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NONPARALLEL STABILITY OF BOUNDARY LAYERS

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The asymptotic formulations of the nonparallel linear stability of incompressible growing boundary layers are critically reviewed. These formulations can be divided into two approaches. The first approach combines a numerical method with either the method of multiple scales, or the method of averaging, or the Wentzel-Kramers-Brillouin (WKB) approximation; all these methods yield the same result. The second approach combines a multi-structure theory with the method of multiple scales. Proponents of the second approach have claimed that their approach is rational and the first approach is not rational. The first approach yields results that are in excellent agreement with all available experimental data, including the growth rates as well as the neutral stability curve. On the other hand, the second approach cannot even yield the neutral curve for the Blasius flow.

Introduction

This paper addresses the linear stability of incompressible growing boundary layers. For two-dimensional mean flows, the streamwise velocity component $U(x,y)$ is a function of the transverse coordinate y as well as the streamwise coordinate x. However, the rate of variation of U with respect to x (i.e., $\partial U/\partial x$) is small compared with the rate of variation of U with respect to y (i.e., $\partial U/\partial y$). Moreover, the transverse velocity component $V(x,y)$ is small compared with U and is a function of y as well as x. For three-dimensional flows, the velocity components $U(x,y,z)$ and $W(x,y,z)$ in the plane of the body are much larger than the transverse velocity component $V(x,y,z)$. Moreover, $\partial U/\partial x$, $\partial U/\partial z$, $\partial W/\partial x$, and $\partial W/\partial z$ are small compared with $\partial U/\partial y$ and $\partial W/\partial y$.

To determine the linear stability of a three-dimensional mean flow, we superimpose on it a small disturbance $u(x,y,z,t)$ $v(x,y,z,t)$, $w(x,y,z,t)$, and $p(x,y,z,t)$. Substituting the total flow into the Navier-Stokes equations, subtracting the mean-flow quantities, and linearizing the resulting equations, we obtain

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + W\frac{\partial u}{\partial z} + v\frac{\partial U}{\partial y} + \frac{\partial p}{\partial x} - \frac{1}{R}\nabla^2 u + \left[u\frac{\partial U}{\partial x} + V\frac{\partial u}{\partial y} + w\frac{\partial W}{\partial z}\right] = 0 \tag{2}$$

$$\frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x} + W\frac{\partial v}{\partial z} + \frac{\partial p}{\partial y} - \frac{1}{R}\nabla^2 v + \left[u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + V\frac{\partial v}{\partial y} + w\frac{\partial V}{\partial z}\right] = 0 \tag{3}$$

$$\frac{\partial w}{\partial t} + U \frac{\partial w}{\partial x} + W \frac{\partial w}{\partial z} + v \frac{\partial W}{\partial y} + \frac{\partial p}{\partial z} - \frac{1}{R} \nabla^2 w$$

$$+ \left[u \frac{\partial W}{\partial x} + V \frac{\partial w}{\partial y} + w \frac{\partial W}{\partial z}\right] = 0 \tag{4}$$

where velocities, lengths, and time were made dimensionless using the free-stream velocity U_∞, a characteristic length δ, and a characteristic time δ/U_∞. Here, $R = U_\infty\delta/\nu$ is the Reynolds number. The boundary conditions are

$$u = v = w = 0 \quad \text{at} \quad y = 0 \tag{5}$$

$$u,\ v,\ w,\ p \to 0 \quad \text{as} \quad y \to \infty \tag{6}$$

The terms in the square brackets in Eqs. (2)-(4) are due to the growth of the boundary layer (nonparallel terms).

Parallel Problem

Considering the parallel problem, one neglects the terms in square brackets in Eqs. (2)-(4) and considers U and W to be functions of y only. Then, one seeks a normal mode solution of the form

$$u = \zeta_1(y)E,\ v = \zeta_3(y)E,\ w = \zeta_5(y)E,\ p = \zeta_4(y)E \tag{7}$$

where

$$E = \exp[i(\alpha x + \beta z - \omega t)] \tag{8}$$

Substituting Eqs. (7) and (8) into Eqs. (1)-(6) and neglecting the terms in square brackets yields

$$i\alpha\zeta_1 + D\zeta_3 + i\beta\zeta_5 = 0 \tag{9}$$

$$i(\alpha U + \beta W - \omega)\zeta_1 + \zeta_3 DU + i\alpha\zeta_4 - \frac{1}{R}(D^2 - \alpha^2 - \beta^2)\zeta_1 = 0 \tag{10}$$

$$i(\alpha U + \beta W - \omega)\zeta_3 + D\zeta_4 - \frac{1}{R}(D^2 - \alpha^2 - \beta^2)\zeta_3 = 0 \tag{11}$$

$$i(\alpha U + \beta W - \omega)\zeta_5 + \zeta_3 DW + i\beta\zeta_4 - \frac{1}{R}(D^2 - \alpha^2 - \beta^2)\zeta_5 = 0 \tag{12}$$

$$\zeta_1 = \zeta_3 = \zeta_5 = 0 \quad \text{at} \quad y = 0 \tag{13}$$

$$\zeta_n \to 0 \quad \text{as} \quad y \to \infty \tag{14}$$

where $D = \partial/\partial y$. For a given U(y) and W(y), Eqs. (9)-(14) constitute an eigenvalue problem, which yields a dispersion relation of the form

$$\omega = \omega(\alpha,\beta,R) \tag{15}$$

A number of techniques have been developed for solving this eigenvalue problem. These include shooting techniques, finite-difference methods, Galerkin methods, and collocation techniques using Chebyshev or Jacobi polynomials.

For the case of a two-dimensional mean flow and a two-dimensional disturbance, $W = 0$, $\zeta_5 = 0$ and $\beta = 0$, and Eqs. (9)-(14) reduce to

$$i\alpha\zeta_1 + D\zeta_3 = 0 \tag{16}$$

$$i(\alpha U - \omega)\zeta_1 + \zeta_3 DU + i\alpha\zeta_4 - \frac{1}{R}(D^2 - \alpha^2)\zeta_1 = 0 \tag{17}$$

$$i(\alpha U - \omega)\zeta_3 + D\zeta_4 - \frac{1}{R}(D^2 - \alpha^2)\zeta_3 = 0 \tag{18}$$

$$\zeta_1 = \zeta_3 = 0 \quad \text{at} \quad y = 0 \tag{19}$$

$$\zeta_n \to 0 \quad \text{as} \quad y \to \infty \tag{20}$$

Equations (16)-(20) can be combined to yield the Orr-Sommerfeld equation

$$(i\alpha R)^{-1}(D^2 - \alpha^2)^2\zeta_3 = (U - c)(D^2 - \alpha^2)\zeta_3 - \zeta_3 D^2U \tag{21}$$

subject to the boundary conditions

$$\zeta_3 = D\zeta_3 = 0 \quad \text{at} \quad y = 0 \tag{22}$$

$$\zeta_3,\ D\zeta_3 \to 0 \quad \text{as} \quad y \to \infty \tag{23}$$

where $c = \omega/\alpha$. The neutral stability curve calculated using either Eqs. (16)-(20) or Eqs. (21)-(23) are in good agreement with available experimental data as shown in Figure 1.

Recently, Smith[1] claimed the above methods to be "irrational" and developed multi-structured theories for treating this problem. He used a result from an "irrational theory" for the Blasius flow to observe that "the typical wavelength of the neutrally stable modes on the lower branch increases proportionally to $Re^{1/8}$ as $Re \to \infty$" and concluded that disturbances at the lower branch vary on a streamwise length $O(Re^{-3/8})$ and a time scale $O(Re^{-1/4})$ and hence they are governed by a triple-deck structure. Consequently, he let

$$x = 1 + \varepsilon^3 X, \quad t = \varepsilon^2 T \tag{24}$$

and

$$u,v,p \propto E = \exp[i\theta(x) - i\Omega T] \tag{25}$$

where $\varepsilon = Re^{-1/8}$ and

$$\Omega = \omega_1 + \varepsilon\omega_2 + \varepsilon^2\omega_3 + \varepsilon^3 \ln\varepsilon\ \omega_{4L} + \ldots \tag{26}$$

$$\frac{d\theta}{dX} = \alpha_1 + \varepsilon\alpha_2 + \varepsilon^2\alpha_3 + \varepsilon^3 \ln\varepsilon\ \alpha_{4L} + \ldots \tag{27}$$

Then, he expanded the variables in the three decks as follows:

Main Deck

$$u = [u_1 + \varepsilon u_2 + \varepsilon^2 u_3 + \varepsilon^3 \ln\varepsilon\ u_{4L} + \ldots]E \tag{28}$$

$$v = [\varepsilon v_1 + \varepsilon^2 v_2 + \varepsilon^3 v_3 + \varepsilon^4 \ln\varepsilon\ v_{4L} + \ldots]E \tag{29}$$

$$p = [\varepsilon p_1 + \varepsilon^2 p_2 + \varepsilon^3 p_3 + \varepsilon^4 \ln\varepsilon\ p_{4L} + \ldots]E \tag{30}$$

where

$$y = \varepsilon^4 Y, \quad Y = O(1)$$

Lower Deck

$$u = [U_1 + \varepsilon U_2 + \varepsilon^2 U_3 + \varepsilon^3 \ln\varepsilon\ U_{4L} + \ldots]E \tag{31}$$

$$v = [\varepsilon^2 V_1 + \varepsilon^3 V_2 + \varepsilon^4 V_3 + \varepsilon^5 \ln\varepsilon\ V_{4L} + \ldots]E \tag{32}$$

$$p = [\varepsilon P_1 + \varepsilon^2 P_2 + \varepsilon^3 P_3 + \varepsilon^4 \ln\varepsilon\ P_{4L} + \ldots]E \tag{33}$$

where

$$y = \varepsilon^5 Z, \quad Z = O(1)$$

Upper Deck

$$u = [\varepsilon\bar{u}_1 + \varepsilon^2\bar{u}_2 + \varepsilon^3\bar{u}_3 + \varepsilon^4 \ln\varepsilon\ \bar{u}_{4L} + \ldots]E \tag{34}$$

$$v = [\varepsilon\bar{v}_1 + \varepsilon^2\bar{v}_2 + \varepsilon^3\bar{v}_3 + \varepsilon^4 \ln\varepsilon\ \bar{v}_{4L} + \ldots]E \tag{35}$$

$$p = [\varepsilon\bar{p}_1 + \varepsilon^2\bar{p}_2 + \varepsilon^3\bar{p}_3 + \varepsilon^4 \ln\varepsilon\ \bar{p}_{4L} + \ldots]E \tag{36}$$

where

$$y = \varepsilon^3\bar{y}, \quad \bar{y} = O(1)$$

To account for the nonparallel effects, Smith had to include the next term in each of the expansions in Eqs. (26)-(36).

Substituting the above expansions into the parallel part of the disturbance equations (1)-(4) and boundary conditions (5) and (6), dropping the terms in square brackets, putting W = 0, separating coefficients of like powers of ε, and solving the resulting 36 equations, one obtains expressions for u, v, and p in the different decks. Matching the resulting expressions provides asymptotic expansions for u, v, and p. For the neutral stability curve, Smith obtained

$$F_n = \omega_n = 0.995\, R_\delta^{-3/2} \left[1 + 1.597\, R_\delta^{-1/4} + 10.02\, R_\delta^{-1/2} + 0.988\, R_\delta^{-3/4} \ln R_\delta + \ldots\right]$$

where

$$R_\delta = 1.7208\sqrt{x\mathrm{Re}}$$

This expression is in fair agreement with the lower branch of the neutral stability curve for large Re. However, its accuracy deteriorates as Re decreases. In fact, it does not predict a minimum critical Reynolds number.

Bodonyi and Smith[2] inspected the results of the "irrational theory" to observe that "the stability properties of the Blasius boundary layer are governed by the behavior on the streamwise length scale $O(\mathrm{Re}^{-9/20})$ as far as the upper branch of the neutral curve is concerned". Consequently, they used $\sigma = \mathrm{Re}^{-1/20}$ as their perturbation parameter and used the streamwise scale X defined by $x = 1 + \sigma^9 X$ and the time scale $t = \sigma^8 T$. This choice leads to a five-zoned structure. To account for the nonparallel effects, one needs to carry out the expansion to $O(\sigma^9)$. In view of the logarithmic terms, one needs 13 terms in the expansion. With three variables and five decks, one needs to derive and solve 195 equations and then match the results. Bodonyi and Smith gave up after four terms. Their calculated neutral stability curve, which is intended to approximate the upper branch, is below the lower branch!! We note that for an $\mathrm{Re} = 10^6$, $\sigma \approx 0.5$, which is not small.

For the case of an accelerating boundary layer, Smith and Bodonyi[3] assumed a streamwise variation $O(\mathrm{Re}^{-5/12})$ and a time scale $O(\mathrm{Re}^{-1/3})$ near the upper branch of the neutral stability curve. Using this streamwise variation leads to a five-zoned structure, with the nonparallel effects appearing at $O(\mathrm{Re}^{-5/12})$.

It should be noted that the parallel flow assumption breaks down miserably for the case of Görtler instability. Floryan and Saric[4] and Ragab and Nayfeh[5] derived the appropriate equations for Görtler instability for the cases of zero and nonzero pressure gradients, respectively. Hall[6] questioned the solution of the resulting equations using a normal-mode approach and suggested solving them as an initial-value problem.

Nonparallel Problem

A better agreement between the theoretical and experimental results can be obtained by accounting for the influence of the nonparallel terms[7-10]. To this end, we can use either the method of averaging or the WKB approximation or the method of multiple scales[11,12]. In this paper, we use the method of averaging and let

$$u = A(x,z,t)\zeta_1(y,x)e^{i\theta}, \quad v = A(x,z,t)\zeta_3(y,x)e^{i\theta} \tag{37}$$

$$p = A(x,z,t)\zeta_4(y,x)e^{i\theta}, \quad w = A(x,z,t)\zeta_5(y,x)e^{i\theta} \tag{38}$$

where A is a slowly varying function of x and t,

$$\frac{\partial\theta}{\partial x} = \alpha(x), \quad \frac{\partial\theta}{\partial z} = \beta, \quad \frac{\partial\theta}{\partial t} = -\omega \tag{39}$$

and the ζ_n are given by Eqs. (9)-(14). After a straightforward but lengthy algebra, one obtains[13-17]

$$h_1 \frac{\partial A}{\partial t} + h_2 \frac{\partial A}{\partial x} + h_3 \frac{\partial A}{\partial z} = h_4 A \tag{40}$$

where

$$h_1 = \int_0^\infty (\zeta_1\zeta_1^\star + \zeta_3\zeta_3^\star + \zeta_5\zeta_5^\star)dy \tag{41}$$

$$h_2 = \int_0^\infty [\zeta_1\zeta_4^\star + \zeta_4\zeta_1^\star + U(\zeta_1\zeta_1^\star + \zeta_3\zeta_3^\star + \zeta_5\zeta_5^\star)]dy \tag{42}$$

$$h_3 = \int_0^\infty [\zeta_5\zeta_4^\star + \zeta_4\zeta_5^\star + W(\zeta_1\zeta_1^\star + \zeta_3\zeta_3^\star + \zeta_5\zeta_5^\star)]dy \tag{43}$$

$$\begin{aligned} h_4 = - \int_0^\infty & [\frac{\partial\zeta_1}{\partial x}\zeta_4^\star + \frac{\partial\zeta_5}{\partial z}\zeta_4^\star + \frac{\partial\zeta_4}{\partial x}\zeta_1^\star + \frac{\partial\zeta_4}{\partial z}\zeta_5^\star + U(\frac{\partial\zeta_1}{\partial x}\zeta_1^\star \\ & + \frac{\partial\zeta_3}{\partial x}\zeta_3^\star + \frac{\partial\zeta_5}{\partial x}\zeta_5^\star) + W(\frac{\partial\zeta_1}{\partial z}\zeta_1^\star + \frac{\partial\zeta_3}{\partial z}\zeta_3^\star + \frac{\partial\zeta_5}{\partial z}\zeta_5^\star) \\ & + (\zeta_1\frac{\partial U}{\partial x} + VD\zeta_1 + \zeta_5\frac{\partial W}{\partial z})\zeta_1^\star + (VD\zeta_3 + \zeta_3 DV + \zeta_3\frac{\partial V}{\partial x} \\ & + \zeta_5\frac{\partial V}{\partial z})\zeta_3^\star + (\zeta_1\frac{\partial W}{\partial x} + VD\zeta_5 + \zeta_5\frac{\partial W}{\partial z})\zeta_5^\star]dy \end{aligned} \tag{44}$$

where the $\zeta_n^\star$ are solutions of the adjoint homogeneous problem. Equation (40) can be rewritten as[13]

$$\frac{\partial A}{\partial t} + \omega_\alpha \frac{\partial A}{\partial x} + \omega_\beta \frac{\partial A}{\partial z} = hA \tag{45}$$

where

$$\omega_\alpha = \frac{h_2}{h_1} \, , \; \omega_\beta = \frac{h_3}{h_1} \, , \; h = \frac{h_4}{h_1}$$

Here, ω_α and ω_β are the components of the group velocity in the streamwise direction.

Equation (45) describes the propagation of a wavepacket centered at the frequency ω_r and the wavenumbers α_r and β_r, where the subscript r stands for the real part. Nayfeh[14] showed that for a physical problem, ω_α and ω_β in Eq. (45) must be real.

For a monochromatic wave, $\partial A/\partial t = 0$ and Eq. (45) reduces to

$$\omega_\alpha \frac{\partial A}{\partial x} + \omega_\beta \frac{\partial A}{\partial z} = hA \tag{46}$$

For a physical problem, Nayfeh[14] showed that $\omega_\beta/\omega_\alpha$ must be real. For the case of a parallel mean flow, this condition reduces to $d\alpha/d\beta$ being real, which was obtained by Nayfeh[14] and Cebici and Stewartson[18] using the saddle-point method.

Two-Dimensional Mean Flows

For the case of a monochromatic wave, $\partial A/\partial t = 0$ and Eq. (40) yields

$$A = A_o \exp[\int_{x_o} (h_4/h_2)dx] \tag{47}$$

where A_o is a constant. Hence,

$$u \approx A_o \zeta_1(y,x) \exp[i(\int \alpha dx - \omega t) + \int (\frac{h_4}{h_2})dx] \tag{48}$$

Consequently, the growth rate

$$\sigma = \text{Real } [\frac{\partial}{\partial x} (\ln u)]$$

is given by

$$\sigma = -\alpha_i + \text{Real } (\frac{h_4}{h_2}) + \text{Real } [\frac{\partial}{\partial x} (\ln \zeta_1)] \tag{49}$$

The first term is the quasiparallel contribution, whereas the last two terms are due to nonparallelism. It should be noted that the last term produces a variaton in the growth rate across the boundary layer.

Since ζ_1 is a function of y and, in general, distorts with streamwise distance, one may term stable disturbances unstable or vice versa. Morever, a different growth rate would be obtained if one replaces u with another variable. For example, using v or p or w, one obtains the growth rates

$$\sigma = -\alpha_i + \text{Real } (\frac{h_4}{h_2}) + \text{Real}[\frac{\partial}{\partial x} (\ln \zeta_m)] \tag{50}$$

where m = 3, 4, and 5, respectively. This raises the questions "What is meant by stability of a boundary layer?" If the stability criterion is based on σ, then which σ should be used? If one uses an N factor to compare the stabilizing or destabilizing influences of certain modifications to the boundary layer, then the contribution of the last term will not be significant.

In the case of parallel flows, the last terms in Eqs. (49) and (50) vanish and the growth rate is unique and independent of the variable being used. Consequently, one can speak of neutral disturbances or neutral stability curve given by the locus of $\alpha_i(R,\omega) = 0$. However, in the case of a nonparallel flow, the neutral stability is given by $\sigma(R,\omega) = 0$ and depends on the flow variable used to calculate the growth rate and the distance from the wall. To compare the analytical results with experimental data, one needs to make the calculations in the same manner in which the measurements are taken. Available experimental stability studies almost exclusively use hot-wire aneomometers. Usually, they measure the rms value $|u|$ of the streamwise velocity component u and use it to define the growth rate. Figure 2 compares the neutral stability curves calculated using $|u|$ and $\bar{u}^2 + \bar{v}^2$ with the experimental data of Kachanov, Kozlov and Levchenko[19*]. Since the experiment measured $|u|$, the calculations of Saric and Nayfeh[10], which were based on $|u|$, are in better agreement with the experimental data than the calculations of Bouthier[7], which were based on $\bar{u}^2 + \bar{v}^2$. Moreover, the growth rate is singular at the locations where $|u| = 0$. Figure 2 shows also that the calculated locations of the singular growth rates are in good agreement with the experimental results.

Some of the available experimental studies follow the maxima of $|u|$ whereas others follow a constant boundary-layer similarity variable η. Saric and Nayfeh[9,10] found that the contribution of the last terms in Eqs. (49) and (50) are significant if one follows a constant η whereas their contributions are negligible if one follows the maxima of $|u|$, yielding

$$\sigma = -\alpha_i + \text{Real}\left(\frac{h_4}{h_2}\right) \tag{51}$$

The neutral stability curve calculated by Saric and Nayfeh[9] using Eq. (51), and shown in Figure 1, is in very good agreement with the experimental data that follow the maxima of $|u|$, except near the minimum critical Reynolds number where the data may be suspect. However, in the case of experiments conducted by following trajectories of constant η such as those of Ross et al.[20], the effect of the distortion of the eigenfunction cancels the nonparallel effects, resulting in a better agreement between their data and the results of quasiparallel theory.

Saric and Nayfeh[10] made other comparisons of the growth rates calculated using Eq. (51) with the experimental data of Strazisar, Prahl and Reshotko[21] and Kachanov, Kozlov and Levchenko[19]. Strazisar et al.

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* Also, private communication, June 1976.

conducted their experiments in a water tunnel and performed their measurements at the maxima of $|u|$, thereby minimizing the effects of the distortion of the eigenfunction. They measured the amplification rate as a function of the frequency at different locations on the plate, corresponding to different Reynolds numbers. Figure 3 shows a good agreement between the theoretical and experimental results. Kachanov et al. also followed the maxima of $|u|$ and measured the amplification factor $a = |u|/|u_0|$, where $|u_0|$ is the rms value of u at the first neutral point. Figure 4 shows a good agreement between the theoretical results calculated by Saric and Nayfeh[10] using Eq. (51) and the experimental results.

The present nonparallel analysis was extended by El-Hady and Nayfeh[22] to the case of two-dimensional compressible boundary layers, by Nayfeh[14] to the case of three-dimensional compressible boundary layers, and by Nayfeh and El-Hady[23] and Asrar and Nayfeh[24] to heated boundary layers.

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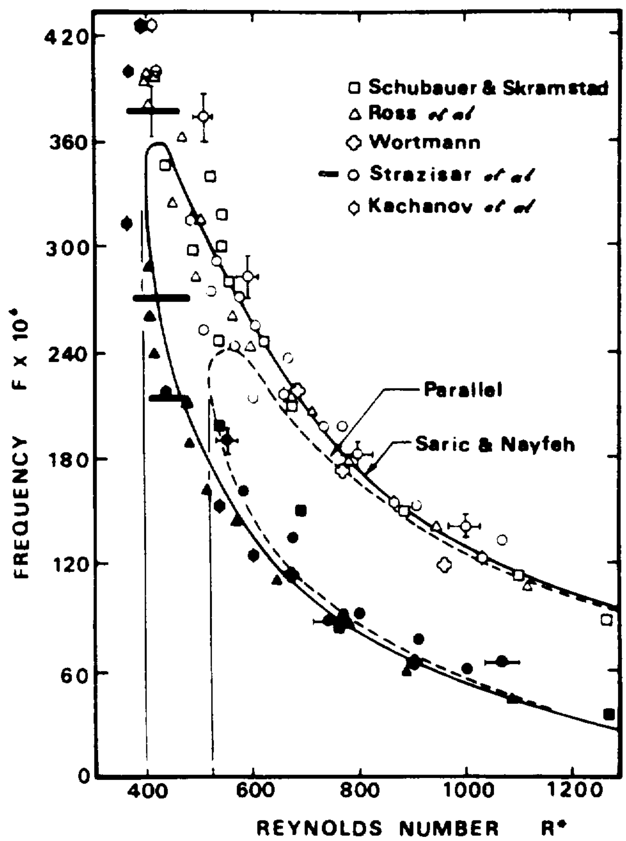

Figure 1. Neutral stability curve for Blasius boundary layer. Solid symbols are Branch I experimental points. Open symbols are Branch II. The critical Reynolds number is 400 for nonparallel calculations, 520 for parallel calculations (Saric and Nayfeh[9]).

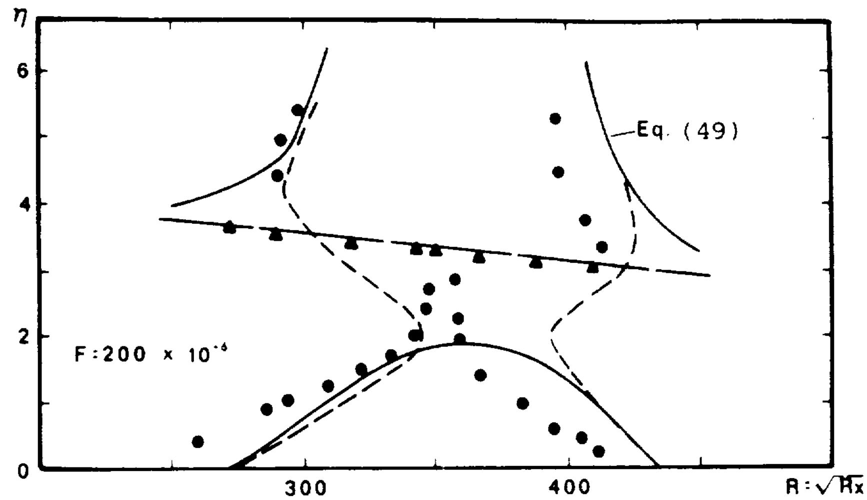

Figure 2. Vertical variation of neutral stability points at F = 200×10^{-6}. Experimental points from Kachanov, Kozlov and Levchenko[17]. Dashed lines are calculations of Bouthier[7] based on energy. Solid lines are calculations of Saric and Nayfeh[10] based on $|u|$. Streamwise position is the Reynolds number based on δ_r which is the ξ of Refs. 7 and 17. Solid triangles give the locus of $|u| = 0$ and the broken line is the calculation[10] for $|u| = 0$.

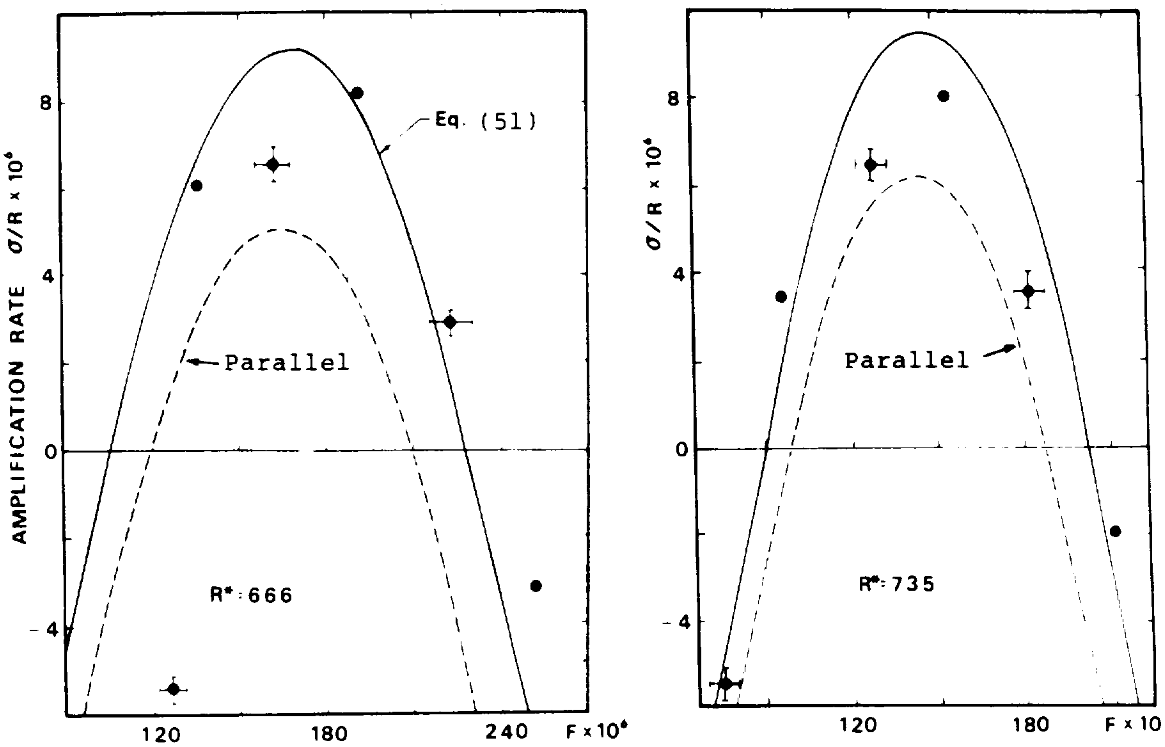

Figure 3. Amplification rate as a function of frequency: Theory of Saric and Nayfeh; Experimental data of Strazisar, Prahl and Reshotko following maximum of $|u|$.

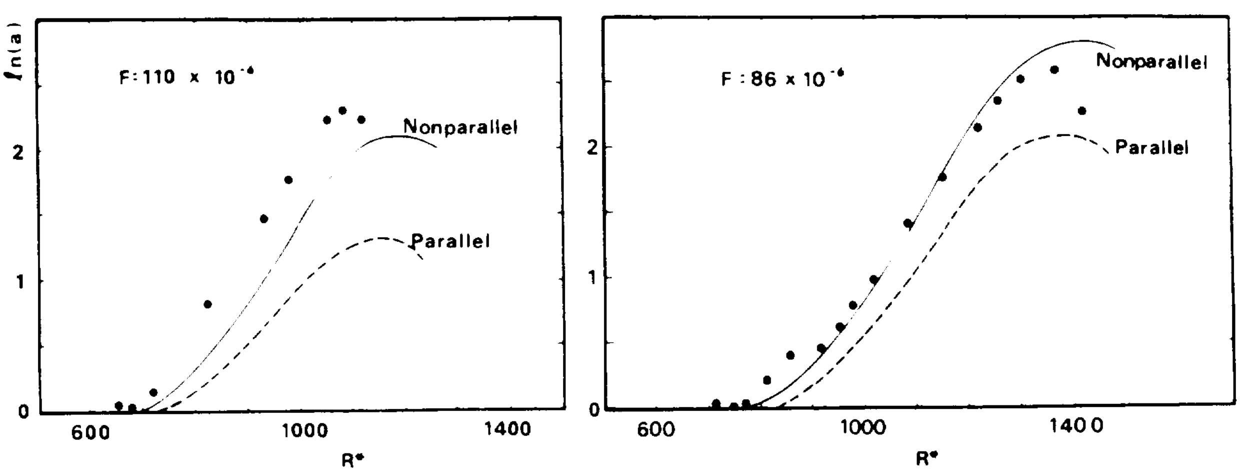

Figure 4. Amplification factor a as a function of streamwise position at F = 110×10^{-6} and F = 86×10^{-6}. Experimental points from Kachanov, Kozlov and Levchenko following maximum of $|u|$ and nonparallel results of Saric and Nayfeh based on following maximum of $|u|$.