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KALMAN ESTIMATION FOR SEDS MEASUREMENTS

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ABSTRACT

The first on-orbit experiment of the Small Expendable Deployer System (SEDS) for tethered satellites will collect telemetry data for tether length, rate of deployment, and tether tension. The post-flight analysis will use this data to reconstruct the deployment history and determine payload position and tether shape.

Two Kalman estimator algorithms were written, and output using simulated measurement data was compared. Both estimators exhibited the same estimated state histories, indicating that numerical instability in the traditional algorithm was not the cause of filter divergence. Estimation of acceleration biases was added, which reduced the error but did not correct the divergence. An "add-a-bead" estimator that adds lumped masses as the tether is deployed was written, which provides a state model that matches the BEADSIM simulation providing the "true" measurements and states. This twenty-one bead estimator produced state histories similar to those of the two-bead estimator, indicating that the filter divergence was not caused by a reduced-order model.

The noise models used to date are relatively simple and may be the source of estimator divergence. The investigation of colored noise models, cross-correlated measurement and process covariances, and noise-adaptive filter techniques is recommended.

ACKNOWLEDGEMENTS

The summer program participant would like to thank Program Development at Marshall Space Flight Center for a most enjoyable summer of research. The SEDS analysis provided a "real-life" problem that has demonstrated the importance of statisical characteristics in Kalman filter design.

INTRODUCTION

A Kalman estimator post-flight data reduction program has been written to process measurement data from the Small Expendable-tether Deployment System (SEDS) experiment to be flown in 1991. The tether will be deployed from an expendable launch vehicle to a length of 20 km; it will swing to vertical and then be cut. Measurement data will be collected and relayed to the ground to be processed after flight.

The estimator processes length, length rate, and deployer position and velocity measurements, and estimates velocities and positions of tether endpoints and points between. Two algorithms has been developed for both two and twenty-one bead models, and simulated measurements have been processed.

OBJECTIVES

The objective of the summer faculty research was to continue development of the post-flight data reduction algorithms started the previous summer.

STATE MODEL

Several computer simulations of tether deployment dynamics are available, ranging from planar simple-pendulum representations to three-dimensional partial-differential-equation models. Last summer the investigator had chosen Energy Science Laboratories (ESL) BEADSIM model to provide the tether dynamics state equations, since it is relatively simple and yet still produces results that are comparable to more complex models. BEADSIM is a lumped mass model, in which masses or "beads" are added as the tether becomes longer (Fig. 1). No out-of-orbit-plane motion is modelled, and the external forces on each bead are the gravity gradient, aerodynamic drag, and Coriolis and centripetal accelerations. The equations are written using a Cartesian coordinate frame with an origin at the center of mass and moving at orbit speed.

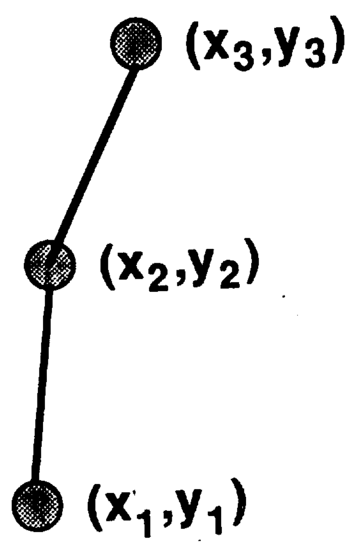

Fig. 1. BEADSIM lumped mass model.

Each bead's motion is governed by a second-order nonlinear differential equation, with a uniform tether tension providing the coupling between beads. Four states represent the motion of each bead: the x and y positions, and the x and y velocities. If we define z as a vector containing positions and velocities for each bead, then the state equations may be written as

$$\dot{z} = F(z,t) + \rho \tag{1}$$

where F(z,t) contain the gravity gradient, aerodynamic drag, and tension forces on each bead, and ρ is a vector of process noise. The process noise has been modelled as white random noise with zero mean and covariance matrix

$$Q = E(\rho\ \rho^T) = \begin{bmatrix} \sigma_1{}^2 & 0 & 0 \dots & 0 \\ 0 & \sigma_2{}^2 & 0 \dots & 0 \\ & & & \\ 0 & 0 & 0 \dots & \sigma_N{}^2 \end{bmatrix} \tag{2}$$

where E is the expected value.

The SEDS measurements consist of tether length l, length rate $\dot{l}$, and the deployer position and velocity. These measurements are nonlinear functions of the states, represented by the following measurement equations

$$l = G(z) + \nu \tag{3}$$

where l represents the vector of measurements at a given time, z the corresponding states, and ν a vector of measurement noise. The measurement noise has been modelled as white with zero mean and covariance matrix

$$R = E(\nu\ \nu^T) = \begin{bmatrix} \gamma_1{}^2 & 0 & 0 \dots & 0 \\ 0 & \gamma_2{}^2 & 0 \dots & 0 \\ & & & \\ 0 & 0 & 0 \dots & \gamma_m{}^2 \end{bmatrix} \tag{4}$$

STATE ESTIMATORS

The measurements are not the same variables as the states, so that payload position and tether shape cannot be directly determined from the measurements. A state estimator uses the measurements as input, filter the measurement noise, and then output the values of the states. Two estimator algorithms have been developed:

the traditional Kalman estimator algorithm, and the U-D square-root factorization technique. Both estimators process simulated measurements generated by BEADSIM to which white noise is added.

Estimators for dynamic systems were developed primarily for linear systems; for nonlinear systems represented by equations (1) and (3), an extended Kalman estimator is used. The nonlinear equations are used for state and measurement prediction at the next time step, and the state correction is calculated using a local linearization of these equations. The linearization of equations (1) and (3) are

$$\dot{z} = Az + \rho \tag{5}$$

and

$$l = Hz + \nu \tag{6}$$

where

$$H = \left.\frac{\partial G}{\partial z}\right|_{z_{current}} \tag{7}$$

$$A = \left.\frac{\partial F}{\partial z}\right|_{z_{current}} \tag{8}$$

The estimators correct the predicted states at each time step by multiplying the error between the actual measurement and the predicted measurement by a gain matrix. This gain matrix is calculated using the measurement covariance matrix and an extrapolated state estimate covariance, which is based on the linearized system's state transition matrix. The state transition matrix $\Phi(t_2,t_1)$ for equation (5) is the transformation that takes the state z_1 at time t_1 into the later state z_2

$$z_2 = \Phi(t_2,t1)z_1 \tag{9}$$

For linear systems, the state transition matrix obeys the same differential equation as the states themselves

$$\dot{\Phi}(t) = A\Phi(t) \tag{10}$$

and can be integrated forward in time along with the state equations. For nonlinear systems, however, equation (10) is a linearized approximation valid only in a neighborhood of the current state value. A comparison of the linearized and nonlinear state and measurement values indicates that a time-step size of one second produces four decimal digit agreement during deployment, and hence measurements will be processed at one-second intervals.

SIMULATION RESULTS

Traditional Kalman algorithm

The traditional Kalman algorithm using a two-bead model was developed in the summer of 1988. Payload position and velocity estimates are shown in Figs. 2 and 3, in which the solid line is the BEADSIM deployment and the dashed line is the estimator output. Fig. 4 contains the tether length and length rate during deployment, and Fig. 5 shows the rms position and velocity errors for the payload. This algorithm uses the identity matrix for the initial state covariance estimate, and propagates the covariance estimates with no checks on element values. Note that the payload position error reaches a maximum of 1500 m using this algorithm.

Filter divergence in many cases is due to overly optimistic covariance estimates, in which the algorithm depends heavily on the state prediction, and tends to reject the measurement data corrections. One way to treat filter divergence is to monitor the covariance estimates and reinitialize this matrix if the diagonal elements fall below a set threshold. Figs. 6 and 7 show the deployer and payload position and velocity errors using reinitialization of the state covariance matrix. The payload position error now has a maximum of 300 m. Similar results can be attained by increasing the estimates of the process noise.

Criticism of the traditional Kalman algorithm includes numerical instability, which can produce filter divergence such as that shown in Figs. 2 through 5, and that process-noise adjustments or reinitialization of the covariance matrix may tune the filter to produce any output desired, whether it is reasonable or not. The U-D square-root factorization algorithm provides an alternative method for Kalman estimation, and is considered a more numerically-stable method.

U-D Factorization Algorithm

The U-D factorization algorithm is computationally slower than the traditional Kalman algorithm, but in many cases produces more reliable results. The same two-bead model was used for state and covariance propagation, and the U-D algorithm produced estimates similar to those of the untuned traditional algorithm (Figs. 2 through 5). The estimation of acceleration biases due to aerodynamic drag on the deployer and payload improved these results. This algorithm produced the tether length and length rate history shown in Fig. 8, and payload position and deployer position rms errors shown in Fig. 9.

To determine the magnitude of errors associated with the two-bead model, a twenty-one bead model and estimator was developed. This estimator starts with two beads, and adds beads for every 1000 m of tether deployed, similar to the BEADSIM simulation. Figs. 10 and 11 show the length, length rate, payload position and deployer position rms errors for the twenty-one bead estimator. Comparison between Figs. 9 and 11 indicates that little improvement was obtained by using the twenty-one bead estimator, and that the filter errors do not occur due to the reduced-order model.

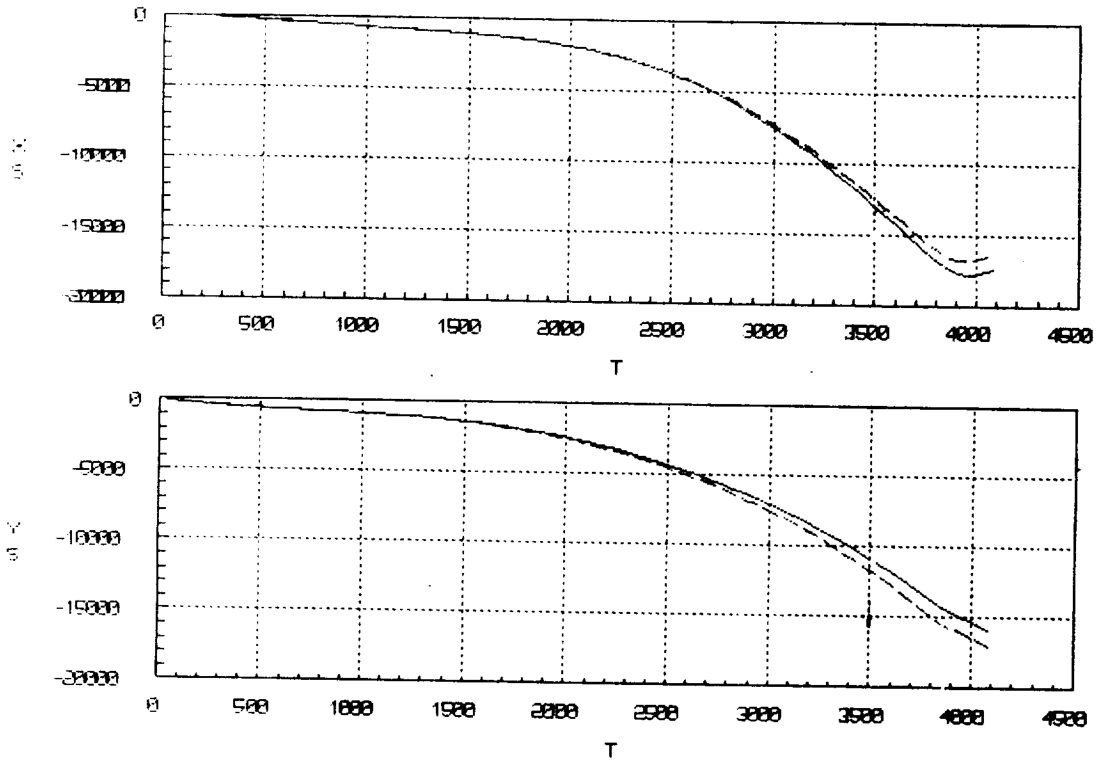

Fig. 2. Estimated payload position using untuned traditional Kalman algorithm.

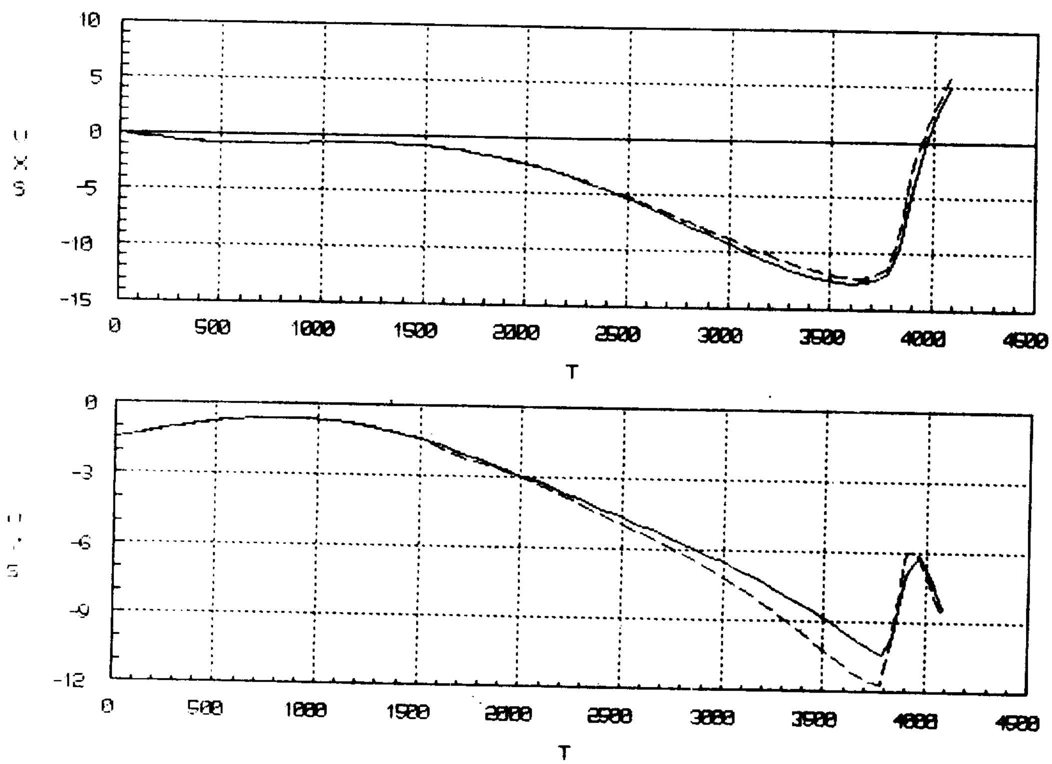

Fig. 3. Estimated payload velocity using untuned traditional Kalman algorithm.

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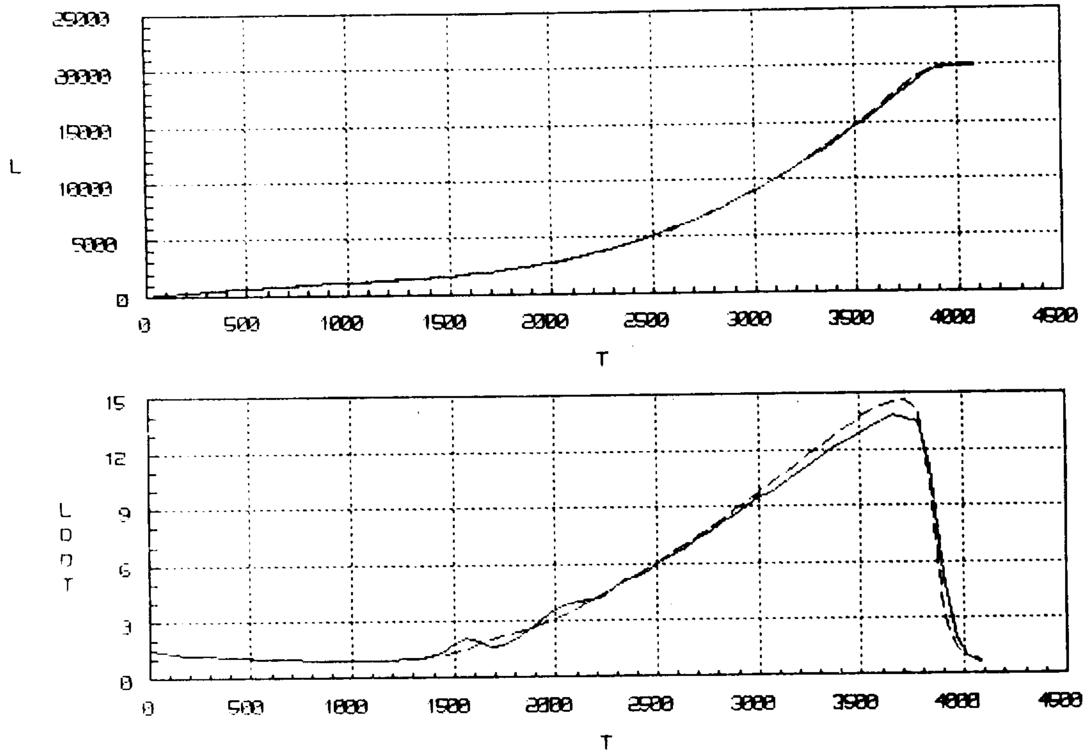


Fig. 4. Tether length,length rate using
untuned traditional Kalman algorithm.

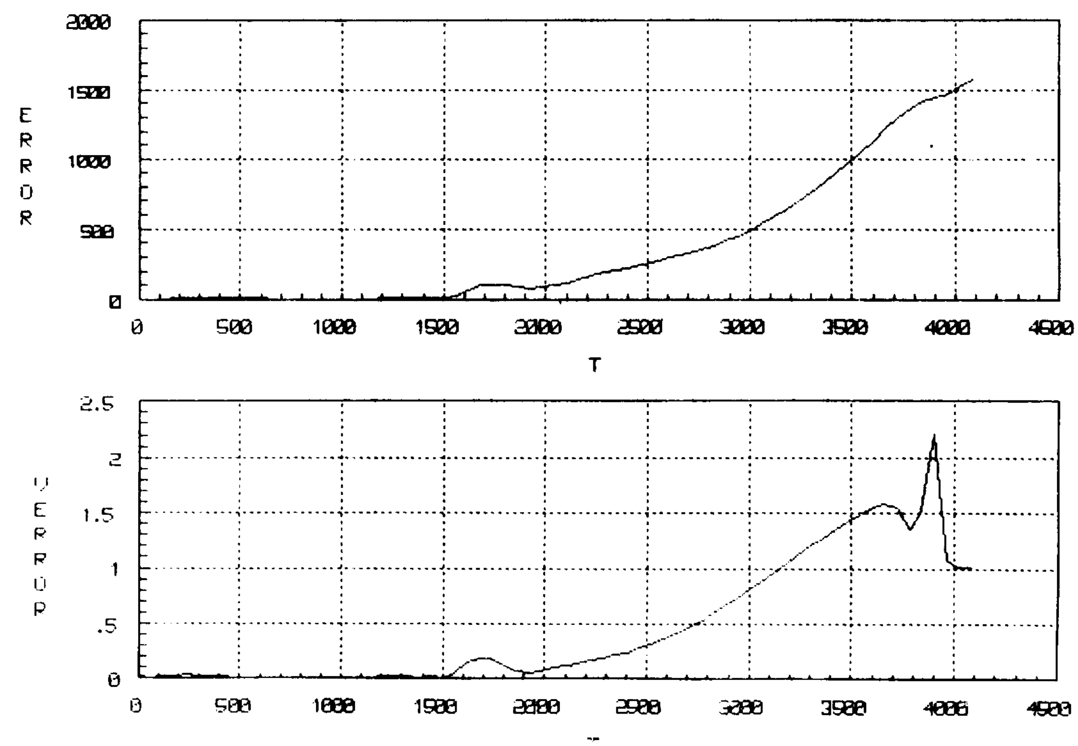


Fig. 5. Payload position and velocity rms errors
using untuned traditional Kalman algorithm.

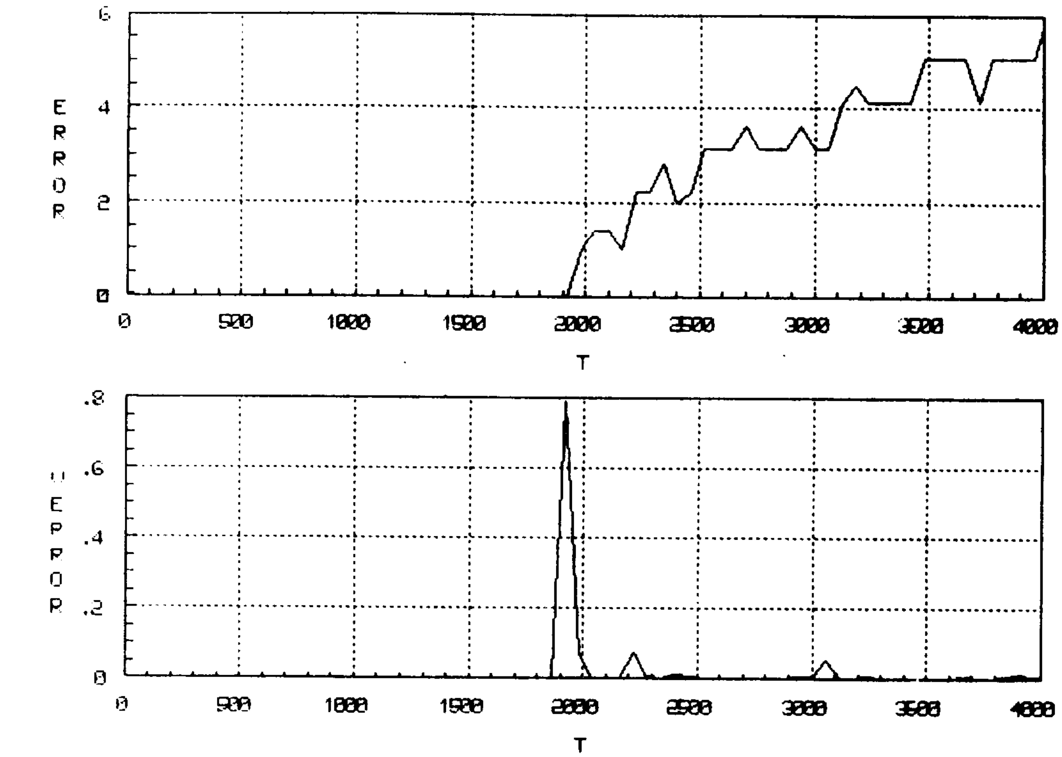


Fig. 6. Deployer position and velocity rms errors using tuned traditional Kalman algorithm.

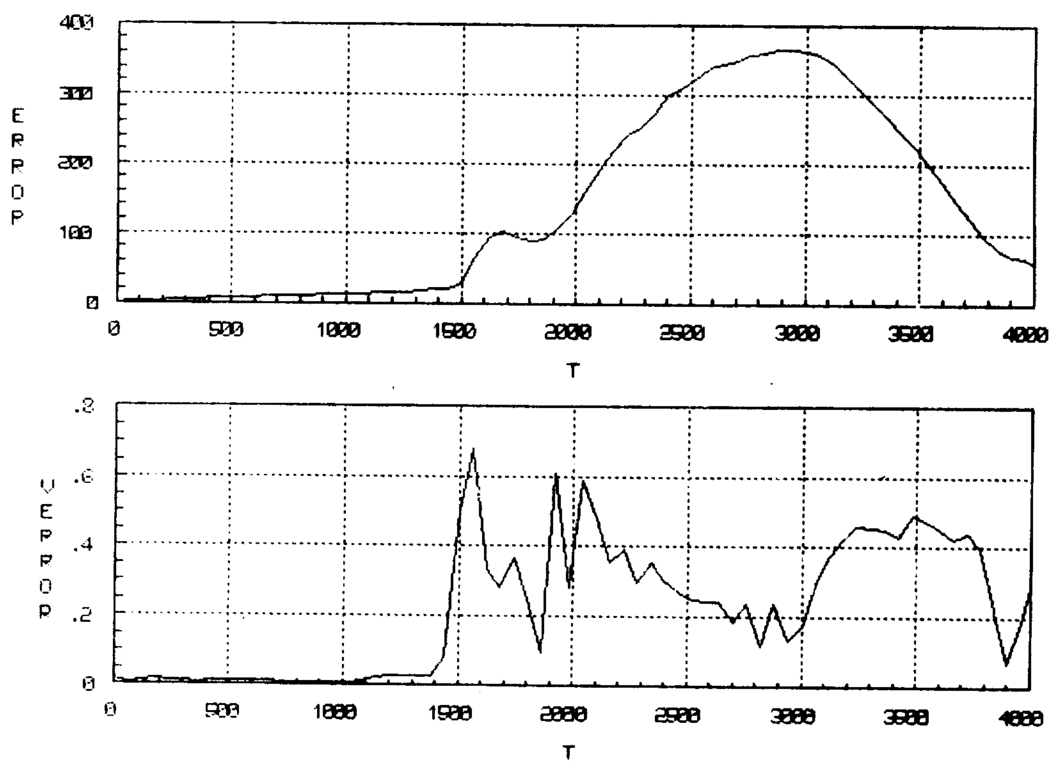
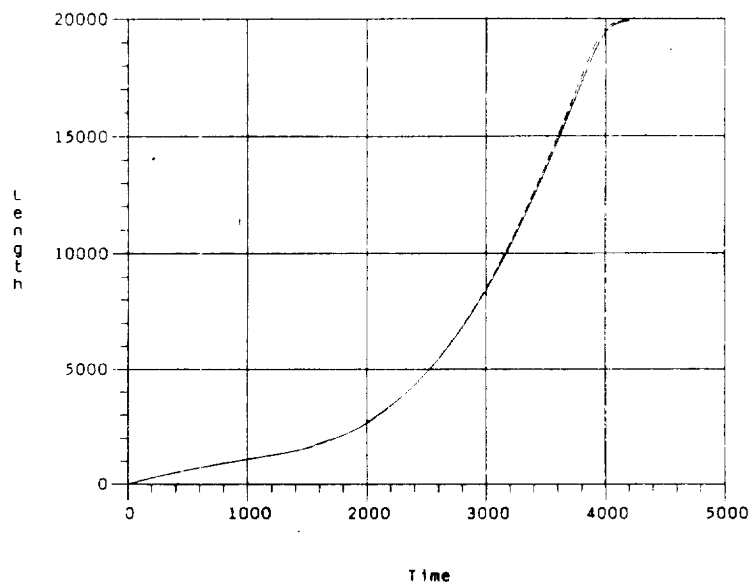


Fig. 7. Payload position and velocity rms errors using tuned traditional Kalman algorithm.



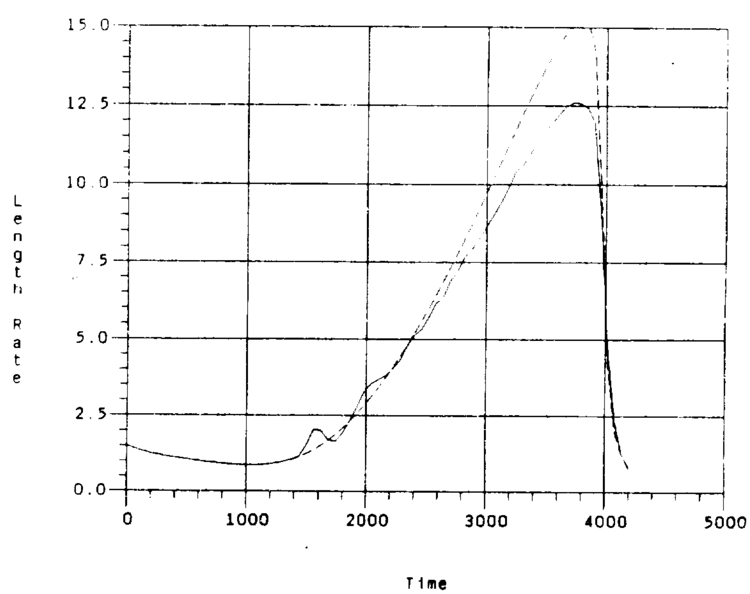


Fig. 8. Length and length rate using U-D factorization with two-beads and estimation of acceleration biases.

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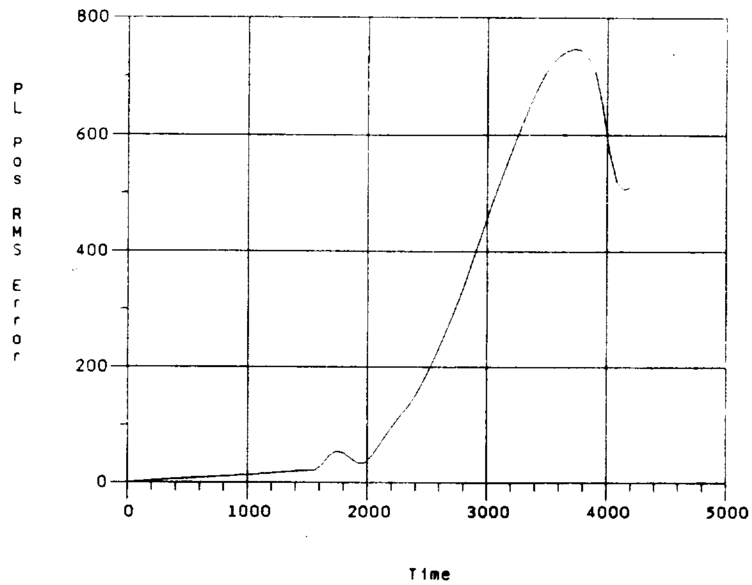

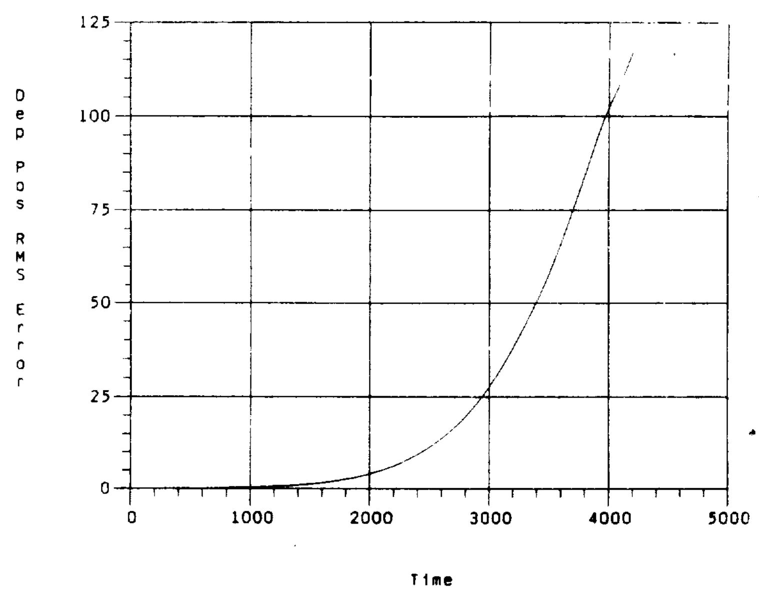

Fig. 9. Payload and deployer position rms errors using U-D factorization with two-beads and estimation of acceleration biases.

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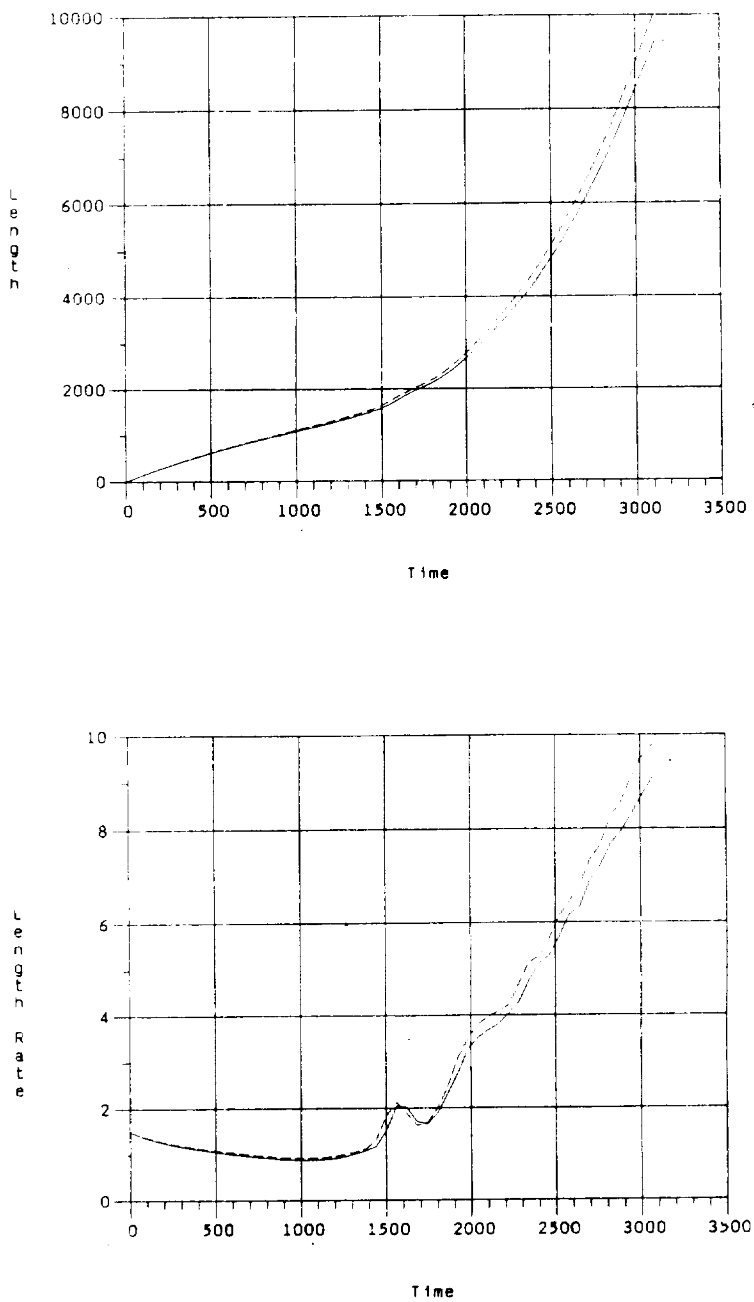

Fig. 10. Length and length rate using U-D factorization with "add-a-bead": up to twenty-one beads.

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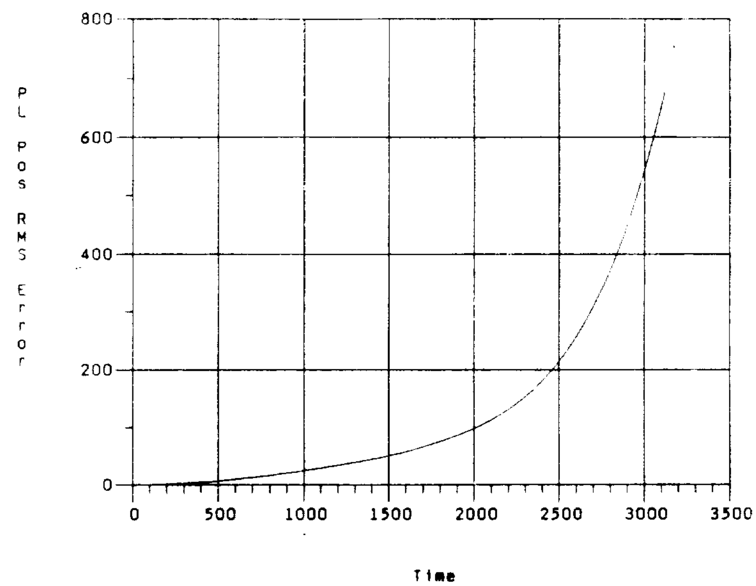

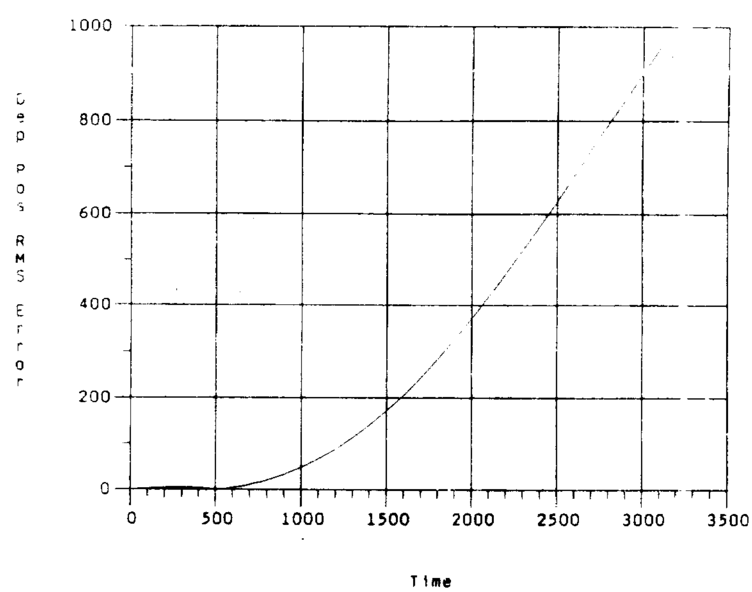

Fig. 11. Payload and deployer positon rms errors using U-D factorization with "add-a-bead": up to twenty-one beads.

CONCLUSIONS AND RECOMMENDATIONS

The untuned traditional Kalman estimator using two-beads produced state estimates that diverged from the "true" state values during deployment. By monitoring the state covariance matrix, we found that the estimated variances became unacceptably small, so that measurement information was being ignored by the algorithm, and the estimates from the state equations were favored. Tuning this algorithm by adjustment of the process noise levels or by reinitialization of the covariance matrix during deployment reduced this divergence, but these techniques may be unreliable for flight-data use. The estimator divergence may come from a variety of sources: numerical instability in the traditional algorithm, errors from ignoring acceleration biases, modelling errors from using a two-bead model, linearization errors, measurement and process noise models that are not accurate, or incorrect apriori covariance estimates.

To determine if numerical instability was a primary cause of filter divergence, the U-D factorization algorithm was programmed. Similar estimates were obtained from both methods using the two-bead model, indicating that errors in the traditional algorithm are not due to numerical instability. Estimation of acceleration biases, which occur primarily due to aerodynamic drag, reduced the estimator errors. The "add-a-bead" estimator that reproduces the BEADSIM state model produced estimates that were very similar to those of the two-bead model, indicating that filter divergence is not due to a reduced-order model.

Linearization errors in the A and H matrices of equations (7) and (8) have been monitored by comparison with the actual states and their derivatives, and calculations have agreed to four decimal digits. This linearization directly impacts the gain calculation and the propagation of the state covariance matrix, and could be the source of filter divergence. The measurement and process noise models used in these calculations have been white, Gaussian, with zero mean; the linearization errors are by definition colored, since they are the product of a dynamic process. A correlation calculation of both the measurements and states produced by the estimator indicates that they are both correlated, and yet these algorithms have not used off-diagonal elements in the covariance matrices. Both estimators are sensitive to changes in the apriori covariance estimates; in some cases, the U-D factorization algorithm overflows early in deployment when large initial covariance estimates are used.

Recommendations include the use of cross-correlation terms in the covariance matrices, which is easily accomodated by the traditional Kalman algorithm. Since the linearization of the state and measurement equations most likely pollute the white noise models for the process and measurement noise, a Gauss-Markov model should be developed for these processes. The time-sequence of linearization errors for both the measurements and state derivatives could be saved, and the correlation matrices calculated for both. The time-varying correlation matrix for the process noise would provide the transition matrix A_{k-1} for a shaping filter of the form

$$\rho_k = A_{k-1}\rho_{k-1} + \xi_{k-1} \tag{11}$$

where ρ_k is the colored process noise sequence and ξ_k is white-noise. The same calculations could be done to provide a colored noise model for the measurement linearization errors. (see Stengel).

Noise-adaptive filtering techniques may be necessary when apriori covariance estimates are not accurate. These techniques sample the measurement residuals and calculate measurement biases and covariances; similar techniques can be used to give better estimates of an initial state covariance matrix. (see Myers and Tapley).

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