552502
P22

NASA Contractor Report 181931
ICASE Report No. 89–74

ICASE

NUMERICAL STUDY OF THREE-DIMENSIONAL SPATIAL INSTABILITY OF A SUPERSONIC FLAT PLATE BOUNDARY LAYER

L. Maestrello
A. Bayliss
R. Krishnan

Contract Nos. NAS1–18107, NAS1–18605
October 1989

Institute for Computer Applications in Science and Engineering
NASA Langley Research Center
Hampton, Virginia 23665–5225

Operated by the Universities Space Research Association

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665-5225

NUMERICAL STUDY OF THREE-DIMENSIONAL SPATIAL INSTABILITY OF A SUPERSONIC FLAT PLATE BOUNDARY LAYER

L. Maestrello
NASA Langley Research Center, Hampton, Virginia

A. Bayliss
Northwestern University, Evanston, Illinois

and

R. Krishnan
Vigyan Research Assoc., Inc., Hampton, Virginia

ABSTRACT

The behavior of spatially growing three-dimensional waves in a supersonic boundary layer is studied. The objectives are to validate our computer code by comparing with experiments and parallel and non-parallel linear stability theories for $M_\infty = 4.5$, $\sqrt{\mathrm{Re/ft}} = 1550$ and at $T_{t,\infty} = 100°\mathrm{F}$ and then use the program to study nonlinear effects. The three-dimensional unsteady Navier-Stokes equations are solved by a finite difference method which is fourth-order and second-order accurate in the convection and viscous terms respectively, and second-order accurate in time. Spanwise periodicity is assumed. The inflow disturbance is composed of eigenfunctions from linear stability theory. Computed results for small amplitudes of this disturbance agree well with linear theory and experiment for several frequencies. By increasing the amplitude of the inflow disturbance, nonlinear effects in the form of a relaxation type oscillation of the time signal of ρu are observed.

[1]This research was supported in part by the National Aeronautics and Space Administration under NASA Contract Nos. NAS1-18107 and NAS1-18605 while the second author was in residence at the Institute for Computer Applications in Science and Engineering (ICASE), NASA Langley Research Center, Hampton, VA 23665.

1. INTRODUCTION

The objective of this paper is to study the linear and nonlinear behavior of spatially growing waves in a supersonic flat plate boundary layer. The behavior of these waves will be obtained by numerically solving the three-dimensional, unsteady Navier-Stokes equations using a fourth-order accurate finite difference scheme. The numerical results will demonstrate a linear regime where the growth is well predicted by linear stability theory and a nonlinear regime characterized by the generation of higher harmonics associated with the appearance of relaxation type oscillations.

The numerical computations will be conducted using a two-dimensional mean flow over a flat plate. The parameters of the mean flow will be chosen to agree with experimental measurements (Kendall [1967]). The mean flow will be perturbed by a disturbance at the inflow boundary. This disturbance will be taken as an eigenfunction obtained from linear stability theory (El Hady [1981]). Due to the spatial growth of the waves, nonlinear behavior can be obtained in two ways: either by increasing the amplitude of the disturbance at inflow or by following the disturbance for a longer distance downstream. Both mechanisms will be noted here.

An extensive discussion of linear stability theory can be found in Mack [1969]. The results in Mack [1969] were obtained by assuming parallel flow and solving the Orr-Sommerfeld equations for the vertically varying eigenfunction. Nonparallel effects can be modeled by allowing the growth rate to vary with the longitudinal coordinate x. Nonlinear effects cannot be accounted for in this theory. The growth rates predicted by linear stability theory were compared with experimentally measured growth rates of Kendall [1967] and found to be in fair agreement for a flow at Mach number 4.5 (the mean flow used in the numerical computations). An approximation to the nonparallel effect of the same Mach number has been investigated by El Hady [1981] using a multiple scale expansion. His results are also in fair agreement with Kendall's experiment and Mack's analysis.

Nonlinear spatial instability was studied for subsonic flows in two dimensions by Bayliss, et al. [1985, 1968], in three dimensions by Parikh et al [1987], and for incompressible flows in three dimensions by Fasel, et al. [1987] and Murdock [1986]. The computations in Bayliss, et al. [1986] demonstrated a progressive sharpening of the time signals as the disturbance propagates downstream. At a sufficiently large downstream location, the fluctuating disturbances take on the character of relaxation oscillations. Similar results are shown here for the three- dimensional computations in supersonic flow. The computations also show a progressive clustering of vorticity contours as the wave propagates downstream and an uplifting of vorticity away from the wall. Ultimately the nonlinear growth will saturate and the flow will undergo transition.

In this paper we study the nonlinear distortion, occurring prior to transition, with increasing spatial distance. Computations of temporally growing waves assuming periodicity in the streamwise direction have been reported in Erlebacher and Hussaini [1986]. The study of spatially growing instability waves as conducted here will permit the study of nonparallel effects on the nonlinear growth. In addition, the growth of the disturbance will be generated in a more physically realizable manner. The additional resolution requirements needed to resolve the spatial growth are severe and this limits the degree of nonlinearity that can be reliably computed. In Section 2, we describe the numerical procedure, and in Section 3, we discuss our numerical results.

2. COMPUTATIONAL METHOD

The numerical method uses an explicit finite difference scheme to solve the unsteady Navier-Stokes equations in conservation form. The finite difference scheme is fourth order accurate in the spatial coordinates and second order accurate in time. The scheme is a generalization of MacCormack's scheme due to Gottlieb and Turkel [1976] and has been previously used in the study of two-dimensional spatially unstable waves (Bayliss, et al. [1985, 1986]).

The Navier-Stokes equations can be written as

$$w_t = F_x + G_y + H_z \tag{2.1}$$

where w is the vector $(\rho, \rho u, \rho v, \rho w, E)^T$, ρ is the density, u, v, and w are the streamwise, vertical, and spanwise components of the velocity vector and E is the internal energy. The functional form of the flux functions F, G, and H are well-known and for the sake of brevity will not be given here. For the scalar equation

$$u_t = f_y$$

we consider the predictor-corrector scheme to compute $u_j^{n+1} \sim u((n+1)\Delta t, jx)$

$$\begin{aligned} \hat{u}_j &= u_j^n + \frac{\Delta t}{6\Delta x}[-7f_j + 8f_{j+1} - f_{j+2}] \\ u_j^{n+1} &= \frac{1}{2}[u_j^n + \hat{u}_j + \frac{\Delta t}{6\Delta x}[7\hat{f}_j - 8\hat{f}_{j-1} + \hat{f}_{j-2}]] \end{aligned} \tag{2.2}$$

where $\hat{f}_j = f(\hat{u}_j)$. It is known that if (2.2) is alternated with its symmetric variant, the resulting scheme is fourth order accurate provided $\Delta t = 0(\Delta x^2)$. For the case of flux functions which depend on derivatives of u, (e.g., as in (2.2)), the scheme is fourth order accurate on the inviscid terms and second order accurate on the viscous terms.

In the numerical method used here, the scheme (2.2) is implemented with operator splitting, Strang [1968]. That is, if L_x denoted the solution (via 2.2)) to the one-dimensional equation

$$w_t + f_x = 0 \tag{2.3}$$

and L_y, L_z are defined similarly, then the solution is advanced two timesteps via the formula

$$w^{n+2} = BL_xL_yL_zBL_zL_yL_xw^n \tag{2.4}$$

where the operator B denotes the boundary conditions (discussed below). The resulting scheme is fourth order accurate provided the timestep is sufficiently small. A comparison of this scheme with the second order MacCormack schemes for two-dimensional, spatially-unstable waves was presented in Bayliss, et al. [1985] and indicated a significant improvement in accuracy for the higher order scheme.

The solution is obtained in a rectangular box illustrated by Figure 1. Spanwise periodicity is assumed. At the wall, the no-slip condition and a wall temperature T_w are imposed, i.e.,

$$\begin{aligned} u &= v = 0 \\ T &= T_w. \end{aligned} \tag{2.5}$$

At the inflow boundary, the field is imposed. The inflow data is of the form

$$w = w_0 + \varepsilon \text{ Real Part } [\rho^{i(\omega t+\beta z)}\xi(y)]. \tag{2.6}$$

Here, w_0 is the mean state, a spreading two-dimensional boundary layer computed in a previous calculation. The eigenfunction $\xi(y)$ is obtained from the stability code of El Hady [1981] and the frequency ω and spanwise wave number β are specified. The parameter ϵ measures the strength of the inflow disturbance. At the outflow boundary, the solution is extrapolated from the interior. At both boundaries there is a small subsonic region near the wall where one less boundary condition should be imposed at the inflow and one boundary condition should be imposed at the outflow. These segments are treated by using one-dimensional normal characteristics described in Bayliss and Turkel [1982] and similar to the treatment of the upper boundary described immediately below.

The upper boundary is treated as a subsonic outflow boundary as v is small and positive. At this boundary, the outgoing normal characteristic variable $(p - \rho cv)$ based on linearizing the inviscid equations about the current state is prescribed as its initial value. The remaining characteristic variables are extrapolated from the interior. This approach was used in Bayliss and Turkel et al. [1982] and found to be effective in allowing the upper boundary to be brought closer to the edge of the boundary layer.

For the computations presented here, the vertical extent of the computational domain is approximately $5\delta_I$ where δ_I is the boundary layer thickness at the inflow.

All of the computations presented here were obtained with a mean flow with freestream Mach number $M_\infty = 4.5$, total temperature $T_{t_\infty} = 100°\text{F}$ and Reynolds number per foot, Re/ft $= 2.425 \times 10^6$. This corresponded to the experimental conditions of Kendall. The inflow boundary was one foot from the leading edge. The Reynolds number based on boundary layer thickness Re_{δ^*} was 17900. The wall temperature T_w was 110.8°R.

3. NUMERICAL RESULTS AND DISCUSSION

We first consider inflow disturbances of small amplitude so as to remain in the linear regime. This enables a study of complete nonparallel effects and a comparison or our results to linear stability theory and also to the experimental measurements of Kendall [1967].

We first consider an inflow disturbance level of .075% of the free stream. One measure of the solution is the growth of the disturbance downstream. We compute our growth from the fluctuating mass flux

$$(\rho u)' = \rho u - < \rho u > \tag{3.1}$$

where $< \rho u >$ is the computed mean in time.

In order to quantify the growth of the disturbance, we compute the root mean square (RMS) in t of $(\rho u)'$. This is then integrated in y and z, i.e.,

$$G(x) = \int (\text{RMS}(\rho u)') dz dy. \tag{3.2}$$

The growth rate σ is then obtained from an approximation to $1/x \log(G(x)/G(x_0))$. We have computed $\sigma(F)$ for a range of non-dimensional frequencies F. Two modes, the first and second modes are of interest. The first mode is three-dimensional and for the parameters considered here the most amplified wave propagates at an angle of 55°. It is known, Mack [1969], that for the parameters considered here the most unstable second mode is two dimensional. The most amplified second mode has an amplification rate approximately three times greater than the most amplified first mode.

In Figure 2, we indicate the computed values of growth rate $\sigma(F)$ for several frequencies and for both first and second modes. The data is compared to the growth rates predicted from linear theory (Mack [1969]), experiment (Kendall [1967]) and nonparallel theory (El Hady [1981]). In all cases, the peak inflow disturbance was .075% of the free stream (in terms of u). We note that both the nonparallel results and the experimental data are for the growth of the maximum disturbance, not for the integrated R.M.S. disturbance as in (3.2).

The numerical results show that for all frequencies considered the computed growth rate lies close to both the parallel and nonparallel results. Where experimental results are available, the computed growth rates are close to the experimental measurements. The results show that the magnitude of full nonparallel effects (i.e., as obtained from the Navier-Stokes equation) are of the same order as that predicted from nonparallel theory.

The computational domain extends approximately 8.3 wavelengths in the streamwise direction (based on the value of λ_x at the inflow determined from linear theory. There are 28 points per wavelength in the streamwise direction and 150 points in the vertical direction (using an exponential stretching to concentrate points in the boundary layer). When $\varepsilon = 0.075\%$ and 2%, 15 points per wavelength in the spanwise direction were used. When $\varepsilon = 3\%$, 32 points per wavelength were used. A case with $\varepsilon = 1.5\%$ was run with 15 and 30 points per wavelength in the spanwise direction and virtually identical results were obtained in the early stages, i.e., before pronounced nonlinear effects became apparent. However, the spatial resolution in the spanwise direction, even with 32 points, will not be sufficient when nonlinear effects become pronounced as in the case with $\epsilon = 3\%$. An effort to change the spanwise approximation to one using a Fourier spectral method is now underway.

In Figure 3, we present the time signal of $\rho u(t)$ (non-dimensional) at a fixed location in the boundary layer and for several downstream stations. This is for the case where the inflow disturbance is 3% and nonlinear effects should be expected . Close to the inflow boundary, the oscillations are sinusoidal. With increasing distance downstream, weak nonlinear effects first emerge in the form of relaxation oscillations. There is a broadening of the valleys of the wave form and a narrowing of the peaks. (The opposite pattern was noted at other points in the boundary layer). At $x = 5\lambda_x$ from the inflow ($\mathrm{Re}_\delta = 2.3x10^4$) what may be the initial stages of breakdown are noticeable. At a lower value of the inflow perturbation, these effects are not as pronounced. In Figure 4, for $\epsilon = 1\%$, at $3\lambda_x$ from the inflow, $\rho u(t)$ at the same location in the boundary layer as in Figure 3 is plotted. The oscillations are not of the relaxation type. In Figure 5, spanwise-vorticity contours are plotted. In these figures δ_i denotes the inflow boundary layer thickness. This is for the case with $\epsilon = 3\%$ and $F = 1.5x10^{-5}$ and at a time $t = 0.89$. Note that the disturbance has not reached the outflow boundary. Figure 6 has two such plots at an earlier time. They do not show as advanced a state. Figure 7, where $t = 0.99$ is close to the point where numerical instability prevented us from continuing our computations any further. It shows a much more advanced state than $t = 0.89$. The spatial resolution here is suspect.

Several physical features of a boundary layer approaching transition are noticed in Figure 5. There is a lifting of vorticity away from the wall towards the edge of the boundary layer. Also, the bending downward of these vortices is evident. This phenomenon, the one spike stage, is one of the early stages of transition.

In Figure 8, contours of vorticity in the $x - z$ plane are presented. Streamwise vorticity contours at three downstream locations are shown in Figure 8. Both these sets of figures also show the progresion (with increasing downstream distance) towards breakdown. The time development of this process is also apparent from Figure 9.

Finally in Figure 10, the instantaneous velocity on the plane $z = \frac{\lambda_z}{2}$ are shown. The perturbation imposed at the inflow only modifies the profile slightly but the distortion becomes more pronounced with increasing distance downstream. At a downstream location where nonlinear effects are apparent from spanwise vorticity plots, i.e., between $4\lambda_x$ and $5\lambda_x$ (Re_δ between 2.2 x 10^4 and 2.3 x 10^4), the velocity profiles indicate a strong distortion.

4. CONCLUSION

The three-dimensional stability of a growing, supersonic flat-plate boundary layer was studied numerically by solving the complete Navier-Stokes equations. Satisfactory comparison with linear parallel and non-parallel stability theories, and experiment are obtained when a small amplitude inflow disturbance is used. For higher inflow disturbance amplitudes, nonlinear effects are apparent. The nonlinear effects obtained in the present study are only the early stages of transition. Several modifications to improve the accuracy of the code will be necessary before it can reliably compute the more advanced stages of transition. These would include a variable grid spacing in the streamwise direction and a spectral approximation in the spanwise direction.

Work in progress includes the incorporation of some of these changes as well as the study of the nonlinear interaction of two- and three-dimensional waves.

References

[1] Bayliss, A.; Maestrello, L.; Parikh, P.; and Turkel, E., "Wave Phenomena in a High Reynolds Number Compressible Boundary Layer," Proceedings of the Symposium on the Stability of Time Dependent and Spatially Varying Flows, August 1985, Springer-Verlag.

[2] Bayliss, A.; Parikh, P.; Maestrello, L.; and Turkel, E., "A Fourth-Order Scheme for the Unsteady, Compressible Navier-Stokes Equations," AIAA Paper No. 85-1694, July 1985.

[3] Bayliss, A.; Maestrello, L.; Parikh, P.; and Turkel, E., "Numerical Simulation of Boundary-Layer Excitation by Surface Heating/Cooling," AIAA J., Vol. 24, No. 7, 1986, pp. 1095-1101.

[4] El Hady, N., "On the Effect of Boundary-Layer Growth on the Stability of Compressible Flows," NASA Contractor Report 3474, October 1981.

[5] Erlebacher, G. and Hussaini, M. Y., "Incipient Transition Phenomena in Compressible Flows over a Flat Plate," ICASE Report No. 86-39, June 1986.

[6] Fasel, H.; Rist, V.; and Konzelman, V., "Numerical Investigation of the Three-Dimensional Development in Boundary-Layer Transition," AIAA Paper No. 87-1203, June 1987.

[7] Gottlieb, D. and Turkel, E., "Dissipative Two-Four Methods for Time Dependent Problems," Math. Comp., Vol. 30, 1976.

[8] Kendall, L. M., "Supersonic Boundary-Layer Stability Experiments," in Proceedings of Boundary-Layer Transition Study Group Meeting, Vol. II, W. D. McCauley (ed.), Aerospace Corp. Report No. TR-0158 (53816-63)-1, August 1967.

[9] Mack, L. M., "Boundary-Layer Stability Theory," Rep. 900-277 Rev. A, Jet Propulsion Laboratory, Pasadena, CA, 1969.

[10] Murdock, J. W., "Three-Dimensional, Numerical Study of Boundary-Layer Stability," AIAA Paper No. 86-0434, 1986.

[11] Parikh, P., L. Maestrello, A. Bayliss and R. Krishnan, "Study of Instability and Active Control of Boundary Layers Using Navier-Stokes Equations," AIAA Paper No. 87-1260, 1987.

[12] Schlichting, H., "Boundary-Layer Theory," 7th Edition, McGraw-Hill, 1979.

[13] Strang, W. G., "On the Construction and Comparison of Difference Schemes," SIAM J. Numer. Anal., Vol. 5, 1968, pp. 506-517.

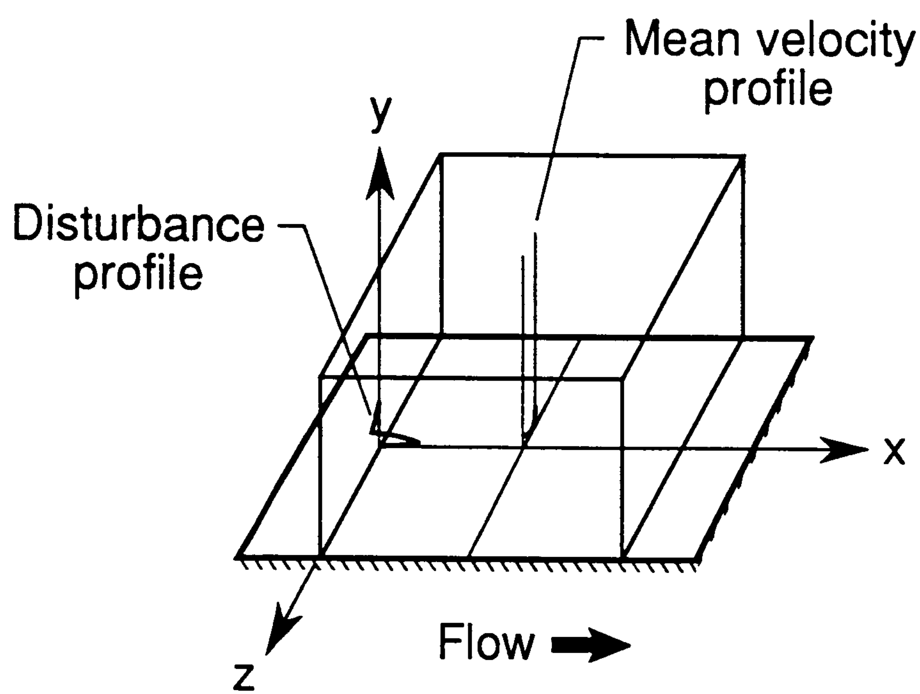

Figure 1. Schematic of three-dimensional computational domain used.

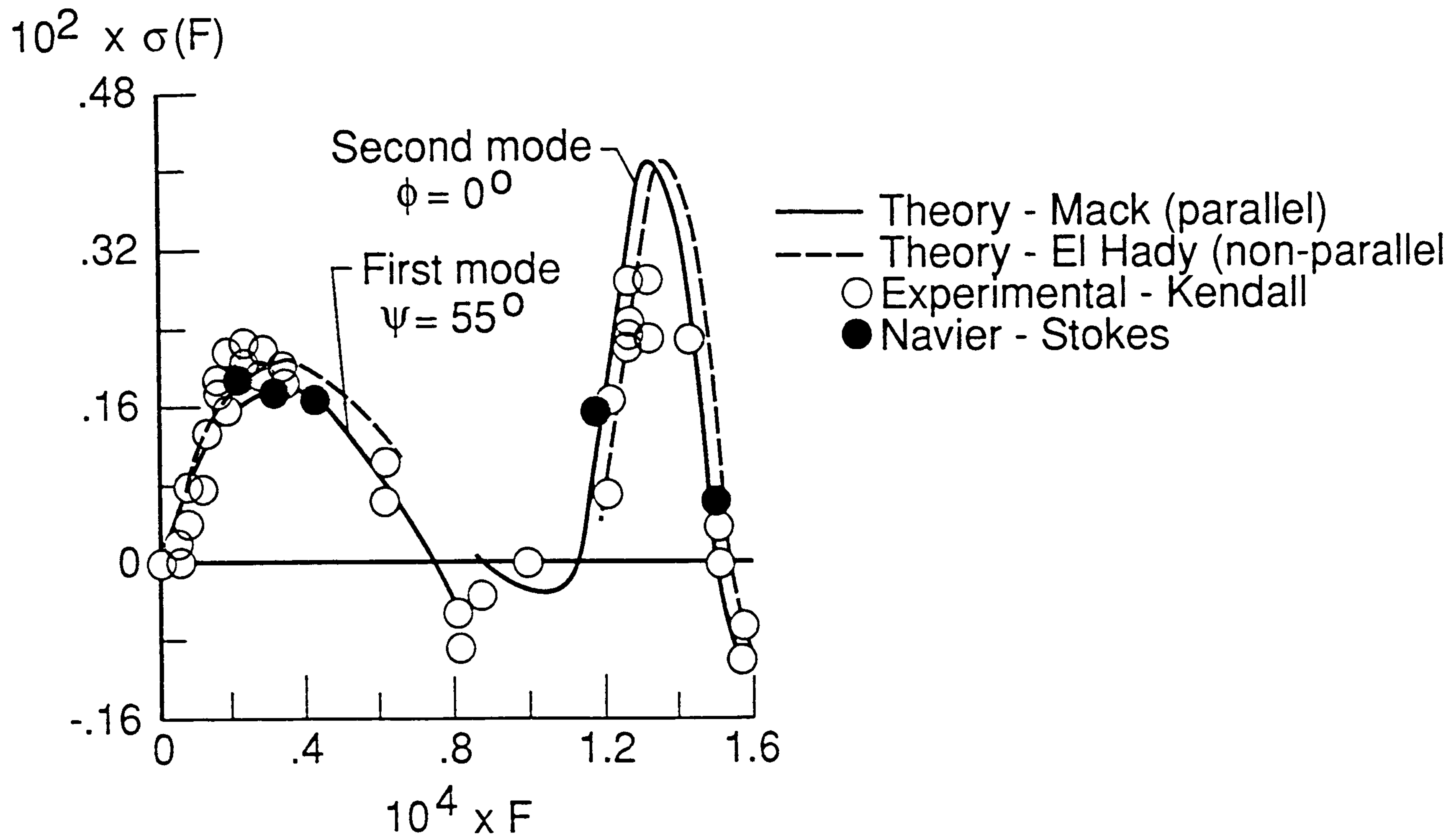

Figure 2. Comparison of computed growth rates with theory and experiment.

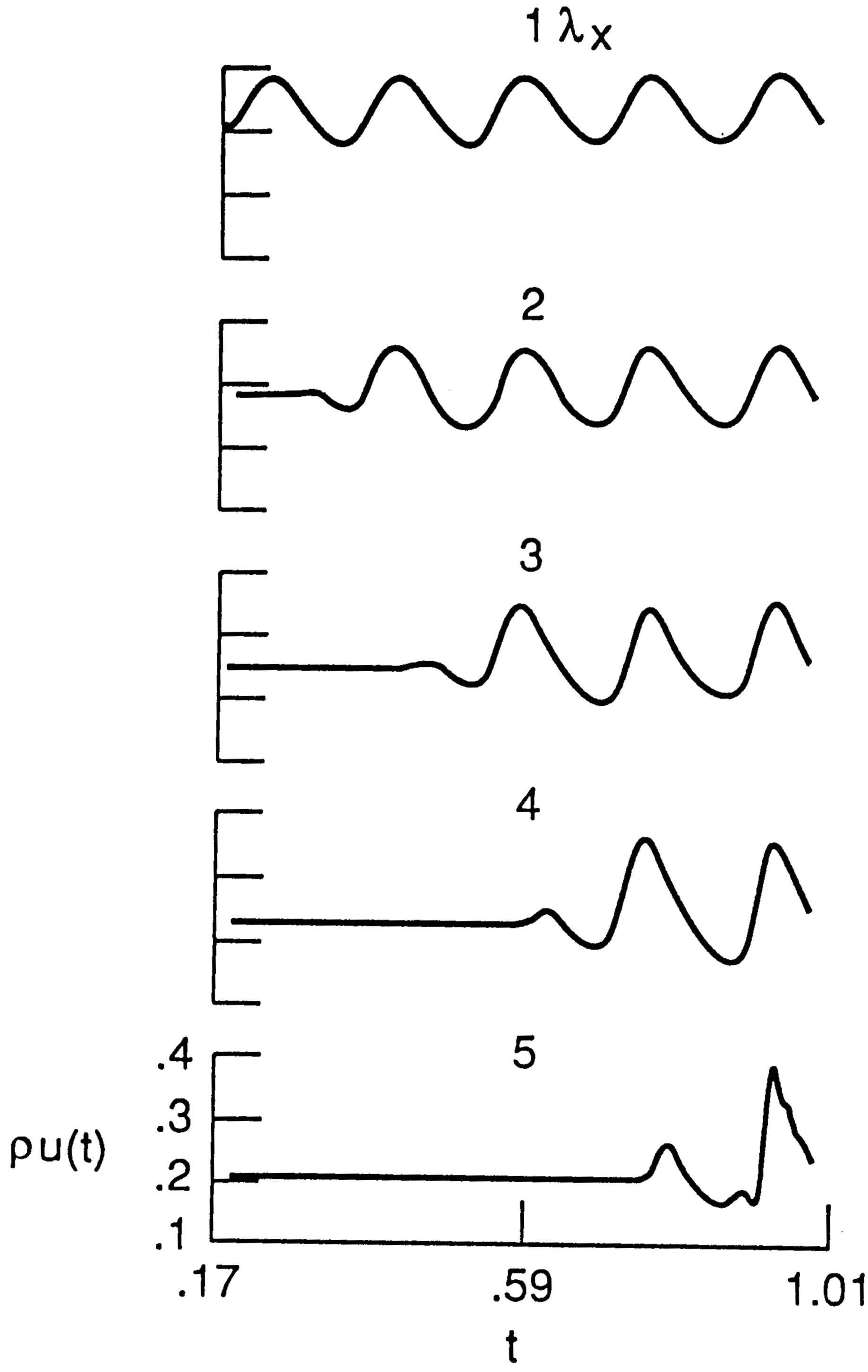

Figure 3. Time series of $\rho\, u$ at $\frac{y}{\delta} = 0.795$ for $\epsilon = 3\%$ case. This shows the development of relaxation oscillations with distance downstream of inflow.

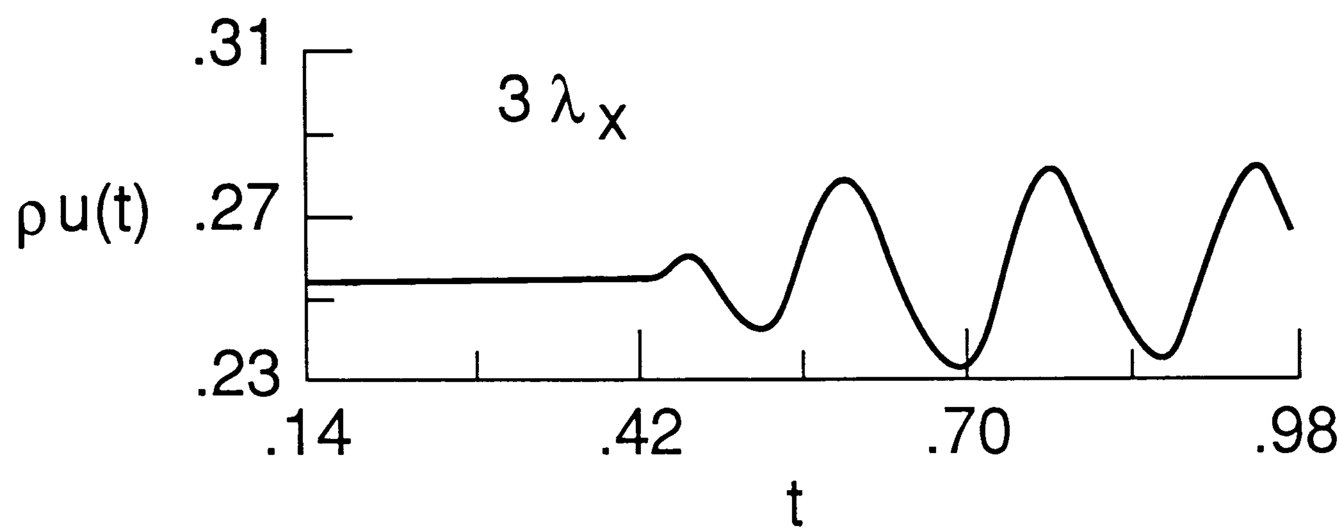

Figure 4. Time series of ρu at $\frac{y}{\delta} = 0.795$ for $\epsilon = 1\%$. Comparison with fig. 4 shows that relaxation oscillations are not as pronounced in this case.

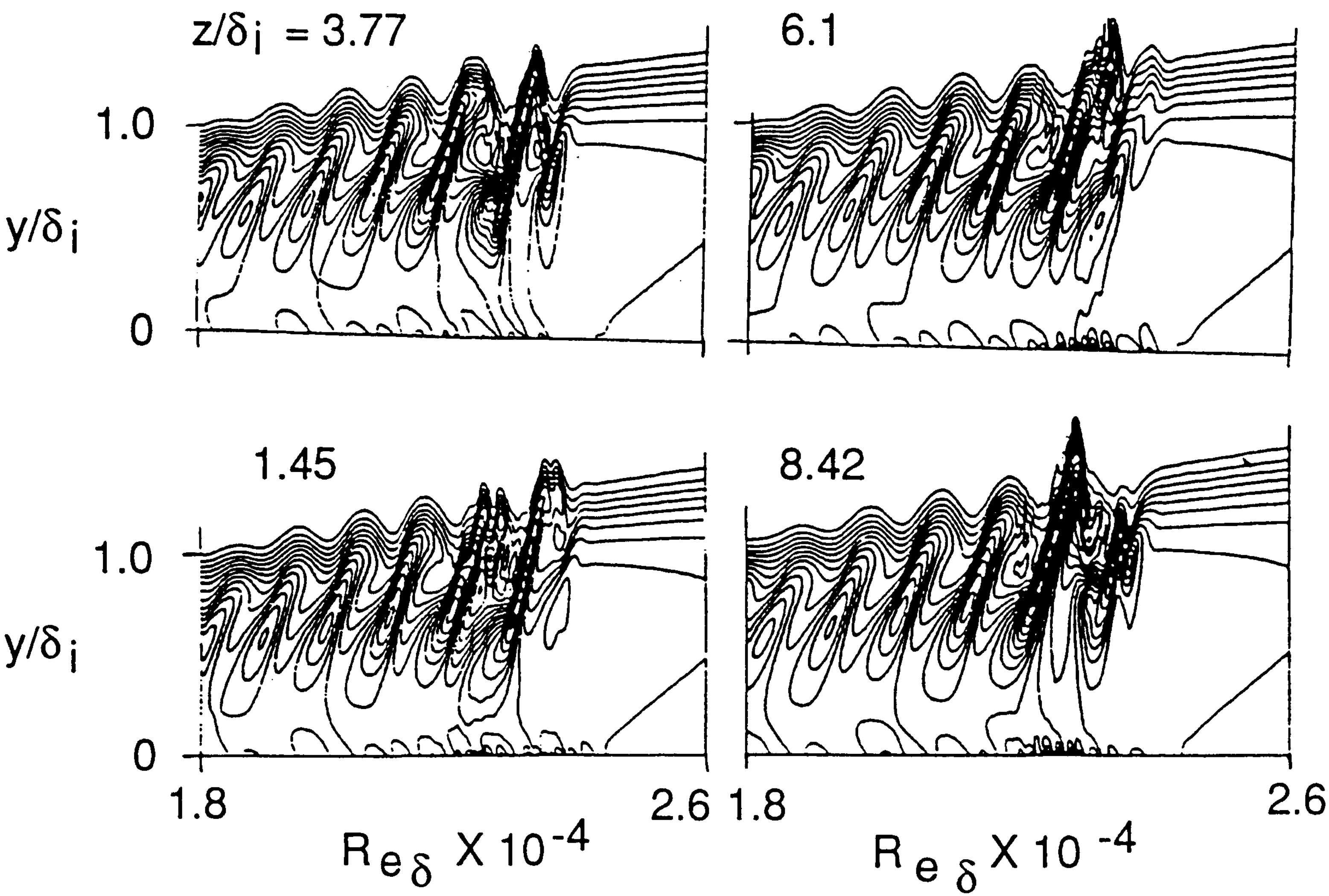

Figure 5. Spanwise vorticity contours at $t = 0.89$.

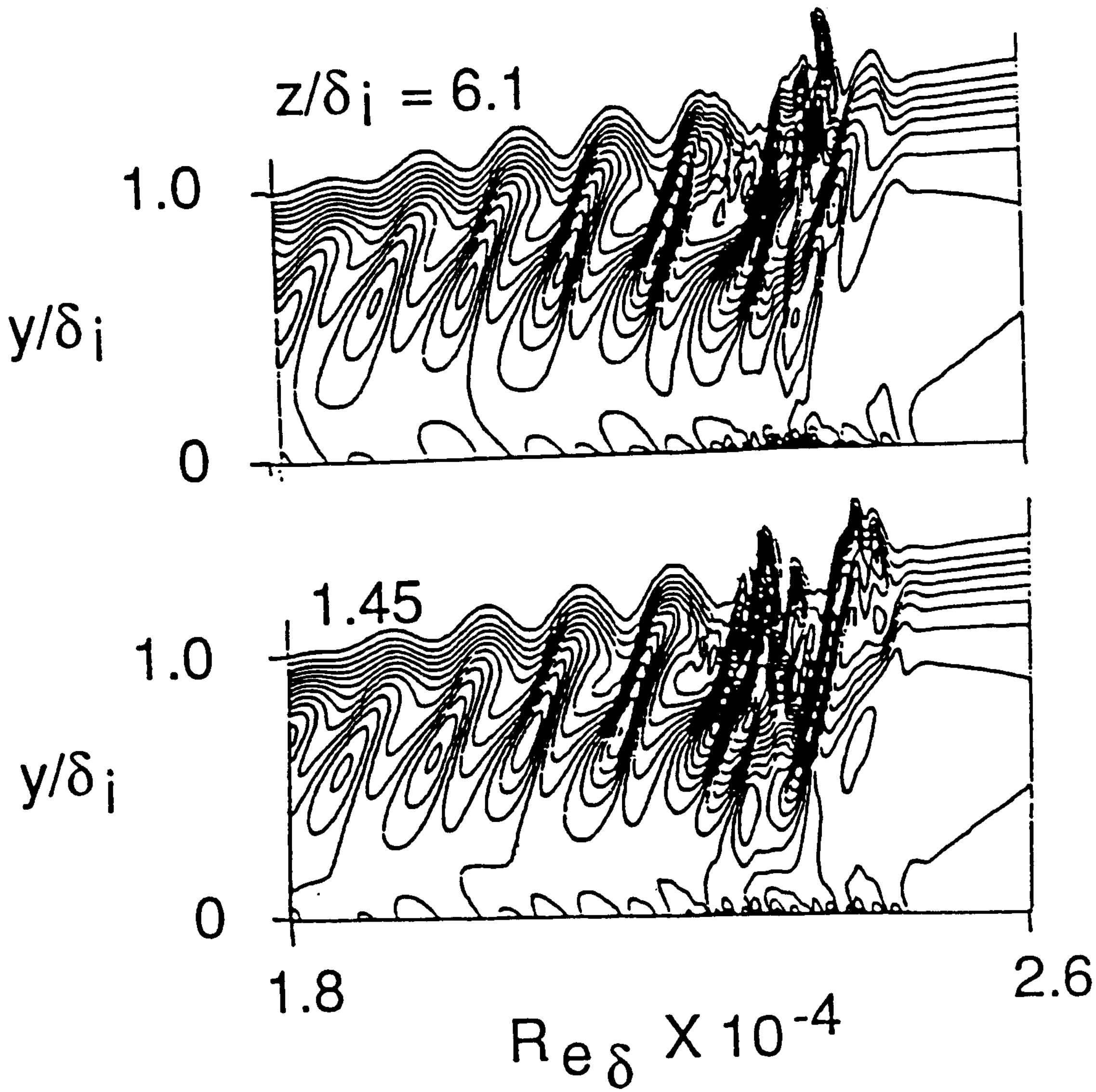

Figure 6. Spanwise vorticity contours at $t = 0.79$.

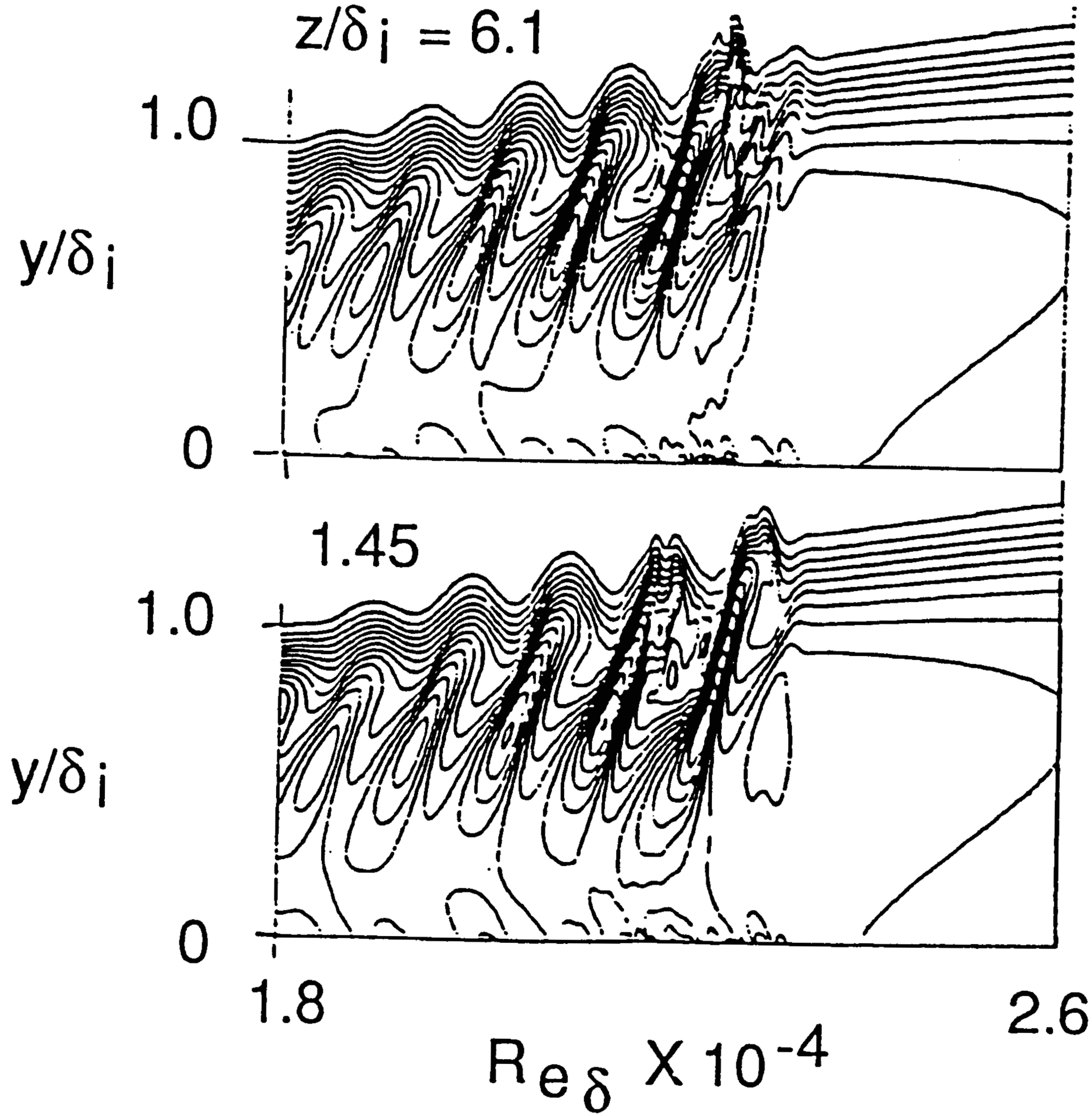

Figure 7. Spanwise vorticity contours at $t = 0.99$.

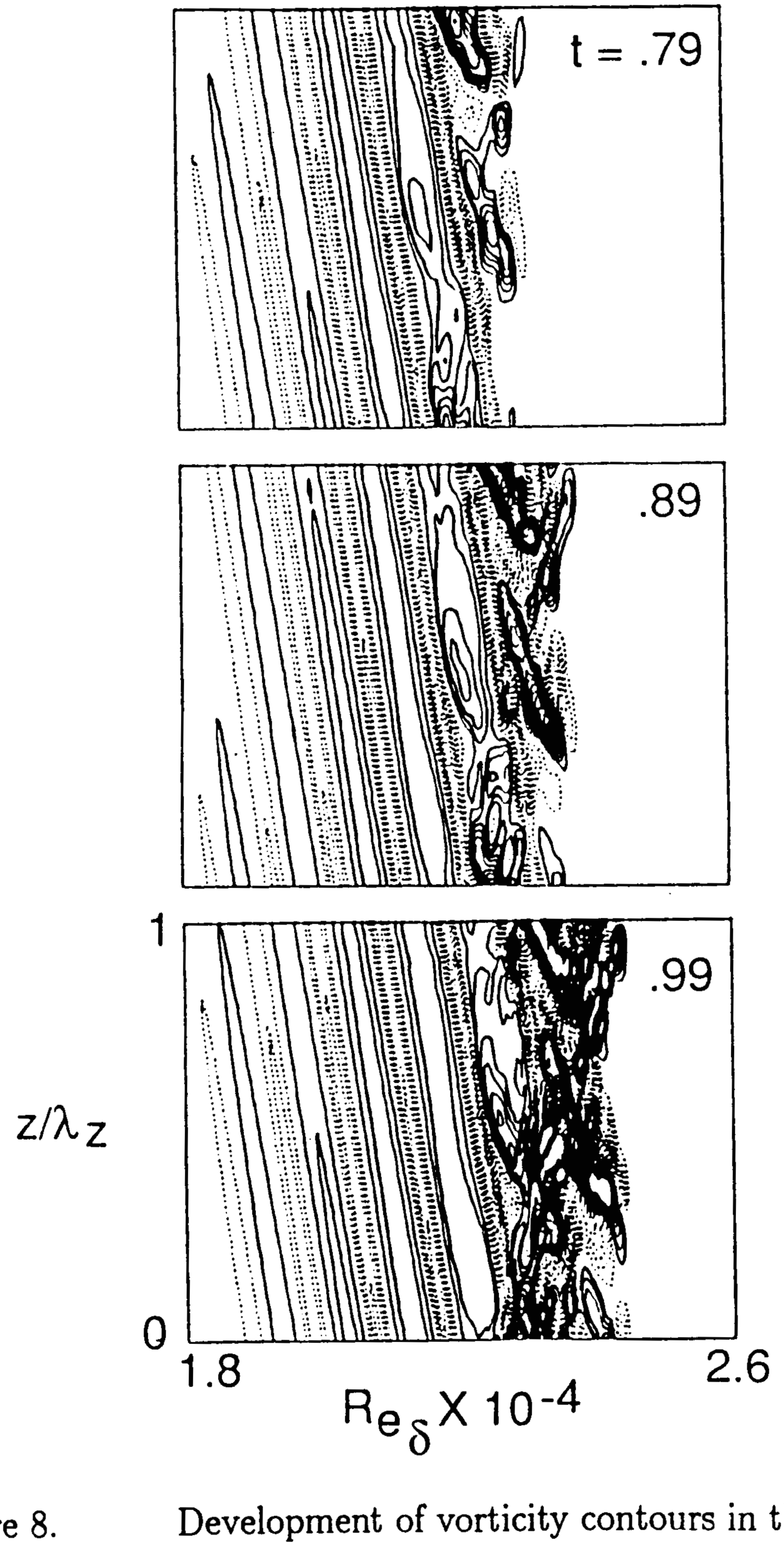

Figure 8. Development of vorticity contours in the $x - z$ plane at $y/\delta_i = 0.795$.

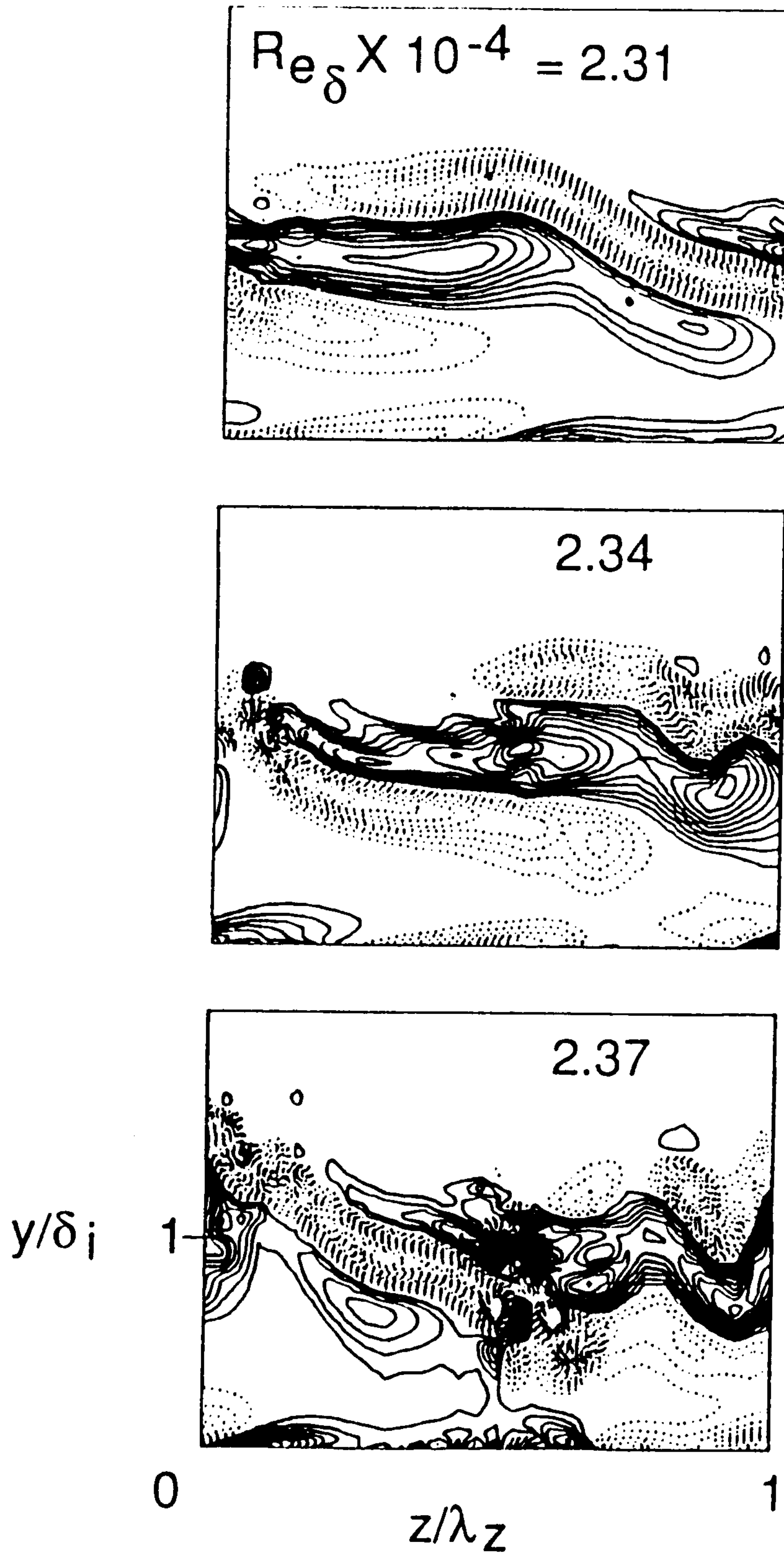

Figure 9. Streamwise vorticity contours at $t = 0.89$. The three stations show progressively increasing non-linear effects.

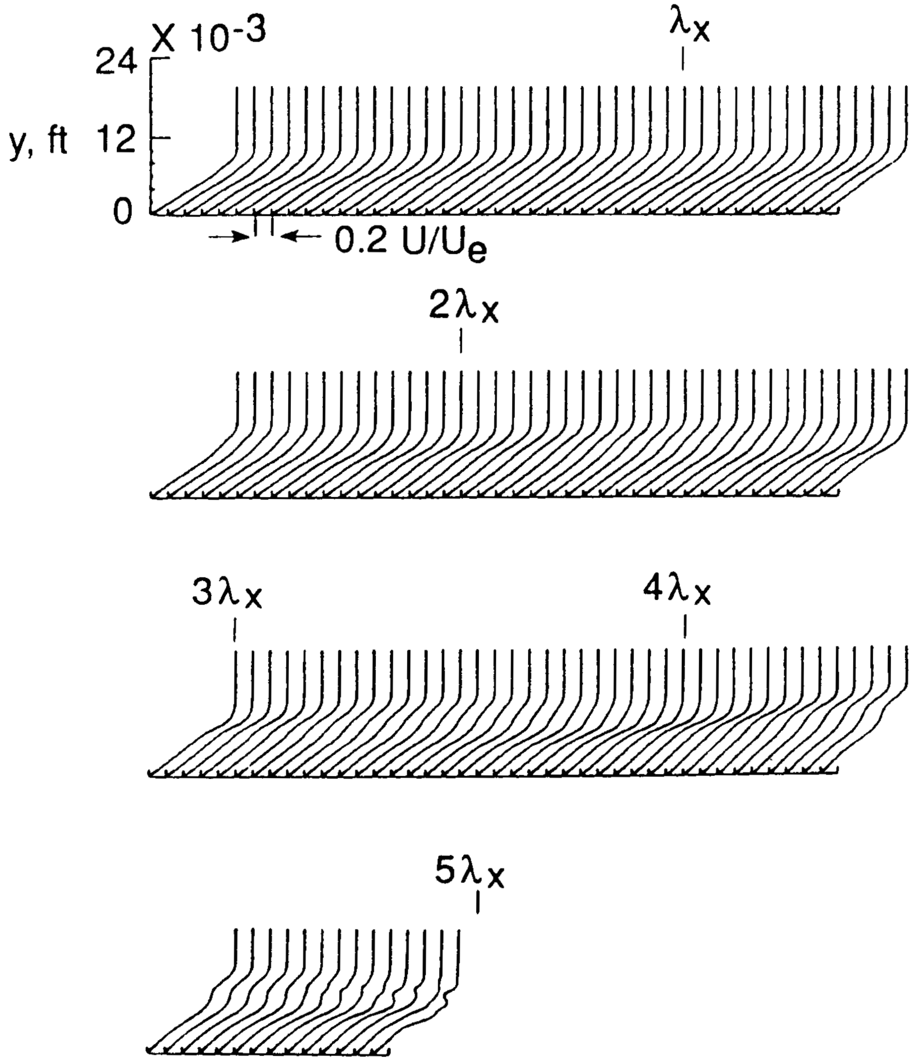

Figure 10. Instantaneous velocity profiles on the $\frac{z}{\lambda_z} = \frac{1}{2}$ plane.

NASA

National Aeronautics and Space Administration

Report Documentation Page

1. Report No. NASA CR-181931 ICASE Report No. 89-74	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle NUMERICAL STUDY OF THREE-DIMENSIONAL SPATIAL INSTABILITY OF A SUPERSONIC FLAT PLATE BOUNDARY LAYER		5. Report Date October 1989
		6. Performing Organization Code
7. Author(s) L. Maestrello, A. Bayliss, R. Krishnan		8. Performing Organization Report No. 89-74
		10. Work Unit No. 505-90-21-01
9. Performing Organization Name and Address Institute for Computer Applications in Science and Engineering Mail Stop 132C, NASA Langley Research Center Hampton, VA 23665-5225		11. Contract or Grant No. NAS1-18107, NAS1-18605
		13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Langley Research Center Hampton, VA 23665-5225		14. Sponsoring Agency Code

15. Supplementary Notes

Langley Technical Monitor: Richard W. Barnwell

Physics of Fluids

Final Report

16. Abstract

The behavior of spatially growing three-dimensional waves in a supersonic boundary layer is studied. The objectives are to validate our computer code by comparing with experiments and parallel and non-parallel linear stability theories for $M_\infty = 4.5$, $\sqrt{\mathrm{Re/ft}} = 1550$ and at $T_{t,\infty} = 100°\mathrm{F}$ and then use the program to study nonlinear effects. The three-dimensional unsteady Navier-Stokes equations are solved by a finite difference method which is fourth-order and second-order accurate in the convection and viscous terms respectively, and second-order accurate in time. Spanwise periodicity is assumed. The inflow disturbance is composed of eigenfunctions from linear stability theory. Computed results for small amplitudes of this disturbance agree well with linear theory and experiment for several frequencies. By increasing the amplitude of the inflow disturbance, nonlinear effects in the form of a relaxation type oscillation of the time signal of ρu are observed.

17. Key Words (Suggested by Author(s)) flow, fluid mechanics, instability	18. Distribution Statement 34 - Fluid Mechanics and Heat Transfer; 01 - Aeronautics; Unclassified - Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	19	A03

NASA FORM 1626 OCT 86

NASA-Langley, 1989