscillation of the loop. The length of the cusp region of the loop can be estimated[3] as $\ell_{cusp} \sim L^{2/3}\omega^{1/3}$, where L is the total length of the loop and $\omega \sim \mu^{-1/2}$ is the width of the string. (The length L of the string is defined such that μL is equal to the total energy of the string). The energy released due to cusp evaporation will be in the form of bursts with time-scale $\Delta t_{burst} \sim \ell_{cusp}$. The period of oscillation, T_{osc}, for a loop of length L is[5] $\frac{L}{2}$. Thus, $\Delta t_{burst}/T_{osc} \sim \left(\frac{\omega}{L}\right)^{1/3} << 1$. Thus the rate of energy released due to cusp

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reproduces the particle multiplicity growth as seen in GeV–TeV jets in colliders. This gives,[11]

$$\frac{dN}{dx} = \frac{15}{16} x^{-3/2} (1 - x)^2,\tag{4}$$

where $x = E/E_{jet} = \frac{E}{E_X/3} \leq 1$, E being the energy of a hadron in the jet. A small fraction($\sim 3\%$) (Ref. 11) of the hadrons in the jet will be nucleons and antinucleons which ultimately end up as protons and antiprotons. Observationally, since the primary particles at the high energies involved here are not detected directly, one cannot distinguish between protons and antiprotons. We shall, therefore, in the following, collectively refer to them simply as protons. Let $\Phi(E_i, t_i)$ denote the injection spectrum of the protons, i.e., the number density of injected protons per unit energy interval at an injection energy E_i per unit time at an injection time t_i due to cusp evaporation from all CS loops. Then using eqs.(4) and (2) we get,

$$\Phi(E_i, t_i) \simeq 2 \times 0.03 \times \frac{15}{16} x^{-\frac{3}{2}} (1 - x)^2 \frac{3}{E_X} \gamma_c E_X^{-1} \mu^{\frac{5}{6}} \int dL \frac{dn}{dL}(L, t_i) L^{-\frac{1}{3}},\tag{5}$$

where $x = 3E_i/E_X = 3E_i f^{-1} \epsilon^{-\frac{1}{2}} M_{Pl}^{-1}$ and $\frac{dn}{dL}(L, t_i)$ is the CS loop length distribution function, i.e., $dn(L, t_i)$ is the number density of CS loops with lengths in the interval $[L, L + dL]$ at the time t_i. The factor of 2 in eq.(5) takes care of the fact that we have assumed two quarks in the decay products of each X and each quark produces one hadronic jet. Thus eq.(4) yields an injection spectrum $\propto E_i^{-3/2}$ for $E_i << E_X$.

IV. GENERAL EXPRESSION FOR THE FLUX

Let $j(E_0)$ denote the number of protons per unit energy interval at energy E_0 in the present epoch(t_0) crossing per unit area per unit solid angle per unit time due to the source $\Phi(E_i, t_i)$. Then, assuming an isotropic distribution of the CS loops in an Einstein-deSitter "flat"($\Omega_0 = 1$) universe, we get

$$j(E_0) = \frac{1}{4\pi} \int\limits_0^\infty 4\pi a^3(t_i) r^2 dr \left[(1 + z_i)^{-1} \Phi(E_i, t_i) \right] \left(\frac{dE_i}{dE_0} \right)_{E_0} \frac{1}{4\pi a^2(t_0) r^2},\tag{6}$$

where t_i is the injection time, z_i is the corresponding redshift, $E_i \equiv E_i(E_0, t_i)$ is the energy at the time of injection t_i, $a(t)$ is the scale-factor of the universe, and r is the comoving radial coordinate of the source. The factor $(1 + z_i)^{-1} = a(t_i)/a(t_0)$ in eq.(6) is due to the cosmological "redshift" of the frequency of emission.[14] Now for a $\Omega_0 = 1$ universe, $r = c \int_{t_i}^{t_0} dt/a(t)$ (assuming that the particles are ultrarelativistic, so that they travel almost with the speed of light, c), so that $a(t_i)dr = -cdt_i$. Furthermore, $t_i > t_{eq}$ (in fact, as we shall see below, for all values of energy E_0, all injection times t_i satisfy $t_i >> t_{eq}$) so that $(1 + z_i)^{-1} = a(t_i)/a(t_0) = (t_i/t_0)^{2/3}$, giving $a(t_i)dr = -cdt_i = \frac{3}{2}ct_0(1 + z_i)^{-5/2}dz_i$. Putting all these together, eq.(6) becomes

$$j(E_0) = \frac{3}{8\pi} ct_0 \int_0^\infty dz_i (1 + z_i)^{-11/2} \left(\frac{dE_i(E_0, z_i)}{dE} \right)_{E_0} \Phi(E_i, z_i).\tag{7}$$

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cosmic-rays, while not entirely devoid of controversies, do seem to indicate[19] the existence of a cutoff as predicted.

Now, given the full knowledge of the energy-loss functions $\beta_{0,pair}(E)$ and $\beta_{0,pion}(E)$, one can solve eq. (11) numerically to find the energy E_i of a proton at any injection redshift z_i corresponding to a given value of its energy in the present epoch(E_0). One can then evaluate the injection spectrum $\Phi(E_i, z_i)$ using eq. (5)(with a given CS loop length distribution function, see Sec. VI) and obtain the flux by evaluating the z_i-integral in eq. (7). The full numerical calculation according to this procedure is described in Ref. 20 in the context of another particle production process involving CS. Here we undertake an approximate calculation which essentially yields the same result, but it allows us to avoid the full numerical solution of eq. (11). The approximation is based on the use of the arguments that lead to the prediction of the Greisen-Zatsepin-Kuz'min[18] cutoff mentioned above.

To see this, let us consider the energy-range $5 \times 10^{18} eV \lesssim E_0 \lesssim 6 \times 10^{19} eV$, in which, as mentioned above, $\beta_{0,pair}$ is dominant over $\beta_{0,pion}$ and the former is weakly energy-dependent remaining roughly constant at $\beta_0 \approx 2.13 \times 10^{-10} yr^{-1}$. In this case, as long as $(1 + z_i)E_i < 6 \times 10^{19} eV$, eq. (11) has the analytic solution, namely,

$$E_i(z_i) = E_0(1 + z_i) \exp\left[\frac{2}{3}\frac{\beta_0}{H_0}\left((1 + z_i)^{\frac{3}{2}} - 1\right)\right], \qquad \text{for} \quad 5 \times 10^{18} eV \lesssim E_0 \lesssim 6 \times 10^{19} eV.$$

(12)

Thus in the above energy-range, if we consider a proton at energy E_0 today, its energy E_i at any injection redshift z_i rises exponentially with z_i. If for any given value of E_0, we define the injection redshift $z_{i,max}$ such that

$$(1 + z_{i,max})E_i(z_i = z_{i,max}, E_0) \simeq 6 \times 10^{19} eV,$$

(13)

then for $z_i \geq z_{i,max}$, the proton would be in the photopion energy-loss regime. In this regime the energy-loss itself rises sharply(roughly[17] exponentially) with energy and so the energy E_i of the proton at the injection redshifts $z_i > z_{i,max}(E_0)$ rises evan faster[21] with increasing values of z_i. As a result,the rapid fall of the injection spectrum $\Phi(E_i, z_i)$ (which goes as $\sim E_i^{-3/2}$) with increasing value of z_i dominates over the power-law rise of Φ with z_i coming from the fact that the number-density of the CS loops increases with redshift(see Sec. VI–VII). This in fact ensures that the z_i-integral in eq. (7) converges fast. In other words, for a given value of E_0, the contribution to the flux $j(E_0)$ of eq. (7) from injection redshifts $z_i > z_{i,max}(E_0)$ are negligible compared to those from the injection redshifts $z_i < z_{i,max}(E_0)$. The quantity $z_{i,max}$ defined by eq. (13) can, therefore, be taken as an effective cutoff for the integral in eq. (7). Actually, since the maximum energy of a particle in our case cannot exceed $\frac{1}{3}E_X$, the cutoff redshift should be determined from the condition

$$(1 + z_{i,max})E_i(z_i = z_{i,max}, E_0) = \min\left(\frac{1}{3}E_X(1 + z_{i,max}), 6 \times 10^{19} eV\right),$$

(14)

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(ii) for $t > t_*$,

$$
dn(L,t) \simeq
\begin{cases}
\beta\alpha^3 L^{-4}\left(\dfrac{a(L/\alpha)}{a(t)}\right)^3 dL, & \text{if } \Gamma\epsilon t \leq L \leq \alpha t, \\[4mm]
\beta\alpha^3(\Gamma\epsilon t)^{-4}\left(\dfrac{a(\Gamma\epsilon t/\alpha)}{a(t)}\right)^3 dL, & \text{if } L_{min}^{cusp}(t) \leq L < \Gamma\epsilon t, \\[4mm]
0, & \text{if } L < L_{min}^{cusp}.
\end{cases}
\tag{19}
$$

In deriving eq.(18)–(19) we have assumed that the loops survive with their lengths essentially unchanged till the end of their lifetime at which they instantaneously disappear.

VII. CALCULATION OF THE FLUX

We are now ready to evaluate the flux from eq. (7). First let us define $I_L \equiv t_0^{10/3} \int dL \dfrac{dn}{dL}(L,t_i)L^{-1/3}$. Using (18) and (19) together with the appropriate forms for the scale factor of the universe in the matter- and radiation dominated epochs, these L-integrals are easily carried out. After some algebra, and expressing t_i in terms of z_i by the relation $t_i = t_0(1 + z_i)^{-3/2}$, we get

(i) for $t_i < t_*$,

$$
I_L =
\begin{cases}
\kappa_1(1 + z_i)^{9/2} - \kappa_2(1 + z_i)^5, & \text{for } (1 + z_i) \leq Z_1, \\[2mm]
\kappa_3(1 + z_i)^{81/16} + \kappa_4(1 + z_i)^3 - \kappa_5(1 + z_i)^5, & \text{for } (1 + z_i) > Z_1,
\end{cases}
\tag{20}
$$

where $Z_1 = \gamma_c^{2/3}\epsilon^{-1/9}\alpha^{-8/9}(t_{Pl}/t_0)^{2/9}(1 + z_{eq})^{4/3}$, and

$$
\kappa_1 = \frac{3}{4}\beta\alpha\gamma_c^{-1}\epsilon^{1/6}\left(\frac{t_{Pl}}{t_0}\right)^{-1/3},
$$

$$
\kappa_2 = \frac{3}{4}\beta\alpha^{-1/3}, \qquad\qquad \kappa_4 = \frac{9}{44}\beta\alpha^{-1/3}\left(\frac{t_{eq}}{t_0}\right)^{-4/3},
$$

$$
\kappa_3 = \frac{6}{11}\beta\alpha^{3/2}\gamma_c^{-11/8}\epsilon^{11/48}\left(\frac{t_{eq}}{t_0}\right)^{1/2}\left(\frac{t_{Pl}}{t_0}\right)^{-11/24}, \qquad \kappa_5 = \kappa_2;
\tag{21}
$$

and

(ii) for $t_i \geq t_*$,

$$
I_L =
\begin{cases}
A_1(1 + z_i)^5 - A_2(1 + z_i)^{21/4}, & \text{for}(1 + z_i) \leq Z_2, \\[2mm]
B_1(1 + z_i)^{23/4} - B_2(1 + z_i)^6 + B_3(1 + z_i)^3 - B_4(1 + z_i)^5, & \text{for } (1 + z_i) > Z_2,
\end{cases}
\tag{22}
$$

where $Z_2 = (\Gamma\epsilon/\alpha)^{2/3}(1 + z_{eq})$, and

$$
A_1 = \frac{9}{4}\beta\alpha(\Gamma\epsilon)^{-4/3}, \qquad\qquad B_2 = \frac{3}{2}\beta\alpha^{3/2}\gamma_c^{1/2}\Gamma^{-\frac{5}{2}}\epsilon^{-\frac{31}{12}}\left(\frac{t_{eq}}{t_0}\right)^{\frac{1}{2}}\left(\frac{t_{Pl}}{t_0}\right)^{\frac{1}{6}},
$$

$$
A_2 = \frac{3}{2}\beta\alpha\gamma_c^{1/2}\Gamma^{-2}\epsilon^{-\frac{25}{12}}\left(\frac{t_{Pl}}{t_0}\right)^{\frac{1}{6}}, \qquad B_3 = \frac{9}{44}\beta\alpha^{-1/3}\left(\frac{t_{eq}}{t_0}\right)^{-\frac{4}{3}},
\tag{23}
$$

$$
B_1 = \frac{45}{22}\beta\alpha^{3/2}(\Gamma\epsilon)^{-\frac{11}{6}}\left(\frac{t_{eq}}{t_0}\right)^{\frac{1}{2}}, \qquad B_4 = \frac{3}{4}\beta\alpha^{-1/3}.
$$

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One might think, by looking at eq. (24), that decreasing the value of f(i.e.,E_X; see eq. (3)) may give a higher value of the flux at any given energy. This is true as long as the values of f and ϵ are such that $E_X > 6 \times 10^{19} eV$. However, if f is made too small, eventually one gets $E_X < 6 \times 10^{19} eV$, in which case eq. (14) yields a smaller value of $z_{i,max}$(than what one would get for the case $E_X > 6 \times 10^{19} eV$; see Fig. 1)resulting in a smaller flux. Moreover, for too small values of f and ϵ one gets $E_X < E_0$, in which case, obviously, no particles of the given energy can be produced in the first place. Explicit calculation shows that the peak flux always remains below the value obtained with $f = 1$ and $\epsilon \approx 10^{-15}$.

Note also that in all the above calculations we have assumed that the cusp evaporation process occurs at the maximum efficiency($\gamma_c = 1$). If $\gamma_c << 1$, then all the above values of the fluxes will be correspondingly lower.

Now, consider the case of neutrinos. First note that for $\epsilon \le 5.43 \times 10^{-16}$, eq. (17) gives $t_* \ge t_0 \approx 2.67 \times 10^{17} sec$ (for $\Omega_0 = 1$, $h = 0.75$). In this case, obviously, all the injection times t_i satisfy $t_i < t_*$ irrespective of whether one is considering neutrinos or protons. Eqs. (20),(21) then imply that the values of the flux at all energies will decrease with further decrease of the value of ϵ for $\epsilon \le 5.43 \times 10^{-16}$. So the lower limit, $\epsilon \ge 10^{-17}$, found in Ref. 4 will probably disappear when the correct form of the loop LDF is used. Similarly, for the case $\epsilon = 10^{-15}$, we have $t_* \approx 1.7 \times 10^{16} sec$, and with $E_X = 10^{15} GeV$ and for $E_0 = 10^{19} eV$, say, we have for neutrinos,[4,25] $1 + z_{i,max} = E_X/E_0 = 10^5$, implying that the earliest possible time of injection($t_{i,min}$) satisfies $t_{i,min} << t_*$. So,the contribution to the present-day flux from the t_i's in the range $t_{i,min} \le t_i \le t_*$, when calculated by using the loop LDF as determined by cusp evaporation itself(eqs. 20–21), will give a lower value of the flux(at the given energy) than what is obtained in Ref. 4. Further reduction of the value of ϵ will then reduce the flux further. Thus the lower bound, $\epsilon \ge 10^{-15}$, found in Ref. 4 will also, it seems, disappear, unless the values of some other parameters(e. g., β) are significantly different from their currently favored values.

In summary, we have estimated the UHE proton flux resulting from CS cusp evaporation process and found that the flux at all energies remains below the observed flux, and that there is no lower limit on the temperature of a CS-forming phase transition in the early universe as far as high-energy particle production from CS cusp evaporation is concerned.

[23] N. Turok and P. Bhattacharjee, *Phys. Rev.* **D 33**, 1557(1984); T. W. B. Kibble, *Nucl. Phys.* **B252**, 227(1985).

[24] T. Vachaspati and A. Vilenkin, *Phys. Rev.* **D 31**, 3052(1985).

[25] Neutrinos do not lose any significant amount of enargy in collision with the background medium. So the maximum possible injection redshift, $z_{i,max}$, for any given value of E_0 is primarily determined by the energy-loss due to redshift.
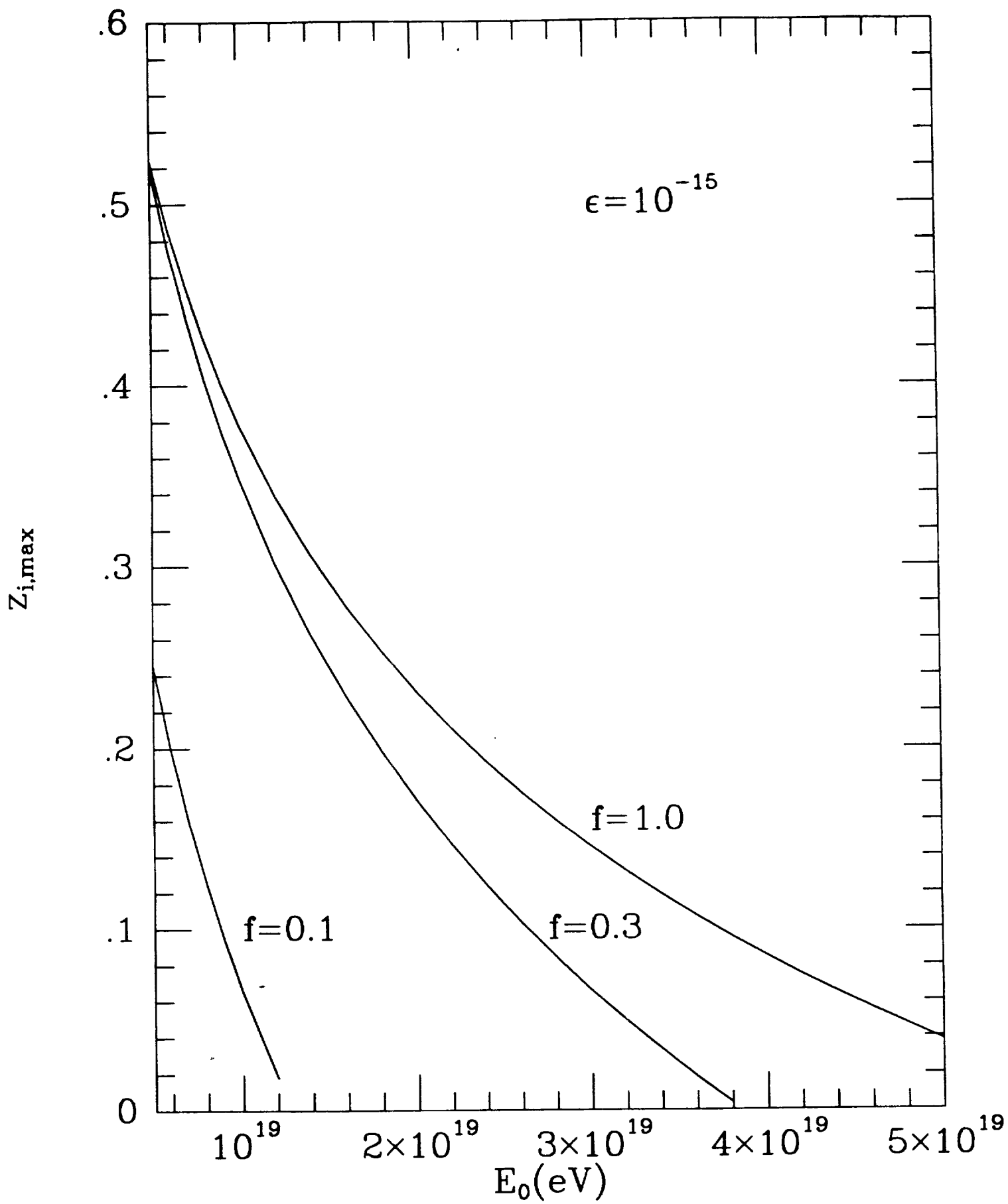
Fig. 1