N90-18988

ANTIHYDROGEN FORMATION IN COLLISIONS OF POSITRONIUM WITH ANTIPROTONS

J.W. Humberston
Department of Physics and Astronomy
University College London, Gower Street, London WC1E 6BT, U.K.

INTRODUCTION

Antihydrogen, consisting of a positron orbiting around an antiproton, is the simplest few body system consisting entirely of antimatter and as such is of considerable importance in providing additional tests of the validity of charge conjugation invariance. In addition, the nature of the gravitational interaction between matter and antimatter might more readily be investigated for an electrically neutral system than one which is charged.

Before such studies can be undertaken the antihydrogen must, of course, be produced by attachment of a positron to an antiproton. Unfortunately, both constituents can only be produced in very small quantities and the viability of a particular method of antihydrogen production therefore depends critically on the efficiency of the attachment process.

Several production mechanisms have been proposed, the two most favoured of which are radiative capture (spontaneous or stimulated)

$$e^+ + \overline{p} \rightarrow \overline{H} + \gamma, \tag{1}$$

$$e^+ + \overline{p} + n\gamma \rightarrow \overline{H} + \gamma + n\gamma, \tag{2}$$

and charge exchange in positronium-antiproton collisions

$$Ps + \overline{p} \rightarrow \overline{H} + e^-. \tag{3}$$

Both methods are being actively investigated.$^{(1-3)}$

The cross section for radiative capture is very much less than that for charge exchange, so that it might be thought that the latter process is greatly to be preferred. However, in the proposed experiment using the radiative capture process the positrons will be confined in a storage ring (as also will be the antiprotons in both methods) and, therefore, those positrons which do not undergo capture by antiprotons on the first occasion will be recycled until capture occurs. In this way nearly all the antiprotons can be converted into antihydrogen.

The charge exchange process (equation (3)) is a single pass method because positronium is neutral, but the somewhat simpler nature of the proposed apparatus (see figure 1) and the much larger antihydrogen formation cross section are advantages which may, nevertheless, make this an effective method of production also. We shall, therefore, now consider various calculations of the cross section for the charge exchange process.

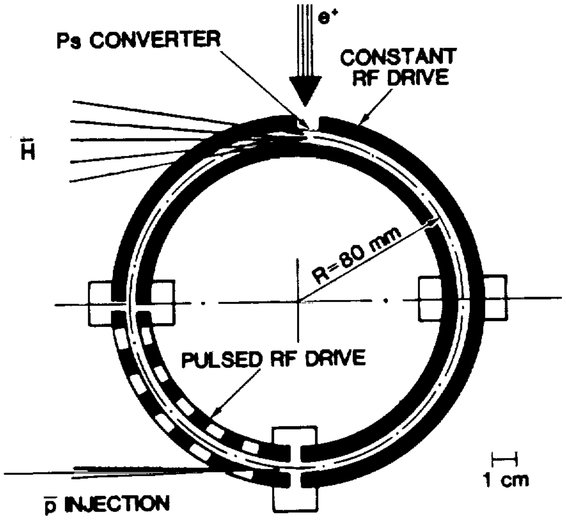

Figure 1. Proposed experimental arrangement for the formation of antihydrogen$^{(3)}$.

CALCULATIONS OF THE CHARGE EXCHANGE CROSS-SECTION

Until very recently the prospect of forming antihydrogen in positronium-antiproton collisions seemed so remote that no calculations of the cross-section had been considered. However, the required cross-section is rather simply related by charge conjugation and time reversal invariance to the cross-section for positronium formation in positron-hydrogen collisions, a process which has received extensive theoretical attention for several years.$^{(4-6)}$

Starting with the process of interest

$$Ps + \overline{p} \rightarrow \overline{H} + e^- \qquad (\text{ cross-section } \sigma_{\overline{H}}), \tag{4}$$

the charge conjugate system is

$$Ps + p \rightarrow H + e^+ \qquad (\text{cross-section } \sigma_H), \tag{5}$$

and charge conjugation invariance requires $\sigma_H = \sigma_{\overline{H}}$. The time reversed process is then

$$H + e^+ \rightarrow Ps + p \qquad (\text{cross-section } \sigma_{Ps}). \tag{6}$$

Time reversal invariance implies the symmetry of the S-matrix which provides the following connection between the cross-sections $\sigma_{\bar{H}}$ and σ_{Ps}. Consider a positron colliding with a hydrogen atom in a state with energy and orbital angular momentum quantum numbers n_1 and ℓ_1 respectively, to produce positronium with corresponding quantum numbers n_2 and ℓ_2. If the cross-section for this process is $\sigma_{Ps}(n_2,\ell_2;n_1,\ell_1)$ then the cross-section for the time reversed process is $\sigma_H(n_1,\ell_1;n_2,\ell_2)$, and

$$\sigma_{\bar{H}}(n_1,\ell_1;n_2,\ell_2) = \sigma_H(n_1,\ell_1;n_2,\ell_2) \qquad (7)$$

$$= \frac{k^2}{\kappa^2}\frac{(2\ell_1+1)}{(2\ell_2+1)}\sigma_{Ps}(n_2,\ell_2;n_1,\ell_1), \qquad (8)$$

where k and κ are the wave numbers of the positron and positronium respectively. They are related by energy conservation such that (in atomic units)

$$E_{H+e^+} = E_{Ps+p} = \frac{k^2}{2} - \frac{1}{n_1^2} = \frac{\kappa^2}{4} - \frac{1}{2n_2^2}. \qquad (9)$$

If the initial positronium and residual antihydrogen are both in their ground states

$$\sigma_{\bar{H}} = \sigma_H = \frac{k^2}{\kappa^2}\sigma_{Ps}, \qquad (10)$$

where

$$k^2 - 1 = \frac{1}{2}\kappa^2 - \frac{1}{2}. \qquad (11)$$

This simple rescaling formula may be applied to the various positronium formation cross sections already calculated.

As we shall see, the peak in $\sigma_{\bar{H}}$ occurs at low kinetic energies of the positronium relative to the antiproton, corresponding to positronium formation in, and just above, the Ore gap (6.8 - 10.2 eV for hydrogen). In the Ore gap, where the only two open channels are elastic scattering and positronium formation, the most accurate results are probably those of Humberston$^{(4)}$, and Brown and Humberston$^{(5)}$. They used a two channel version of the Kohn variational method of the form

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} K_{11}^t & K_{12}^t \\ K_{21}^t & K_{22}^t \end{bmatrix} - \begin{bmatrix} (\Psi_1,\mathcal{L}\Psi_1) & (\Psi_1,\mathcal{L}\Psi_2) \\ (\Psi_2,\mathcal{L}\Psi_1) & (\Psi_2,\mathcal{L}\Psi_2) \end{bmatrix} \qquad (12)$$

where $\mathcal{L} = 2(H-E)$.

The two components of the trial wave function, Ψ_1 and Ψ_2, represent positron elastic scattering plus positronium formation and positronium elastic scattering plus hydrogen formation respectively. Each component contains many short range correlation terms with associated variational parameters to allow adequately for the strong electron-positron correlations. Investigations of the variations of the values of the K-matrix elements with respect to systematic improvements in the trial functions suggest that the most accurate results obtained in this manner have converged to within a few per cent of the exact values.

The partial wave cross-section for positronium formation is then (in units of πa_o^2)

$$\sigma_{Ps} = \frac{4(2\ell+1)}{k^2}\left|\left(\frac{K}{1-iK}\right)_{12}\right|^2 \qquad (13)$$

Results were obtained for s-, p- and d-wave scattering and compared with results from other simpler methods of calculation. The accurate variational values are several orders of magnitude smaller than the results of the Born approximation for s-wave scattering but only a factor of three smaller for p-wave scattering, and for the d-wave the difference between the two methods is rather small. It is therefore reasonable to suppose that the Born approximation provides quite accurate values for the positronium formation cross-section for all higher partial waves. With this assumption, the total positronium formation cross section in the Ore gap is as given in figure 2. The Born results were first calculated by Omidvar (unpublished).

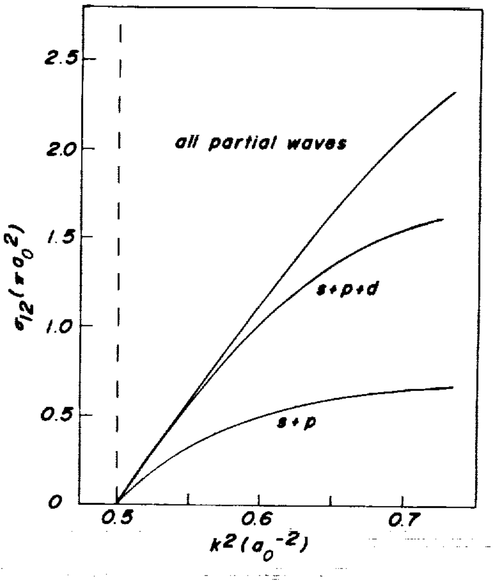

Figure 2. Partial wave cross sections for ground state positronium formation in positron collisions with atomic hydrogen$^{(4,5)}$. The s-wave contribution is too small to be shown on this scale.

At positron energies somewhat above the top of the Ore gap, where the Born approximation is still not sufficiently accurate for the low partial waves, the most reliable calculations of the positronium formation cross section are probably those of Shakeshaft and Wadehra(6), who used a distorted wave Born approximation and obtained results in the positron energy range 13.6 - 200eV. These results match on reasonably well at the lower end of the range to the more accurate variational results.

Beyond 200eV the Born results of Omidvar may be used.

These three sets of positronium formation cross sections produce, upon rescaling according to equation (10), the hydrogen (antihydrogen) formation cross-section in collisions of positronium with protons (antiprotons) shown in figure 3(7). The peak value of the cross-section is approximately $3.5 \times 10^{-16} cm^2$ at a positronium energy of ~ 3.5eV relative to the antiproton, corresponding to an antiproton energy relative to the positronium of ~ 3keV.

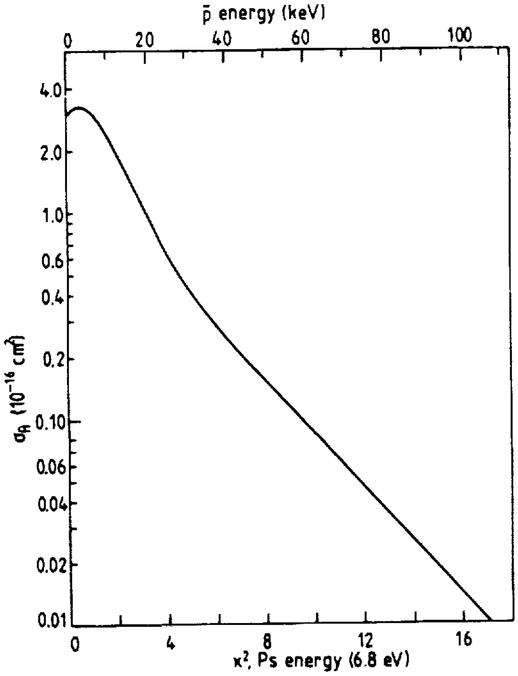

Figure 3. Theoretical estimates by Humberston et al(7). of the ground state antihydrogen formation cross section from ground state positronium.

There is an interesting feature of the antihydrogen formation cross section at very low positronium energies, as is shown on the expanded scale in figure 4, with the cross section tending to infinity as $\kappa^2 \rightarrow 0$. The zero orbital angular momentum contribution to $\sigma_{\overline{H}}$ is, from equation (13),

$$\sigma_{\overline{H}}^{(0)} = \frac{4}{\kappa^2}\left[\frac{K_{12}^2}{(1 - K_{11}K_{22} + K_{12}^2)^2 + (K_{11} + K_{22})^2}\right] \tag{14}$$

and the K-matrix elements have the following behaviour as the positronium threshold is approached from above:

$$\left.\begin{array}{ll} K_{11} \rightarrow & \text{const.} \\ K_{12} \rightarrow & \kappa^{\frac{1}{2}} \\ K_{22} \rightarrow & \kappa \end{array}\right\} \quad \kappa \rightarrow 0 \tag{15}$$

so that

$$\sigma_{\overline{H}}^{(0)} \propto \frac{1}{\kappa} \quad \text{as } \kappa \rightarrow 0, \tag{16}$$

as is to be expected for an exothermic reaction such as this. However, the rate of production of $\overline{H}$ is $\upsilon\sigma_{\overline{H}} \propto \kappa\sigma_{\overline{H}} \rightarrow$ finite const. as $\kappa \rightarrow 0$.

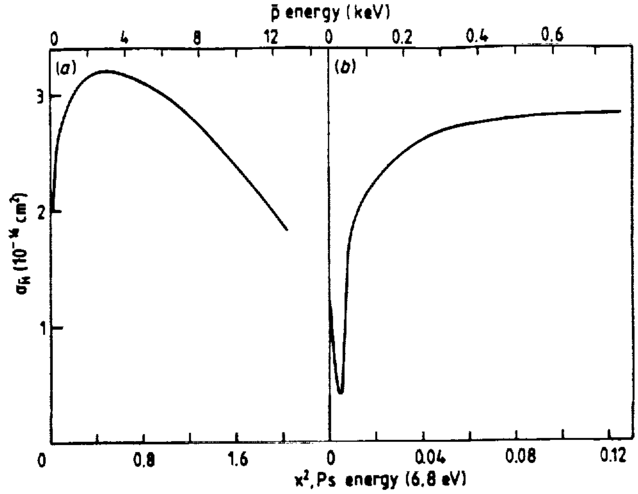

Figure 4. Low energy portions of figure 3 on expanded scales.

Thus far we have considered the formation of antihydrogen in its ground state, using positronium also in its ground state. It would be equally acceptable to form antihydrogen in one of its excited states, and several authors have recently considered this process. Darewych(8) used the first Born approximation to calculate the cross sections for antihydrogen formation into the $1s, 2s, 2p$ and $3s$ states and his results are shown in figure 5. As can be seen, these excited state formation cross sections are comparable to the ground state formation cross section at low positronium energies and, therefore, make a very significant contribution to the total formation cross section. Almost certainly the Born approximation significantly overestimates the s-wave contributions to each of these cross-sections, as it does for the ground state, but even if this partial wave contribution is totally supressed the excited formation cross sections remain significant, as shown in figure 5.

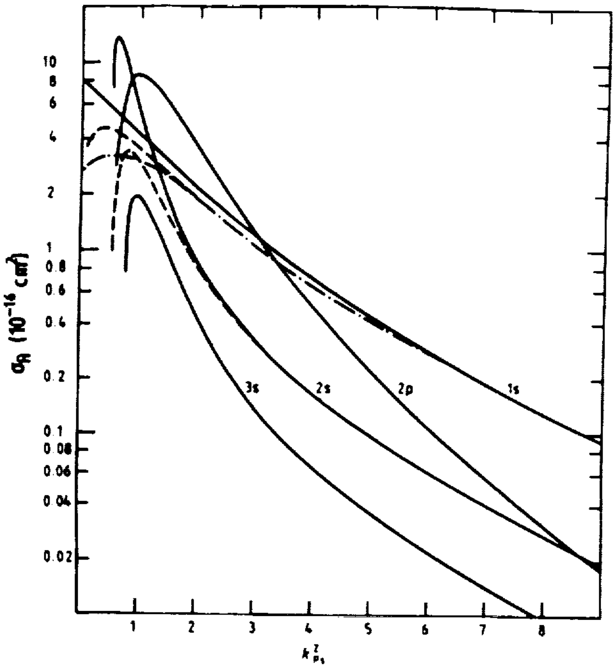

Figure 5. Cross sections for antihydrogen formation in various states from ground state positronium according to Darewych$^{(8)}$. Full curves; Born approximation: broken curves; s-wave subtracted Born approximation: chain curve; 'exact' results of Humberston *et al.*$^{(7)}$

Similar investigations, also using the first Born approximation, have recently been conducted by Nahar and Wadehra$^{(9)}$ and are shown in figure 6. They obtained good agreement with Darewych for the s-state but not quite so good for the p-state, where Darewych had used an approximate method to evaluate some of the angular integrals. These authors attempted to include the contribution to the total antihydrogen formation cross section from even higher energy states by making use of the fact that, at sufficiently high energy, the Born cross section for antihydrogen formation into a state with quantum numbers n, ℓ is

$$\sigma_{\overline{H}}(n, \ell) \propto \frac{1}{n^3}. \tag{17}$$

However, such a scaling formula is almost certainly not valid in the low energy range of interest here.

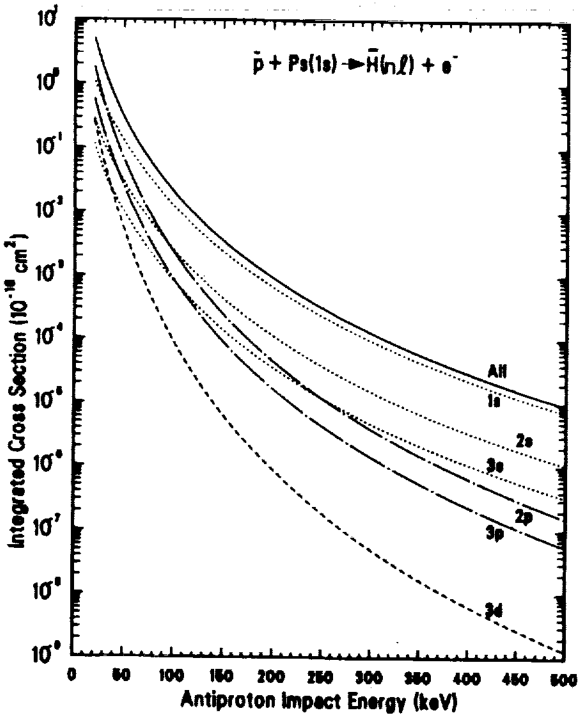

Figure 6. Cross sections for antihydrogen formation in various states from ground state positronium according to the Born calculations of Nahar and Wadehra.$^{(9)}$

Ermolaev *et al.*$^{(10)}$ used classical and semiclassical methods to investigate the same formation cross sections, and their total results for formation of antihydrogen into all states with $n \leq 3$ are shown in figure 7. Although the low mass of the incident positronium must cast some doubt on the validity of such methods for this system, the results are, nevertheless, in tolerably good agreement with the quantum mechanical results.

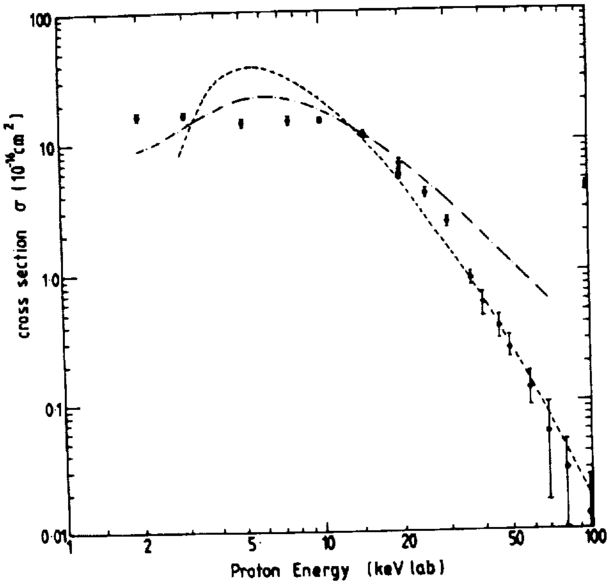

Figure 7. Total antihydrogen formation cross-sections from ground state positronium according to Ermolaev(10) –·–·–·; impact parameter method: •; classical trajectory Monte-Carlo: – – – –; Born approximation (Darewych(8)).

In all the investigations described so far, the incident positronium has been assumed to be in its ground state, but antihydrogen formation from positronium in excited states should also be considered.

Nahar and Wadehra(9) have used the Born approximation to investigate the formation of ground state antihydrogen in collisions of excited state positronium with antiprotons. They included the positronium states with n=1 and 2 explicitly and again used the scaling law given in equation (17) to estimate the effect of all other states. Their results are given in figure 8. .

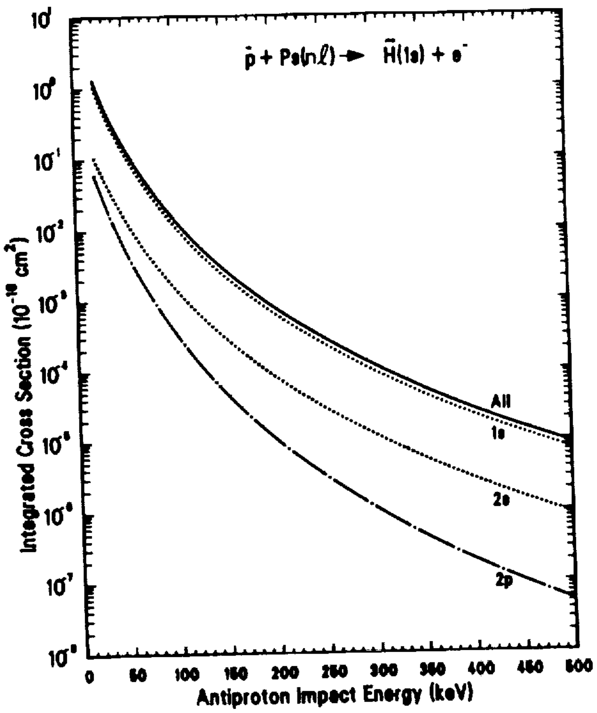

Figure 8. Cross sections for ground state antihydrogen formation from various states of positronium according to the Born calculations of Nahar and Wadehra.(9)

These Born calculations were not conducted at very low energies around the peak in the antihydrogen formation cross section and, even had they been, they would probably have suffered again from a similar overestimation of the low partial wave contributions to that mentioned previously. Nevertheless, it is very likely that antihydrogen formation from positronium in excited states will make a further very significant contribution to the total antihydrogen formation cross section.

Taking all these contributions into account, it would not seem unduely optimistic to predict a total antihydrogen formation cross section of at least $20\pi a_o^2$. With the expected currents of positronium and antiproton, this cross section suggests a possible rate of production of antihydrogen atoms of a few per second.

I should like to thank Prof. R.P. McEachran and his colleagues, Profs. J.W. Darewych and A.D. Stauffer for their hospitality at York University, Toronto during the writing of this article.

REFERENCES

1. H. Poth, "Atomic Physics with Positrons", edited by J.W. Humberston and E.A.G. Armour (Plenum, New York, 1987) p. 307.
2. A. Rich, R. Conti, W. Frieze, D.W. Gidley, H. Griffin, M. Skalsey, T. Steiger, J. Van House, W. Zheng and P.W. Zitzewitz, ibid, p. 321.
3. F.M. Jacobsen, L.H. Andersen, B.I. Deutsch, P. Hvelplund, H. Knudsen, M. Charlton, G. Laricchia and M. Holzscheiter, ibid, p. 333.
4. J.W. Humberston, Can. J. Phys. **60**, 591 (1982).
   --------, J. Phys. B. **17**, 2353 (1984).
   --------, Adv. At. Mol. Phys. **22**, 1 (1986).
5. C.J. Brown and J.W. Humberston, J. Phys. B. **17**, L423 (1984).
   --------, J. Phys. B. **18**, L401 (1985).
6. R. Shakeshaft and J.M. Wadehra, Phys. Rev. A**22**, 968 (1980).
7. J.W. Humberston, M. Charlton, F.M. Jacobsen and B.I. Deutch, J. Phys. B**20**, L25 (1987).
8. J.W. Darewych, J. Phys. B20, 5917 (1987).
9. S.N. Nahar and J.M. Wadehra, Phys. Rev. A**37**, 4118 (1988).
10. A.M. Ermolaev, B.H. Bransden and C.R. Mandal, Phys. Lett. A**125**, 44 (1987).