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INTERACTION OF TOLLMIEN-SCHLICHTING WAVES AND GÖRTLER VORTICES

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List of Symbols

$\underline{\mu}_B(x,y,z)$	mean velocity
$\delta\underline{\mu}'\exp[i\{\alpha x + \beta y - \Omega t\}]$	perturbation velocity
α	complex streamwise wavenumber
β	spanwise wavenumber of Görtler perturbation
k	spanwise wavenumber of TS wave
Ω	temporal frequency
T	Taylor number
TS	Tollmien-Schlichting
M^{o}	angle between TS wave propagation and the mean flow direction

There are many fluid flows of practical interest where transition can be caused by competing hydrodynamic instabilities. Thus in three-dimensional boundary-layer flows over curved walls, instability might be caused by Tollmien-Schlichting waves, Görtler vortices or crossflow vortices. If a particular type of instability is suppressed by some means, there is the possibility that another one might be stimulated. Hence it is important to understand the mechanisms by which these different instabilities interact. Here we shall discuss some properties of the interaction which can take place between Görtler vortices and Tollmien-Schlichting waves.

INTERACTION OF TOLLMIEN-SCHLICHTING WAVES AND GÖRTLER VORTICES

1. Large amplitude Görtler vortices, small linear Tollmien-Schlichting (TS) waves

2. Weakly nonlinear interaction of small amplitude Görtler vortices and small amplitude Tollmien-Schlichting waves

3. Large amplitude TS waves, 3-D breakdown induced by unsteady Görtler instability

We discuss the linear stability of large amplitude Görtler vortices to Tollmien-Schlichting waves. In order to avoid technical difficulties associated with boundary-layer growth, we shall concentrate on fully developed flows in curved channels. However, the corresponding external flow problem can be treated in essentially the same way and gives similar results. Some discussion will also be given about the secondary instability of large amplitude Tollmien-Schlichting waves to Görtler vortices. In this case, instability occurs in the presence of convex or concave curvature.

Secondary instabilities of large Görtler vortices

- Basic state is now a spatially periodic flow in z direction. We calculate this flow by integrating the Navier-Stokes equations numerically
- In external flows basic state is a function of x, y, z
- Now perturb the basic state by writing

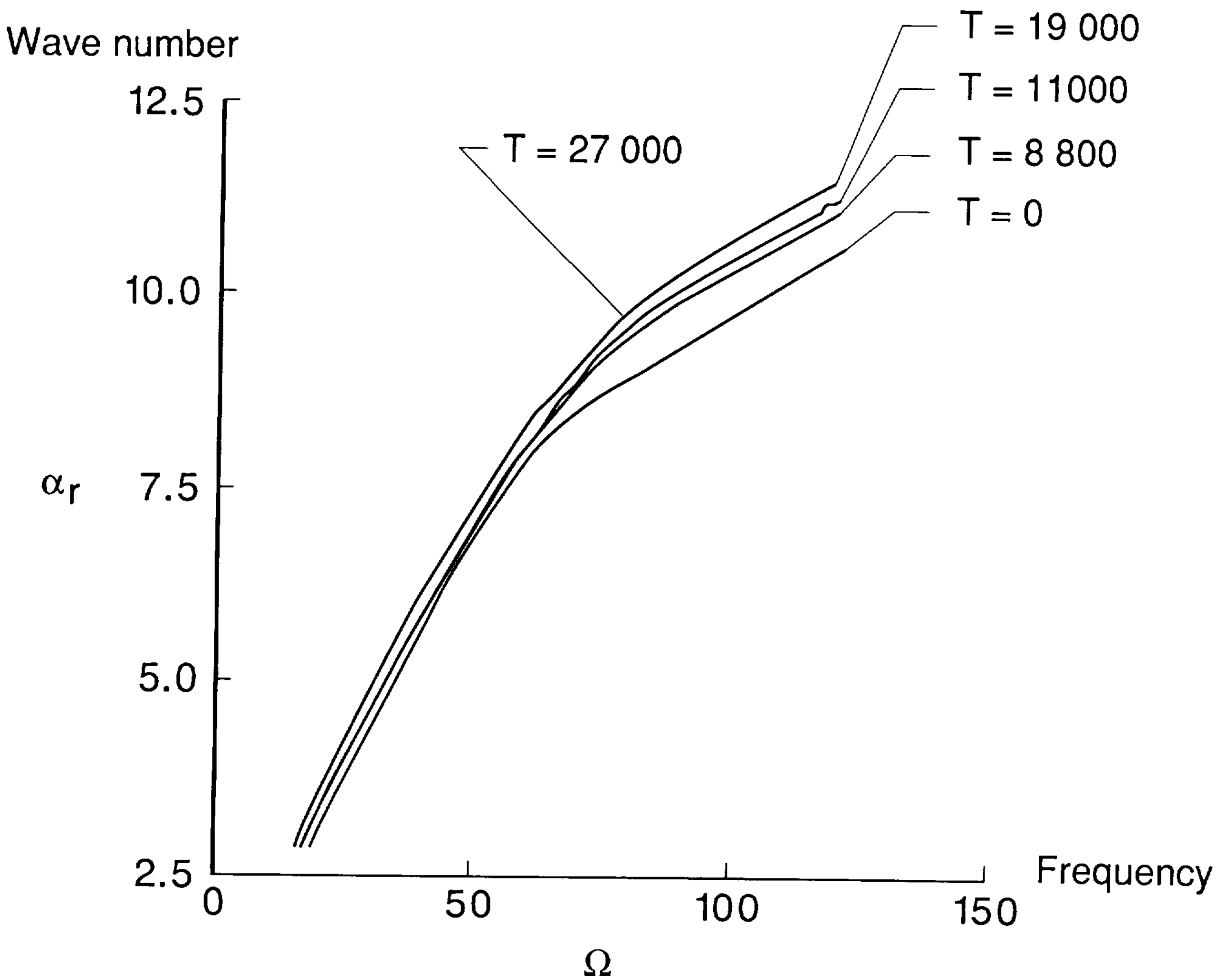
$$\underline{u} = \underline{u}_B(x, y, z) + \delta \underline{u}' \exp\left[i\{\alpha x + \beta y - \Omega t\}\right]$$

- Solve the linearized equations at high Reynolds numbers using Triple Deck Theory
- For spatially varying flows write

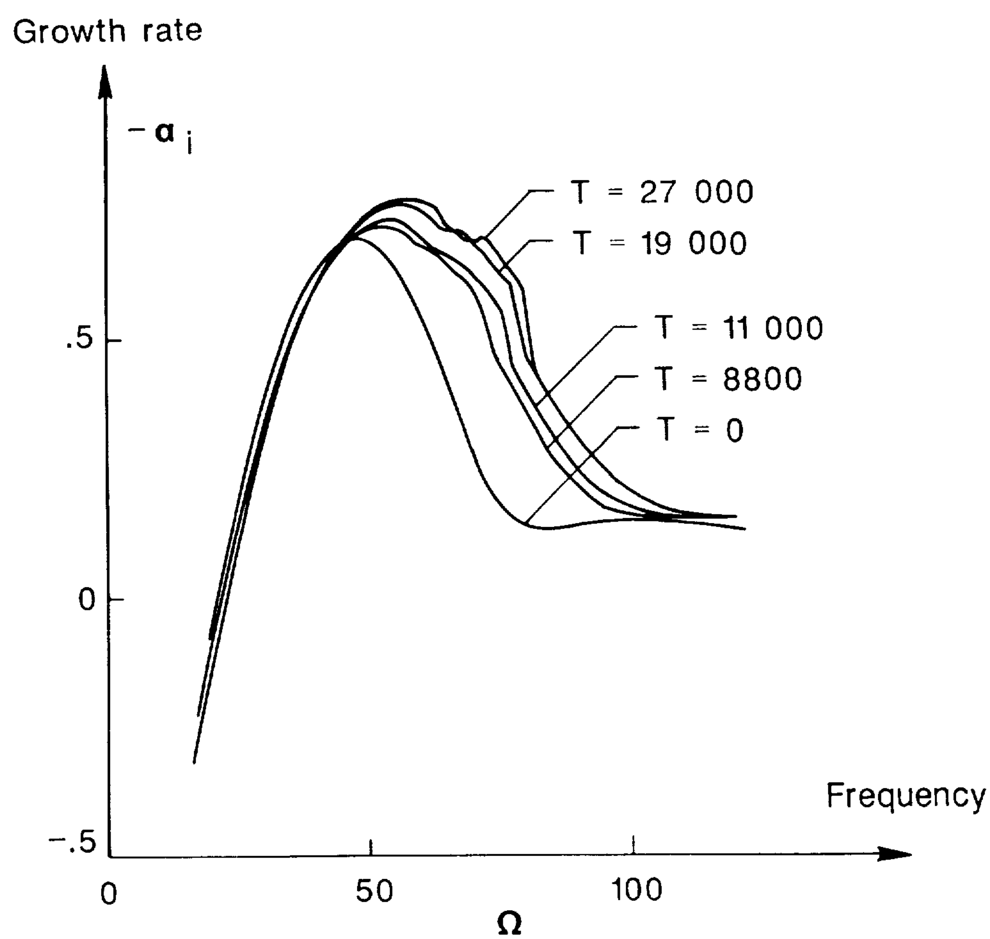
$$\alpha x = \int^{x} \alpha(x')dx'$$

and calculate α as the disturbance moves downstream

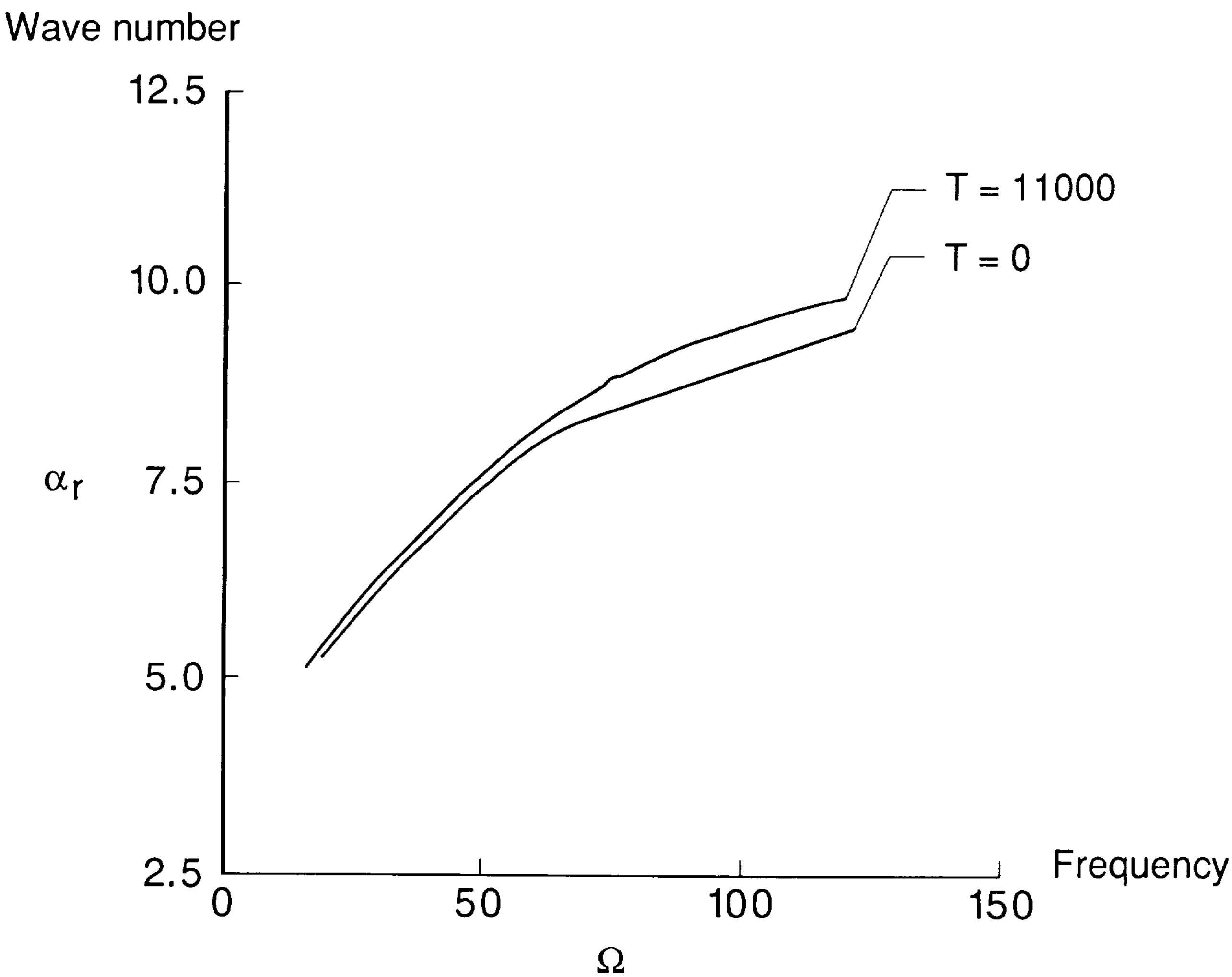
Here we show the dependence of the wave number of neutral TS waves on frequency at different values of the Taylor number T. The results for T = 0 correspond to zero curvature. The wave number at a given frequency increases monotonically with the curvature.



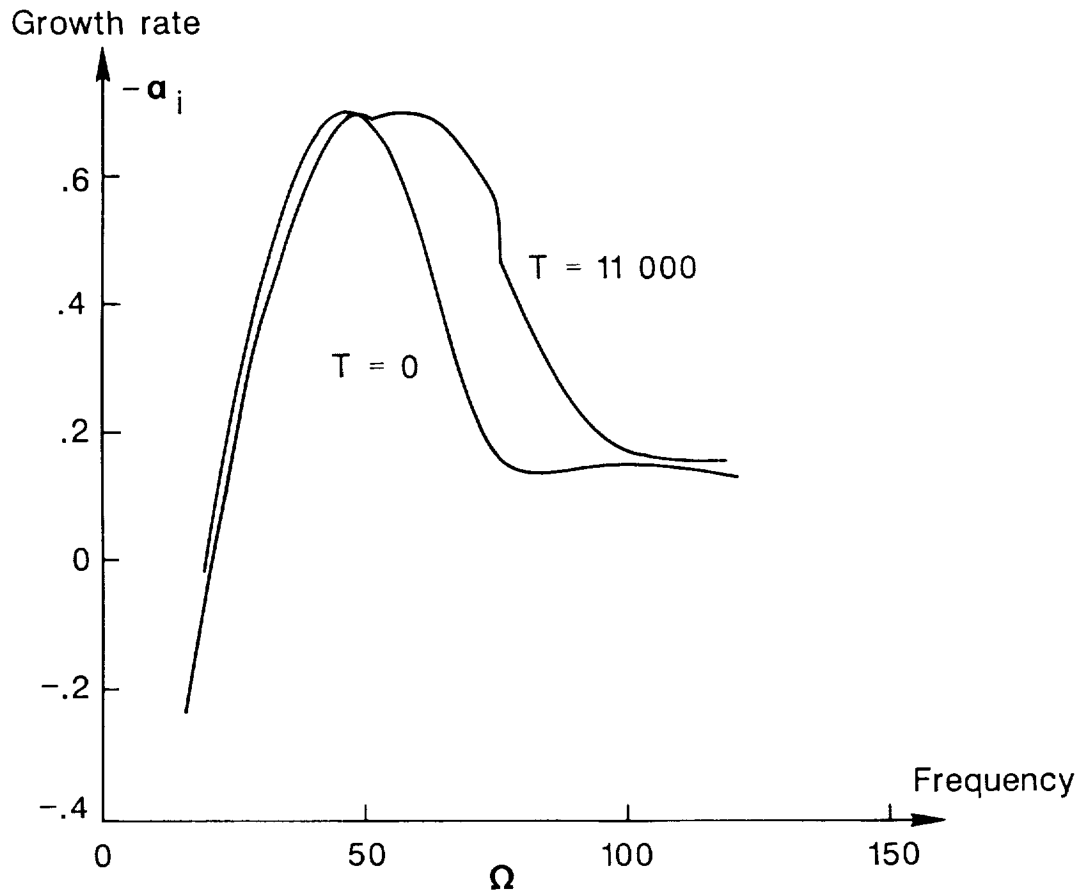
This figure shows the growth rate of unstable TS waves as a function of frequency at different Taylor numbers. Note the significant destabilization effect of the vortices on the growth rate. At the larger values of T the area under the unstable part of the curve is typically increased by 40-50%.



The wave numbers for a 3-D TS wave at $T = 0$ and $T = 11000$ are shown below. Note the increase in the wave number produced by the curvature. Calculations at different values of T produce similar results.



This figure shows the growth rate of a 3-D TS wave at $T = 11000$ at different values of the frequency. The vortex flow again destabilizes the TS wave and the unstable area under the curve is again increased by 40-50%. This result is typical of the effect of vortices on 3-D TS waves.

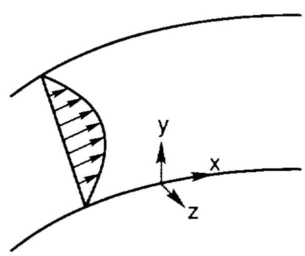


The interaction of a Görtler vortex of spanwise wave number β with a pair of skewed Tollmien-Schlichting waves with wave numbers α and $\pm k$ in the x and z directions was considered. A particularly strong interaction was found to occur when $\beta = k/2$. In fact there is a "resonant triad" interaction between the different modes in this case. The amplitudes a, b, and c of the Tollmien-Schlichting waves over the Görtler vortex were found to satisfy the equations

$$\frac{da}{dt} = e\,a + f_0 b_c,$$
$$\frac{db}{dt} = g_0 b + h_0 a\bar{c},$$
$$\frac{dc}{dt} = g_1 c + h_1 a\bar{b},$$

where e, f, g_0, h_0, g_1, h_1 are constants. These constants were calculated numerically and determine the nature of the solutions to these equations. For the values of these constants appropriate to channel flows, we find that any solution of these equations terminates in a singularity at a finite time. Physically this means that the disturbance amplitude becomes unbounded at that time.

Weakly nonlinear interaction of TS and Görtler



TS waves ~ $e^{i\{\alpha x \pm kz - \Omega t\}}$

Görtler vortices ~ $e^{i\beta z}$

Triad interactions involving 2 TS waves and a Görtler vortex dominate nonlinear growth. TS waves are inclined at an angle M° to flow direction. Interaction governed by triple deck theory

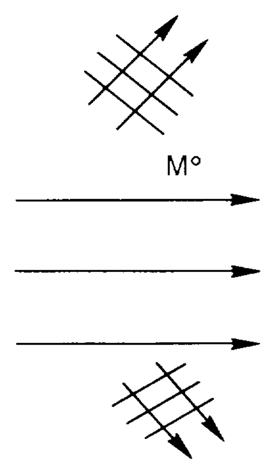
In fact, at this stage a much stronger interaction takes place. The interaction is governed by a coupled partial differential system and an ordinary integro-differential equation. The nature of the solutions to this system again depends on numerical values of the constants appearing in this equation. The resulting behavior is characterized in terms of M, the angle between the direction of propagation of the waves and the flow direction. If M is less than 41.6°, a much weaker blow-up occurs in an infinite time. Thus, the system stays in the smaller amplitude state for a much longer time if $M < 41.6$. Indeed the strong interaction for $M > 41.6^{\circ}$ can take place in the absence of curvature. We conclude that in shear flows this is a nonlinear interaction mechanism involving two skewed Tollmien-Schlichting waves and a longitudinal vortex which produces unbounded growth of the disturbance after a finite time in a channel flow or after a finite distance in an external flow.

Stage 1

- Small amplitude TS and Görtler interact and develop a finite time singularity

Stage 2

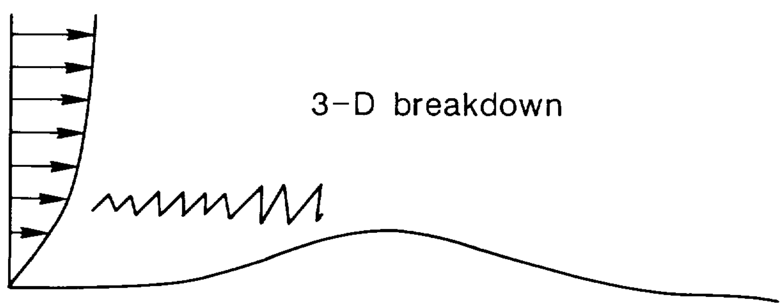
- Large amplitude disturbances, blow up if $M > 41.6$
- Curvature not needed, mechanism occurs in straight channels & flat plate boundary layers



Now let us consider the instability of large amplitude Tollmien-Schlichting waves to Görtler vortices. The linearized form of the Görtler equations applies to interacting boundary layers or Triple-Deck flows. Thus since it has been shown that large amplitude two and three-dimensional Tollmien-Schlichting waves are governed by Triple Deck theory, we can use these equations to investigate the instability of these flows.

The surprising feature of the large amplitude structure of Tollmien-Schlichting waves is that they have a wall layer essentially identical to a Stokes layer induced by oscillating a flat plate in a viscous fluid. However, it was shown that in the presence of curvature, Stokes layers are unstable to Görtler vortices. The vortices are confined to the Stokes layer and have axes aligned with the flow direction. Thus this instability mechanism occurs for large amplitude Tollmien-Schlichting waves. The instability can occur for either convex or concave curvature since for time-periodic flows there is no analogue of Rayleigh's criterion for the centrifugal instability of curved flows. It suffices to say that at moderate value of the curvature even relatively small amplitude Tollmien-Schlichting waves break up in this way.

Sublayer instabilities of large amplitude TS waves interacting with surface curvature. Convex or concave curvature causes breakdown.



Use Smith-Burggraf theory to calculate large amplitude 2- or 3-D TS waves. The Stokes sublayer of these waves is unstable in presence of convex or concave curvature.