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FATIGUE LIFE ESTIMATES FOR HELICOPTER LOADING SPECTRA

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Abstract

Helicopter loading histories applied to notched metal samples are used as examples, and their fatigue lives are calculated by using a simplified version of the local strain approach. This simplified method has the advantage that it requires knowing the loading history in only the reduced form of ranges and means and number of cycles from the rain-flow cycle counting method. The calculated lives compare favorably with test data.

Introduction

The Palmgren-Miner (P-M) theory of damage has long been used to predict the time to crack initiation in metals. This rule states that the fatigue failure occurs when the summation of life fractions reaches unity. The successful use of this rule requires proper handling of cycle counting, overstrain effects, and local notch mean stress effects. In this paper, the P-M rule is used, and the above three complexities are included by using the rain-flow cycle counting method, by basing the life calculations on data for specimens that have been prestrained [1-3], and by using the local strain approach [4-6], respectively. The local strain approach focuses attention on the stresses and strains that occur locally at a stress raiser of interest, and the S-N curve used is a strain versus life curve.

The rain-flow cycle counting method is a procedure for interpreting an irregular load versus time history as a collection of events (called cycles) to which fatigue damage can be assigned. In this method [7], cycles are counted depending on the comparison of two adjacent ranges as illustrated in Fig. 1, which also defines the range and mean of a cycle. If the first range is less than or equal to the second, a cycle is counted and the corresponding peak and valley are discarded for the purposes of further cycle counting.

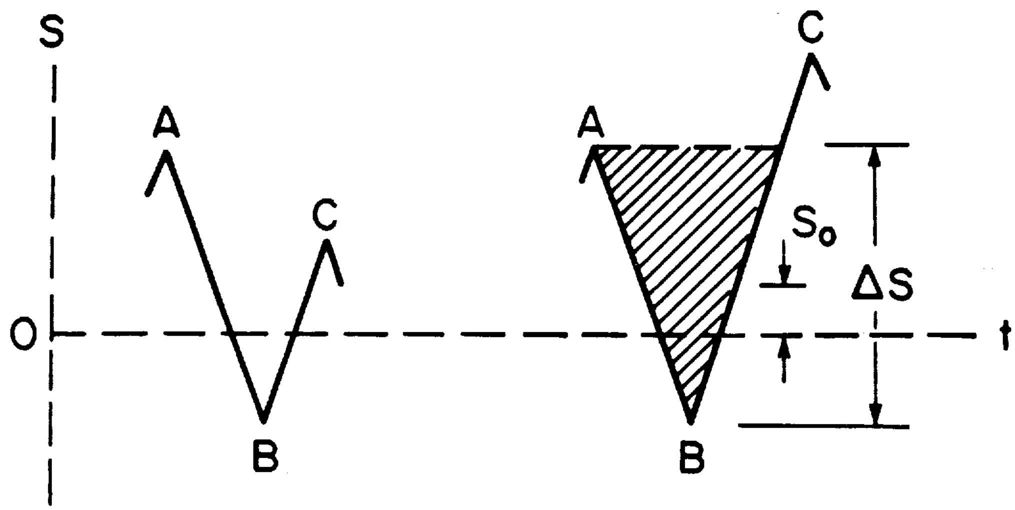
Figure 2 illustrates this process for a simple loading history. First, the history is reordered to start with the highest peak or lowest valley as in (b). Cycle counting then proceeds by moving forward in the history. If a cycle is counted, as in (c), its range and mean are recorded and its peak and valley are removed from the history. This procedure, as described in detail in Ref. [7], always yields a major cycle between the highest peak and lowest valley, and smaller cycles temporarily interrupt larger ones.

By using the rain-flow cycle counting method, a service loading history can be reduced to a convenient compact form. The compact description is in the form of a matrix giving range, and mean, and numbers of rain-flow cycle. Table 1 illustrates such a matrix for the loading history of Fig. 2. As a second example, a portion (about one-third) of an actual helicopter loading history is shown in Fig. 3, and the resulting rain-flow matrix in Fig. 4.

As noted above, an area of importance in predicting fatigue life is the overstrain effect. This effect, which is caused by the higher stress levels, needs to be considered since it increases the damage done by the lower stress levels [1,2]. Figure 5 illustrates this effect for titanium 6Al-4V. The lower curve represents test data for specimens which have been plastically strained, but only to a life fraction of a few percent. There is nevertheless a large effect on the life for subsequent testing at a lower level due to the prestrain causing damage to the material at the microstructural level.

A known load versus time history is necessary for analysis of fatigue life using the local strain approach. But the history may be lengthy, and there are no restrictions on the degree of irregularity of the time variation. The local strain approach predicts crack initiation life and assumes that fatigue life is controlled primarily by the local notch surface strain and mean stress, not the nominal (average) stress. Emphasis on local notch behavior is crucial as this permits rational analysis of local notch yielding and its effect on the local notch mean stress. This method in its complete form requires knowing the loading history in full length.

A simplified method for calculating fatigue crack initiation life based on the local strain approach can be used [5,8]. This method has two very distinct advantages. First, only a rain-flow matrix in the compact form of range-mean values is required as the input information. Although some detail is lost, such a matrix can be used with the local strain approach to place upper and lower bounds on the life that would result from the analysis of the original, unsummarized history. This principle is explained in detail in Refs. [3,9]. Note that this compact form is much easier to handle and store than the full history in the form of a time sequence of peaks and valleys. Secondly, the life calculations are simpler and more economical.



$\overline{BC} < \overline{AB}$

No cycle

$\overline{BC} \geq \overline{AB}$

AB = cycle

For cycle A - B

Peak = S_A

Valley = S_B

Range = $\Delta S = S_A - S_B$

Mean = $S_0 = (S_A + S_B)/2$

Figure 1. Condition for recording an event during rain-flow cycle counting

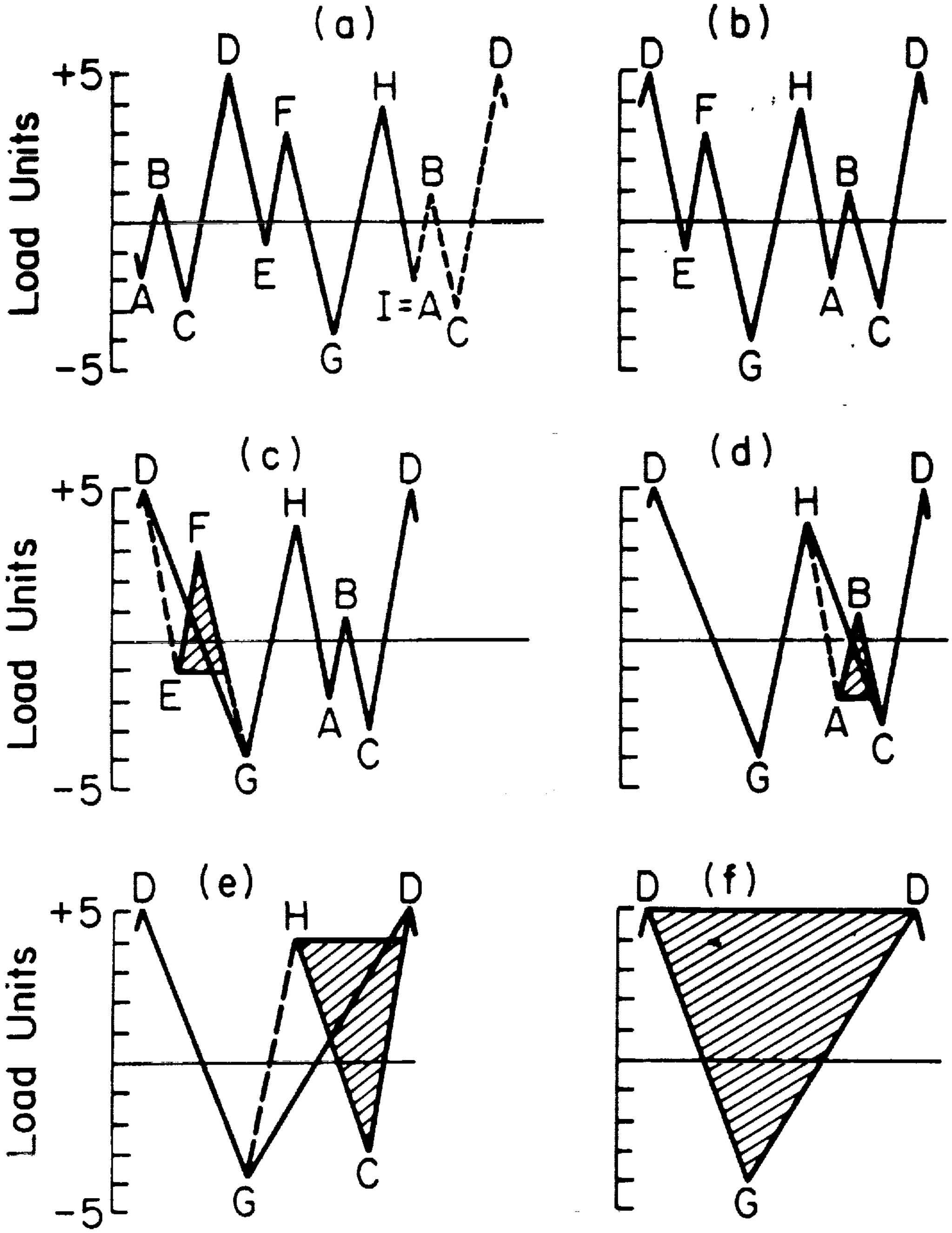


Figure 2. **Example of rain-flow cycle counting from the ASTM standards [7]:** Before the cycle counting begins, the most extreme point in the history should be located, and the history arranged to start and finish at this point as shown in (b).

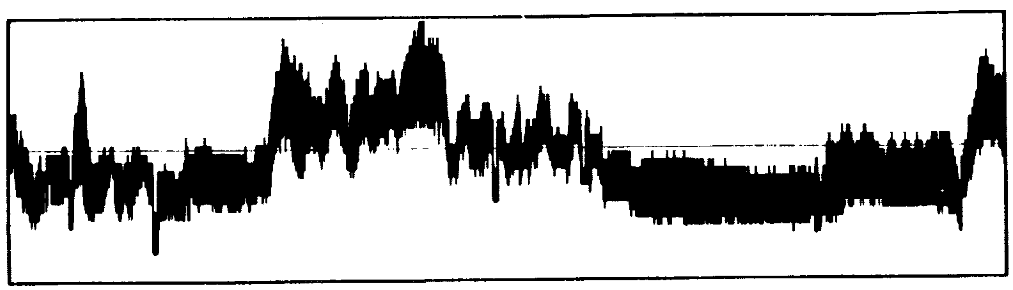


Figure 3. Portion of the modified maneuver history

RANGE \ MEAN	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1				7	17						6	271	404	268	178	7	2															
2			9	28	16							77	36	64	124	23	20	4														
3			10	12	10								21	69	106	107	70	39	26	20	6	4										
4				6	9	2	2	1	16	66	99	74	97	103	99	96	93	96	88	66	27	24	2									
5				8	17	4			6	148	160	193	226	130	167	99	110	96	88	82	80	56	20	5	1							
6				4	3				4	78	202	166	236	158	171	68	94	82	68	68	109	83	49	26	7	1						
7					1					13	140	80	146	116	77	31	44	46	33	49	74	60	71	42	17	4						
8										3	32	30	65	56	37	20	14	30	22	31	53	48	60	58	27	1						
9											3	14	22	26	14	12	7	14	11	13	29	34	39	43	26	2						
10												4	4	6	10	6	7	7	11	12	22	18	15	29	21	6						
11													2	3	7	9	8	6	5	3	11	6	6	15	14							
12													1		3	6	2	5	3	6	5	2	1	5	6							
13												1		1	1	4	1		4	1	6	1										
14															1	4	3	1	2	1		1										
15																1	2		1													
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Figure 4. Range-mean matrix from rain-flow cycle counting of the modified maneuver history

Table 1. Rain-flow matrix for the loading history of Fig. 2 [7]

Event	Range (units)	Mean (units)							
		-1.5	-1.0	-0.5	0	0.5	1.0	1.5	All
--	0.5	-	-	-	-	-	-	-	-
--	1.0	-	-	-	-	-	-	-	-
--	1.5	-	-	-	-	-	-	-	-
--	2.0	-	-	-	-	-	-	-	-
--	2.5	-	-	-	-	-	-	-	-
AB	3.0	-	-	1	-	-	-	-	1
--	3.5	-	-	-	-	-	-	-	-
EF	4.0	-	-	-	-	-	1	-	1
--	4.5	-	-	-	-	-	-	-	-
--	5.0	-	-	-	-	-	-	-	-
--	5.5	-	-	-	-	-	-	-	-
--	6.0	-	-	-	-	-	-	-	-
--	6.5	-	-	-	-	-	-	-	-
HC	7.0	-	-	-	-	1	-	-	1
--	7.5	-	-	-	-	-	-	-	-
--	8.0	-	-	-	-	-	-	-	-
--	8.5	-	-	-	-	-	-	-	-
DG	9.0	-	-	-	-	1	-	-	1
--	9.5	-	-	-	-	-	-	-	-
--	10.0	-	-	-	-	-	-	-	-

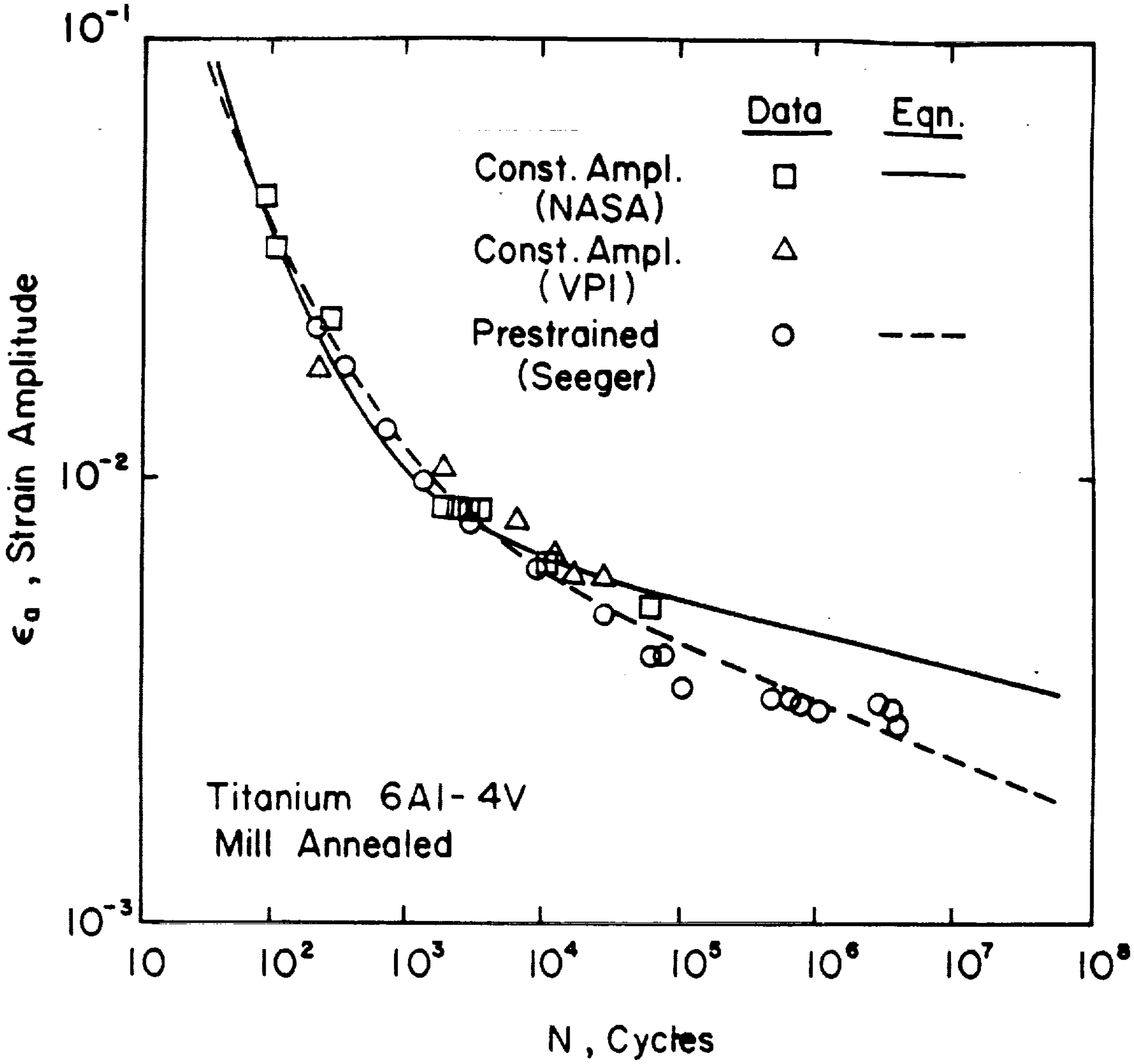

Figure 5. Strain vs. life test data and curves for Ti-6Al-4V

In this paper, two helicopters loading histories are chosen for analysis, and their fatigue lives are calculated using the simplified method. Also, the calculated lives are compared with test data. All life calculations are based on a strain-life curve, which represents failure of small unnotched axial test specimens. Lives so predicted for notched members correspond to initiation of an easily detectable crack. In general, the remaining life for crack growth also needs to be calculated, but this was relatively short and so was neglected for the particular specimens analyzed here.

Life Calculation Methodology

The simplified version of the local strain approach used assumes that fatigue life is controlled primarily by the notch surface strain, and it considers plasticity and mean stress effects in a fairly complete manner. Note that the traditional S-N approach crudely handles plasticity, and also neglects the special mean stress effects which arise from loading sequence. Life calculations by the local strain approach consists of two steps. First, the local notch stress and strain histories must be predicted, and second the life corresponding to the local stress and strain histories must be estimated.

Local Strain Approach

Figure 6 illustrates the initial and most difficult step of estimating the strain-stress history. This step is difficult because it requires specific handling of the complex nonlinearity relating load, strain, and stress. In order to achieve the above task, a cyclic stress curve [10] for the material is needed:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{A}\right)^{\frac{1}{s}} \tag{1}$$

where ε_a, σ_a are amplitudes of strain and stress, respectively, E is the elastic modulus, and A and s are material constants. Next, by employing Eq.1 and with the aid of Neuber's rule, a curve relating nominal stress, S, and the local notch stress and strain is obtained:

$$\sigma_a \varepsilon_a = \frac{(k_t S_a)^2}{E} \tag{2}$$

where k_t is the elastic stress concentration factor.

Figure 6(b) shows these two curves. These curves are then used to estimate the local stress-strain response at the notch by following the loading history while modeling the hysteresis looping behavior of the material. For the example of Fig. 6, the irregular loading history of (c) results in S versus ε and σ versus ε as shown in (d) and (e). Note that there is a set of closed hysteresis loops, such as 2-3-2′, 6-7-6′, 5-8-5′, and 1-4-1′ for this example. Each such loop is identified as a cycle, and the cycles so defined are the same as would be obtained from applying rain-flow cycle counting to the load (S) versus time history.

Each cycle now has a known strain range, $\Delta\varepsilon = 2\varepsilon_a$, and mean stress, σ_0, as shown for cycle 6-7-6′ in Fig. 6 (e). The life, N, corresponding to each combination of ε_a, σ_0 can be obtained from a strain-life curve [10]:

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N')^b + \varepsilon_f'(2N')^c \tag{3}$$

where ε_a is the strain amplitude corresponding to a closed loop, N' is the life in cycles for zero mean stress, and σ_f', b, ε_f' and c are additional constants for the material. If the rule of Morrow [11] is used to account for mean stress effects, the fatigue life as adjusted for mean stress σ_0 can be estimated by:

$$N = N'(1 - \frac{\sigma_o}{\sigma_f'})^{-\frac{1}{b}} \qquad (4)$$

where N is the final adjusted life.

The final step is then to apply the P-M rule. For a loading history that is assumed to repeat until failure occurs, the rule takes the form:

$$B\left[\sum_{per\ block} \frac{n_i}{N_i}\right] = 1 \qquad (5)$$

where n_i is the number of occurrences per block of a cycle corresponding to life N_i, and B is the unknown number of blocks (repetitions) to failure for the irregular history.

Simplified Method

As mentioned earlier, the local strain approach generally requires a knowledge of the full sequence of the loading history. However, in the simplified method, the loading history may be used in concise matrix form. The first step is then to determine the rain-flow matrix that contains the information on range and mean loads of rain-flow cycles. Using this information, upper and lower bounds on life can be calculated [3,9]. As long as these bounds are reasonably tight, which will be the case in most practical situations, the more detailed simulations as in Fig. 6 are unnecessary.

The principle behind this bounding is shown in Fig. 7 for one cycle, namely 6-7 from Fig. 6. The guiding principle is that both load-strain loop 6-7 and stress-strain loop 6-7 must lie within the corresponding loop for the largest cycle in the history, namely 1-4. The load limits S_6 and S_7 are known; therefore limits can be placed on the mean strain of cycle 6-7. As shown in Fig.

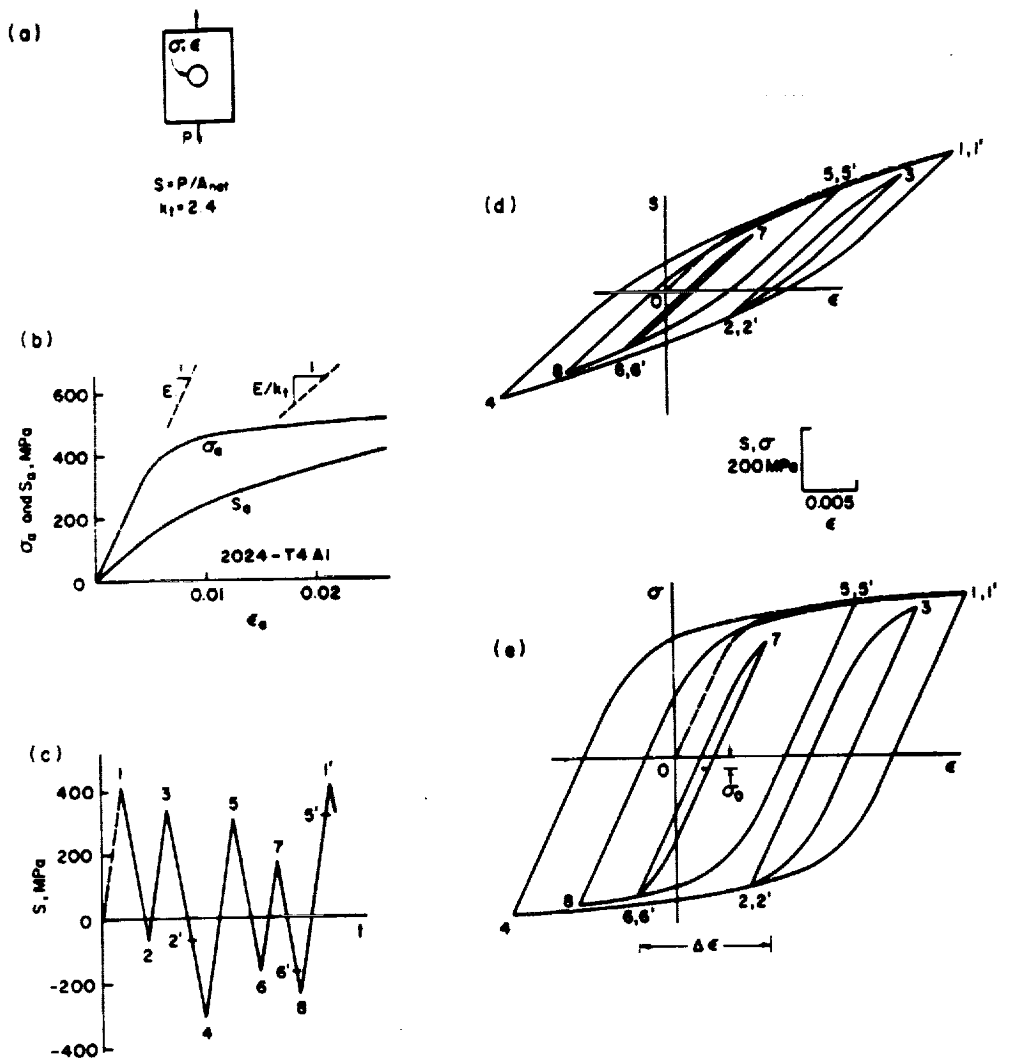

Figure 6. **Illustration of local-strain approach for an irregular history:** Notched member (a), having cyclic stress-strain and load-strain curves as in (b), is subjected to load history (c). The resulting load-strain response is shown in (d), and (e) is the local- notch stress strain response [6].

7(a), loop 6-7 can be so far to the right that it is attached at A, or so far to the left that is attached at B. Similarly, the same line of reasoning can be applied to a load-stress loop as in Fig. 7(c), where loop 6-7 could be so low that it is attached at A, or so high that is attached at B. Figure 7(b) shows the extreme stress-strain loops which satisfy both sets of constraints, so that these correspond to the upper and lower bounds on local notch mean stresses for cycle 6-7, σ_{0B} and σ_{0A}.

Knowing the upper and lower bounds on the mean stress for cycle 6-7 of the example allows bounds to be placed on the life, N, from Eq. 4. The upper bound on N is similarly obtained for all cycles in the history, and these are used with the P-M rule in the following form of Eq. 5 to obtain the upper bound on life for the irregular history. The same procedure, but using the lower bound N's for each cycle, can be used to obtain the lower bound on life for the irregular history. The above procedure is explained in detail in Refs. [3,9].

If the local notch plastic strains are small during the history, the stress-strain loop for the major cycle, such as 1-4 in the example, is reduced to a straight line, and therefore the upper and lower bounds on life are then identical. Also, at high load levels, the cycles causing most of the damage may not have significant mean stresses due to the large plastic strains, and the bounds will then be close. Hence, the widest separation between the bounds is expected at intermediate load levels. Note that if most of the damage is done by low-level cycles, then the degree of separation of the bounds will be greatest. Also, if all but a negligible fraction of the damage is done by the major cycle , then the two bounds are again identical [3,9].

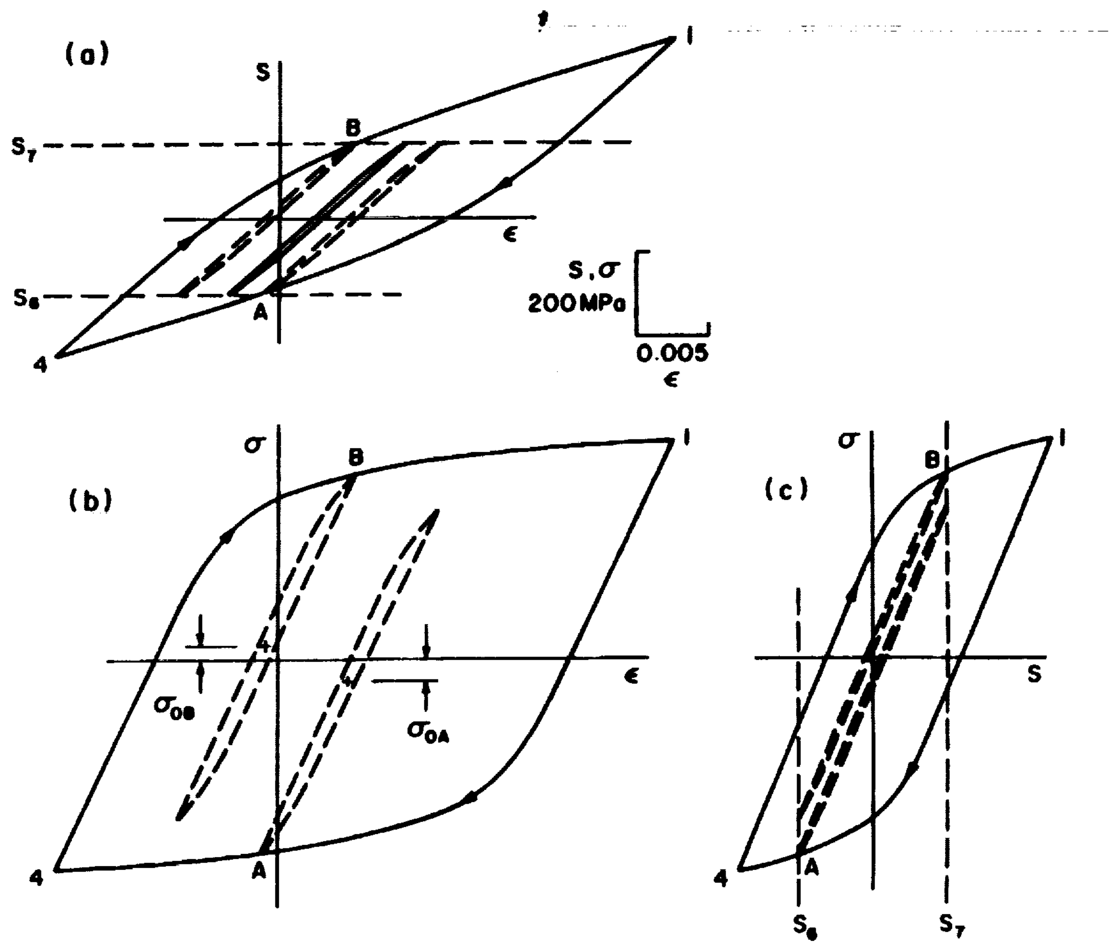

Figure 7. Illustration of placing bounds on the mean stress of a subcycle: This illustration is based on Fig. 6. Note that the sequence of the applied loads is not known, and the mean stress for 6-7 must lie between the σ_{0A} and σ_{0B} values shown.

Material, Specimens and Loading Histories Analyzed

Two helicopter loading histories are analyzed. Fatigue life data for both Helix and a severe maneuver history, applied to plate-with-hole specimens having elastic stress concentration factors between 2.4 and 3.92, are available in Ref. [12] and from tests done at the University of Dayton Research Institute (UDRI). These data include several levels of maximum nominal stress, S_{max}, for titanium 6Al-4V.

Two loading histories, Helix and maneuver, are used in fatigue life calculations. Helix is a standard helicopter loading spectrum obtained from Ref. [12], and the maneuver history was obtained by the University of Dayton Research Institute (UDRI). These two histories are further explained below.

Materials Properties Used

Figures 5 and 8 show strain vs. life and cyclic stress vs. strain data for the titanium 6Al-4V material used, and also curves corresponding to Eqs. 1 or 3 fitted to the data. Constants corresponding to these curves are given in Table 2 for constant amplitude tests and also for tests on prestrained material. Some constant amplitude tests were done at Virginia Tech on the same material. These data are also shown in Figs. 5 and 8, and they agree well with the data obtained from Ref. [13] and also with the fitted constants. In the case of the prestrained material, the data and constants were obtained from Ref. [14].

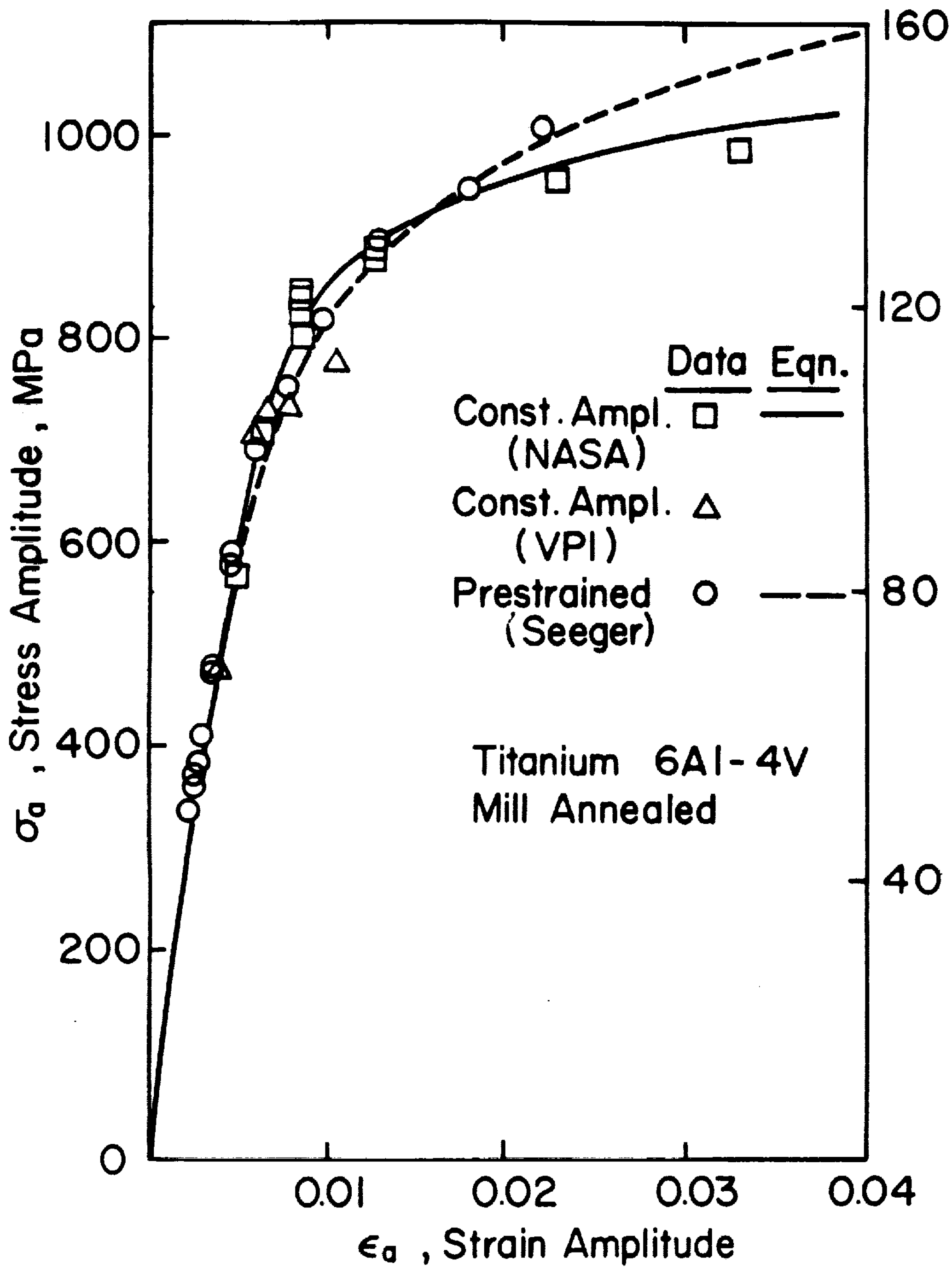

Figure 8. Cyclic stress-strain test data and curves for Ti-6Al-4V

Table 2. Cyclic stress-strain and strain-life constants for Ti 6Al-4V

Symbols, Units	Titanium 6Al-4V	
	Const. Ampl.	Prestrained
E, GPa (ksi)	113.8 (16500)	113.8(16500)
A, MPa (ksi)	1327 (192.4)	1702 (246.9)
s	0.0755	0.127
ε'_f	6.22	2.802
c	-1.01	-0.860
σ'_f, MPa (ksi)	1523 (220.9)	2207 (320.0)
b	-0.0763	-0.126

General Description of Helix

Helix [12] is a standard loading sequence for the main rotors of helicopters with articulated rotors. Helix represents a loading history for a 190.5-hour (2,132,024 cycle) sequence of 140 flights. Each flight in the sequence represents one of either training, transport, antisubmarine warfare, or search and rescue. Each of these appears in the sequence in three different lengths of 0.75 hour, 2.25 hours, and 3.75 hours. There are twelve unique flights, which are applied in a specific number of repetitions and sequence to obtain the total 140 flights. Figure 9 shows the load vs. time history for portions of a transport flight in Helix.

Helix is composed of 24 unique maneuvers, which are repeated in various sequences and numbers of repetitions to compose the various flights. The maneuvers such as take off, forward flight of various load levels, turns, etc. each consists of a mean level and a relatively small number of cycles. These cycles occur at one or more stress amplitudes, with the number of cycles being between 1 and 40.

Helix has a relatively high mean levels for the various maneuvers, these mostly ranging from 60 % to 68 % of the maximum nominal stress in the spectrum , S_{max} . Helix reaches 100 % of the S_{max} level at least once in each flight and returns to -20 % at the end of each flight. Hence, Helix has a large ground-air-ground cycle and a large number of cycles at relatively high mean levels. Table 3 gives the range-mean matrix for Helix from rain-flow cycle counting. The matrix entries in Table 3 were obtained from all of Helix by dividing each by 140, the number of flights. These range and mean values correspond to the history scaled so that S_{max}= 100 units.

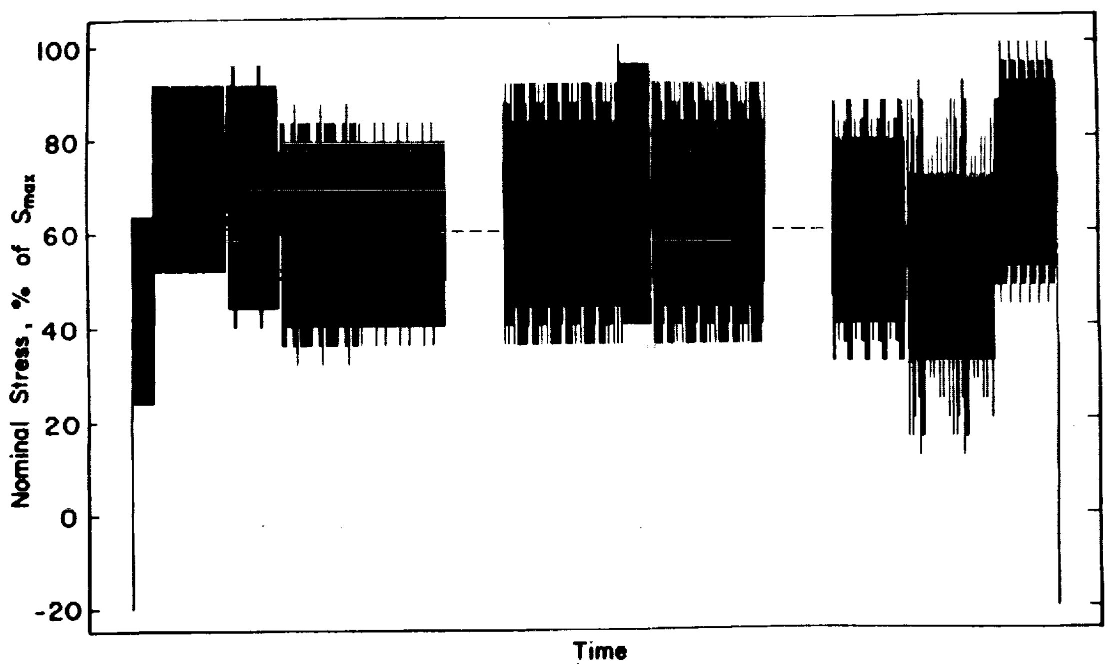

Figure 9. Example of the loading history for portions of a transport flight in Helix [12]

Table 3. Range-mean matrix for Helix from rain-flow cycle counting

	Mean									
Range	40	44	48	52	56	60	64	68	72	ALL
4	0	0	0	1	0	2	16	2	0	21
8	0	0	0	0	1	0	4	0	0	5
12	0	0	0	0	0	0	2	0	0	2
16	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	1	0	0	0	1
24	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	2	0	0	2
32	0	0	0	0	0	0	0	0	0	0
36	0	0	0	1	0	0	0	1	0	2
40	0	12	0	43	30	1742	1348	27	223	3425
44	0	0	0	1	0	1	0	1	0	3
48	0	0	0	14	5	453	2613	155	12	3252
52	0	0	0	0	0	5	20	1	0	26
56	0	0	0	24	6	65	7785	460	4	8344
60	0	0	0	0	1	0	15	0	0	16
64	0	0	0	6	8	30	6	24	0	74
68	0	0	0	1	1	0	0	0	0	2
72	0	0	0	10	4	28	0	0	0	42
76	0	0	0	1	1	1	0	0	0	3
80	0	0	0	1	1	5	0	0	0	7
84	0	0	0	0	0	0	0	0	0	0
88	0	0	0	0	1	0	0	0	0	1
92	0	0	0	0	0	0	0	0	0	0
96	0	0	0	0	0	0	0	0	0	0
100	0	0	0	0	0	0	0	0	0	0
104	0	0	0	0	0	0	0	0	0	0
108	0	0	0	0	0	0	0	0	0	0
112	0	0	0	0	0	0	0	0	0	0
116	0	0	0	0	0	0	0	0	0	0
120	1	0	0	0	0	0	0	0	0	1
ALL	1	12	0	103	59	2333	11811	671	239	15229

General Description of Maneuver History

A loading history for the tail rotor pitch beam of an AUH-76 helicopter was selected as representative, and loading histories from each of 30 distinct severe maneuvers were assumed to occur once each in a specific sequence. This produced a loading history containing 33,470 cycles, which was then modified by UDRI to eliminate minor events, shortening it to 8777 cycles, that is, 8777 peaks and 8777 valleys. This new history is called the modified maneuver history. Figure 3 shows about one third of the modified maneuver history. Figure 4 shows the complete range and mean matrix from the rain-flow cycle counting. Note that the maneuver history is scaled so that the highest load is 1 unit, which results in the lowest load being -0.516 units; therefore, the overall range is 1.516 units.

Fatigue life calculations were done for both the original and modified histories for titanium 6Al-4V. It was found that the differences are small, and the two histories appear to cause the same damage; therefore, all further testing and analysis is based on the modified history.

Upper-Lower Bound Analysis of Helix

Fatigue lives for the plate-with-hole specimens of titanium 6Al-4V subjected to Helix were calculated using the simplified (upper-lower bound) version of the local strain approach. As already discussed, only the rain-flow matrix of the history is required as input information for fatigue life analysis.

Rather than analyzing all of Helix as a single loading history, it is expedient to analyze each of the twelve unique flights separately. Therefore, the first step is to determine the rain-flow matrix of each unique flight. Then, by using these matrices , upper and lower bounds can be determined for each unique flight. These lives then combine to obtain overall bounds by considering the number of times each unique flight is repeated in Helix. Note that this combining of flights works because all of the flights return to the -20% level. This gives a common minimum stresses and strains locally at the notch for each flight, which results in no sequence effects from one flight affecting another.

Another option is also available for predicting the lives. In this option only one matrix is used which is derived from all of Helix. Comparison between these two options shows no significant difference; therefore, the simpler one of a single matrix was adopted. Hence, the rainflow matrix for an average flight in the form of Table 3 is used for the purpose of calculating the upper and lower bounds on life.

The resulting calculated lives are plotted in Fig. 10 and are given in Table 4. Note that one curve is based on constant amplitude strain-life data, and the other on prestrained data. Figure 10 plots $k_t\ S_{max}$ so that the data for various k_t can be shown on the same plot. This is expected to be valid based on Neuber's rule, Eq. 2, as long as net section yielding does not occur, in which case Eq. 2 is not valid.

The data obtained from RAE scatter over a broad range, and the lives tend to be longer than those obtained from UDRI. Also, from Fig. 10, general agreement is obtained between data and analysis.

Upper and Lower Bound Analysis of Maneuver History

Figure 11 shows the calculated results as well as test data. Table 5 gives the calculated lives for notched specimens ($k_t = 2.5$). Reasonable agreement is obtained by comparing the test

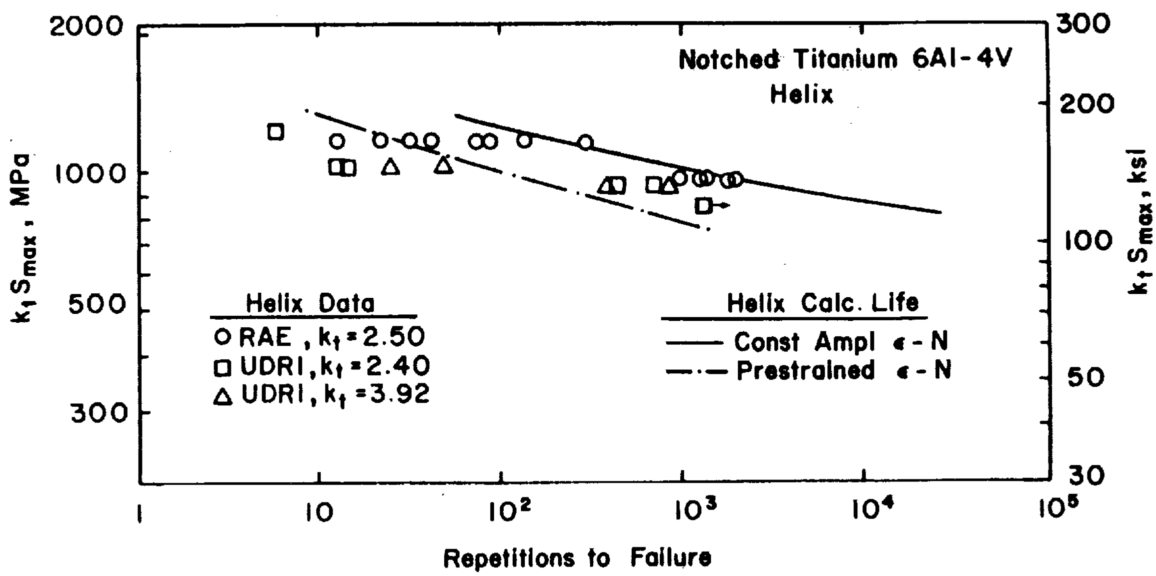

Figure 10. Analysis of Helix compared to test data for Ti 6Al-4V: The lines shown are middles of bounds.

Table 4. Calculated flights to failure for Helix for notched specimens (k_t =2.5)

S_{max}, MPa (ksi) (net area)	Helix Lower Bd.	Upper Bd.
(a)Ti 6Al-4V, Const. Ampl. Strain-Life Curve		
317 (46)	26193	26213
358 (52)	4415	4433
407 (59)	857	876
455 (66)	221	240
517 (75)	46.9	62.8
(b) Ti 6Al-4V, Prestrained Strain-Life Curve		
317 (46)	681	687
455 (66)	32.7	36.0
517 (75)	11.8	13.9
530 (77)	9.59	11.5

data and the calculated values, especially when prestrained data are used. An exception is at the lowest stresses where these calculations are conservative.

Figure 12 shows the distribution of numbers of cycles vs. ranges. During the determination of the fatigue lives, it was noted that most of the fatigue damage was done by the higher range levels, and none by the lower levels. In this regard, it was also noted that a potentially large saving of test time can be realized by eliminating lower level non-damaging stress cycles from the load history. Therefore, a rain-flow filtering was done on the maneuver history.

The basis of this filtering was the fatigue "damage", more properly called the usage fraction, which is obtained from the P-M rule calculation. Usage fraction is defined as:

$$\textit{usage fraction} = \frac{\frac{n_i}{N_i}}{\sum_{i=1}^{I} \frac{n_i}{N_i}} \tag{6}$$

where n_i is the number of cycle applied at a stress level corresponding to life N_i, and I is the number of different discrete stress levels. Figure 13 shows the usage fraction at each level vs. range for the modified history.

As is evident from Figs 12 and 13, most cycles occur at the lower ranges, whereas most damage is estimated to be done at higher ranges. Therefore the modified history was shortened by filtering all the ranges less than 0.45 units, that is, 30 % of the largest rain-flow range of 1.516 units. This filtered history contains 510 cycles, that is, 510 peaks and 510 valleys. Figure 14 shows the filtered history. By filtering the lowest stress levels of the modified history, the usage fraction is decreased only slightly, specifically by about 1 %.

Note that both modified and filtered histories produce similar fatigue lives, since most of the damage is done at higher levels that are not filtered out in the latter. Also, from Fig. 11 com-

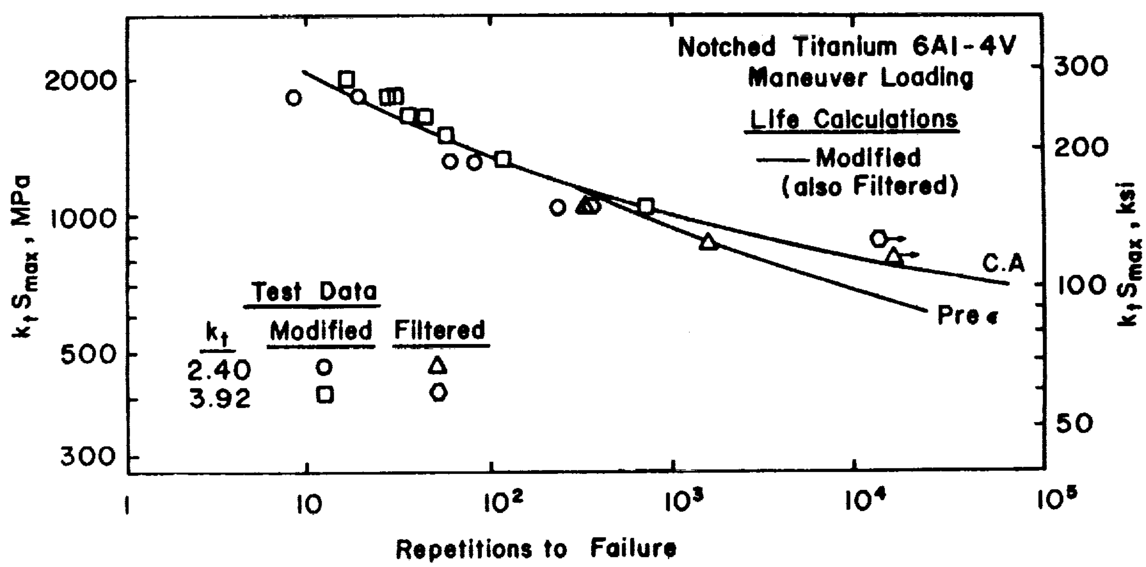

Figure 11. Analysis of maneuver history compared to test data for Ti 6Al-4V: The lines shown are middles of bounds.

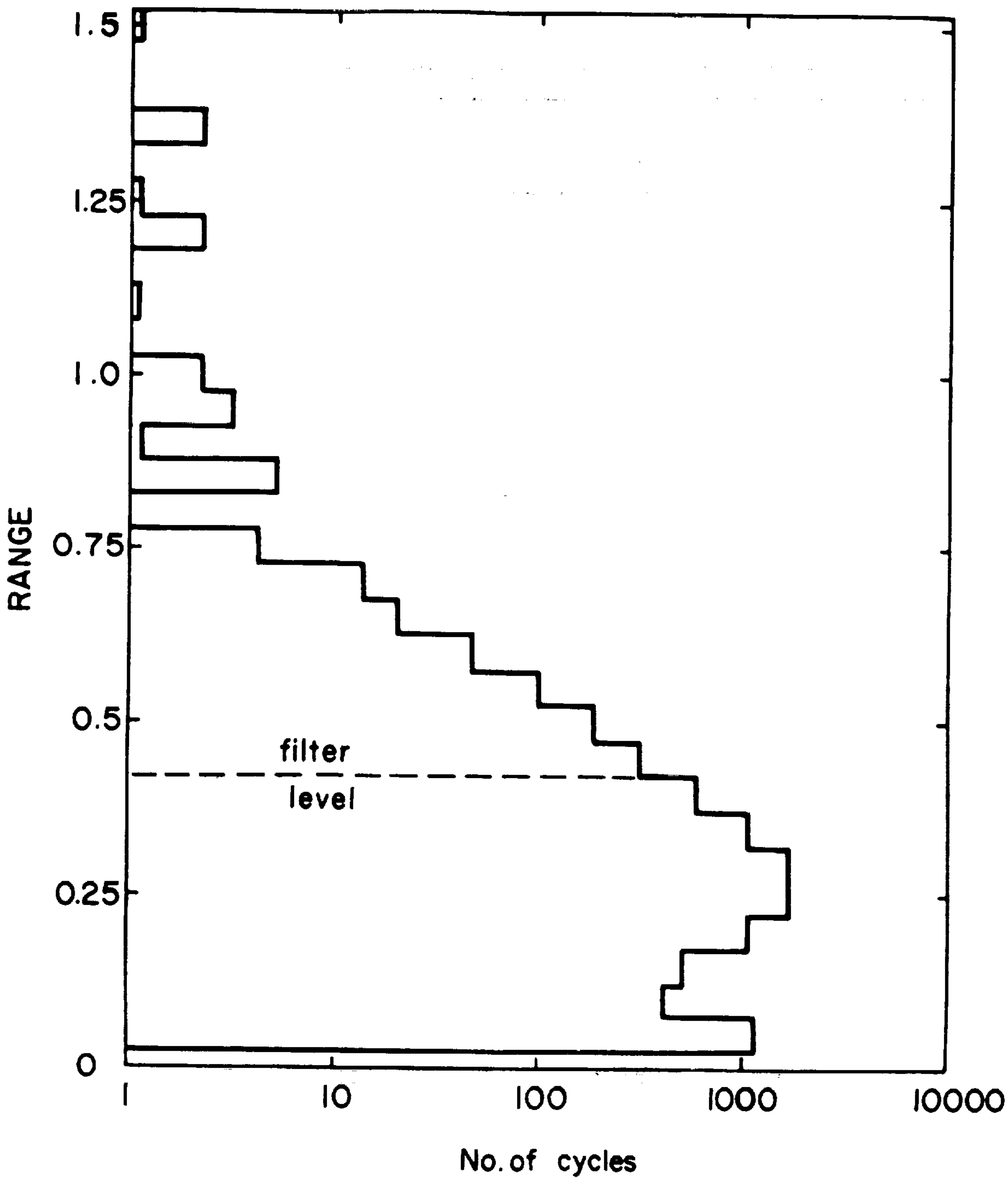

Figure 12. Number of rain-flow cycles vs. range for the modified maneuver history

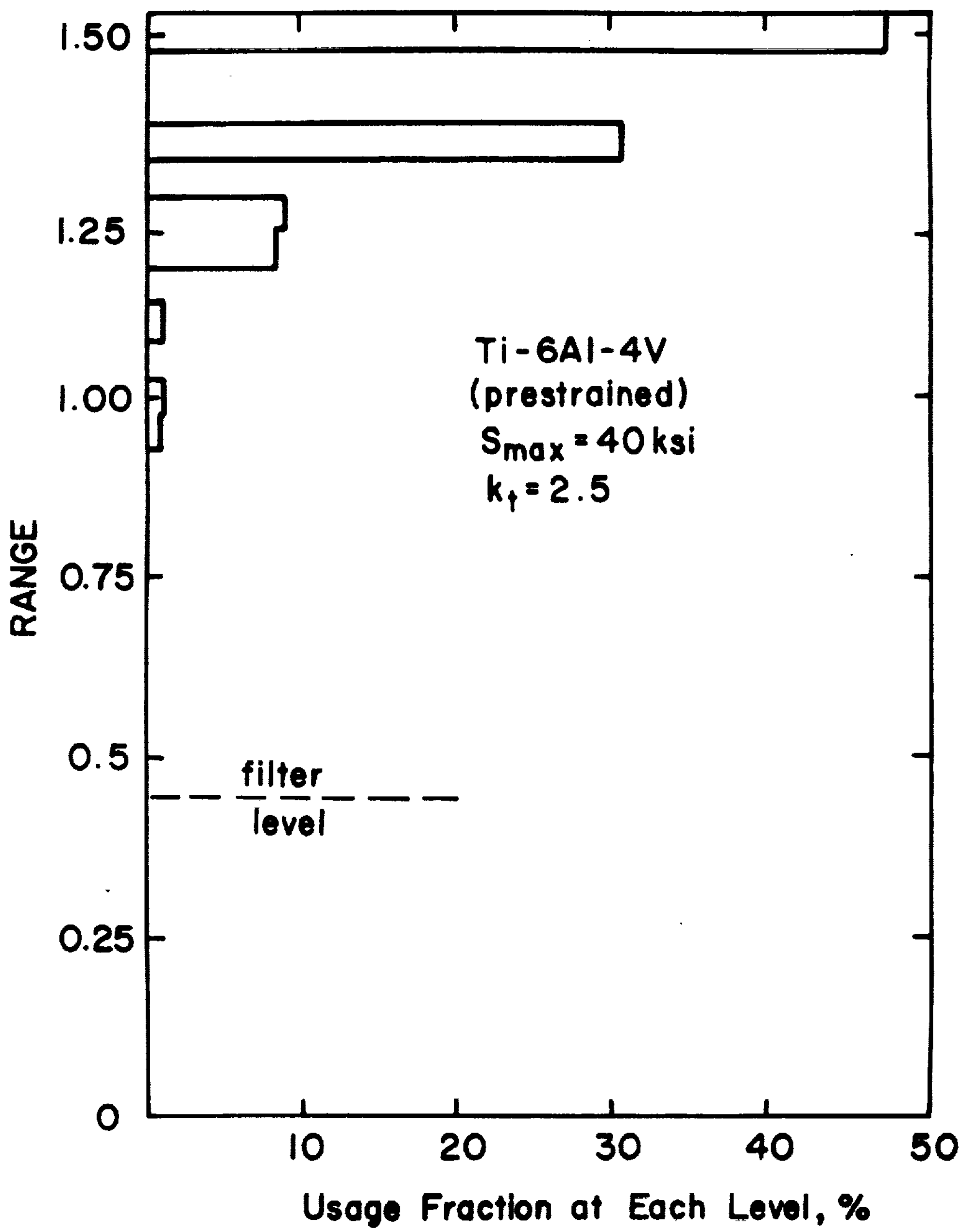

Figure 13. Usage fraction vs. range of rain-flow cycles for the modified maneuver history

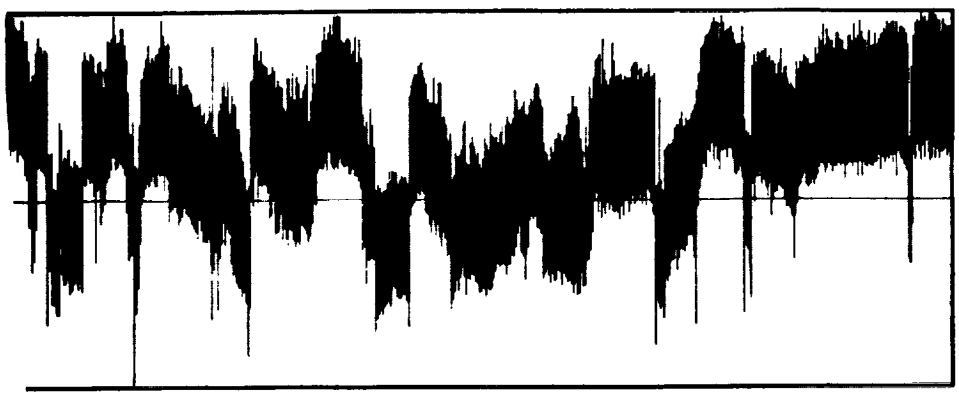

Figure 14. The filtered maneuver loading history

parison of the test data for modified and filtered history at one stress level, where both were tested, shows quite good agreement, which indicates that the filtering was a success.

Discussion

The Helix spectra is typical of a case where the bounds would tend to be relatively widely separated. This is due to most of damage being done at a relatively low level within the spectrum. However, even in this unfavorable situation, the bounds are reasonably tight for both Helix and maneuver history. For real irregular loading histories , wide separation of bounds is not likely to very often be a difficulty with this approach.

From Figs. 10 and 11, it is seen that the calculated lives and the test data are in good agreement when the prestrained data are used. The only exceptions are in the lowest levels, where the data approach or even exceed the calculation based on the constant amplitude strain-life data. The reason for this trend appears to be that these stress levels are so low that even the highest stress level (ground-air-ground) is approaching the endurance limit, and no prestrain effect due to this highest level occurs. The interpretation is made that when the major cycle is sufficiently low, an endurance limit is expected for the spectrum loading. Based on the above discussion, small cycles with amplitudes below the endurance limit can cause fatigue damage if preceded by a major cycle substantially above the endurance limit.

The calculated lives tend to be reasonably accurate at high stress levels, although there is considerable scatter. For Helix on titanium, the highest stress levels used were limited due to static failure, even though the most damaging level was still relatively low. This contrasts with the situation for the maneuver history, where the most damaging levels were calculated to be those near the maximum rain-flow range in the history. (see Fig. 13)

Table 5. Calculated repetitions to failure for maneuver history for notched specimens (k_t = 2.5)

S_{max}, MPa (ksi) (net area)	Modified Sequence Lower Bd.	Upper Bd.
(a)Ti 6Al-4V, Const. Ampl. Strain-Life Curve		
276 (40)	67700	67700
327 (47)	7940	7940
379 (55)	1410	1410
455 (66)	291	315
586 (85)	59.3	68.9
827 (120)	6.14	13.8
(b) Ti 6Al-4V, Prestrained Strain-Life Curve		
276 (40)	8770	8930
379 (55)	888	934
827 (127)	7.89	14.1

Similar comparisons from previous work [3] for Helix applied to notch specimens of an aluminum alloy are shown in Fig. 15. The trends are the same as for titanium, reinforcing the comments just made.

The above discussion and comparisons between test data and calculated lives suggest that the upper and lower bound approach represents a useful method with distinct advantages for predicting fatigue crack initiation.

Conclusions and Recommendations

The following conclusions and recommendations are drawn based on the above analysis and discussion:

1. Overstrain effects caused by the higher stress levels in a load spectrum need to be considered since these increase damage at the lower stress levels. This is especially true for cycles below the endurance limit where some cycles in the load spectrum are above the endurance limit.

2. The simplified version of the local strain approach (upper and lower bounds) should be used more widely, since only a rain-flow matrix is required as input information, and since it is easy to program on a digital computer. This approach is more economical than analysis of the large amount of data involved in full time sequence load histories. Based on the comparison of test data and calculated lives, the accuracy of the simplified method was either reasonably accurate or conservative depending on the S_{max} level.

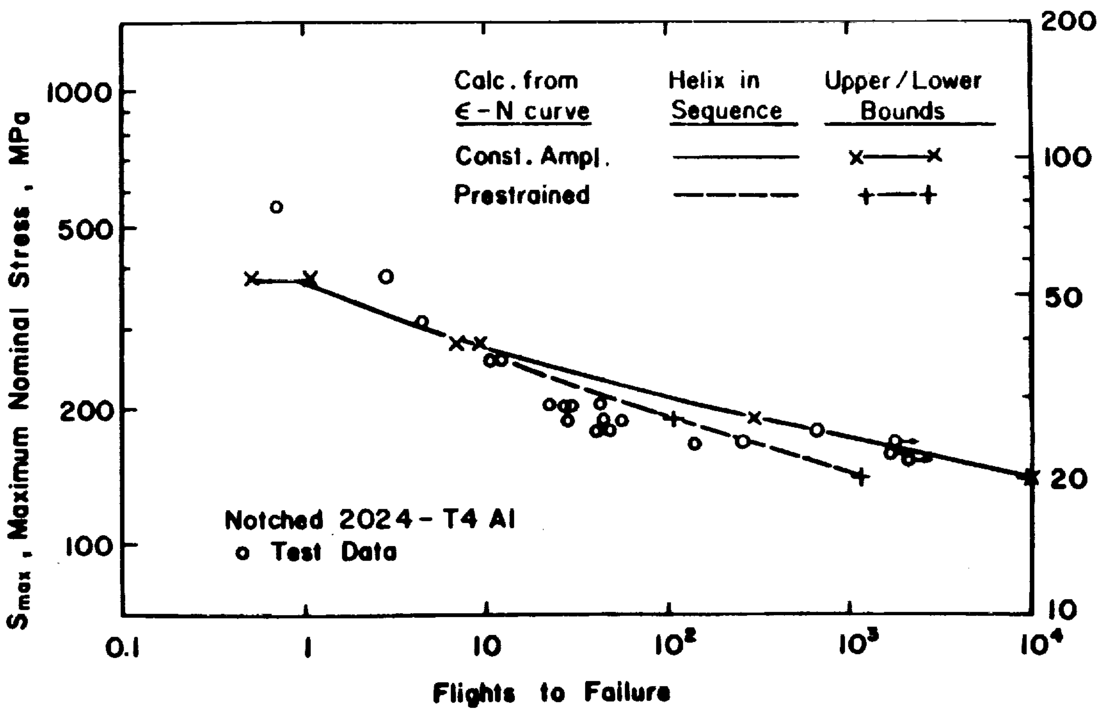

Figure 15. Analysis [3] of Helix compared to test data [12] for 2024-T4 aluminum notched specimens with k_t =2.5: The curves represent full analysis based on local strains, and the bounds from the simplified method are also shown.

Acknowledgements

The experimental work and analysis reported were supported by the U.S. Army Aviation Applied Technology Directorate, Fort Eustis, VA, as part of the Investigation of Fatigue Methodology Program with the University of Dayton Research Institute, Dayton, OH. Virginia Tech participated in this program as a subcontractor to UDRI. The preparation of this paper was supported by the U.S. Army Aerostructures Directorate at the National Aeronautics and Space Administration, Langley Research Center, Hampton, VA, under grant No. NAG-1-822 to Virginia Tech, entitled Helicopter Fatigue Research.

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APPENDIX A

COMPUTER PROGRAM FOR UPPER/LOWER BOUNDS ON LIFE ANALYSIS

- USER'S MANUAL FOR UPLO -

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(WP:computer-1)

ABSTRACT

A computer program (UPLO) for predicting upper/lower bounds on life is provided. The program takes the result of rainflow counting in the form of a matrix, that is, given numbers of cycles at each combination of range and mean values, and then uses this information by appling a local strain approach method to place upper and lower bounds on life. The input values are defined, and one example problem is attached.

CONTENTS

INTRODUCTION

Consider a notch member as in Figure A-1a, subjected to an irregular variation of nominal stress, S, with time. The goal is to predict the upper and lower bounds on life. A simplified version of local strain approach described in Reference A-1 was used to predict the upper and lower bounds on life. The first step is to summarize a lengthy history, using the rain-flow counting method, into a compact form of a matrix giving combinations of range and mean values. This step must be done using a rain-flow computer (RAINF) program which feeds its output to the upper and lower bounds program as data. The above matrix is then used with the local strain approach to place upper and lower bounds on life that would result from the analysis of the original unsummarized history. The principle behind this bounding is illustrated by Figure A-2 with the aid of Figure A-1.

The computer program explained in this manual is the same program that was previously used in Reference A-2.

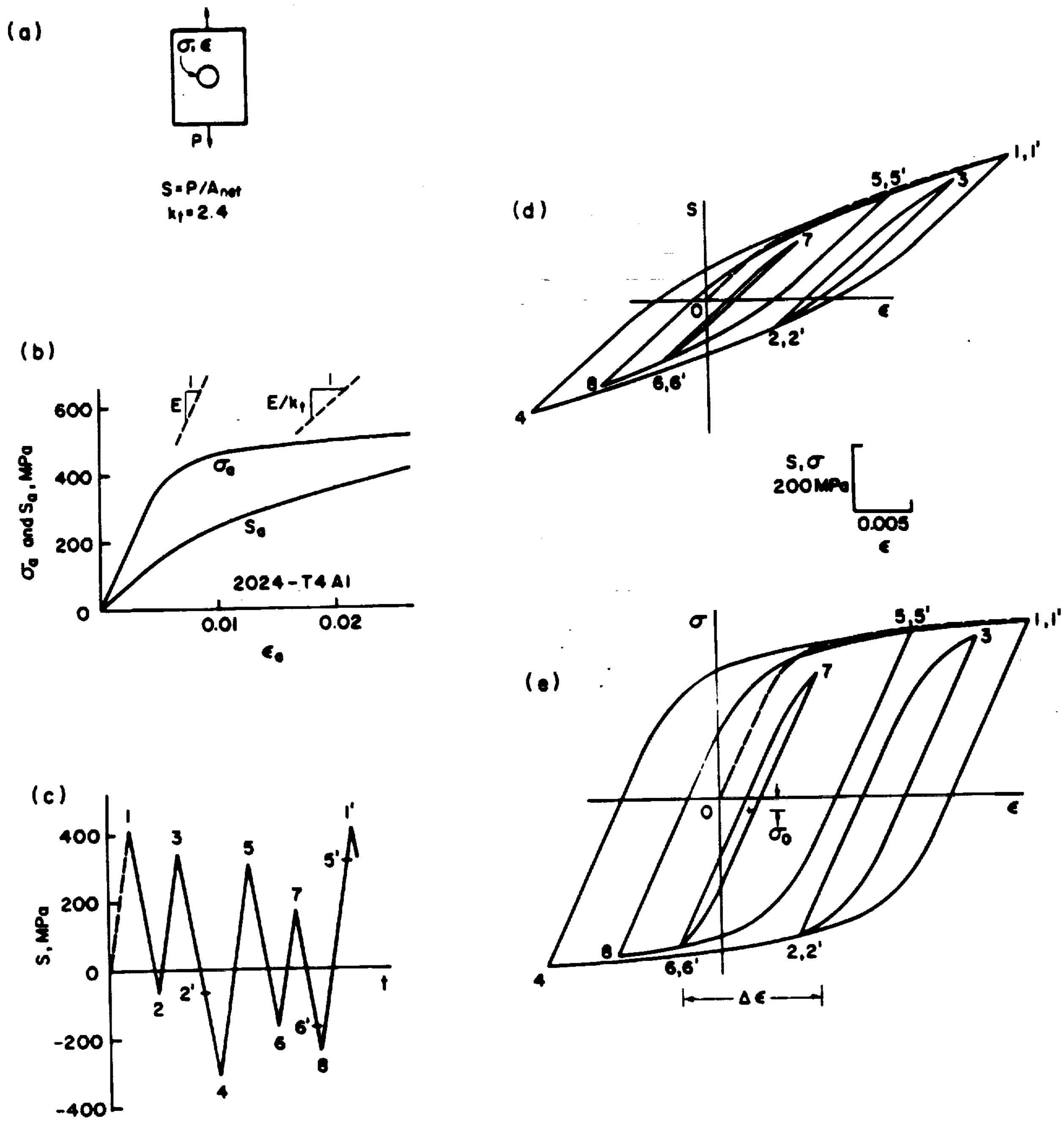

Figure A-1 - Illustration of local strain approach for an irregular load vs. time history. Notched member (a), having cyclic stress-strain and load-strain curves as in (b), is subjected to load history (c). The resulting load-strain response is shown in (d) and the local notch stress-strain response in (e).

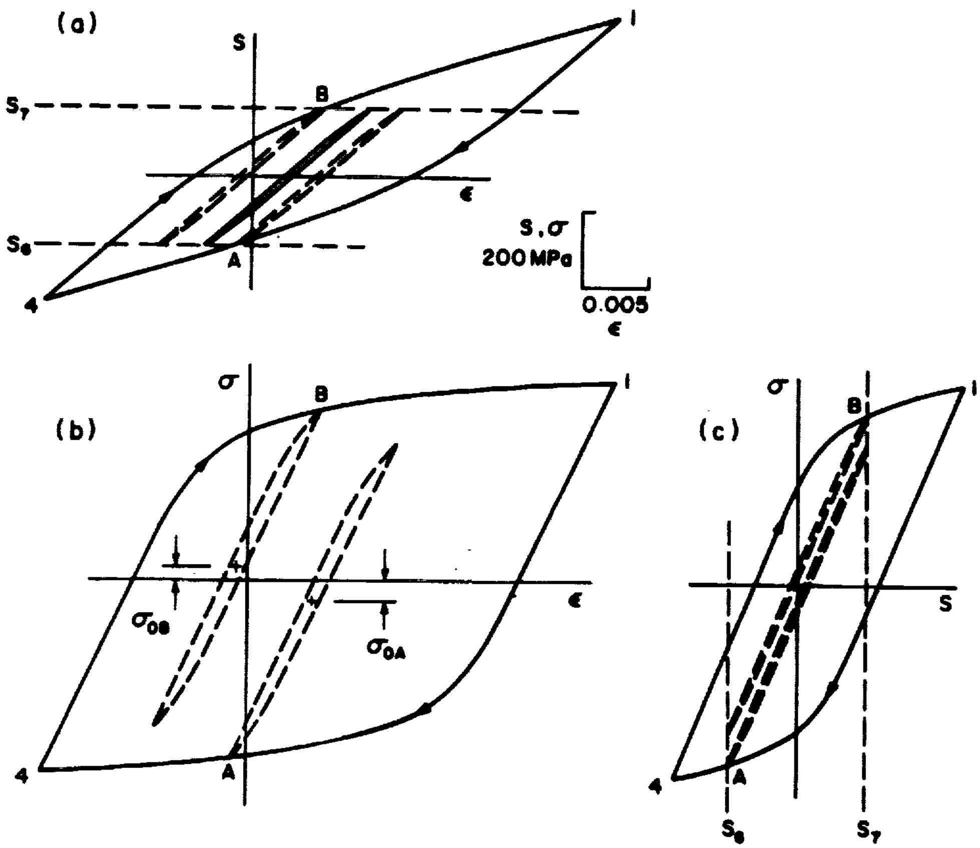

Figure A-2 - Illustration based on Fig. 1 of placing bounds on the mean stress of a subcycle when the sequence of the applied loads is not known. The mean stress for cycle 6-7 must lie between the values of σ_{0A} and σ_{0B} .

DETAILS OF EQUATIONS

The equations associated with the program are listed below to provide necessary information for the user.

1. Cyclic stress-strain curve:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{A}\right)^{1/s} \qquad (1)$$

where

σ_a = stress amplitude

ε_a = strain amplitude

E = elastic modulus

A = cyclic strength coefficient

s = cyclic strain hardening exponent

2. General strain-life curve:

$$\varepsilon_a = \frac{\sigma'_f}{E} (2N^*)^b + \varepsilon'_f (2N^*)^c \qquad (2)$$

where

σ'_f = fatigue strength coefficient

b = fatigue strength exponent

ε'_f = fatigue ductility coefficient

c = fatigue ductility exponent

N^* = life in cycles for zero mean stress

3. Life equation due to Morrow, considering the effect of mean stress:

$$N = N^* \left(1 - \frac{\sigma_o}{\sigma'_f}\right)^{-1/b} \qquad (3)$$

where

N = final life

σ_o = mean stress

4. Palmgren-Miner rule:

$$B \sum_{\text{per block}} \frac{ni}{N_i} = 1 \qquad (4)$$

where

n_i = number of occurrences of a cycle corresponding to life N_i

B = unknown number of blocks to failure

5. Neuber's rule:

$$\sigma_a \varepsilon_a = \frac{(k_t\ S_a)^2}{E} \qquad (5)$$

where

k_t = stress concentration factor

S_a = nominal stress amplitude

PROGRAM PROCEDURE OF UPPER/LOWER BOUND CALCULATIONS

The detailed procedure of the upper/lower program is given below. The example of cycle 6-7-6' of Figure A-1 is further employed as an example with the aid of Figure A-2.

It is convenient to write Equations 1 and 5 in general form without subscripts:

$$\varepsilon = \sigma/E = (\sigma/A)^{1/s} \tag{6}$$

$$\sigma\varepsilon = \frac{(k_t S)^2}{E} \tag{7}$$

The values of the constants E, A, s and k_t are of course unchanged.

Combining Equations 6 and 7 gives a relationship involving only strain, ε, and nominal stress, S.

$$\varepsilon = \left[\frac{k_t S}{E}\right]^2 \frac{1}{\varepsilon} + \left[\frac{(k_t S)^2}{E\varepsilon A}\right]^{1/s} \tag{8}$$

Considering Figure A-2, the goal is to determine the bounds on mean stress, such as σ_{oA} and σ_{oB}. Point 1 corresponds to the maximum load in the history and 4 to the minimum load in the history. As a convenience, it is assumed that the largest absolute value of load is positive. If not, then what follows will need to be modified with appropriate sign changes. Note that the load history is known, which implies that S values are known for all calculations, so that the unknowns are the σ and ε values. These calculations take advantage of the fact that various loop curves in either Figure A-2a or b have the same shape, which is that of the corresponding curve from Figure A-1 expanded with a scale factor of two.

To obtain the unknowns for point 1 let

$$S = S_1 \tag{9a}$$

$$\varepsilon = \varepsilon_1 \tag{9b}$$

$$\sigma = \sigma_1 \tag{9c}$$

Substitute Equation 9 into Equation 6 and 8, and solve for ε_1 from Equation 8. Then using Equation 6, solve for σ_1 . Equations 6 and 8 are solved by Newton's method, as direct solutions are not possible. To analyze the range of major cycle 1-4-1', let

$$S = \frac{\Delta S_{1-4}}{2} \tag{10a}$$

$$\varepsilon = \frac{\Delta\varepsilon_{1-4}}{2} \tag{10b}$$

$$\sigma = \frac{\Delta\sigma_{1-4}}{2} \tag{10c}$$

Using a parallel procedure to that just described, $\Delta\varepsilon_{1-4}$ and $\Delta\sigma_{1-4}$ are obtained. Then the stress and strain at point 4 are

$$\varepsilon_4 = \varepsilon_1 - \Delta\varepsilon_{1-4} \tag{11a}$$

$$\sigma_4 = \sigma_1 - \Delta\sigma_{1-4} \tag{11b}$$

To analyze the range of minor cycles, such as 6-7, let

$$S = \frac{\Delta S_{6-7}}{2} \tag{12a}$$

$$\varepsilon = \frac{\Delta\varepsilon_{6-7}}{2} \qquad (12b)$$

$$\sigma = \frac{\Delta\sigma_{6-7}}{2} \qquad (12c)$$

Using the same procedure, $\Delta\varepsilon_{6-7}$ and $\Delta\sigma_{6-7}$ are determined.

Once the stress and strain at points 1 and 4 are obtained, the points of attachment of loops A and B in Figure A-2 must be determined. In order to determine point of attachment of loop A, let

$$S = \frac{S_1 - S_A}{2} \qquad (13a)$$

$$\varepsilon = \frac{\varepsilon_1 - \varepsilon_A}{2} \qquad (13b)$$

$$\sigma = \frac{\sigma_1 - \sigma_A}{2} \qquad (13c)$$

Then substitute Equation 13 into Equations 6 and 8 and obtain ε_A and σ_A. Then to find the point of attachment of loop B, let

$$S = \frac{S_B - S_4}{2} \qquad (14a)$$

$$\varepsilon = \frac{\varepsilon_B - \varepsilon_4}{2} \qquad (14b)$$

$$\sigma = \frac{\sigma_B - \sigma_4}{2} \qquad (14c)$$

Again substituting into Equations 6 and 8, obtain ε_B and σ_B.

The bounds on mean stresses are then

$$\sigma_{oB} = \sigma_B - \frac{\Delta\sigma_{6-7}}{2} \tag{15a}$$

$$\sigma_{oA} = \sigma_A + \frac{\Delta\sigma_{6-7}}{2} \tag{15b}$$

Next, into Eq. 2 substitute

$$\varepsilon_a = \frac{\Delta\varepsilon_{6-7}}{2} \tag{16}$$

and obtain N, the life for zero mean stress. Finally, substitute this N^* and σ_{oB} into Equation 3 to obtain the lower bound in life, N, for cycle 6-7-6'. Similarly, substitute N^* and σ_{oA} to get the upper bound on N.

Following a similar procedure for all cycles smaller than the major one then allows the P-M rule, Equation 4, to be employed once with all of the lower bound N values, and a second time with all of the upper bound N values, to obtain bounds on the calculated number of blocks (repetitions) to failure, B.

DEFINITION OF INPUT DATA

Line No. Of Read Statement	Variable Name	Explanation	Comment
3	L	(Number of columns)x(number of rows) in the range/mean matrix	
4	KT	Stress concentration factor, k_t	
5	AMAX	Load scale factor	Factor which multiplies by the range and mean values from the matrix to give nominal stresses, S in ksi, defined consistently with k_t.
6	RMMIN LEVLM XIM RMIN LEVLR XIR	Lowest mean value in the matrix Number of columns (mean values) in the range/mean matrix Constant increment between mean values Lowest range value in the matrix Number of rows (range values) in the range/mean matrix Constant increment between range values	All these values should be the same as the values in the range/mean rainflow matrix used as input.
7	EM	E	ksi
	A	A	ksi
	SH	s	
	SFP	σ'_f	ksi
	BS	b	
	EFP	ε'_f	
	CS	c	
9	MM ( )	Elements of the matrix in one dimensional form	For Example: If a matrix has 10 columns, then MM(1) = A(1,1) MM(2) = A(1,2) MM(10) = A(1,10) MM(11) = A(2,1), etc.

Note: Other consistent stress units may replace ksi for E, A, σ'_f, and AMAX. All other inputs are dimensionless.

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EXAMPLE PROBLEM

A rainflow matrix of combinations of range and mean values for the standard helicopter load spectrum Helix [A-3] is used to determine the upper and lower bounds on life. Table A-1 shows this matrix which contains 30 rows and 9 columns. Table A-2 shows the input for this example. The entire program listing and output are also attached as Table A-3. The output lists the upper bound mean stresses, such as σ_{oB} in Figure A-2b and also the lower bound mean stresses, such as σ_{oA} in Figure A-2b. It also gives the percentage of the total "damage," that is, the Palmgren-Miner usage factor, corresponding to the upper and lower bound mean stress cases for each particular combination of nominal stress range and mean.

The load scale factor AMAX is selected to scale the load history to any desired magnitude. To select a value, it is necessary to know how the original rainflow matrix used as input is scaled. AMAX is specifically the constant which the values in the input matrix are multiplied by to obtain the desired nominal stresses, S. For this example, the largest cycle has a mean value from the matrix of 0.40 and a range of 1.20. Hence, its maximum is 1.00 and its minimum is -0.20. If a load scale factor of AMAX = 59 is chosen, this causes the analysis to be done for S_{max} = 59x1.00 = 59.0 ksi, which corresponds to the largest range in the load history being ΔS = 59x1.20 = 70.8 ksi.

Table A-1. Range-Mean Matrix for Helix from Rain-Flow Cycle Counting (Cycles per Flight, Average)*

	Mean									
Range	.40	.44	.48	.52	.56	.60	.64	.68	.72	ALL
.04	0	0	0	1	0	2	16	2	0	21
.08	0	0	0	0	1	0	4	0	0	5
.12	0	0	0	0	0	0	2	0	0	2
.16	0	0	0	0	0	0	0	0	0	0
.20	0	0	0	0	0	1	0	0	0	1
.24	0	0	0	0	0	0	0	0	0	0
.28	0	0	0	0	0	0	2	0	0	2
.32	0	0	0	0	0	0	0	0	0	0
.36	0	0	0	1	0	0	0	1	0	2
.40	0	12	0	43	30	1742	1348	27	223	3425
.44	0	0	0	1	0	1	0	1	0	3
.48	0	0	0	14	5	453	2613	155	12	3252
.52	0	0	0	0	0	5	20	1	0	26
.56	0	0	0	24	6	65	7785	460	4	8344
.60	0	0	0	0	1	0	15	0	0	16
.64	0	0	0	6	8	30	6	24	0	74
.68	0	0	0	1	1	0	0	0	0	2
.72	0	0	0	10	4	28	0	0	0	42
.76	0	0	0	1	1	1	0	0	0	3
.80	0	0	0	1	1	5	0	0	0	7
.84	0	0	0	0	0	0	0	0	0	0
.88	0	0	0	0	1	0	0	0	0	1
.92	0	0	0	0	0	0	0	0	0	0
.96	0	0	0	0	0	0	0	0	0	0
.100	0	0	0	0	0	0	0	0	0	0
1.04	0	0	0	0	0	0	0	0	0	0
1.08	0	0	0	0	0	0	0	0	0	0
1.12	0	0	0	0	0	0	0	0	0	0
1.16	0	0	0	0	0	0	0	0	0	0
1.20	1	0	0	0	0	0	0	0	0	1
ALL	1	12	0	103	59	2333	11811	671	239	15229

*The matrix entries were obtained from those for all of Helix by dividing each by 140, the number of flights.

Table A-2. Input for Example Problem

```
270
2.5
59.
0.4,9,0.04,0.04,30,0.04
15838.,192.4,.0755,220.9,-.0763,6.216,-1.0101
0,0,0,1,0,2,16,2,0,
0,0,0,0,1,0,4,2,0,
0,0,0,0,0,0,2,0,0,
0,0,0,0,0,0,0,0,0,
0,0,0,0,0,1,0,0,0,
0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,2,0,0,
0,0,0,0,0,0,0,0,0,
0,0,0,1,0,0,0,1,0,
0,12,0,43,30,1742,1348,27,223,
0,0,0,1,0,1,0,1,0,
0,0,0,14,5,453,2613,155,12,
0,0,0,0,0,5,20,1,0,
0,0,0,24,6,65,7785,460,4,
0,0,0,0,1,0,15,0,0,
0,0,0,6,8,30,6,24,0,
0,0,0,1,1,0,0,0,0,
0,0,0,10,4,28,0,0,0,
0,0,0,1,1,1,0,0,0,
0,0,0,1,1,5,0,0,0,
0,0,0,0,0,0,0,0,0,
0,0,0,0,1,0,0,0,0,
0,0,0,0,0,0,0,0,0,
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0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,
1,0,0,0,0,0,0,0,0,
```

Table A-3. Program Listing and Output for Example

```
     C$JOB          A1336,P=100
     C          UPPER/LOWER LIFE PREDICTION PROGRAM (UPLO)
     C
     C
     C      INPUT
     C      DATA LINE 1.      L=NO OF COLUMNS*NO OF ROWS IN THE RANGE/
     C                          MEAN MATRIX.
     C      DATA LINE 2.     KT=STRESS CONCENTRATION FACTOR.
     C      DATA LINE 3.   AMAX=LOAD SCALE FACTOR .(KSI)
     C      DATA LINE 4.   RMMIN=LOWEST MEAN VALUE IN THE MATRIX.
     C                     LEVLM=NO OF COLUMNS(MEAN VALUES) IN THE
     C                           RANGE/MEAN MATRIX.
     C                      XIM=CONSTANT INCREMENT BETWEEN MEAN VALUES.
     C                     RMIN=LOWEST RANGE VALUE IN THE MATRIX.
     C                     LEVLR=NO OF ROWS(RANGE VALUES)IN THE RANGE/
     C                           MEAN MATRIX.
     C                       XIR=CONSTANT INCREMENT BETWEEN RANGE VALUES.
     C             THE ABOVE VALUES SHOULD BE THE SAME AS THE VALUES
     C             IN THE RANGE/MEAN RAINFLOW MATRIX USED AS INPUT.
     C      DATA LINE 4.      EM=ELASTIC MODULUS(KSI)
     C                         A=CYCLIC STRENGTH COEFF(KSI)
     C                        SH=CYCLIC STRAIN HARDENING EXPONENT
     C                       SFP=FATIGUE STRENGTH COEFF(KSI)
     C                        BS=FATIGUE STRENGTH EXPONENT
     C                       EFP=FATIGUE DUCTILITY COEFF
     C                        CS=FATIGUE DUCTILITY EXPONENT
     C      DATA LINE 5.    MM( )=ELEMENTS OF MATRIX IN ONE DIMEN-
     C                            SIONAL FORM.
     C                            FOR EXAMPLE IF A MATRIX HAS 10
     C                            COLUMNS THEN MM(1)=A(1,1)
     C                            MM(2)=A(1,2),MM(11)=A(2,1),ETC.
     C
     C
 1          DIMENSION VV(300),SLL(300),XNF(300),XNNU(300),XNNL(300),
           *SIGL(300),SIGU(300),UU(300),STRN1(300),STRN2(300),STMEN1(300
           *),STMEN2(300),RQ(300),RM(300)
           *,SMAX1(300),SMIN1(300),SMXMN(2)
           *,MM(900),RA(32),CMM(32),X2(900),Y2(900),BU(900),BL(90
           *0),X3(1999),Y3(1999),MMM(1999),X4(1999),Y4(1999),SD(300)
           *,STRS1(300),STRS2(300),DMU(300),DML(300)
 2          REAL KT,KT1
 3          READ(5,*)L
 4          READ(5,*)KT
 5          READ(5,*)AMAX
 6          READ(5,*)RMMIN,LEVLM,XIM,RMIN,LEVLR,XIR
 7          READ(5,*)EM,A,SH,SFP,BS,EFP,CS
 8          WRITE(6,300)EM,SFP,BS,EFP,CS,A,SH,AMAX
 9          READ (5,*)(MMM(I),I=1,L)
10          CMM(1)=RMMIN
11          DO 904 L=2,LEVLM
12          LL=L-1
13          CMM(L)=CMM(LL)+XIM
14   904    CONTINUE
15          RA(1)=RMIN
16          DO 905 L=2,LEVLR
17          LL=L-1
18   905    RA(L)=RA(LL)+XIR
19          R=1/SH
20          IKK=1
21          DO 907 I=1,LEVLR
```

Table A-3 (2nd page)

```
      DO 908 J=1,LEVLM
      X4(IKK)=((2.*CMM(J))+RA(I))/2.
      Y4(IKK)=(2.*CMM(J))-X4(IKK)
      X3(IKK)=X4(IKK)*AMAX
      Y3(IKK)=Y4(IKK)*AMAX
      IKK=IKK+1
908   CONTINUE
907   CONTINUE
      IKK=IKK-1
      IK=1
      DO 960 I=1,IKK
      IF(MMM(I).EQ.0) GO TO 960
      X2(IK)=X3(I)
      Y2(IK)=Y3(I)
      MM(IK)=MMM(I)
      IK=IK+1
960   CONTINUE
      IK=IK-1
      SMAX=X2(IK)
      SMIN=Y2(IK)
      DO 909 I=1,IK
909   SLL(I)=(X2(I)-Y2(I))/2.
      NLOAD=IK
      DO 911 I=1,IK
      SMAX1(I)=X2(I)
      IF(SMAX1(I).GT.Y2(I)) GO TO 912
      SMAX1(I)=Y2(I)
      SMIN1(I)=X2(I)
      GO TO 911
912   SMIN1(I)=Y2(I)
911   CONTINUE
C********************************
C     DETERMINING MAX,MIN LOCAL STRAIN AND STRESS
      IF(ABS(SMIN).LT.ABS(SMAX))THEN
      C=((SMAX*KT)**2)/EM
      CALL NEUBER(C,R,A,V,U,EM)
      FMAX=U
      STMAX=V
      SSS=SLL(IK)
      C=((SSS*KT)**2)/EM
      CALL NEUBER(C,R,A,V,U,EM)
      FMIN=FMAX-(2.*U)
      STMIN=STMAX-(2.*V)
      ELSE
      C=((SMIN*KT)**2)/EM
      CALL NEUBER(C,R,A,V,U,EM)
      FMIN=U
      STMIN=V
      SSS=SLL(IK)
      C=((SSS*KT)**2)/EM
      CALL NEUBER(C,R,A,V,U,EM)
      WRITE(6,*)V,U
      FMAX=FMIN+(2.*U)
      STMAX=STMIN+(2.*V)
      END IF
      WRITE(6,310)FMAX,STMAX
      WRITE(6,320)FMIN,STMIN
C********************************
C     DETERMINING UPPER AND LOWER BOUNDS ON MEAN STRESS
      DO 1100 I=1,NLOAD
```

Table A-3 (3rd page)

```
        SPRM1=(SMAX1(I)-SMIN)/2.
        C=((SPRM1*KT)**2)/EM
        CALL NEUBER (C,R,A,V,U,EM)
        STRN1(I)=(2.*V)+STMIN
        STRS1(I)=(2.*U)+FMIN
        SPRM2=(SMAX-SMIN1(I))/2.
        C=((SPRM2*KT)**2)/EM
        CALL NEUBER (C,R,A,V,U,EM)
        STRN2(I)=STMAX-(2.*V)
        STRS2(I)=FMAX-(2.*U)
        SL=SLL(I)
        C=((SL*KT)**2)/EM
        CALL NEUBER(C,R,A,V,U,EM)
        VV(I)=V
        UU(I)=U
        STMEN1(I)=STRN1(I)-VV(I)
        STMEN2(I)=STRN2(I)+VV(I)
 1100   CONTINUE
        DO 1310 I=1,NLOAD
        SIGU(I)=STRS1(I)-UU(I)
        SIGL(I)=STRS2(I)+UU(I)
 1310   CONTINUE
        DO 1099 I=1,IK
 1099   SD(I)=2.*SLL(I)
        WRITE(6,330)
        DO 1400 I=1,NLOAD
        ER=VV(I)
C*****************************************
C       UPPER/LOWER LIFE PREDICTION
        CALL  LIFE (ER,SFP,BS,EFP,CS,EM,XNFF)
        XNF(I)=XNFF
 1400   CONTINUE
        DO 1500 I=1,NLOAD
        XNNU(I)=(XNF(I))/((1-(SIGL(I)/SFP))**(1/BS))
        XNNL(I)=(XNF(I))/((1-(SIGU(I)/SFP))**(1/BS))
 1500   CONTINUE
        DO 1600 I=1,IK
        BU(I)=MM(I)/XNNL(I)
        BL(I)=MM(I)/XNNU(I)
 1600    CONTINUE
        BUU=0.
        BLL=0.
        DO 1800 I=1,IK
        BUU=BU(I)+BUU
        BLL=BL(I)+BLL
 1800    CONTINUE
        BLLL=1./BLL
        BUUU=1./BUU
        DO 2001 I=1,IK
        DML(I)=(BL(I)/BLL)*100.
        IF(DML(I).LT.0.01)DML(I)=0.
 2001   CONTINUE
        DO 2200 I=1,IK
         DMU(I)=(BU(I)/BUU)*100.
        IF(DMU(I).LT.0.01)DMU(I)=0.
 2200   CONTINUE
        DO 1700 I=1,IK
        RQ(I)=ABS(X2(I)-Y2(I))
        RM(I)=(X2(I)+Y2(I))/2.
        WRITE(6,34)RQ(I),RM(I),VV(I),MM(I),SIGL(I),SIGU(I),DML(I),DMU(I)
```

```
 1700 CONTINUE
      WRITE(6,350)BUUU
      WRITE(6,360)BLLL
 300  FORMAT('1',/12X,'ELASTIC MODULUS(KSI)=',F7.0//12X,
     *'FATIGUE STRENGTH COEFFICIENT(KSI) =',F6.1//12X,
     *'FATIGUE STRENGTH EXPONENT         =',F7.4//12X,
     *'FATIGUE DUCTILITY COEFFICIENT     =',F8.4//12X,
     *'FATIGUE DUCTILITY EXPONENT        =',F7.4//12X,
     *'CYCLIC STRENGTH COEFFICIENT(KSI)  =',F6.1//12X,
     *'CYCLIC STRAIN HARDENING EXPONENT  =',F6.3//12X,
     *'LOAD SCALE FACTOR                 =',F6.3)
 310  FORMAT('1',//,12X,'MAX LOCAL STRESS(KSI)=',F9.3,//,12X,
     *'MAX LOCAL STRAIN=',F9.5)
 320  FORMAT(//,12X,'MIN LOCAL STRESS(KSI)=',F9.3,//,12X,
     *'MIN LOCAL STRAIN',F9.5)
 330  FORMAT(/12X,'NOMINAL',3X,'NOMINAL',21X,'LOWER',4X,'UPPER',3X,
     *'PERCENT',3X,'PERCENT',
     */13X,'STRESS',4X,'STRESS',21X,'MEAN',5X,'MEAN',4X,'OF TOTAL',
     *2X,'OF TOTAL',
     */14X,'RANGE',5X,'MEAN',4X,'STRAIN',4X,'NO OF',3X,'STRESS',3X,
     *'STRESS',2X,'LOWER',5X,'UPPER',
     */14X,'(KSI)',5X,'(KSI)',2X,'AMPLITUDE',2X,'CYCLES',2X,'(KSI)',
     *4X,'(KSI)',3X,'DAMAGE',4X,'DAMAGE')
 34   FORMAT(12X,F7.2,3X,F7.2,3X,F8.5,2X,I5,2X,F7.3,2X,F7.3,3X,F5.2,
     *5X,F5.2)
 350  FORMAT(//,12X,'LOWER LIFE BOUND=',E11.3)
 360  FORMAT(//,12X,'UPPER LIFE BOUND=',E11.3)
      STOP
      END
C*************************************

      SUBROUTINE NEUBER(C,R,A,V,U,EM)
      DIMENSION E(100)
      S1=(EM/A**R)**(1/(1-R))
      EP1=(S1/A)**R
      C1=S1*EP1
      IF(C.GT.C1) GO TO 10
      E(1)=(C/EM)**.5
      GO TO 20
 10   E(1)=(C/A)*(R/(1+R))
 20   DO 1 I=1,100
      P=C/EM/E(I)
      Q=(C/A/E(I))**R
      E(I+1)=E(I)-(E(I)-P-Q)/(1+P/E(I)+R*Q/E(I))
      X=E(I+1)/E(I)
      IF(X.GT.0.999.AND.X.LT.1.001) GO TO 30
 1    CONTINUE
 30   V=E(I+1)
      U=C/V
      RETURN
      END
C********************************************

      SUBROUTINE LIFE(ER,SFP,BS,EFP,CS,EM,XNFF)
      DIMENSION XN(100)
      XNT=.5*(SFP/EM/EFP)**(1/(CS-BS))
      EPT=EFP*(2*XNT)**CS
      IF(ER.GT.EPT)GO TO 40
      XN(1)=.5*(EM*ER/SFP)**(1/BS)
      GO TO 50
```

Table A-3 (5th page)

```
40    XN(1)=.5*(ER/EFP)**(1/CS)
50    DO 210 K=1,100
      G=SFP/EM*2**BS
      H=EFP*2**CS
      XN(K+1)=XN(K)-(G*XN(K)**BS+H*XN(K)**CS-ER)/(G*BS*XN(K)**(BS-1)+H
     &*CS*XN(K)**(CS-1))
      Z=XN(K+1)/XN(K)
      IF(Z.GT.0.999.AND.Z.LT.1.0010) GO TO 60
210   CONTINUE
60      XNFF=XN(K+1)
      RETURN
      END

C$ENTRY
```

Table A-3 (6th page)

ELASTIC MODULUS(KSI)= 15838.	
FATIGUE STRENGTH COEFFICIENT(KSI)	= 220.9
FATIGUE STRENGTH EXPONENT	=-0.0763
FATIGUE DUCTILITY COEFFICIENT	= 6.2160
FATIGUE DUCTILITY EXPONENT	=-1.0101
CYCLIC STRENGTH COEFFICIENT(KSI)	= 192.4
CYCLIC STRAIN HARDENING EXPONENT	= 0.076
LOAD SCALE FACTOR	=59.000

Table A-3 (7th page)

MAX LOCAL STRESS(KSI)= 124.578

MAX LOCAL STRAIN= 0.01103

MIN LOCAL STRESS(KSI)= -51.903

MIN LOCAL STRAIN -0.00018

NOMINAL STRESS RANGE (KSI)	NOMINAL STRESS MEAN (KSI)	STRAIN AMPLITUDE	NO OF CYCLES	LOWER MEAN STRESS (KSI)	UPPER MEAN STRESS (KSI)	PERCENT OF TOTAL LOWER DAMAGE	PERCENT OF TOTAL UPPER DAMAGE
2.36	30.68	0.00019	1	53.778	54.296	0.00	0.00
2.36	35.40	0.00019	2	65.578	66.093	0.00	0.00
2.36	37.76	0.00019	16	71.478	71.990	0.00	0.00
2.36	40.12	0.00019	2	77.378	77.885	0.00	0.00
4.72	33.04	0.00037	1	59.678	60.194	0.00	0.00
4.72	37.76	0.00037	4	71.478	71.988	0.00	0.00
4.72	40.12	0.00037	2	77.378	77.881	0.00	0.00
7.08	37.76	0.00056	2	71.478	71.985	0.00	0.00
11.80	35.40	0.00093	1	65.578	66.085	0.00	0.00
16.52	37.76	0.00130	2	71.478	71.960	0.00	0.00
21.24	30.68	0.00168	1	53.778	54.285	0.00	0.00
21.24	40.12	0.00168	1	77.378	77.793	0.00	0.00
23.60	25.96	0.00186	12	41.979	42.492	0.00	0.00
23.60	30.68	0.00186	43	53.778	54.281	0.00	0.00
23.60	33.04	0.00186	30	59.678	60.169	0.00	0.00
23.60	35.40	0.00186	1742	65.578	66.049	0.11	0.11
23.60	37.76	0.00186	1348	71.478	71.916	0.14	0.14
23.60	40.12	0.00186	27	77.378	77.765	0.00	0.00
23.60	42.48	0.00186	223	83.278	83.584	0.07	0.07
25.96	30.68	0.00205	1	53.778	54.276	0.00	0.00
25.96	35.40	0.00205	1	65.578	66.034	0.00	0.00
25.96	40.12	0.00205	1	77.378	77.729	0.00	0.00
28.32	30.68	0.00224	14	53.779	54.269	0.00	0.00
28.32	33.04	0.00224	5	59.678	60.149	0.00	0.00
28.32	35.40	0.00224	453	65.578	66.016	0.30	0.31
28.32	37.76	0.00224	2613	71.478	71.865	2.89	2.92
28.32	40.12	0.00224	155	77.378	77.684	0.29	0.29
28.32	42.48	0.00224	12	83.278	83.461	0.04	0.04
30.68	35.40	0.00242	5	65.578	65.993	0.00	0.00
30.68	37.76	0.00242	20	71.478	71.829	0.06	0.06
30.68	40.12	0.00242	1	77.378	77.629	0.00	0.00
33.04	30.68	0.00261	24	53.779	54.249	0.05	0.05
33.04	33.04	0.00261	6	59.679	60.116	0.02	0.02
33.04	35.40	0.00261	65	65.578	65.965	0.33	0.33
33.04	37.76	0.00261	7785	71.478	71.784	64.88	65.23
33.04	40.12	0.00261	460	77.378	77.561	6.50	6.47
33.04	42.48	0.00261	4	83.278	83.278	0.10	0.10
35.40	33.04	0.00279	1	59.679	60.093	0.00	0.00
35.40	37.76	0.00279	15	71.478	71.729	0.31	0.31
37.76	30.68	0.00298	6	53.780	54.216	0.07	0.07
37.76	33.04	0.00298	8	59.679	60.065	0.14	0.14
37.76	35.40	0.00298	30	65.578	65.884	0.87	0.87
37.76	37.76	0.00298	6	71.478	71.661	0.29	0.29
37.76	40.12	0.00298	24	77.378	77.378	1.95	1.91
40.12	30.68	0.00317	1	53.781	54.194	0.02	0.02

Table A-3 (8th page)

40.12	33.04	0.00317	1	59.680	60.029	0.04	0.04
42.48	30.68	0.00335	10	53.782	54.165	0.52	0.52
42.48	33.04	0.00335	4	59.680	59.984	0.33	0.33
42.48	35.40	0.00335	28	65.579	65.762	3.78	3.76
44.84	30.68	0.00354	1	53.784	54.129	0.11	0.11
44.84	33.04	0.00354	1	59.681	59.930	0.17	0.17
44.84	35.40	0.00354	1	65.579	65.679	0.27	0.27
47.20	30.68	0.00373	1	53.785	54.085	0.21	0.21
47.20	33.04	0.00373	1	59.681	59.863	0.33	0.33
47.20	35.40	0.00373	5	65.579	65.579	2.69	2.63
51.92	33.04	0.00410	1	59.682	59.682	1.15	1.13
70.80	23.60	0.00560	1	36.337	36.337	10.96	10.73

LOWER LIFE BOUND= 0.851E 03

UPPER LIFE BOUND= 0.869E 03

APPENDIX B

COMPUTER PROGRAM FOR RAIN-FLOW CYCLE COUNTING ANALYSIS

- USER'S MANUAL FOR RAINF -

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(WP:computer-2)

ABSTRACT

A computer program (RAINF) for rain-flow cycle counting analysis is provided. The program can take a lengthy load history and reduce it to the compact form of a matrix giving combinations of range and mean or peak and valley values. This information can be used for fatigue analysis. The input values are defined and five examples using different options of the program are provided.

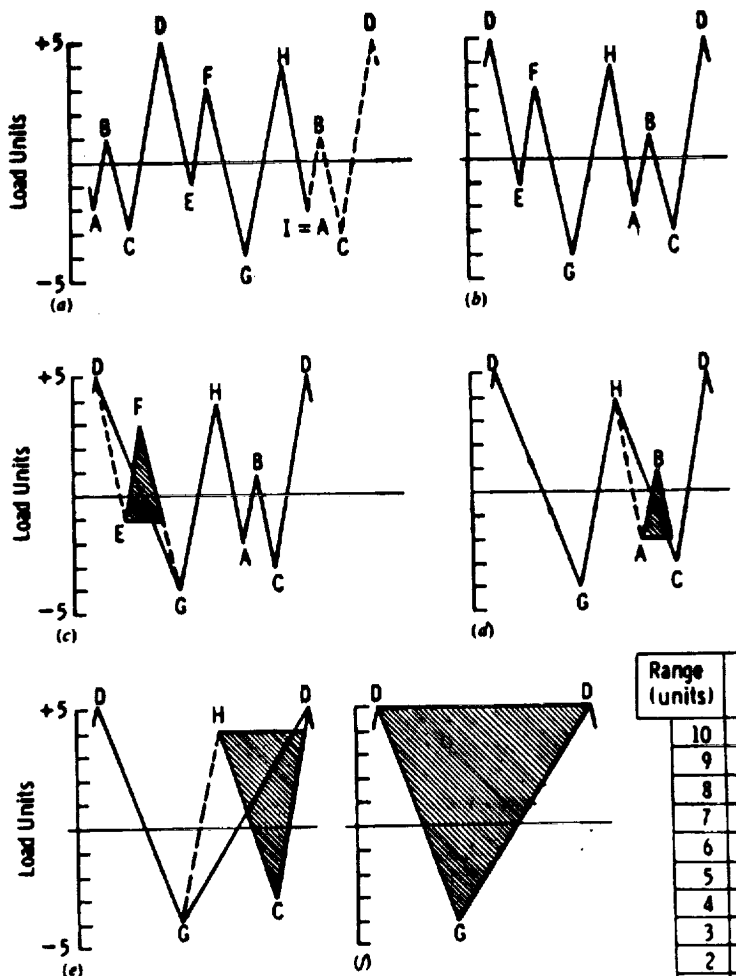
CONTENTS

INTRODUCTION

Rain-flow cycle counting is a method that exists for reducing service load history records to a compact description so that the information can be used in analysis for fatigue. The compact description is in the form of a matrix giving combinations of range and mean, or peak and valley values. This method is widely accepted as the most accurate cycle counting method for predicting fatigue life based on a cumulative damage type of approach. The rain-flow method cannot be misled by any synthetic load sequences and will always count the cycles correctly, based on the fact that closed hysteresis loops are most representative of a fatigue damage event.

Figure B-1 illustrates rain-flow cycle counting. As shown in this example, the rain-flow method has the important characteristic that it counts the major load excursions as cycles, while also counting the minor events. This feature allows it to realistically handle real service loading where there are low level vibratory loads, etc., superimposed on major cycles associated with the usage of the machine, vehicle or structure, such as ground-air-ground cycles in aircraft.

The computer program explained in this manual is the same program that was previously used in Reference B-1.



Range (units)	Cycle Counts	Events
10	0	
9	1	D-G
8	0	
7	1	H-C
6	0	
5	0	
4	1	E-F
3	1	A-B
2	0	
1	0	

FIG. 7 Example of Simplified Rainflow Counting for a Repeating History

TABLE X1.4 Simplified Rainflow Counting for Repeating Histories (Fig. 7)

Range Units	Mean Units								
	−2.0	−1.5	−1.0	−0.5	0	+0.5	+1.0	+1.5	+2.0
10	...	...	...	...	...	...	...	...	...
9	...	...	...	...	...	1	...	...	...
8	...	...	...	...	...	...	...	...	...
7	...	...	...	...	...	1	...	...	...
6	...	...	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	1	...	...
3	...	...	...	1	...	...	...	...	...
2	...	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...	...

Figure B-1. Illustration of rain-flow cycle counting from the ASTM Standards [B-2].

PROGRAM LOGIC

The following logic is used consistent with the ASTM standard [B-2]: Let x denote the absolute value of the range under consideration, and y previous absolute range adjacent to x.

Step 1: Determine the maximum absolute value in the history. (Note that this value can be either a peak or a valley.)

Step 2: Arrange the history to start with the maximum absolute value. Move all peaks and valleys which occur prior to the maximum to the end as illustrated in Figure B-1a.

Step 3: Read the next value. If out of data, go to step 9.

Step 4: Three points are needed to define x and y. If there are less than three points, go back to step 3. Define x and y using the three most recent peaks and valleys that have not been discarded.

Step 5: Compare the two ranges, namely x and y.

a) If $x < y$, go to step 3. b) If $x \geq y$, go to step 6.

Step 6: If a rain-flow filtered history is not desired, go to step 8.

Step 7: If $y \leq$ filter level specified in the program Input, then discard the peak and valley of the range y in the array in memory which is the original history of step 2.

Step 8: Count range y as one cycle, determine the mean value of the peak and valley of y, discard the peak and valley of y in the array set up in step 3, and go to step 4.

Step 9: Stop.

DEFINITION OF INPUT DATA

Line No. of READ Statement	Variable Name	Explanation	Comment
3	OPTION	= 1 List filtered history as peak/valley sequence; also print range/mean matrix of rain-flow cycles for original history. = 2 List range, mean, min, and a max of rain-flow cycles not in matrix form. = 3 Print range/mean matrix of rain-flow cycles. = 4 Print max/min 32x32 matrix of rain-flow cycles.	Integer The history is converted to a min value of 1 and max value of 32.
4	FL	Filter level as a range value	Required only for OPTION = 1.
5	NN	Number of peak/valley points in history	
8	XIM	Constant increment between mean values in the range/mean matrix	Required only for OPTION = 1 or 3.
	XIR	Constant increment between range values in the range/mean matrix	Required only for OPTION = 1 or 3.
9	P( )	Input load history as peaks and valleys in sequence	History must start and end with the same value.

REFERENCES

B-1. Berens, A. P., Gallagher, J. P., Dowling, N. E., Khosrovaneh, A., K., and Thangjitham, S., "Helicopter Fatigue Methodology, Vols. I and II," Report No. USAAVSCOM TR 87-D-13A and 13B, U. S. Army Aviation Applied Technology Directorate, Ft. Eustis, VA, 1987.

B-2. "Standard Practice for Cycle Counting in Fatigue Analysis," 1986 Annual Book of ASTM Standards, Vol. 03.01, Standard No. 1049, pp. 836-848.

B-3. Fatigue Under Complex Loading: Analyses and Experiments, R. M. Wetzel, Editor, The Society of Automotive Engineers, Warrendale, PA, Vol. AE-6, 1977.

EXAMPLE 1

The history of Figure B-1 is used for this rain-flow cycle counting example. Option 2 of the program is used; therefore, the result is shown as a list of range, mean, minimum, and maximum values. Table B-1 shows the input for this example. The entire program listing and program output for this example are attached as Table B-2.

Table B-1. Input for Example 1

```
2
9
-2.,1.,-3.,5.,-1.,3.,-4.,4.,-2.
```

Table B-2. Program Listing and Output for Example 1

```
C$JOB           NPV1336,P=100
C                 RAIN-FLOW COUNTING PROGRAM (RAINF)
C      NOTE THAT THE HISTORY MUST START AND END WITH THE SAME VALUE.
C
C      INPUT
C      DATA LINE 1.  OPTION=1    LIST FILTER HISTORY AS PEAK/VALLEY
C                                SEQUENCE.ALSO PRINT RANGE/MEAN
C                                MATRIX OF RAINFLOW CYCLES FOR
C                                ORIGINAL HISTORY.
C                          =2    LIST RANGE,MEAN,MIN,AND MAX OF RAIN-
C                                FLOW CYCLES NOT IN MATRIX FORM.
C                          =3    PRINT RANGE/MEAN MATRIX OF RAINFLOW
C                                CYCLES.
C                          =4    PRINT MAX/MIN MATRIX OF RAINFLOW
C                                CYCLES.
C      DATA LINE 2.      FL=FILTER VALUE AS A RANGE
C      DATA LINE 3.      NN=NUMBER OF PEAK/VALLEY POINTS IN HISTORY
C      DATA LINE 4.      XIM=CONSTANT INCREMENT BETWEEN MEAN VALUES
C                            IN THE RANGE/MEAN MATRIX.
C                        XIR=CONSTANT INCREMENT BETWEEN RANGE VALUES
C                            IN THE RANGE/MEAN MATRIX.
C                  NOTE THAT XIM,AND XIR REQUIRED FOR OPTION=1 OR 3
C      DATA LINE 5.      P( )=INPUT LOAD HISTORY AS PEAKS AND VALLEYS
C                            IN SEQUENCE.
C
C
C
       REAL P(10000),PE(10000),PP(10000),PC(10000),PCC(10000),MM(64),
      *RA(64),PI(10000),MEAN(5005),R(5005)
       INTEGER M(64,64),SUM(64),SUMM,MI(32,32),OPTION
       READ(5,*)OPTION
       IF(OPTION.EQ.1)READ(5,*)FL
       READ(5,*)NN
       IF(OPTION.EQ.4)GO TO 40
       IF(OPTION.EQ.2)GO TO 40
       READ(5,*)XIR,XIM
 40    READ(5,*)(P(I),I=1,NN)
       N=NN
C
C
C      DETERMINATION OF LARGEST PEAK OR VALLEY
       LCOUNT=1
       DO 100 I=1,N
       PE(I)=P(I)
 100   CONTINUE
       PMAX=ABS(P(1))
       DO 200 I=2,N
       IF(PMAX.GE.PE(I)) GO TO 200
       PMAX=ABS(PE(I))
       LCOUNT=I
 200    CONTINUE
       IF(OPTION.EQ.4)THEN
       SMAX=P(1)
       SMIN=P(1)
       DO 301 I=2,NN
       IF(P(I).GT.SMAX)SMAX=P(I)
       IF(P(I).LT.SMIN)SMIN=P(I)
 301   CONTINUE
       CF1=SMIN
       CF2=SMAX
```

```
      CF3=SMAX-SMIN
      END IF
C
C
C     ARRANGE THE PEAK OR VALLEY
      JK=LCOUNT+1
      J=N-JK+1
      KKK=LCOUNT
      DO 300 I=1,J
      PP(I)=P(KKK)
      KKK=KKK+1
300   CONTINUE
C     JJJ=N-LCOUNT-1
      J=J+1
C     DO 130 I=1,JJJ
      DO 350 I=1,LCOUNT
      PP(J)=P(I)
      J=J+1
350   CONTINUE
      DO 500 I=1,NN
      PC(I)=PP(I)
500   CONTINUE
      NNN=N+1
      IF(OPTION.EQ.2)WRITE(6,210)
C
C
C     FINDING THE CYCLE
      AA=3.1422
      DO 194 I=1,32
      DO 195 J=1,32
      MI(I,J)=0
195   CONTINUE
194   CONTINUE
      I=0
      K=1
      IF(OPTION.EQ.2)WRITE(6,107)
      J=1
2     I=I+1
      IF(I.LT.3) GO TO 2
      J=J+1
      IF(I.EQ.NNN) GO TO 400
50    IF(PP(J).EQ.AA) THEN
      J=J-1
      GO TO 50
      END IF
      JM1=J-1
60    IF(PP(JM1).EQ.AA) THEN
      JM1=JM1-1
      GO TO 60
      END IF
70    IF(I.GT.NNN) GO TO 400
      X=ABS(PP(I)-PP(J))
      Y=ABS(PP(J)-PP(JM1))
      XX=(PP(J)+PP(JM1))/2.
5     IF(X.GE.Y)THEN
      IF(OPTION.NE.1)GO TO1600
      IF(Y.LE.FL) THEN
      PC(J)=AA
      PC(JM1)=AA
      END IF
```

Table B-2 (3rd page)

```
1600  IF(OPTION.EQ.2)GO TO 41
      IF(OPTION.EQ.4)THEN
      PI(J)=((32.*(-CF1+PP(J)))+CF2-PP(J))/CF3
      PI(JM1)=((32.*(-CF1+PP(JM1)))+CF2-PP(JM1))/CF3
      PMAX=PI(J)
      PMIN=PI(JM1)
      IF(PMAX.LT.PMIN)THEN
      PMAX=PI(JM1)
      PMIN=PI(J)
      END IF
      END IF
      IF(OPTION.EQ.4)THEN
      EIJ=PMAX+.5
      EJI=PMIN+.5
      IJ=INT(EIJ)
      JI=INT(EJI)
      MI(IJ,JI)=MI(IJ,JI)+1
      END IF
      IF(OPTION.EQ.4)GO TO 42
      R(K)=Y
      MEAN(K)=XX
      K=K+1
      GO TO 42
41    PMAX=PP(J)
      PMIN=PP(JM1)
      IF(PMAX.LT.PMIN)THEN
      PMAX=PP(JM1)
      PMIN=PP(J)
      END IF
      WRITE(6,108)Y,XX,PMAX,PMIN
42    PP(J)=AA
      PP(JM1)=AA
      J=J-1
3     J=J-1
      IF(J.LT.1) GO TO 11
      IF(PP(J).EQ.AA) GO TO 3
      IF(I.EQ.4) THEN
      I=5
      J=I-1
      END IF
      JM1=J-1
      IF(JM1.LT.1)THEN
      JM1=J
      J=I
      I=I+1
      GO TO 70
      END IF
6     IF(PP(JM1).EQ.AA) GO TO 4
      X=ABS(PP(I)-PP(J))
      Y=ABS(PP(J)-PP(JM1))
      XX=(PP(J)+PP(JM1))/2.
      GO TO 5
4     JM1=JM1-1
      IF(JM1.LT.1)THEN
      JM1=J
      J=I
      I=I+1
      END IF
      GO TO 6
11    I=I+2
```

Table B-2 (4th page)

```
      J=I-1
      JM1=J-1
      GO TO 70
      ELSE
      XX=(PP(J)+PP(JM1))/2.
      J=I-1
      END IF
      GO TO 2
C
C
C
400   IF(OPTION.EQ.2) GO TO 999
      IF(OPTION.EQ.4)GO TO 998
      K=K-1
      RMAX=R(1)
      RMIN=R(1)
      RMMAX=MEAN(1)
      RMMIN=MEAN(1)
      DO 1800 I=2,K
      IF(R(I).GT.RMAX)RMAX=R(I)
      IF(R(I).LT.RMIN)RMIN=R(I)
      IF(MEAN(I).GT.RMMAX)RMMAX=MEAN(I)
      IF(MEAN(I).LT.RMMIN)RMMIN=MEAN(I)
1800  CONTINUE
      DIFR=RMAX-RMIN
      DIFM=RMMAX-RMMIN
      ER=DIFR/XIR
      EM=DIFM/XIM
      ER=ER+2
      EM=EM+2
      LEVLM=INT(EM)
      LEVLR=INT(ER)
      DO 192 L=1,LEVLR
      DO 193 LL=1,LEVLM
      M(L,LL)=0
193   CONTINUE
192   CONTINUE
      YA=RMIN-XIR
      XA=RMMIN-XIM
      WRITE(6,111)RMIN,RMAX,RMMIN,RMMAX
      XB=.50
      YB=.50
      DO 1900 I=1,K
      EI=((R(I)-YA)/XIR)+YB
      EJ=((MEAN(I)-XA)/XIM)+XB
      II=INT(EI)
      JJ=INT(EJ)
      IF(II.EQ.0)II=1
      IF(JJ.EQ.0)JJ=1
      M(II,JJ)=M(II,JJ)+1
1900  CONTINUE
C
C
C     FILTERING PROCESS
      IF(OPTION.NE.1)GO TO 1102
      KN=1
      DO 1000 II=1,NN
      IF(PC(II).EQ.AA) GO TO 1000
      PCC(KN)=PC(II)
      KN=KN+1
```

```
1000  CONTINUE
      KN=KN-1
      WRITE(6,112)
112   FORMAT('1',//15X,'FILTER HISTIRY-PEAK/VALLEY SEQUENCE')
      WRITE(6,113)FL
113   FORMAT(//15X,'FILTER LEVEL=',F7.3)
      WRITE(6,1103) KN
1103  FORMAT(//15X,'NUMBER OF POINTS IN FILTER HISTORY=',I5,//)
      WRITE(6,1001)(PCC(I),I=1,KN)
1001  FORMAT(1X,8(2X,F6.1))
C
C
C     MATRIX PREPRATION
      GO TO 1102
998   LEVLM=32
      LEVLR=32
      XIR=1
      XIM=1
      RMMIN=1
      RMIN=1
      DO 201 I=1,32
      DO 202 J=1,32
      M(I,J)=MI(I,J)
202   CONTINUE
201   CONTINUE
1102  IF(OPTION.EQ.2)GO TO 999
      MM(1)=RMMIN
      DO 900 L=2,LEVLM
      LL=L-1
      MM(L)=MM(LL)+XIM
900   CONTINUE
      RA(1)=RMIN
      DO 1100 L=2,LEVLR
      LL=L-1
1100   RA(L)=RA(LL)+XIR
      I=0
99    I=I+1
      IF(I.GT.LEVLR) GO TO 1153
      SUM(I)=0.
      DO 98 J=1,LEVLM
      SUM(I)=SUM(I)+M(I,J)
98    CONTINUE
      GO TO 99
1153  CONTINUE
999   IF(OPTION.EQ.2)GO TO 997
1151  L=1
      LB=8
1152  IF(OPTION.EQ.1)GO TO 996
      WRITE(6,116)
      GO TO 1154
996   WRITE(6,114)
      WRITE(6,115)FL
1154  IF(OPTION.EQ.4)GO TO 604
      GO TO 1150
604    WRITE(6,605)
605   FORMAT(77X,'TOTAL',/1X,'PEAK  /*******************************VALLEY
     **********************************',4X,'CYCLES',/)
      GO TO 2100
1150  WRITE(6,600)
600   FORMAT(77X,'TOTAL',/1X,'RANGE  /***********************************MEA
```

```
        *N******************************',3X,'CYCLES',/)
 2100   WRITE(6,101)(MM(LL),LL=L,LB)
        DO 1300 I=1,LEVLR
        WRITE(6,102)RA(I),SUM(I)
        WRITE(6,103)(M(I,J),J=L,LB)
 1300   CONTINUE
        IF(LB.EQ.LEVLM)GO TO 1400
        L=L+8
        LB=LB+8
        IF(LB.GT.LEVLM)LB=LEVLM
        GO TO 1152
 1400   SUMM=0
        DO 1500 I=1,LEVLR
 1500   SUMM=SUMM+SUM(I)
        WRITE(6,104)SUMM
 101    FORMAT(12X,8(F6.1,2X))
 102    FORMAT(2X,F6.1,69X,I4)
 103    FORMAT('+',11X,8(I6,2X))
 104    FORMAT(//,5X,'TOTAL NO OF CYCLES=',3X,I5)
 997    CONTINUE
 107    FORMAT(15X,'RANGE',15X,'MEAN',15X,' MAX',15X,'    MIN')
 108    FORMAT(14X,F7.3,12X,F7.3,13X,F7.3,13X,F7.3)
 111    FORMAT('1',//15X,'MIN RANGE=',F8.3,//15X,'MAX RANGE=',F8.3,
       *//15X,'MIN MEAN=',F8.3,//15X,'MAX MEAN=',F8.3)
 116    FORMAT('1',//35X,'RAINFLOW CYCLES ')
 114    FORMAT('1',//20X,'RAINFLOW CYCLES FOR ORIGINAL HISTORY')
 115    FORMAT(/5X,'NO RANGE LESS THAN OR EQUAL TO FILTER LEVEL=',F7.3
       *,3X,'OCCUR IN FILTER HISTORY')
 210    FORMAT('1',//10X,'RANGES COUNTED AS CYCLES BY RAINFLOW CYCLE COUNT
       *ING METHOD.')
        STOP
        END

C$ENTRY
```

Table B-2 (7th page)

RANGES COUNTED AS CYCLES BY RAINFLOW CYCLE COUNTING METHOD.

RANGE	MEAN	MAX	MIN
4.000	1.000	3.000	-1.000
3.000	-0.500	1.000	-2.000
7.000	0.500	4.000	-3.000
9.000	0.500	5.000	-4.000

EXAMPLE 2

A history containing 41 peak/valley points is used with Option 2. Tables B-3 and B-4 show the input and output of this program, respectively.

Table B-3. Input for Example 2

```
2
41
-8.6,1.11,-6.2,-1.71,-2.95,2.10,-6.6,5.43,-2.58,-.4,-2.9,6.51,-1.12,.97,
-1.65,1.88,-4.47,2.01,-2.5,7.3,-1.08,5.36,-5.87,-.18,-3.5,.85,-1.89,
5.3,.1,4.45,-3.82,1.89,-2.04,3.9,-4.97,7.78,-3.99,4.61,-5.52,3.1,-8.6
```

Table B-4. Output for Example 2

RANGES COUNTED AS CYCLES BY RAINFLOW CYCLE COUNTING METHOD.

RANGE	MEAN	MAX	MIN
1.240	-2.330	-1.710	-2.950
7.310	-2.545	1.110	-6.200
8.700	-2.250	2.100	-6.600
2.180	-1.490	-0.400	-2.580
8.330	1.265	5.430	-2.900
2.090	-0.075	0.970	-1.120
3.530	0.115	1.880	-1.650
4.510	-0.245	2.010	-2.500
10.980	1.020	6.510	-4.470
6.440	2.140	5.360	-1.080
3.320	-1.840	-0.180	-3.500
2.740	-0.520	0.850	-1.890
4.350	2.275	4.450	0.100
3.930	-0.075	1.890	-2.040
7.720	0.040	3.900	-3.820
10.270	0.165	5.300	-4.970
13.170	0.715	7.300	-5.870
8.600	0.310	4.610	-3.990
8.620	-1.210	3.100	-5.520
16.380	-0.410	7.780	-8.600

EXAMPLE 3

A history containing 1709 peak/valley points, called the SAE Transmission History, is used with Option 3. Therefore, the results are given in the form of a compact matrix containing range and mean values. The results are in agreement with the published values from Reference B-3. Tables B-5 and B-6 show the input and output for this example, respectively.

Note that the size of the matrix is determined by the XIM and XIR increment values, for mean and range, respectively, which are given to the program as input data. The program determines the smallest range and the lowest mean value. The increments are then added by the program a sufficient number of times to include the largest range and highest mean. Hence, the program automatically makes the matrix large enough to include the largest range and highest mean. In this particular case, the matrix size resulting from XIM = XIR = 50 is 27x20. Since the smallest range is 200

$$200 + (27 - 1)50 = 1500$$

which is sufficiently large to include the largest range of 1494. Similarly, since the lowest mean value is -160,

$$-160 + (20 - 1)50 = 790$$

which is sufficiently large to include the highest mean of 753.

Note that the number of cycles counted also appears in the output. This is always related to the number of peak/valley points in the history, NN, as follows:

$$\text{No. of cycles} = \frac{NN - 1}{2}$$

NN is always odd, since the last peak or valley, P( ), is a repetition of the first one, in this particular case

$$\text{No. of cycles} = \frac{1709 - 1}{2} = 854$$

Table B-5. Input for Example 3

```
   3
1709
 50. 50.
   0  513  292  562  267  585  314  524  299  513  299  569  272  535
 298  523  291  680  444  683  396  678  374  701  276  607  331  542
 339  572  274  500  285  565  274  491  237  496  -76  142 -276   71
-309   19 -249  137  -93  658  275  534  240  612  360  690  315  689
 306  698  417  666  392  699  374  701  368  659  345  650  403  610
 393  637  400  618  285  630  385  616  374  703 -174  136 -331  334
  98  510  151  542   84  642  398  651 -197   44 -197  125 -121  228
 -17  631  387  657  408  623  218  553  291  640  408  693  453  657
 356  642  400  698  330  571  337  689  395  647  377  594  227  631
 299  682  411  663  404  629  306  631  306  614  409  648  414  683
 374  575  324  691  379  585  368  621  371  610  373  582   -8  240
-227  248 -205  285  -89  470  248  460  136  519  317  517  278  503
 179  615  324  559  219  679  421  683 -254   84 -222  243  -19  495
 270  629  291  560  351  657  243  608  373  663  269  653  156  663
 443  690  249  612  229  602  339  738  231  498  237  614 -167   39
-261  437  192  503  103  682  201  589  226  480  135  561  347  548
 149  494  260  661  158  467 -184  161 -105  555  248  449  132  712
 276  529  301  524  309  564  195  467   35  378 -294  188 -308  245
-211  607   41  526  205  506  218  513  249  603  261  712  506  727
 194  400 -244  173 -276  610  405  615  360  642  403  641  353  556
 340  655  433  669  422  662  457  796  403  671   78  549  285  616
 376  690   18  731  108  578  282  629  170  748  181  578  342  604
 261  585  330  694  349  614  313  521  290  533  266  565  291  645
-228    1 -286  157  -87  725  183  714  393  621  194  426  152  561
 235  551  213  691  136  487  164  549  244  598  162  507  249  716
 339  733  194  566 -191  120 -179  108 -119  181  -74  722  422  639
 303  671  369  715  350  605  153  765  409  762  355  712  398  718
 362  679  448  718  446  698  342  605  188  498  276  537  283  522
 222  546  345  570  191  500  280  658  269  685  341  551  344  618
 303  612  285  666  228  534  303  546 -110  110 -362   88 -199  173
-130  612  333  680  341  603  283  619   30  406  206  470  260  717
 218  487  190  573  270  476  265  607  310  784  573  822  496  825
 508  887  491  769  486  716  342  594  365  578  254  593  355  561
 324  573  153  696  443  651  372  653 -256   39 -173  620  213  560
 217  559  274  491  211  571  266  640  169  572  223  496  221  573
 264  758  358  743  501  777 -161   57 -146  210  -13  216  -15  373
 133  371  151  624  313  628  405  698 -202  146  -92  483  141  551
 229  551  207  533  271  554  218  532  210  630  179  489  266  618
 408  650  328  599 -211  158 -204   68 -179  256 -108  114 -114  476
 228  494  178  658  218  612  245  652  416  658  355  678  428  722
 437  747  542  776 -201  405  188  707  408  640  422  648  415  727
 393  610  387  722  417  720  476  688  484  691  360  570 -350  528
 216  679  475  689  274  562  286  653  233  453  253  573  282  582
 356  603  234  871  494  771  511  773  481  727  491  836  566  782
 398  659  329  685  387  629  218  604  329  628 -248  274 -206  710
 463  663  365  695  406  803  437  739  449  679  452  695  454  703
 260  602 -330   34 -350  188 -180  309   -7  648  417  647  439  683
 438  699  451  712  382  698  296  625  385  680  405  718  457  673
 372  744  414  618   82  597  191  570  211  522  212  517  211  495
 232  439  235  616  256  512  270  575  314  791  369  787  502  769
 468  785  444  798  501  732  522  723  462  680 -264   68 -154  188
 -14  242   13  582  291  618  227  537  285  555  224  545  329  588
 374  647  242  561  353  694  433  753 -237  313 -147  226 -163  705
 329  636  376  680  383  641  404  712  367  732  290  610  318  629
 315  588  353  577  265  612  368  723 -183  267 -208  111 -153  473
 271  576  365  618  395  605 -283  235 -172  687  242  521  299  532
```

Table B-5 (2nd page)

283	596	306	629	392	639	326	583	368	591	360	601	373	632
409	640	272	496	175	470	251	549	301	573	313	517	317	565
322	529	265	586	353	555	-215	251	-133	419	63	535	175	490
164	517	126	560	165	726	408	671	399	662	325	596	331	583
378	744	309	635	417	677	373	650	368	673	-189	242	-17	610
197	548	181	523	296	582	113	560	137	739	366	725	362	717
341	607	317	604	351	567	141	653	452	709	158	404	-280	253
-183	199	-28	425	167	620	400	669	381	706	401	652	382	725
365	812	395	695	416	758	469	671	374	763	514	757	265	519
27	637	212	566	275	526	325	593	141	511	135	693	412	657
281	533	-204	199	-185	136	-86	186	-18	206	-8	739	110	453
253	506	275	511	108	516	169	420	116	383	-8	299	66	292
-259	-22	-242	244	1	222	-17	221	-276	233	-194	226	-11	560
358	567	356	599	361	607	394	652	378	632	342	637	276	557
299	583	190	506	235	570	299	557	334	582	255	544	240	636
270	619	228	583	371	593	373	589	377	626	426	648	250	647
293	717	417	701	320	709	385	701	427	682	286	601	175	559
285	533	310	535	319	555	326	564	-181	138	-219	251	7	443
102	490	263	534	45	693	398	678	329	671	433	696	283	554
189	489	-367	207	-276	-34	-263	149	-86	474	73	363	-419	-17
-240	-18	-219	181	-470	-19	-293	277	-259	292	-145	276	-1	227
1	566	117	516	148	415	-406	-19	-301	205	-330	632	231	433
178	582	331	835	490	748	355	693	475	722	441	742	437	730
419	722	416	769	442	775	501	701	449	685	431	632	425	646
415	683	341	769	485	738	421	762	485	696	374	591	356	639
356	675	426	705	449	688	479	770	251	484	45	282	-251	255
-320	438	222	639	251	647	431	707	469	789	400	669	334	546
342	705	374	684	427	698	368	645	358	625	373	652	409	634
322	624	368	659	392	748	355	675	362	667	447	755	349	683
315	693	330	607	226	744	443	754	503	709	427	703	274	516
258	588	141	457	221	428	-164	78	-251	526	313	725	314	582
212	733	318	629	357	752	458	791	492	717	403	727	369	603
277	636	358	562	229	770	535	840	544	792	318	522	-318	110
-233	288	-100	265	-120	703	14	592	162	530	229	577	217	613
164	593	319	632	398	623	250	789	256	475	-387	232	-347	133
-141	140	-129	544	303	644	358	642	365	673	336	889	365	669
447	813	506	817	537	776	524	749	542	868	545	871	415	690
471	789	487	706	452	669	192	512	-227	93	-189	518	293	594
89	576	362	789	191	449	151	478	238	501	104	786	299	832
610	829	530	734	-318	728	83	441	-7	511	46	895	405	877
609	876	630	888	527	797	-239	163	-102	632	11	811	390	855
271	578	324	675	221	632	362	733	420	803	-495	141	-296	100
-140	510	251	661	366	695	161	453	235	459	117	691	428	764
-247	217	-301	149	-207	157	-74	222	-66	280	-204	561	197	561
153	492	270	607	240	645	202	571	213	695	-448	103	-312	129
-138	508	304	603	67	626	342	601	286	576	221	721	315	636
304	669	337	605	367	567	280	514	-286	94	-133	355	36	458
146	506	294	577	-11	222	-341	94	-227	50	-156	329	19	585
165	398	176	822	561	801	553	776	-258	192	-125	464	211	730
152	834	352	642	285	587	-222	233	-136	251	18	528	33	449
234	626	226	480	35	594	189	518	47	589	63	449	205	645
-229	336	87	798	529	753	51	412	188	999	411	942	486	757
394	830	464	732	330	800	452	689	329	712	421	690	378	870
462	782	116	653	-223	54	-223	130	-237	-33	-254	551	274	549
251	658	192	640	323	578	281	530	294	575	240	478	234	519
296	645	353	664	318	575	178	594	374	598	308	581	330	739
412	615	344	601	328	549	324	645	381	639	393	645	377	592
294	538	297	535	251	503	286	656	427	635	394	640	378	653
392	614	358	608	313	513	244	844	365	607	406	620	387	609
346	572	259	543	269	498	269	570	309	513	151	506	221	582

Table B-5 (3rd page)

347	553	-277	50	-204	109	-256	164	-65	313	49	342	124	365
162	716	34	325	-249	507	-15	322	66	360	-158	83	-281	314
43	511	280	682	292	503	218	705	197	486	221	458	202	608
-325	-20	-242	508	254	478	192	631	408	716	350	567	258	592
367	658	408	623	264	587	371	589	285	632	368	599	318	589
309	512	260	571	346	725	261	487	192	593	293	539	189	554
0													

Table B-6. Output for Example 3

```
MIN RANGE= 200.000
MAX RANGE=1494.000
MIN MEAN=-160.000
MAX MEAN= 753.000
```

Table B-6 (2nd page)

RAINFLOW CYCLES

RANGE	MEAN -160.0	-110.0	-60.0	-10.0	40.0	90.0	140.0	190.0	TOTAL CYCLES
200.0	0	5	5	2	2	5	0	0	173
250.0	2	2	5	8	9	9	5	5	244
300.0	1	0	5	3	2	1	3	3	134
350.0	1	1	3	3	2	3	0	1	91
400.0	0	1	1	1	2	1	0	1	68
450.0	0	0	1	1	5	1	0	0	33
500.0	0	0	1	3	2	1	0	0	21
550.0	0	0	1	1	0	0	0	0	18
600.0	0	1	1	0	0	0	0	0	10
650.0	0	0	0	0	0	0	0	0	3
700.0	0	0	0	0	0	0	0	0	3
750.0	0	0	0	0	0	0	1	0	4
800.0	0	0	0	0	0	0	0	0	0
850.0	0	0	0	0	1	0	1	2	8
900.0	0	0	0	0	0	0	0	0	4
950.0	0	0	0	0	0	1	0	0	7
1000.0	0	0	0	0	0	0	0	3	9
1050.0	0	0	0	0	0	0	0	2	9
1100.0	0	0	0	0	0	0	0	0	5
1150.0	0	0	0	0	0	0	0	0	3
1200.0	0	0	0	0	0	0	1	0	3
1250.0	0	0	0	0	0	0	0	0	2
1300.0	0	0	0	0	0	0	0	0	0
1350.0	0	0	0	0	0	0	0	1	1
1400.0	0	0	0	0	0	0	0	0	0
1450.0	0	0	0	0	0	0	0	0	0
1500.0	0	0	0	0	0	0	0	0	1

Table B-6 (3rd page)

RAINFLOW CYCLES

RANGE	240.0	290.0	340.0	390.0	440.0	490.0	540.0	590.0	TOTAL CYCLES
	/****************************MEAN							*****************************	
200.0	3	5	11	24	30	33	25	14	173
250.0	3	6	13	32	37	41	39	14	244
300.0	1	4	10	15	24	23	20	11	134
350.0	2	3	6	15	17	11	14	6	91
400.0	0	3	7	10	21	9	7	4	68
450.0	0	2	0	8	9	2	2	2	33
500.0	0	2	2	2	3	2	1	0	21
550.0	0	0	4	2	6	2	0	1	18
600.0	0	0	2	2	3	1	0	0	10
650.0	0	0	2	1	0	0	0	0	3
700.0	0	0	2	0	1	0	0	0	3
750.0	0	0	1	1	1	0	0	0	4
800.0	0	0	0	0	0	0	0	0	0
850.0	3	1	0	0	0	0	0	0	8
900.0	3	1	0	0	0	0	0	0	4
950.0	4	2	0	0	0	0	0	0	7
1000.0	5	1	0	0	0	0	0	0	9
1050.0	3	4	0	0	0	0	0	0	9
1100.0	2	3	0	0	0	0	0	0	5
1150.0	3	0	0	0	0	0	0	0	3
1200.0	1	1	0	0	0	0	0	0	3
1250.0	2	0	0	0	0	0	0	0	2
1300.0	0	0	0	0	0	0	0	0	0
1350.0	0	0	0	0	0	0	0	0	1
1400.0	0	0	0	0	0	0	0	0	0
1450.0	0	0	0	0	0	0	0	0	0
1500.0	1	0	0	0	0	0	0	0	1

Table B-6 (4th page)

RAINFLOW CYCLES

RANGE	640.0	690.0	740.0	790.0	TOTAL CYCLES
	/*****************************MEAN*****************************				
200.0	6	2	1	0	173
250.0	10	2	2	0	244
300.0	5	3	0	0	134
350.0	3	0	0	0	91
400.0	0	0	0	0	68
450.0	0	0	0	0	33
500.0	2	0	0	0	21
550.0	0	1	0	0	18
600.0	0	0	0	0	10
650.0	0	0	0	0	3
700.0	0	0	0	0	3
750.0	0	0	0	0	4
800.0	0	0	0	0	0
850.0	0	0	0	0	8
900.0	0	0	0	0	4
950.0	0	0	0	0	7
1000.0	0	0	0	0	9
1050.0	0	0	0	0	9
1100.0	0	0	0	0	5
1150.0	0	0	0	0	3
1200.0	0	0	0	0	3
1250.0	0	0	0	0	2
1300.0	0	0	0	0	0
1350.0	0	0	0	0	1
1400.0	0	0	0	0	0
1450.0	0	0	0	0	0
1500.0	0	0	0	0	1

EXAMPLE 4

The same history as Example 3 is analyzed using Option 1. Tables B-7 and B-8 show the input and output of this program, respectively. The output includes the filtered history as a peak/valley sequence and also the rain-flow cycles for the original history.

In this case, filtering at a range of 400 reduced the length of the history from 1709 peaks and valleys to 361. The listing of the filtered history alternates between peaks and valleys, or valleys and peaks, and starts and ends with the largest absolute value. Note that range/mean matrix printed is the same as for Example 3.

Table B-7. Input for Example 4

```
   1
400.0
1709
 50.  50.
   0  513  292  562  267  585  314  524  299  513  299  569  272  535
 298  523  291  680  444  683  396  678  374  701  276  607  331  542
 339  572  274  500  285  565  274  491  237  496  -76  142 -276   71
-309   19 -249  137  -93  658  275  534  240  612  360  690  315  689
 306  698  417  666  392  699  374  701  368  659  345  650  403  610
 393  637  400  618  285  630  385  616  374  703 -174  136 -331  334
  98  510  151  542   84  642  398  651 -197   44 -197  125 -121  228
 -17  631  387  657  408  623  218  553  291  640  408  693  453  657
 356  642  400  698  330  571  337  689  395  647  377  594  227  631
 299  682  411  663  404  629  306  631  306  614  409  648  414  683
 374  575  324  691  379  585  368  621  371  610  373  582   -8  240
-227  248 -205  285  -89  470  248  460  136  519  317  517  278  503
 179  615  324  559  219  679  421  683 -254   84 -222  243  -19  495
 270  629  291  560  351  657  243  608  373  663  269  653  156  663
 443  690  249  612  229  602  339  738  231  498  237  614 -167   39
-261  437  192  503  103  682  201  589  226  480  135  561  347  548
 149  494  260  661  158  467 -184  161 -105  555  248  449  132  712
 276  529  301  524  309  564  195  467   35  378 -294  188 -308  245
-211  607   41  526  205  506  218  513  249  603  261  712  506  727
 194  400 -244  173 -276  610  405  615  360  642  403  641  353  556
 340  655  433  669  422  662  457  796  403  671   78  549  285  616
 376  690   18  731  108  578  282  629  170  748  181  578  342  604
 261  585  330  694  349  614  313  521  290  533  266  565  291  645
-228    1 -286  157  -87  725  183  714  393  621  194  426  152  561
 235  551  213  691  136  487  164  549  244  598  162  507  249  716
 339  733  194  566 -191  120 -179  108 -119  181  -74  722  422  639
 303  671  369  715  350  605  153  765  409  762  355  712  398  718
 362  679  448  718  446  698  342  605  188  498  276  537  283  522
 222  546  345  570  191  500  280  658  269  685  341  551  344  618
 303  612  285  666  228  534  303  546 -110  110 -362   88 -199  173
-130  612  333  680  341  603  283  619   30  406  206  470  260  717
 218  487  190  573  270  476  265  607  310  784  573  822  496  825
 508  887  491  769  486  716  342  594  365  578  254  593  355  561
 324  573  153  696  443  651  372  653 -256   39 -173  620  213  560
 217  559  274  491  211  571  266  640  169  572  223  496  221  573
 264  758  358  743  501  777 -161   57 -146  210  -13  216  -15  373
 133  371  151  624  313  628  405  698 -202  146  -92  483  141  551
 229  551  207  533  271  554  218  532  210  630  179  489  266  618
 408  650  328  599 -211  158 -204   68 -179  256 -108  114 -114  476
 228  494  178  658  218  612  245  652  416  658  355  678  428  722
 437  747  542  776 -201  405  188  707  408  640  422  648  415  727
 393  610  387  722  417  720  476  688  484  691  360  570 -350  528
 216  679  475  689  274  562  286  653  233  453  253  573  282  582
 356  603  234  871  494  771  511  773  481  727  491  836  566  782
 398  659  329  685  387  629  218  604  329  628 -248  274 -206  710
 463  663  365  695  406  803  437  739  449  679  452  695  454  703
 260  602 -330   34 -350  188 -180  309   -7  648  417  647  439  683
 438  699  451  712  382  698  296  625  385  680  405  718  457  673
 372  744  414  618   82  597  191  570  211  522  212  517  211  495
 232  439  235  616  256  512  270  575  314  791  369  787  502  769
 468  785  444  798  501  732  522  723  462  680 -264   68 -154  188
 -14  242   13  582  291  618  227  537  285  555  224  545  329  588
 374  647  242  561  353  694  433  753 -237  313 -147  226 -163  705
 329  636  376  680  383  641  404  712  367  732  290  610  318  629
 315  588  353  577  265  612  368  723 -183  267 -208  111 -153  473
```

Table B-7 (2nd page)

271	576	365	618	395	605	-283	235	-172	687	242	521	299	532
283	596	306	629	392	639	326	583	368	591	360	601	373	632
409	640	272	496	175	470	251	549	301	573	313	517	317	565
322	529	265	586	353	555	-215	251	-133	419	63	535	175	490
164	517	126	560	165	726	408	671	399	662	325	596	331	583
378	744	309	635	417	677	373	650	368	673	-189	242	-17	610
197	548	181	523	296	582	113	560	137	739	366	725	362	717
341	607	317	604	351	567	141	653	452	709	158	404	-280	253
-183	199	-28	425	167	620	400	669	381	706	401	652	382	725
365	812	395	695	416	758	469	671	374	763	514	757	265	519
27	637	212	566	275	526	325	593	141	511	135	693	412	657
281	533	-204	199	-185	136	-86	186	-18	206	-8	739	110	453
253	506	275	511	108	516	169	420	116	383	-8	299	66	292
-259	-22	-242	244	1	222	-17	221	-276	233	-194	226	-11	560
358	567	356	599	361	607	394	652	378	632	342	637	276	557
299	583	190	506	235	570	299	557	334	582	255	544	240	636
270	619	228	583	371	593	373	589	377	626	426	648	250	647
293	717	417	701	320	709	385	701	427	682	286	601	175	559
285	533	310	535	319	555	326	564	-181	138	-219	251	7	443
102	490	263	534	45	693	398	678	329	671	433	696	283	554
189	489	-367	207	-276	-34	-263	149	-86	474	73	363	-419	-17
-240	-18	-219	181	-470	-19	-293	277	-259	292	-145	276	-1	227
1	566	117	516	148	415	-406	-19	-301	205	-330	632	231	433
178	582	331	835	490	748	355	693	475	722	441	742	437	730
419	722	416	769	442	775	501	701	449	685	431	632	425	646
415	683	341	769	485	738	421	762	485	696	374	591	356	639
356	675	426	705	449	688	479	770	251	484	45	282	-251	255
-320	438	222	639	251	647	431	707	469	789	400	669	334	546
342	705	374	684	427	698	368	645	358	625	373	652	409	634
322	624	368	659	392	748	355	675	362	667	447	755	349	683
315	693	330	607	226	744	443	754	503	709	427	703	274	516
258	588	141	457	221	428	-164	78	-251	526	313	725	314	582
212	733	318	629	357	752	458	791	492	717	403	727	369	603
277	636	358	562	229	770	535	840	544	792	318	522	-318	110
-233	288	-100	265	-120	703	14	592	162	530	229	577	217	613
164	593	319	632	398	623	250	789	256	475	-387	232	-347	133
-141	140	-129	544	303	644	358	642	365	673	336	889	365	669
447	813	506	817	537	776	524	749	542	868	545	871	415	690
471	789	487	706	452	669	192	512	-227	93	-189	518	293	594
89	576	362	789	191	449	151	478	238	501	104	786	299	832
610	829	530	734	-318	728	83	441	-7	511	46	895	405	877
609	876	630	888	527	797	-239	163	-102	632	11	811	390	855
271	578	324	675	221	632	362	733	420	803	-495	141	-296	100
-140	510	251	661	366	695	161	453	235	459	117	691	428	764
-247	217	-301	149	-207	157	-74	222	-66	280	-204	561	197	561
153	492	270	607	240	645	202	571	213	695	-448	103	-312	129
-138	508	304	603	67	626	342	601	286	576	221	721	315	636
304	669	337	605	367	567	280	514	-286	94	-133	355	36	458
146	506	294	577	-11	222	-341	94	-227	50	-156	329	19	585
165	398	176	822	561	801	553	776	-258	192	-125	464	211	730
152	834	352	642	285	587	-222	233	-136	251	18	528	33	449
234	626	226	480	35	594	189	518	47	589	63	449	205	645
-229	336	87	798	529	753	51	412	188	999	411	942	486	757
394	830	464	732	330	800	452	689	329	712	421	690	378	870
462	782	116	653	-223	54	-223	130	-237	-33	-254	551	274	549
251	658	192	640	323	578	281	530	294	575	240	478	234	519
296	645	353	664	318	575	178	594	374	598	308	581	330	739
412	615	344	601	328	549	324	645	381	639	393	645	377	592
294	538	297	535	251	503	286	656	427	635	394	640	378	653
392	614	358	608	313	513	244	844	365	607	406	620	387	609

Table B-7 (3rd page)

346	572	259	543	269	498	269	570	309	513	151	506	221	582
347	553	-277	50	-204	109	-256	164	-65	313	49	342	124	365
162	716	34	325	-249	507	-15	322	66	360	-158	83	-281	314
43	511	280	682	292	503	218	705	197	486	221	458	202	608
-325	-20	-242	508	254	478	192	631	408	716	350	567	258	592
367	658	408	623	264	587	371	589	285	632	368	599	318	589
309	512	260	571	346	725	261	487	192	593	293	539	189	554
0													

Table B-8. Output for Example 4

```
MIN RANGE= 200.000
MAX RANGE=1494.000
MIN MEAN=-160.000
MAX MEAN= 753.000
```

Table B-8 (2nd page)

FILTER HISTIRY-PEAK/VALLEY SEQUENCE

FILTER LEVEL=400.000

NUMBER OF POINTS IN FILTER HISTORY= 361

999.0	411.0	942.0	394.0	830.0	330.0	800.0	329.0
870.0	116.0	653.0	-254.0	658.0	192.0	640.0	234.0
664.0	178.0	739.0	251.0	656.0	244.0	844.0	151.0
582.0	-277.0	716.0	-249.0	507.0	-281.0	682.0	218.0
705.0	197.0	608.0	-325.0	716.0	258.0	725.0	192.0
593.0	0.0	701.0	-309.0	658.0	240.0	701.0	285.0
703.0	-331.0	542.0	84.0	651.0	-197.0	657.0	218.0
698.0	227.0	691.0	-227.0	248.0	-205.0	683.0	-254.0
657.0	243.0	663.0	156.0	690.0	229.0	738.0	-261.0
682.0	135.0	561.0	149.0	661.0	-184.0	555.0	132.0
712.0	-294.0	188.0	-308.0	245.0	-211.0	607.0	41.0
727.0	-244.0	173.0	-276.0	796.0	78.0	690.0	18.0
731.0	108.0	629.0	170.0	748.0	181.0	694.0	-286.0
725.0	183.0	714.0	152.0	691.0	136.0	598.0	162.0
733.0	-191.0	722.0	303.0	715.0	153.0	765.0	188.0
685.0	-362.0	680.0	30.0	717.0	190.0	887.0	153.0
696.0	-256.0	620.0	211.0	640.0	169.0	777.0	-161.0
698.0	-202.0	630.0	179.0	650.0	-211.0	658.0	218.0
776.0	-201.0	727.0	-350.0	689.0	233.0	871.0	218.0
628.0	-248.0	274.0	-206.0	803.0	-350.0	712.0	296.0
744.0	82.0	597.0	191.0	791.0	369.0	798.0	-264.0
647.0	242.0	753.0	-237.0	313.0	-163.0	732.0	265.0
723.0	-183.0	267.0	-208.0	618.0	-283.0	235.0	-172.0
687.0	175.0	586.0	-215.0	535.0	126.0	726.0	325.0
744.0	-189.0	610.0	181.0	582.0	113.0	560.0	137.0
739.0	141.0	709.0	-280.0	253.0	-183.0	812.0	27.0
637.0	135.0	693.0	-204.0	739.0	110.0	511.0	108.0
516.0	-259.0	244.0	-276.0	233.0	-194.0	652.0	190.0
636.0	228.0	717.0	-219.0	534.0	45.0	696.0	-367.0
207.0	-276.0	474.0	-419.0	181.0	-470.0	277.0	-259.0
292.0	-145.0	566.0	-406.0	205.0	-330.0	632.0	178.0
835.0	355.0	775.0	341.0	769.0	356.0	770.0	-251.0
255.0	-320.0	789.0	322.0	755.0	226.0	754.0	-251.0
725.0	212.0	733.0	318.0	791.0	229.0	840.0	-318.0
288.0	-120.0	703.0	14.0	592.0	162.0	613.0	164.0
789.0	-387.0	232.0	-347.0	889.0	365.0	871.0	-227.0
594.0	89.0	789.0	104.0	786.0	299.0	832.0	-318.0
728.0	-7.0	511.0	46.0	895.0	405.0	888.0	-239.0
632.0	11.0	811.0	390.0	855.0	271.0	675.0	221.0
803.0	-495.0	141.0	-296.0	695.0	117.0	764.0	-247.0
217.0	-301.0	280.0	-204.0	561.0	153.0	645.0	202.0
695.0	-448.0	103.0	-312.0	603.0	67.0	626.0	221.0
721.0	-286.0	577.0	-341.0	585.0	165.0	822.0	-258.0
730.0	152.0	834.0	-222.0	528.0	33.0	626.0	35.0
594.0	47.0	589.0	63.0	645.0	-229.0	798.0	51.0
999.0							

Table B-8 (3rd page)

RAINFLOW CYCLES FOR ORIGINAL HISTORY

NO RANGE LESS THAN OR EQUAL TO FILTER LEVEL=400.000 OCCUR IN FILTER HISTORY

RANGE	/****************************			MEAN	****************************				TOTAL CYCLES
	-160.0	-110.0	-60.0	-10.0	40.0	90.0	140.0	190.0	
200.0	0	5	5	2	2	5	0	0	173
250.0	2	2	5	8	9	9	5	5	244
300.0	1	0	5	3	2	1	3	3	134
350.0	1	1	3	3	2	3	0	1	91
400.0	0	1	1	1	2	1	0	1	68
450.0	0	0	1	1	5	1	0	0	33
500.0	0	0	1	3	2	1	0	0	21
550.0	0	0	1	1	0	0	0	0	18
600.0	0	1	1	0	0	0	0	0	10
650.0	0	0	0	0	0	0	0	0	3
700.0	0	0	0	0	0	0	0	0	3
750.0	0	0	0	0	0	0	1	0	4
800.0	0	0	0	0	0	0	0	0	0
850.0	0	0	0	0	1	0	1	2	8
900.0	0	0	0	0	0	0	0	0	4
950.0	0	0	0	0	0	1	0	0	7
1000.0	0	0	0	0	0	0	0	3	9
1050.0	0	0	0	0	0	0	0	2	9
1100.0	0	0	0	0	0	0	0	0	5
1150.0	0	0	0	0	0	0	0	0	3
1200.0	0	0	0	0	0	0	1	0	3
1250.0	0	0	0	0	0	0	0	0	2
1300.0	0	0	0	0	0	0	0	0	0
1350.0	0	0	0	0	0	0	0	1	1
1400.0	0	0	0	0	0	0	0	0	0
1450.0	0	0	0	0	0	0	0	0	0
1500.0	0	0	0	0	0	0	0	0	1

C-2

Table B-8 (4th page)

RAINFLOW CYCLES FOR ORIGINAL HISTORY

NO RANGE LESS THAN OR EQUAL TO FILTER LEVEL=400.000 OCCUR IN FILTER HISTORY

RANGE	240.0	290.0	340.0	390.0	440.0	490.0	540.0	590.0	TOTAL CYCLES
200.0	3	5	11	24	30	33	25	14	173
250.0	3	6	13	32	37	41	39	14	244
300.0	1	4	10	15	24	23	20	11	134
350.0	2	3	6	15	17	11	14	6	91
400.0	0	3	7	10	21	9	7	4	68
450.0	0	2	0	8	9	2	2	2	33
500.0	0	2	2	2	3	2	1	0	21
550.0	0	0	4	2	6	2	0	1	18
600.0	0	0	2	2	3	1	0	0	10
650.0	0	0	2	1	0	0	0	0	3
700.0	0	0	2	0	1	0	0	0	3
750.0	0	0	1	1	1	0	0	0	4
800.0	0	0	0	0	0	0	0	0	0
850.0	3	1	0	0	0	0	0	0	8
900.0	3	1	0	0	0	0	0	0	4
950.0	4	2	0	0	0	0	0	0	7
1000.0	5	1	0	0	0	0	0	0	9
1050.0	3	4	0	0	0	0	0	0	9
1100.0	2	3	0	0	0	0	0	0	5
1150.0	3	0	0	0	0	0	0	0	3
1200.0	1	1	0	0	0	0	0	0	3
1250.0	2	0	0	0	0	0	0	0	2
1300.0	0	0	0	0	0	0	0	0	0
1350.0	0	0	0	0	0	0	0	0	1
1400.0	0	0	0	0	0	0	0	0	0
1450.0	0	0	0	0	0	0	0	0	0
1500.0	1	0	0	0	0	0	0	0	1

Table B-8 (5th page)

RAINFLOW CYCLES FOR ORIGINAL HISTORY

NO RANGE LESS THAN OR EQUAL TO FILTER LEVEL=400.000 OCCUR IN FILTER HISTORY

RANGE /*******************************MEAN******************************* TOTAL CYCLES

RANGE	640.0	690.0	740.0	790.0	TOTAL CYCLES
200.0	6	2	1	0	173
250.0	10	2	2	0	244
300.0	5	3	0	0	134
350.0	3	0	0	0	91
400.0	0	0	0	0	68
450.0	0	0	0	0	33
500.0	2	0	0	0	21
550.0	0	1	0	0	18
600.0	0	0	0	0	10
650.0	0	0	0	0	3
700.0	0	0	0	0	3
750.0	0	0	0	0	4
800.0	0	0	0	0	0
850.0	0	0	0	0	8
900.0	0	0	0	0	4
950.0	0	0	0	0	7
1000.0	0	0	0	0	9
1050.0	0	0	0	0	9
1100.0	0	0	0	0	5
1150.0	0	0	0	0	3
1200.0	0	0	0	0	3
1250.0	0	0	0	0	2
1300.0	0	0	0	0	0
1350.0	0	0	0	0	1
1400.0	0	0	0	0	0
1450.0	0	0	0	0	0
1500.0	0	0	0	0	1

EXAMPLE 5

A history containing 1021 peak/valley points is used. This is the Filtered version (filter level = .46) of a simulated helicopter combat maneuver loading history. Option 4 is used, so that the results are given in the form of a compact 32 x 32 matrix containing the maximum and minimum (peak and valley) values of the rain-flow cycles. Tables B-9 and B-10 show the input and output of this program, respectively. Note that the history is converted using linear interpolation to have a minimum value of 1 and a maximum value of 32.

Table B-9. Input for Example 5

```
4
1021
1.000  0.430  0.973  0.457  0.978  0.456  0.939  0.467  0.924  0.409
0.919  0.467  0.925  0.442  0.971  0.426  0.884  0.357  0.887  0.334
0.802  0.135  0.721  0.154  0.641 -0.041  0.749  0.001  0.750  0.127
0.835  0.380  0.896  0.397  0.860  0.369  0.832  0.365  0.849  0.350
0.836 -0.258  0.433 -0.033  0.422 -0.033  0.434 -0.174  0.293 -0.180
0.287 -0.216  0.355 -0.222  0.558  0.059  0.528 -0.055  0.398 -0.096
0.420 -0.092  0.388 -0.067  0.449 -0.134  0.381 -0.121  0.356 -0.095
0.406 -0.084  0.393 -0.067  0.387 -0.114  0.371 -0.087  0.379 -0.124
0.804  0.335  0.841  0.306  0.805  0.340  0.796  0.316  0.835  0.324
0.820  0.264  0.758 -0.010  0.748  0.289  0.797  0.333  0.803  0.149
0.765  0.200  0.697  0.230  0.771  0.321  0.886  0.335  0.874  0.373
0.894  0.423  0.983  0.423  0.909  0.369  0.860  0.383  0.879  0.422
0.932  0.395  0.936  0.380  0.867  0.348  0.806  0.197  0.676  0.002
0.514  0.018  0.511  0.046  0.559 -0.516  0.381 -0.215  0.376 -0.109
0.545  0.073  0.726  0.235  0.733  0.260  0.776  0.294  0.814  0.317
0.866  0.304  0.791  0.324  0.819  0.348  0.800  0.301  0.828  0.303
0.834  0.363  0.857  0.342  0.964  0.350  0.904  0.254  0.863  0.350
0.837  0.334  0.876  0.157  0.710  0.199  0.776  0.209  0.704  0.223
0.727  0.192  0.664  0.141  0.785  0.163  0.775  0.179  0.701  0.216
0.706  0.191  0.712  0.183  0.690  0.165  0.674  0.149  0.631  0.127
0.616  0.116  0.621  0.088  0.627  0.165  0.635  0.122  0.581  0.058
0.598  0.080  0.550  0.073  0.564  0.091  0.625 -0.016  0.596 -0.237
0.860 -0.035  0.510  0.032  0.495 -0.201  0.603  0.009  0.631  0.066
0.696  0.157  0.719  0.059  0.657  0.132  0.727  0.109  0.589 -0.011
0.668 -0.071  0.630 -0.048  0.626 -0.091  0.527 -0.112  0.529 -0.109
0.436 -0.146  0.403 -0.124  0.334 -0.171  0.280 -0.382  0.298 -0.193
0.721  0.254  0.722  0.219  0.828  0.335  0.898  0.365  0.871  0.232
0.799  0.303  0.839  0.296  0.788  0.309  0.780  0.303  0.775  0.275
0.742  0.167  0.747  0.183  0.781  0.259  0.760  0.099  0.692  0.108
0.725  0.116  0.700  0.189  0.692  0.076  0.619  0.128  0.600  0.088
0.731  0.051  0.732  0.124  0.656  0.118  0.755  0.079  0.643  0.053
0.739  0.079  0.711  0.063  0.648  0.153  0.614  0.144  0.681  0.128
0.705  0.109  0.785  0.075  0.561  0.098  0.641  0.146  0.771  0.230
0.708  0.220  0.826  0.361  0.832  0.373  0.832  0.375  0.884  0.369
0.939  0.398  0.882  0.365  0.911  0.442  0.947  0.393  0.901  0.373
0.995  0.365  0.974  0.382  0.976  0.371  0.978  0.412  0.883  0.390
0.872  0.362  0.839  0.381  0.882  0.419  0.889  0.346  0.881  0.348
0.826  0.371  0.839  0.342  0.896  0.337  0.790  0.321  0.885  0.287
0.811  0.039  0.583  0.054  0.535 -0.097  0.555  0.005  0.458  0.001
0.611  0.010  0.542  0.000  0.451 -0.294  0.277 -0.237  0.303 -0.215
0.287 -0.201  0.268 -0.219  0.275 -0.190  0.280 -0.174  0.312 -0.163
0.364 -0.185  0.316 -0.135  0.360 -0.103  0.352 -0.115  0.352 -0.148
0.304 -0.155  0.322 -0.153  0.336 -0.137  0.327 -0.142  0.332 -0.169
0.301 -0.266  0.659  0.182  0.702  0.220  0.733  0.231  0.746  0.237
0.742  0.271  0.776  0.259  0.783  0.279  0.757  0.270  0.729  0.196
0.703  0.190  0.642  0.147  0.630  0.106  0.629  0.165  0.730  0.139
0.624  0.099  0.603  0.147  0.710  0.092  0.711  0.047  0.566  0.115
0.577  0.084  0.541  0.015  0.472  0.021  0.512 -0.199  0.321 -0.158
0.337 -0.124  0.430 -0.081  0.412 -0.125  0.417 -0.074  0.442 -0.026
0.540 -0.066  0.561 -0.141  0.387 -0.072  0.382 -0.082  0.468 -0.054
0.497 -0.060  0.452 -0.089  0.411 -0.094  0.430 -0.131  0.382 -0.131
```

Table B-9 (2nd page)

0.356	-0.099	0.405	-0.116	0.488	-0.144	0.460	-0.086	0.440	-0.121
0.553	-0.115	0.501	-0.077	0.487	-0.041	0.495	-0.012	0.470	-0.026
0.519	-0.019	0.532	-0.013	0.459	-0.022	0.542	-0.062	0.575	-0.016
0.549	-0.017	0.542	0.024	0.573	0.012	0.681	0.053	0.640	0.108
0.595	0.139	0.602	0.028	0.613	0.015	0.635	-0.079	0.582	0.013
0.530	0.052	0.620	0.161	0.647	0.180	0.635	0.164	0.685	0.180
0.709	0.129	0.701	0.127	0.643	0.104	0.615	-0.075	0.471	-0.049
0.432	-0.032	0.423	-0.060	0.396	-0.095	0.458	-0.055	0.429	-0.083
0.494	-0.080	0.486	-0.040	0.515	-0.047	0.505	-0.101	0.477	-0.049
0.509	-0.065	0.541	-0.087	0.590	-0.084	0.579	0.077	0.595	-0.098
0.654	-0.133	0.612	-0.087	0.618	-0.137	0.572	-0.134	0.490	-0.043
0.435	-0.107	0.366	-0.099	0.536	0.000	0.456	0.005	0.695	0.042
0.648	0.117	0.731	0.238	0.803	0.275	0.811	0.193	0.825	0.248
0.700	0.171	0.887	0.217	0.762	0.185	0.721	0.236	0.701	0.200
0.732	0.227	0.873	0.197	0.800	0.192	0.684	0.221	0.738	0.194
0.742	0.161	0.750	0.223	0.718	0.181	0.742	0.253	0.732	0.241
0.731	0.194	0.730	0.228	0.776	0.265	0.790	0.226	0.795	0.238
0.717	0.244	0.765	0.270	0.768	0.226	0.732	0.256	0.748	0.216
0.755	0.286	0.787	0.272	0.727	0.232	0.738	0.187	0.749	-0.341
0.299	-0.155	0.311	-0.180	0.650	-0.198	0.318	-0.182	0.281	-0.231
0.442	-0.073	0.453	-0.085	0.471	-0.061	0.467	0.000	0.483	0.002
0.531	0.044	0.546	-0.041	0.541	0.010	0.564	0.047	0.594	0.040
0.534	0.033	0.570	0.059	0.556	0.101	0.562	0.097	0.587	0.110
0.601	0.012	0.625	-0.259	0.683	0.220	0.748	0.287	0.849	0.328
0.858	0.401	0.870	0.389	0.892	0.375	0.900	0.446	0.917	0.436
0.948	0.475	0.971	0.403	0.951	0.460	0.951	0.404	0.884	0.365
0.975	0.447	0.950	0.440	0.914	0.449	0.922	0.414	0.926	0.456
0.941	0.450	0.910	0.408	0.880	0.352	0.824	0.342	0.865	0.358
0.864	0.381	0.936	0.346	0.872	-0.213	0.559	0.061	0.513	0.040
0.509	-0.020	0.466	-0.151	0.775	0.305	0.821	0.229	0.788	0.333
0.834	0.333	0.819	0.362	0.829	0.302	0.833	0.324	0.846	0.290
0.802	0.252	0.813	0.309	0.775	0.175	0.797	0.239	0.780	0.273
0.737	0.276	0.789	0.304	0.846	0.291	0.742	0.285	0.767	0.240
0.775	0.211	0.722	0.221	0.698	0.205	0.707	0.174	0.690	0.002
0.673	0.172	0.790	0.225	0.760	0.294	0.795	0.308	0.789	0.334
0.822	0.351	0.811	0.303	0.802	0.289	0.782	0.301	0.784	0.325
0.783	0.299	0.847	0.324	0.822	0.343	0.896	0.373	0.875	0.394
0.943	0.383	0.857	0.350	0.924	0.372	0.894	0.421	0.874	0.383
0.878	0.357	0.846	0.379	0.908	0.377	0.913	0.434	0.887	0.390
0.879	0.379	0.878	0.388	0.839	0.334	0.842	0.381	0.896	0.395
0.901	0.393	0.872	0.374	0.946	0.398	0.881	0.392	0.897	0.338
0.958	0.402	0.912	0.337	0.877	0.398	0.878	0.379	0.838	0.380
0.920	0.389	0.856	0.383	0.858	0.377	0.910	0.398	0.860	0.399
0.881	0.428	0.901	0.362	0.910	0.417	0.928	0.414	0.897	0.394
0.956	0.422	0.943	0.407	0.949	0.425	0.954	0.394	0.922	0.423
0.955	0.337	0.905	0.444	0.934	0.413	0.932	0.405	0.926	0.383
0.907	0.362	0.860	0.004	0.494	-0.016	0.771	0.141	0.900	0.431
0.952	0.406	0.961	0.470	0.952	0.440	0.995	0.401	0.944	0.465
0.972	0.414	0.952	0.485	0.958	0.437	0.987	0.399	0.990	0.414
0.980	0.457	0.981	0.407	0.962	0.452	0.959	0.407	0.910	0.437
0.966	0.440	0.945	0.453	0.932	0.421	0.939	0.438	0.920	0.000
1.000									

Table B-10. Output for Example 5

RAINFLOW CYCLES

PEAK	/***VALLEY*** 1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	TOTAL CYCLES
1.0	0	0	0	0	0	0	0	0	0
2.0	0	0	0	0	0	0	0	0	0
3.0	0	0	0	0	0	0	0	0	0
4.0	0	0	0	0	0	0	0	0	0
5.0	0	0	0	0	0	0	0	0	0
6.0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0
9.0	0	0	0	0	0	0	0	0	0
10.0	0	0	0	0	0	0	0	0	0
11.0	0	0	0	0	0	0	0	0	0
12.0	0	0	0	0	0	0	0	0	0
13.0	0	0	0	0	0	0	0	0	0
14.0	0	0	0	0	0	0	0	0	0
15.0	0	0	0	0	0	0	0	0	0
16.0	0	0	0	0	0	0	0	0	0
17.0	0	0	0	0	0	0	3	5	8
18.0	0	0	0	0	0	0	1	10	17
19.0	0	0	0	0	0	1	2	1	18
20.0	0	0	0	0	0	0	1	0	20
21.0	0	0	0	0	0	0	0	0	22
22.0	0	0	0	0	0	0	0	0	27
23.0	0	0	0	0	0	0	0	1	33
24.0	0	0	0	0	0	0	1	0	33
25.0	0	0	0	0	0	1	0	0	24
26.0	0	0	0	0	0	0	1	0	48
27.0	0	0	0	0	0	0	0	0	47
28.0	0	0	0	0	0	0	1	0	51
29.0	0	0	0	0	0	0	1	0	50
30.0	0	0	0	0	0	1	0	0	60
31.0	0	0	0	1	0	0	1	0	41
32.0	1	0	0	0	1	1	0	0	11

Table B-10 (2nd page)

RAINFLOW CYCLES

PEAK	VALLEY 9.0	10.0	11.0	12.0	13.0	14.0	15.0	16.0	TOTAL CYCLES
1.0	0	0	0	0	0	0	0	0	0
2.0	0	0	0	0	0	0	0	0	0
3.0	0	0	0	0	0	0	0	0	0
4.0	0	0	0	0	0	0	0	0	0
5.0	0	0	0	0	0	0	0	0	0
6.0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0
9.0	0	0	0	0	0	0	0	0	0
10.0	0	0	0	0	0	0	0	0	0
11.0	0	0	0	0	0	0	0	0	0
12.0	0	0	0	0	0	0	0	0	0
13.0	0	0	0	0	0	0	0	0	0
14.0	0	0	0	0	0	0	0	0	0
15.0	0	0	0	0	0	0	0	0	0
16.0	0	0	0	0	0	0	0	0	0
17.0	0	0	0	0	0	0	0	0	8
18.0	6	0	0	0	0	0	0	0	17
19.0	6	8	0	0	0	0	0	0	18
20.0	5	9	5	0	0	0	0	0	20
21.0	1	9	6	6	0	0	0	0	22
22.0	4	5	6	8	4	0	0	0	27
23.0	4	3	3	9	8	5	0	0	33
24.0	1	5	2	4	4	9	7	0	33
25.0	1	1	1	1	3	6	9	1	24
26.0	1	0	0	1	4	8	7	21	48
27.0	0	0	0	1	2	3	6	12	47
28.0	0	0	0	0	1	1	5	6	51
29.0	0	0	1	1	0	0	2	1	50
30.0	0	0	1	0	1	0	0	0	60
31.0	0	0	1	0	0	0	0	0	41
32.0	0	0	0	1	0	0	0	0	11

Table B-10 (3rd page)

RAINFLOW CYCLES

PEAK	/****VALLEY**** 17.0	18.0	19.0	20.0	21.0	22.0	23.0	24.0	TOTAL CYCLES
1.0	0	0	0	0	0	0	0	0	0
2.0	0	0	0	0	0	0	0	0	0
3.0	0	0	0	0	0	0	0	0	0
4.0	0	0	0	0	0	0	0	0	0
5.0	0	0	0	0	0	0	0	0	0
6.0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0
9.0	0	0	0	0	0	0	0	0	0
10.0	0	0	0	0	0	0	0	0	0
11.0	0	0	0	0	0	0	0	0	0
12.0	0	0	0	0	0	0	0	0	0
13.0	0	0	0	0	0	0	0	0	0
14.0	0	0	0	0	0	0	0	0	0
15.0	0	0	0	0	0	0	0	0	0
16.0	0	0	0	0	0	0	0	0	0
17.0	0	0	0	0	0	0	0	0	0
18.0	0	0	0	0	0	0	0	0	8
19.0	0	0	0	0	0	0	0	0	17
20.0	0	0	0	0	0	0	0	0	18
21.0	0	0	0	0	0	0	0	0	20
22.0	0	0	0	0	0	0	0	0	22
23.0	0	0	0	0	0	0	0	0	27
24.0	0	0	0	0	0	0	0	0	33
25.0	0	0	0	0	0	0	0	0	33
26.0	5	0	0	0	0	0	0	0	24
27.0	16	7	0	0	0	0	0	0	48
28.0	6	22	9	0	0	0	0	0	47
29.0	1	8	25	10	0	0	0	0	51
30.0	0	2	19	25	11	0	0	0	50
31.0	0	4	6	16	12	0	0	0	60
32.0	0	0	2	4	1	0	0	0	41
									11

Table B-10 (4th page)

RAINFLOW CYCLES

PEAK /VALLEY	25.0	26.0	27.0	28.0	29.0	30.0	31.0	32.0	TOTAL CYCLES
1.0	0	0	0	0	0	0	0	0	0
2.0	0	0	0	0	0	0	0	0	0
3.0	0	0	0	0	0	0	0	0	0
4.0	0	0	0	0	0	0	0	0	0
5.0	0	0	0	0	0	0	0	0	0
6.0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0
9.0	0	0	0	0	0	0	0	0	0
10.0	0	0	0	0	0	0	0	0	0
11.0	0	0	0	0	0	0	0	0	0
12.0	0	0	0	0	0	0	0	0	0
13.0	0	0	0	0	0	0	0	0	0
14.0	0	0	0	0	0	0	0	0	0
15.0	0	0	0	0	0	0	0	0	0
16.0	0	0	0	0	0	0	0	0	0
17.0	0	0	0	0	0	0	0	0	8
18.0	0	0	0	0	0	0	0	0	17
19.0	0	0	0	0	0	0	0	0	18
20.0	0	0	0	0	0	0	0	0	20
21.0	0	0	0	0	0	0	0	0	22
22.0	0	0	0	0	0	0	0	0	27
23.0	0	0	0	0	0	0	0	0	33
24.0	0	0	0	0	0	0	0	0	33
25.0	0	0	0	0	0	0	0	0	24
26.0	0	0	0	0	0	0	0	0	48
27.0	0	0	0	0	0	0	0	0	47
28.0	0	0	0	0	0	0	0	0	51
29.0	0	0	0	0	0	0	0	0	50
30.0	0	0	0	0	0	0	0	0	60
31.0	0	0	0	0	0	0	0	0	41
32.0	0	0	0	0	0	0	0	0	11

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16. Abstract

Helicopter loading histories applied to notched metal samples are used as examples, and their fatigue lives are calculated by using a simplified version of the local strain approach. This simplified method has the advantage that it requires knowing the loading history in only the reduced form of ranges and means and number of cycles from the rain-flow cycle counting method. The calculated lives compare favorably with test data.

17. Key Words (Suggested by Author(s))

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