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MULTIBODY MODELING AND VERIFICATION

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MULTIBODY MODELING AND VERIFICATION

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ABSTRACT

A summary of a ten week project on flexible multibody modeling, verification and control is presented. Emphasis was on the need for experimental verification. A literature survey was conducted for gathering information on the existence of experimental work related to flexible multibody systems. The first portion of the assigned task encompassed the modeling aspects of flexible multibodies that can undergo large angular displacements. Research in the area of modeling aspects were also surveyed, with special attention given to the component mode approach. Resulting from this is a research plan on various modeling aspects to be investigated over the next year. The relationship between the large angular displacements, boundary conditions, mode selection, and system modes is of particular interest.

The other portion of the assigned task was the generation of a test plan for experimental verification of analytical and/or computer analysis techniques used for flexible multibody systems. Based on current and expected frequency ranges of flexible multibody systems to be used in space applications, an initial test article was selected and designed. A preliminary TREETOPS computer analysis was run to ensure frequency content in the low frequency range, 0.1 to 50 Hz. The initial specifications of experimental measurement and instrumentation components were also generated. Resulting from this effort is the initial multi-phase plan for a Ground Test Facility of Flexible Multibody Systems for Modeling Verification and Control. The plan focusses on the Multibody Modeling and Verification (MMV) Laboratory. General requirements of the Unobtrusive Sensor and Effector (USE) and the Robot Enhancement (RE) laboratories were considered during the laboratory development.

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TABLE OF CONTENTS

List of Figures

List of Tables

Nomenclature

ACES	Active Control Evaluation for Spacecraft
AGS	Augmented Advanced Gimbal System
BET	Base Excitation Table
CASES	Control, Astrophysics and Structures Experiment for Space
FEM	Finite-Element Modeling
GTF	Ground Test Facility
JPL	Jet Propulsion Laboratory
LADD	Lens Antenna Deployment Demonstration
LSS	Large Space Structure
MACE	Middeck Active Control Experiment
MMV	Multibody Modeling Verification Laboratory
RE	Robot Enhancement Laboratory
RMS	Remote Manipulator System
TREETOPS	Control System Simulation for Structures with a Tree Topology Computer Software Package
USE	Unobtrusive Sensor and Effector Laboratory

INTRODUCTION

The NASA's LSS GTF (Large Space Structure Ground Test Facility) at MSFC (Marshall Space Flight Center) was developed for meeting the desired objectives of complex space projects and to become a national test bed for investigations in dynamics and controls [1]. The topics of this facility can be grouped into control development and synthesis, dynamics verification, dynamic modeling, and hardware flight systems for space structures. Due to the increase in complexity and more stringent requirements on spacecraft structures, investigations of multibody dynamics modeling and control have become essential. Many of the future space missions (such as extremely accurate pointing and tracking systems and the attainment of vibration-free observation image planes) require high performance from the LSS. The required state-of-the-art systems to be or currently under development consist of complex arrangements of interconnected rigid and flexible bodies. Presently, the LSS GTF provides ground test capabilities for many experiments involving large structures with flexible components. Therefore, a natural extension of the laboratory's activities would be to investigate the dynamics and controls of flexible multibody systems. Hence, presented in this report is a plan for addressing these needs and bringing into realization the Multibody Modeling and Verification (MMV) Program at the MSFC/LSS GTF.

Project Overview and Objectives

Since the 1960's, a significant amount of theoretical work has been undertaken in the area of modeling and simulation of multibody systems. However, for systems having flexible components, there still seems to be no well defined method for selecting component modes for systems, in which due to large displacements, the boundary conditions of the original assumed modes varies. Furthermore, there has been very limited experimental verification of the existing modeling and simulation techniques. In view of the last two statements, the summer task definition encompassed the following. First: The modeling aspects of flexible multibodies that can undergo large angular displacements are to be studied. The thrust of the study is for determining the sufficiency of component mode synthesis based on individual flexible component data for ascertaining the system modes. The systems subject of this study are those that exhibit configurations other than the initial one used for determining the component data. Second: A test plan is to be generated so that analytical and/or computer analysis can be verified experimentally. Third: If time permits, control methods for multibody systems are to be surveyed and an experimental test plan generated for multibody control verification. The project's ten week procedure set forth was to achieve as much of the first two tasks' objectives as stated above. The third task was set aside and only taken into account throughout the project execution when it was appropriate.

Research by Others

In an attempt to complete a good portion of the tasks set forth, much time was spent in collecting background information and technical articles on flexible multibody systems. The literature search indicated a recently strong and growing interest emerging for developing experimental verification facilities [1, 2, 3, 4 and 5]. Much of the work in the field of multibody systems was largely motivated by the spacecraft problem [2]. This problem required analysis of systems experiencing large rotations while components (such as antennas and solar panels) were undergoing large relative motions. Another area in which independent developments of the same problems were simultaneously being addressed was in kinematics and machine design [6]. With today's technical advances and demands, researchers in both areas have been brought together by their common interests.

Due to the combination of both rigid body large displacements and small elastic deformations occurring in flexible multibody systems, the dynamic models have complex nonlinearity and model reduction problems. The conventional model reduction (modal coordinate truncation) methods still have not been securely established [7]. Other questions also arise in the representation of energy dissipation characteristics, selection of modes and boundary conditions, just to name a few. Hence, with varying degrees of generality and model complexity, a variety of ways have been developed for deriving the equations of motion for multibody systems [8-39]. These range from employing the Newton-Euler formulations, Hamilton's equations, Kane's method, Lagrange's form of D'Alembert's Principle approach to Component Mode Synthesis techniques directly or with special model variations. The methods and their variations can also be grouped as either an assumed mode approach or a finite-element approach. The modeling variations have been in terms of generalized coordinate selection, joint interfacing and flexibility modeling, representation of model uncertainties, computational bottleneck reduction, control system development, etc. To some degree the basic modeling choice is a matter of preference since the different strategies often produce the same results. Using ones preferred modeling scheme and computational methods, various computer simulation codes have been developed and made available to the research and commercial community. The list of computer codes is endless; MBODY, MFLEXBODY, DISCOS, AFBDAP,

TREETOPS, ADAMS, SADACS, CONTOPS, GRASP, MIDAS . . . . . New versions are continuously being generated as advances in research require updates and corrections to the codes when various problems are encountered. As of yet, no systematic comparison of these codes for accuracy or efficiency has been generated [2, 40]. Presently, JPL (Jet Propulsion Laboratory) is conducting a simulation technical verification survey with plans to develop an experimental verification facility [3].

The experimental research on flexible multibody systems can be found arising out of basically two areas: flexible structures with multiple components [19, 41, 42] and the robotics area. In the robotics area, there is numerous amounts of experimental work focussing on the rigid system. Only in recent years have researchers begun pursuing the issues of flexibility in the arms and joints [4, 20, 43-48]. This has been due to the increase in importance of high-speed operation, high accuracy requirements and lightweight designs for manufacturing and new space missions. Furthermore, with the advent of high speed computers, simulation analyses have become feasible. Also, in the field of flexible multibody systems are problems such as those studying the vibrations effects in high-speed machines and mechanisms which have had some experimental verification [49].

Need for Multibody Modeling, Verification and Control GTF

Although numerous theoretical and numerical research has been undertaken, very little experimental work has been carried out in the multibody dynamics and controls field. With the advantages of low power consumption, high load to weight ratios, large workspaces, and potential for high speed operation because of lower inertia, the currently proposed designs for lightweight high-performance multibody and/or robotic systems for space applications make it essential to analyze the fundamental modeling issues in greater detail. To enhance the understanding of multibody dynamic modeling and control, experimental verification is a key element. Issues which need to be addressed are the dynamic effects such as the interactions between the rigid and flexible dynamics, the sensor and actuator dynamics, and the model and controller dynamics. All these need to be analyzed and correlated with reference simulation models; hence, experimental verification of existing modeling and simulation methods. For future space missions which involve many multibody applications, ground testing is necessary to ensure their in-flight success and the safety of the crew. Furthermore, it is far less expensive to do the major research, analysis, and development of flight experiments in ground tests, readying them and the crew for the mission. Ground testing prior to flight has been the universally insisted upon approach for most aerospace structural systems

[50]. Current and proposed experimental research is now summarized.

The experimental approaches taken by researchers have been to work either in the horizontal plane, in an attempt to avoid gravitational effects in modeling [4, 48]; or the vertical plane, in which gravity must be included in the model and compensated for numerically, or offloading techniques must be used such as bungy suspension cables [5, 20, 44, 45]. A third approach is to perform the flexible multibody experiments in space. Presently, space missions are under development or proposed which involve experiments using the RMS arm on the orbiter [46, 47] and scaled multibody experiments to be conducted in the shuttle's middeck area (MACE), [5]. The orbiter-based experiments have their merits and will eventually need to be executed as future space missions warrant them. However, these in-flight experiments also have associated supporting ground testing laboratory development [5]. Even after orbiter-based testing techniques are fully devised and implemented, ground testing will be required and will usually constitute the highest loading environment [50]. Hence, the need for GTFs is considered essential for the successful execution of the expensive in-flight multibody experiments and future space missions.

Currently, the actual experimental research performed thus far has been limited to single link flexible arms and two links with only one being flexible [4, 20, 44, 48]. An exception to this, Book et al [45] has done extensive investigations using a planar arm with two flexible links. Cannon et al is currently extending his work to include two flexible links.

Inspite of these efforts, there still needs to be more experimental research. Book et al [45] has shown by experimental verification that using strictly simulation methods can result in one missing some of the system modes in their analysis. He has also shown that experimental results assist in the determination of proper boundary conditions for analytical modeling. That is, boundary condition selection affects the accuracy of the analysis significantly; further, substantiating the need for experimental verification of existing modeling and simulation techniques. The maximum reliability and accuracy achieved by the correlation and modeling of dynamic parameters based on experimental and analytical results are also considered important aspects in aerospace engineering [51]. What follows is the 1989 Summer Faculty project's results in modeling aspects of flexible multibody systems and the initial plan for addressing the above foreseen needs of future space missions involving multibody systems (e.g. assembly of the space station, etc.)

MODELING

After surveying the various techniques and focussing on the question at hand (component mode selection versus system modes and large angular displacements), the Lagrangian formulation of the equations of motion using assumed modes [8] was selected. This to a large degree was a matter of personal preference and familiarity with the formulation technique. In addition, this approach for deriving the dynamic equations was selected because the resulting analytical form facilitates the exploration of the coupling relationship between the flexible and rigid body motions of the individual links and that of the total system. Also, this approach is less computationally intensive compared to the FEM approaches. The coupling appears in the off-diagonal matrix terms in the following compact symbolic representation of the system's equations of motion.

$$\begin{bmatrix} \mathbf{m}^i_{RR} & \mathbf{m}^i_{R\theta} & \mathbf{m}^i_{Rf} \\ & \mathbf{m}^i_{\theta\theta} & \mathbf{m}^i_{\theta f} \\ \text{symmetric} & & \mathbf{m}^i_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{R}}^i \\ \ddot{\theta}^i \\ \ddot{\mathbf{q}}^i_f \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathbf{K}^i_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{R}^i \\ \theta^i \\ \mathbf{q}^i_f \end{bmatrix} + \begin{bmatrix} \mathbf{C}^T_{\mathbf{R}^i} \\ \mathbf{C}^T_{\theta^i} \\ \mathbf{C}^T_{\mathbf{q}^i_f} \end{bmatrix} \lambda \tag{1}$$

$$= \begin{bmatrix} (\mathbf{Q}^i_e)_R \\ (\mathbf{Q}^i_e)_\theta \\ (\mathbf{Q}^i_e)_f \end{bmatrix} + \begin{bmatrix} (\mathbf{Q}^i_v)_R \\ (\mathbf{Q}^i_v)_\theta \\ (\mathbf{Q}^i_v)_f \end{bmatrix}, \qquad i = 1, 2, \ldots, n_b$$

Using the component mode synthesis, the component equation of motion for each body in the system is as follows.

$$\begin{bmatrix} \mathbf{m}^i_{rr} & \mathbf{m}^i_{rf} \\ \mathbf{m}^i_{fr} & \mathbf{m}^i_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}^i_r \\ \ddot{\mathbf{q}}^i_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \mathbf{K}^i_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{q}^i_r \\ \mathbf{q}^i_f \end{bmatrix} = \begin{bmatrix} (\mathbf{Q}^i_e)_r \\ (\mathbf{Q}^i_e)_f \end{bmatrix} + \begin{bmatrix} (\mathbf{Q}^i_v)_r \\ (\mathbf{Q}^i_v)_f \end{bmatrix} - \begin{bmatrix} \mathbf{C}^T_{\mathbf{q}^i_r} \\ \mathbf{C}^T_{\mathbf{q}^i_f} \end{bmatrix} \lambda \tag{2}$$

Both forms have their corresponding constraints represented by the C matrices. Analyzing and understanding equations (1) and (2) are important for achieving the main objective of the MMV laboratory; that is, <u>model verification</u>. Being able to relate quantities in the equations of motion in terms of component mode synthesis and system modes with experimental results is critical. One should recall that model verification is a process of experimentally verifying an analytical (or numerical) model to gain confidence in its use for predicting system behavior [52]. If there exists any system misrepresentation, the model must be revised based on the new physical evidence. Note, caution must always be taken to avoid the risk of changing the model to match data which are in error.

Applying the above symbolic formulations, the equations of motion of a planar multibody with three flexible links are derived. The links are assumed to be Euler-Bernoulli beams, in bending only, and attached by pin (revolute) joints. Gravity is included in the modeling since the MMV laboratory test articles are to be tested in the vertical plane. See figure 1. The next steps involve detailed analysis of component mode and boundary condition selection as a function of rigid-body motions. These steps have been left for execution during a proposed continuation project over the next twelve months. The project will be a study on the highly nonlinear functional relationship between rigid body and flexible body motions. The analysis will also include numerical techniques (TREETOPS and NASTRAN) in searching for an explicit and useful form of this relationship.

Next, an initial description of a MMV test article and its physical properties is given. The article is defined to exhibit low frequency content (0.1 Hz to 50 Hz), coupling between rigid and flexible body motions, and multiple configurations (open and closed tree topologies) due to large angular displacements, and to be of a large size to mimic those to be used in space applications. The three link flexible multibody of figure 1 possesses these characteristics. However, in selecting the physical dimensions, sensors, and actuators (torquers), the maximum torque versus weight characteristics of torquer motors significantly limited the feasibility of this multibody in open tree topology. To overcome the limitations, there are two options to choose: gravitational offloading via bungy suspensions or counter-balancing, or to redesign the test article. Since the dynamic interactions of bungy suspensions with test articles is not clearly defined and the use of counter-balancing results in greater system masses, the second approach is selected while keeping the other options still open.

The new test article currently under investigation is one in which the third link is made of S-Glass, an extremely lightweight and highly flexible material. It consists of a double branch formation to enhance the modal density, see figure 2. The other two links are made of a carbon/carbon composite material (lightweight and high strength). To meet the flexibility requirements, link one is selected to have a rectangular tubular cross-section with a wall thickness of t=0.34cm. Link two has a solid rectangular cross-section. The cross-sections are selected to exhibit a large degree of flexibility in only one plane. The objective is to restrict the first experimental testing to a planar flexible multibody system. However, out-of-the-plane motions will be measured during the experimental testing to ensure planar motion is maintained. Future experiments will be conducted for spatial motion systems. The current multibody material and physical properties are listed in Table 1, [53].

To ensure desired frequency content before fabrication, a TREETOPS analysis was performed for the new test article. The component modes selected were:

<u>Link One</u> Clamped-free with a concentrated mass at the free end equal to the mass of motors two and three, and links two and three. See figure 3.

<u>Link Two</u> Clamped-free with a concentrated mass at the free end equal to the mass of motor three and link three. See figure 4.

<u>Link Three</u> Clamped-free. See figure 5.

The resulting component mode shapes and frequencies are given in Table 2. Table 3 shows the corresponding system modal frequencies and definitions of the terms used in Table 2. The lowest system mode was found to be 1.26 Hz. Comparing the system modes and the component modes shown in the tables, there appears to be very little modal coupling between the links. This is indicated by very little changes in the values between the component mode and system mode frequencies. Presently, there is still some uncertainty in whether the actual frequency content found from the TREETOPS analysis contains enough modal density in the low frequency range and if the values are correct. Since only one multibody static configuration (see figure 6) and set of boundary conditions has been analyzed, and the effects of gravity and sensor/actuator weights were not included, further TREETOPS and NASTRAN analyses are planned before finalizing the test article description for fabrication. This will be carried out in the proposed continuation project.

EXPERIMENTAL VERIFICATION (Ground Test Facility)

Moving into the next summer task, the following is a general plan for the Multibody Modeling Verification (MMV), Unobstrusive Sensors and Effectors (USE), and Robot Enhancement (RE) laboratories. The main focus is on the MMV laboratory with USE and RE space and general objectives specified. In defining the specifications for the data acquisition and analysis system, the test article described in figure 2 and Table 1 (or one similar) is assumed. Hence, measurement of the frequency range, 0.1 Hz to 50 Hz, is a specified requirement of the facility. The vertical plane (1-g) testing environment selection is heavily driven by the available space. However, this choice is considered viable based on past success by NASA/MSFC LSS GTF and Book et al [45] in conducting flexible body experiments in the same environment, and that many other experts in the flexible multibody for space applications field have vertical plane testing built into their overall projected laboratory plans [5]. In addition, designing the facility for vertical testing allows easy extension of the experiments to include large spatial motions. The general overall outline of the new multi-laboratory GTF is now given.

Phase I. (MMV) Main objectives are to improve multibody modeling and simulation and to experimentally verify component mode synthesis methods [1]. The test articles are to be planar and spatial in their motion characteristics.

A. Static

The first experiments to be conducted in MMV are to be of static multibody configurations. The results are to be used for static verification of the component mode selection process and to determine if a purely kinematic relationship exists between mode selection and large displacements. Open and closed tree topologies are to be tested, see figure 7.

B. Dynamic

Following static tests, the test articles are to be slewed through large angular displacements. The objective is to verify the dynamic relationship between component mode selection and large angular motions and rates. Plus, the laboratory is to be used for verifying existing dynamic modeling and simulation techniques.

C. Control

After exhaustive testing and analysis of parts A and B, the laboratory activities will involve the development, implementation and experimental verification of control techniques for flexible multibody systems.

Phase II. (USE) Laboratory is to investigate the use of sensors and actuators which are lightweight and have unobtrusive geometries. (The piezoelectric materials currently being tested will be used in the MMV laboratory) [1]. NASA/MSFC LSS Laboratory currently has a bid out for the acquisition of the LADD hardware to be used as a test article, see figure 8 [54].

Phase III. (RE) Laboratory is to involve a combination of (MMV) and (USE) results and to investigate the concept of a robot arm manipulating its highly flexible payload with assistance of the payload's own actuators and sensors by controlling them through electrical contacts in the endeffector. Hence, to prevent the TAIL Wagging the DOG phenomenon. An experiment involving the LADD suspended from a flexible boom is also being considered.

Experimental Procedure

In this project, only the experimental procedures, instrumentation and hardware for the MMV laboratory are specified. The following experimental procedures will describe only the initial planar flexible multibody experiments. The results of these experiments will provide vital information for the future spatial motion tests definitions.

Component Modal Experiment. This first experiment is to determine and verify the component modal selections for each link of the test article, individually. Each link will be suspended from the test stand in its desired orientation with the same boundary conditions as assumed for the analytical model (e.g. clamped-free at a 30 degree angle from the vertical and with a lumped mass at the free end). Using standard experimental modal analysis techniques, the component modal properties will be determined under various excitations. The sensors will be accelerometers, strain-gauges, and/or piezoelectric films.

Assembled Multibody System (Static). This experiment is to determine the system modal properties of the multibody in various static configurations. These will be used to verify the analytical model. If discrepancies are found between the analytical (and/or numerical) and the experimental results, steps will be taken to eliminate them or to formulate an explanation for delineating the nonconvergence. The assembled test article will be suspended from the test stand in various selected static configurations (open to closed tree topology when possible). In each configuration, standard modal testing techniques will again be used to determine the system modal properties. The same sensors as before will be used. The results will be compared with those obtained theoretically. Model and/or experimental adjustments and retesting will be done accordingly.

Assembled Multibody System (Dynamic). Following the above experiments, dynamic tests will be performed to determine the existence of a dynamic relationship between component modal selections and large angular motions and rates. Dynamic modeling and simulation techniques will also be verified. This experiment will involve equipping the static experiment's flexible multibody test article with any additional necessary joint actuators and sensors. The test article will then be commanded (open-loop control) to move through prescribed large angular motions. Simultaneously, sensor readings will be taken to determine the system's total response. Again, a theoretical and experimental correlation will be made followed with any model or experimental adjustments and generated explanations of nonconvergences.

Assembled Multibody System (Control). Utilizing the knowledge gained from the previous experiments, control techniques will be derived and implemented experimentally. Again, the sensors and instrumentation will essentially be the same with only special control features added.

The above experiments will be extended to include other test articles, both planar and spatial motion types.

Laboratory Facility and Layout

Figure 9 illustrates the general facility layout of the three laboratories, MMV, USE and RE. The physical space, already allocated for these laboratories, is located at NASA/MSFC, Huntsville, Alabama in the west high bay area of Building 4619. It consists of a 53.5'x29.0' floor space with approximately a 90' ceiling. The location is just east of the Flight Robotics Laboratory (EB-24) and west of the ACES and CASES control room. There is an existing platform at 42.5' with this control room as its only access. The present major structural layout of this space is shown in figure 10 (showing platform only) and 11 (top view after the removal of unused air-handling units along east wall and below the platform, verbal approval has already been given).

After surveying the facility, it was decided that using the existing platform with its access from the ACES and CASES control room would provide the most expeditious and cost effective approach for implementing the MMV and USE laboratories. The platform allows enough vertical height for suspending both the LADD structure and MMV test article, see figure 9. In addition, the control room has room for setting up the data acquisition and control equipment and is conveniently located. The only initial requirements will be the removal of the unused air-handling units and a few structural extrusions, and cutting a 5'x5' hole in the platform for suspending the MMV test article. And last but not least, the leaks in the roof will need to be fixed in order to maintain the quality of the experimentation. The structural rigidity of the platform appears to be adequate for the initial implementation before final renovations (no tests were done to verify this). If these facility changes can be accomplished within the year, necessary instrumentation purchased, test article fabricated, and installation completed, the MMV and USE laboratories could start testing as early as Summer 1990.

The next stage of renovations will be to extend the platform as indicated in figure 12 and 13. This will provide ample volume for planned spatial flexible multibody experiments. Structural beams and bracing will need to be added or removed to ensure that the experiments' supporting structure's frequency content will not interfere with the experimental testing. These details will be determined in collaboration with the Facilities group. In order to provide alternate access to the platform, stairs are to be located on the north side of the platform. These stairs will also continue up another 40' to a second platform for the RE laboratory (not shown in the figures). The estimated cost for the stairs and MMV platform extension is $150,000 (1989 dollars) and another $250,000 for the second platform. The projected completion of renovation for this stage is sometime during 1990 to 1992, depending if it can be scheduled with the Facilities group. It is anticipated that the platform extension could be completed in 1991. The second platform probably would not be finished before 1992.

In the renovation plans, there are certain restrictions placed on the use of this space. One, the bay area doors must remain accessible and fully operational. Second, general passage for NASA employees and guests to and from the Flight Robotics Laboratory and Vibration Testing Facility must be provided (indicated in figure 13).

Measurement and Instrumentation Specifications

The following is a list of the major components of the measurement and instrumentation equipment and their specifications needed for the MMV laboratory. This covers the static, dynamic and most of the control experiments requirements. It should be noted that this is only an initial list and is subject to change as required. The specific selections of the following items is based on the desired characteristics of low noise-to-signal ratio, high resolution, small physical weight additions to test article, and capability with existing ACES and CASES equipment.

Frequency range of test specimen: 0.1 Hz to 50 Hz

The measurement equipment must be able to detect position, velocity and accelerations within this frequency range, in addition to large displacements.

Base Excitation Table (BET): The BET is to produce excitations via disturbance inputs to the multibody system for determining its dynamic characteristics and the effectiveness of control algorithms. Disturbance types to be included are programmable deterministic, random, sine dwell and sine sweep motions. It must be able to excite frequencies within the 0.1 Hz to 50 Hz range. The directions of excitation are to be along the horizontal x,y axes. The load carrying capacity required is 2.3 kN. (Should be able to support test article, sensors, actuators, and gimbal system.) It is anticipated that the BET will be similar to the one currently used for ACES which has a bandwidth of 10 Hz and a dynamic range of ±15 cm. It is driven by a hydraulic servo-loop position controller.

Augmented Advanced Gimbal System (AGS): Should provide articulation and control about three rotational axes. Bandwidths should be in excess of 50 Hz. The dynamic range in the pitch/yaw axes should be 200 N-m and ±45 degrees. For the roll axes, the dynamic range should be 50 N-m and ±90 degrees. These requirements will allow large angular motions in three dimensions.

Joints and Actuators: For the static testing, frictionless joints which give no relative motion are required. For the dynamic and control testing, torquer motors of various sizes, depending on the outer links' weights, are to be selected to provide large angular displacements (±45 degrees). They should have minimum cogging and friction characteristics (e.g. direct-drive brushless torquers).

Joint Sensors: The joint sensors are to measure positions and rates. They should have a resolution down into the arcminute and arcsecond ranges. Their dynamic range should be ±45 degrees and 70 degrees per second. They should have minimum friction. Plus, the sensors must be lightweight since they are part of the test article. At this time, it appears that the incremental optical encoders may be able to meet these requirements.

Rate Gyroscopes: The gyroscopes are to measure x,y,z rates and positions of the ends of the links. They are to provide information for calculating the absolute angular motions of the following attached link due to rigid body motion and flexible bending of the previous link. In addition, three will be mounted to the underside of the AGS payload mounting plate to measure its input motion. The gyros are to be analog, capable of measuring 25e-3 degrees per second, have a dynamic range of 70 degrees per second, and bandwidth above 50 Hz.

Accelerometers: These sensors are to measure the multibody's response due to flexible modal content. They will be located in triax formations at discrete locations along each link for measuring traverse deflections in-the-plane and out-of-the-plane. Measurement capabilities should be within the 0.1 Hz to 50 Hz frequency range. Resolution should be at least 0.001g with a dynamic range of 5 to 10g. Again, they must be lightweight so as to not alter the system characteristics to a great degree. Two accelerometers will be used to measure x,y acceleration of the BET.

Unobtrusive Sensor: Since weight is a major factor in designing a flexible multibody used in a gravitational field, link three has been selected to be made of a very lightweight material. This requires use of unobtrusive sensors. It is intended to implement a piezoelectric type sensor developed in the USE program. It is capable of measuring traverse deflections in- and out-of-the-plane directions by reading voltage levels which are a function of the piezoelectric film deformations. The USE program is also investigating its use as a sensor/actuator pair.

Data Acquisition and Analysis System: This system has the following preliminary component selections which provide the sampling rates, data storage and analysis capabilities, signal conditioning and compatibility with existing ACES system. The recommended system is a HP9000 Series 300 workstation (32 bit); LMS (Fourier Monitor) data acquisition, University of Cincinnati modal analysis package or Test Data Analysis Software (TDAS) by Structural Dynamics Research Corporation (SDRC); a DIFA Measuring System front end signal conditioning (65 channels); and STRUCTCEL PCB 330A accelerometers. In addition to the BET, it is recommended to purchase a 30 lb, long stroke shaker excitation system to allow excitations at locations other than the base.

Other Data Storage Devices: These include analog strip chart recorders, analog magnetic tape recorders, HP-5423 and GenRad 2515 dynamic analyzers which are available for use in the LSS laboratory.

Vision Systems were considered. But do to the large test article(s) undergoing large displacements, they are not recommended at this time.

Partial Estimated Budget

HP9000, Series 300 workstation (32 bit)	$ 60,000
Software - LMS (Fourier Monitor) data acquisition,	$ 12,000
- University of Cincinnati modal analysis package	$ 0
or	
- Test Data Analysis Software (TDAS) by Structural Dynamics Research Corporation (SDRC)	$ 20,000
DIFA Measuring System, Front end signal conditioning (~ 65 channels), A/D, D/A; amplifiers; filters	$100,000
STRUCTCEL accelerometers & instrumentation 60 PCB Structcel; 20 triax mounting blocks; 20 triax cables; 4 patch panels; 4 extension cables Each additional 20 triax ~ $5,000	$ 11,000 (initial)
Excitation System 30 lb, long stroke shaker; amplifier; filter; load cell; conditioning	$ 14,900
Piezoelectric film, shielding and instrumentation for link three's sensor ($1,150) and actuator (additional $2,650)	$ 3,800
Precision Products Group, FG 313 series gyroscopes 6 gyroscopes and instrumentation (Note, weight may be too large.)	$ 60,000
Base excitation table system	$ 50,000
Augmented advanced gimbal system	$100,000
Stairs and MMV platform extension	$150,000
Second platform	$250,000
Partial List Total	$831,700

CONCLUSIONS

An attempt was made to complete a good portion of the tasks set forth. Much time was spent in collecting background information and technical articles on flexible multibody systems. It was found from the literature search a recently strong and growing interest has emerged for developing experimental verification facilities. In parallel with gathering background material, a general dynamic model analytical formulation of a multibody system, comprising of three flexible bodies connected with revolute joints, was derived symbolically. This was followed up with selecting physical properties and component modes of a three body system for defining a possible test article for the experimental verification plan. An initial TREETOPS analysis was performed to estimate the frequency content of the system. This modeling and analysis completed during the summer project has laid the foundation for a continuation project to be executed during the coming year. The project will involve further analysis and design of the test article via TREETOPS and NASTRAN and analytical techniques. Following the modeling and TREETOPS analysis, an initial laboratory plan for experimental model verification was generated. The first Phase's testing is expected to begin as early as summer 1990. The controls verification was taken into consideration during the plan development of the model verification laboratory. However, time did not permit a thorough search of existing control methods and test plan generation.

RECOMMENDATIONS

Based on the growing interest and significant need of understanding flexible multibody systems, it is recommended that the development of the MMV, USE, and RE laboratories and their associated research be continued. Due to the complexities and the dependence on existing numerical code, advances in the area of flexible multibody systems modeling and experimental verification have become essential for the success of future space missions. Hence, with the currently available space, the feasibility is there for beginning work on the MMV and USE laboratories as soon as possible. In addition, there still needs to be more work performed on modeling, analyzing and defining the fabrication specifications for the MMV test article(s).

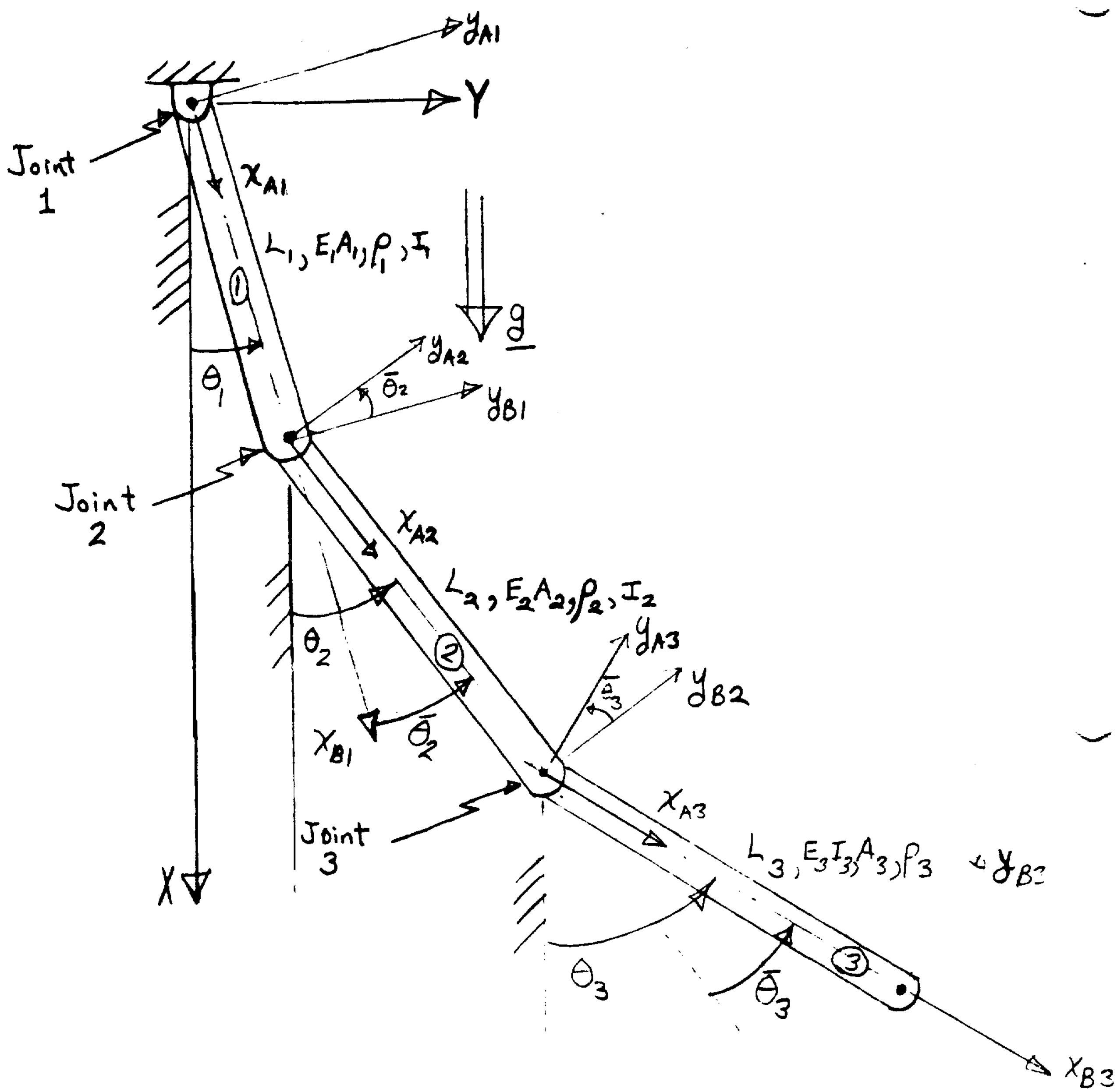

Figure 1. Planar Multibody with Three Flexible (Euler-Bernoulli) Links and Three Pinned Joints

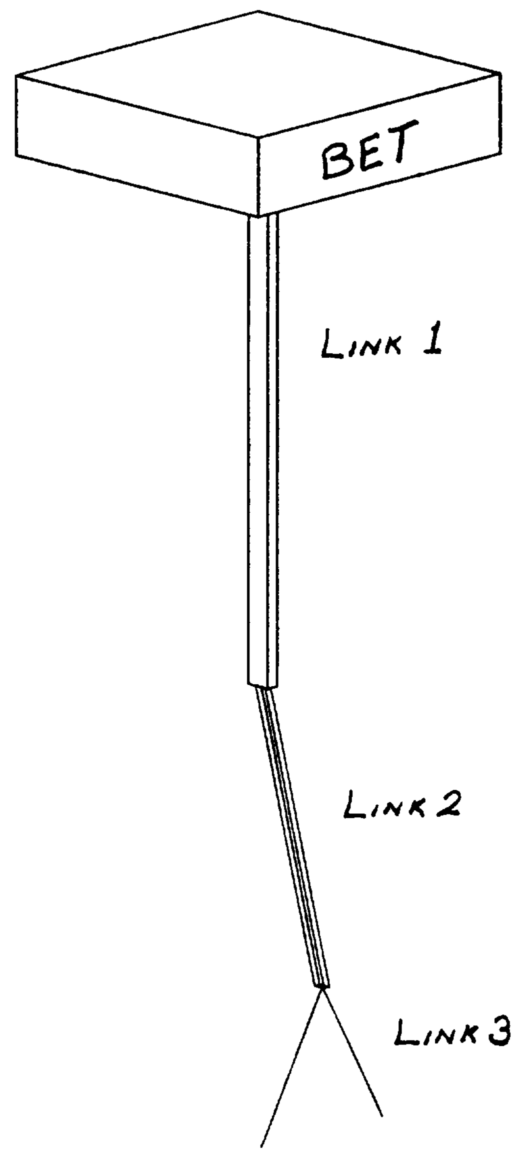

Figure 2. Proposed Flexible Multibody Test Article

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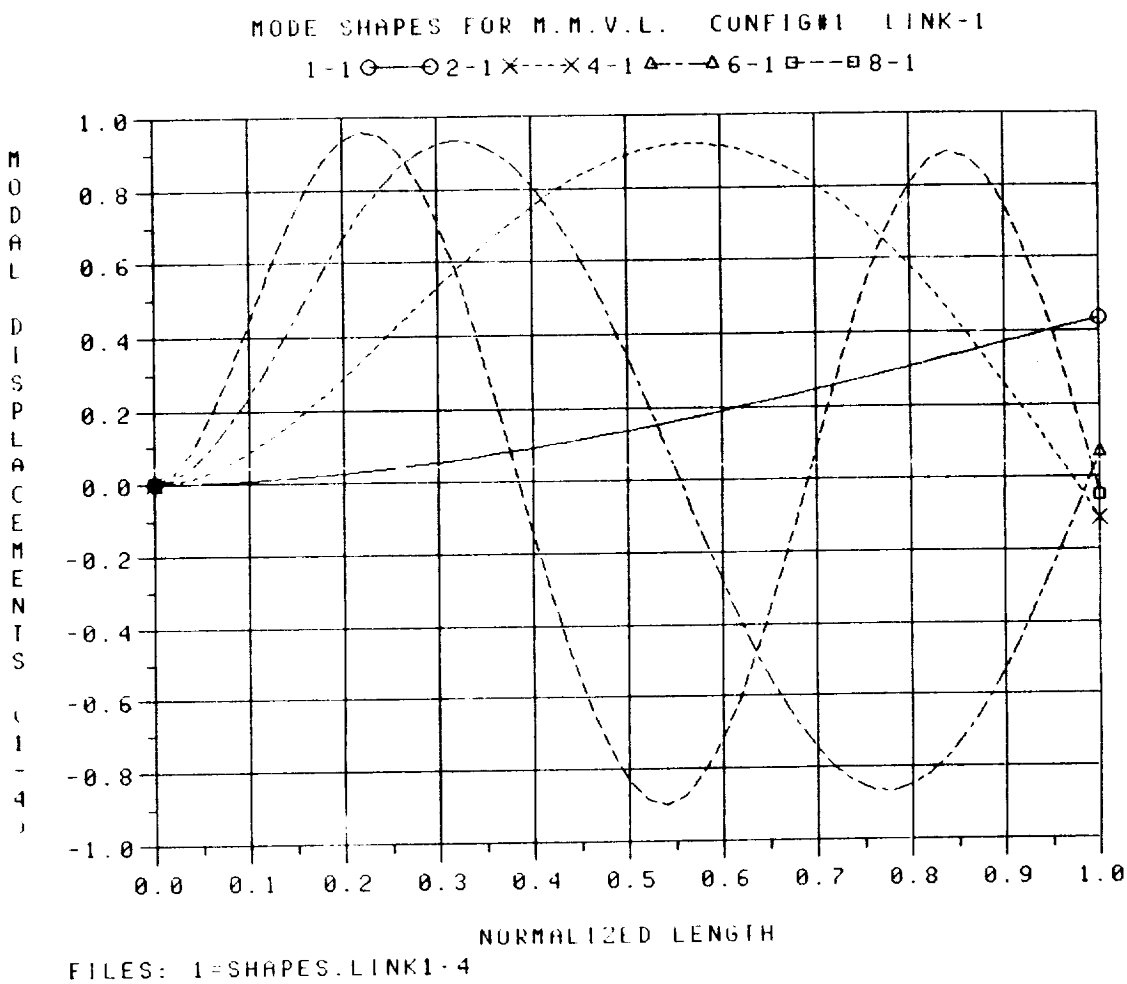

Figure 3. Normalized Mode Shapes for Link One

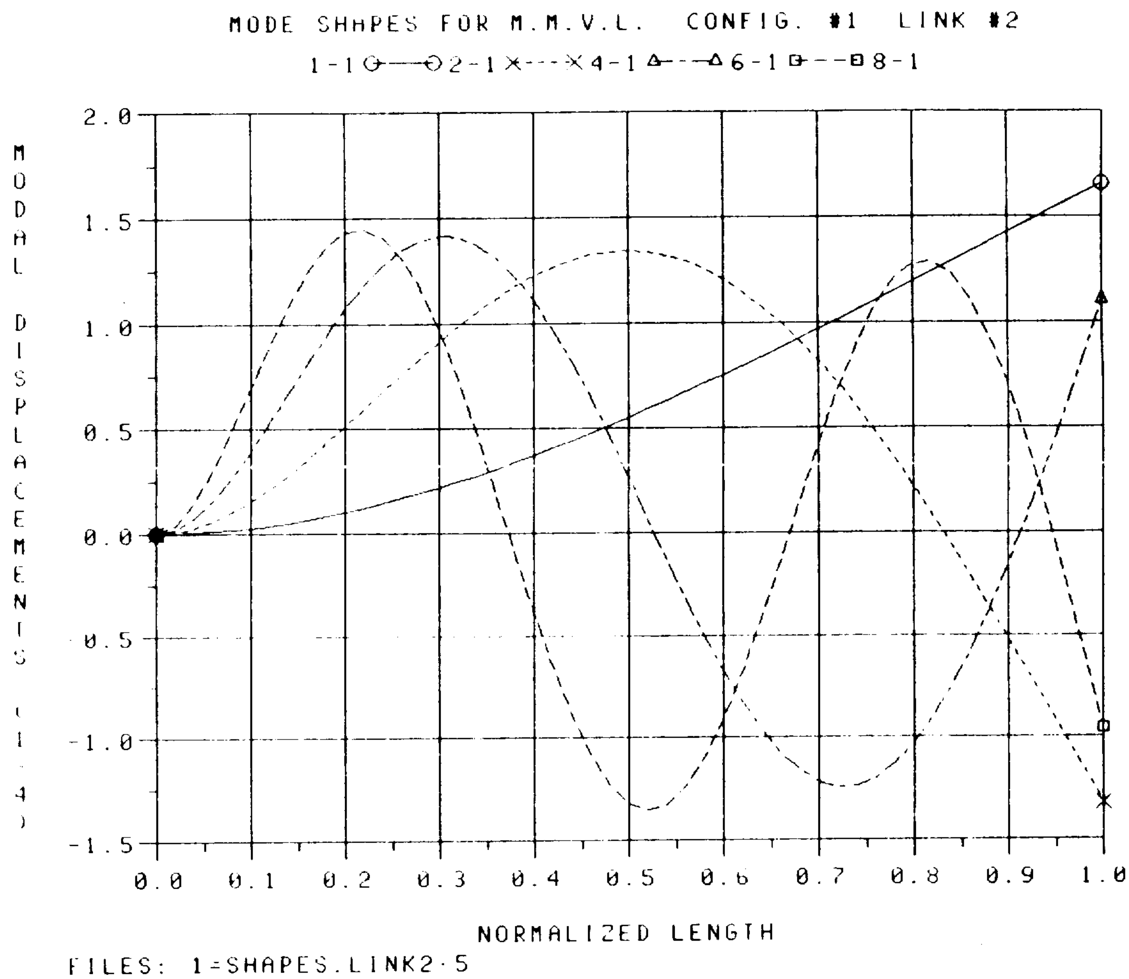

Figure 4. Normalized Mode Shapes for Link Two

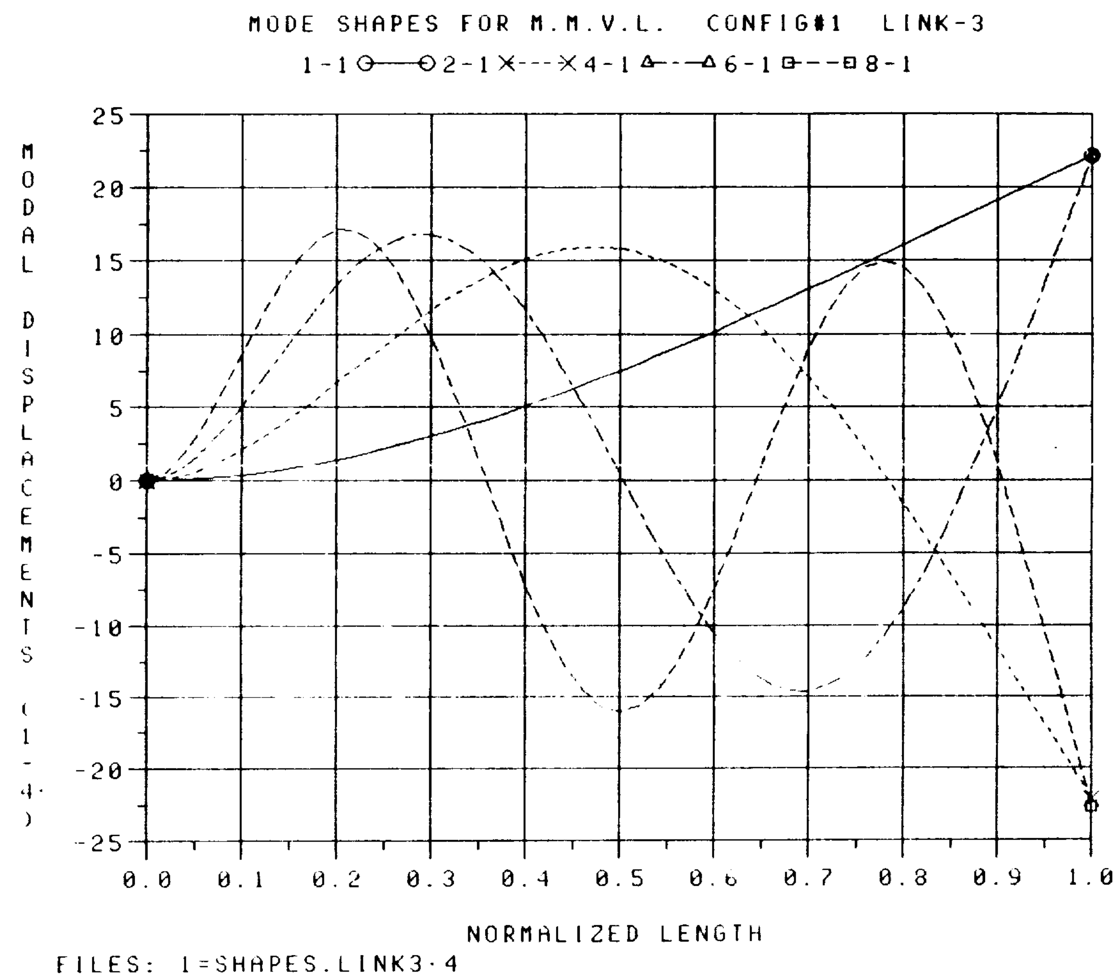

Figure 5. Normalized Mode Shapes for Link Three

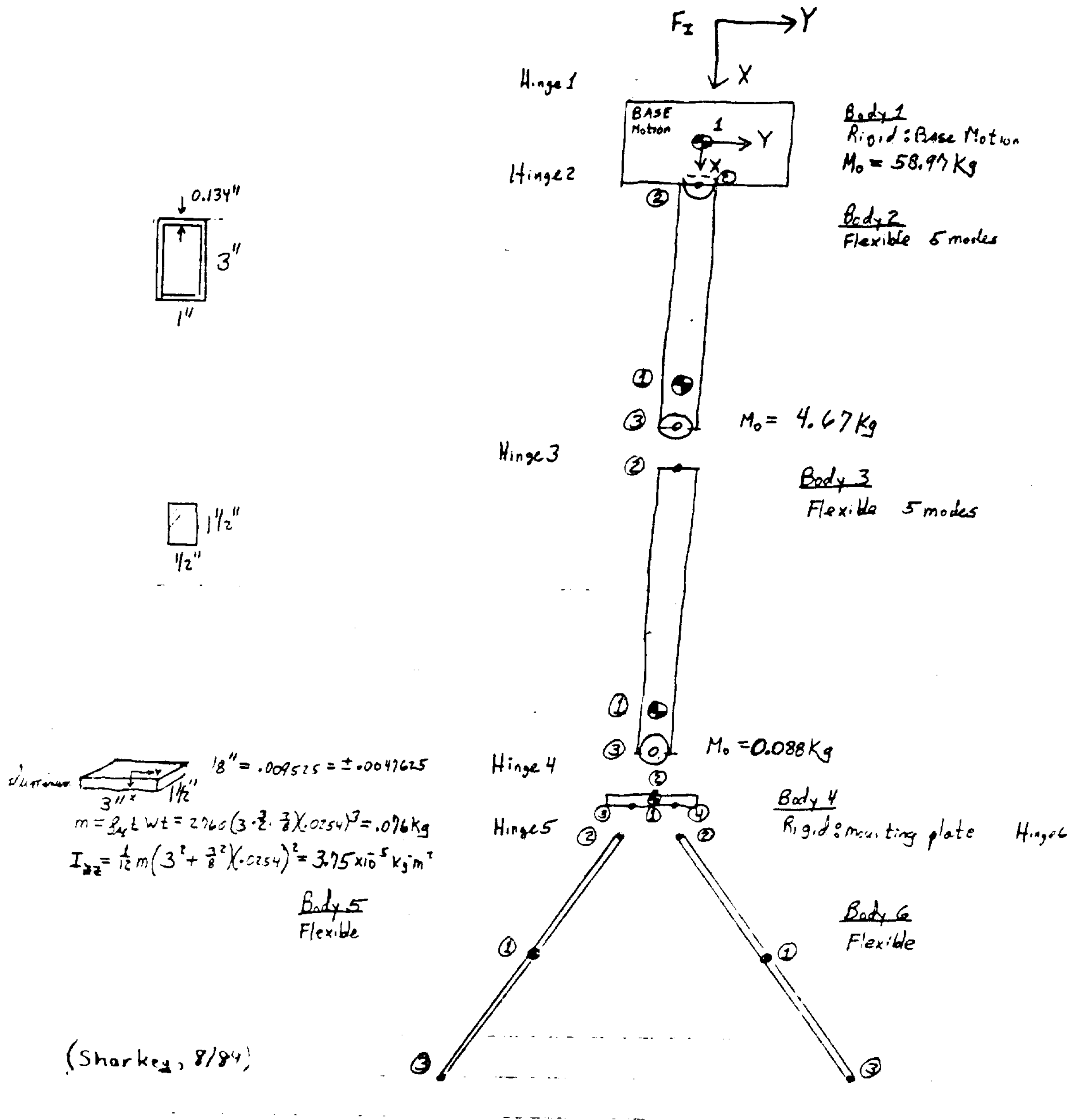

Figure 6. TREETOPS Configuration of Test Article

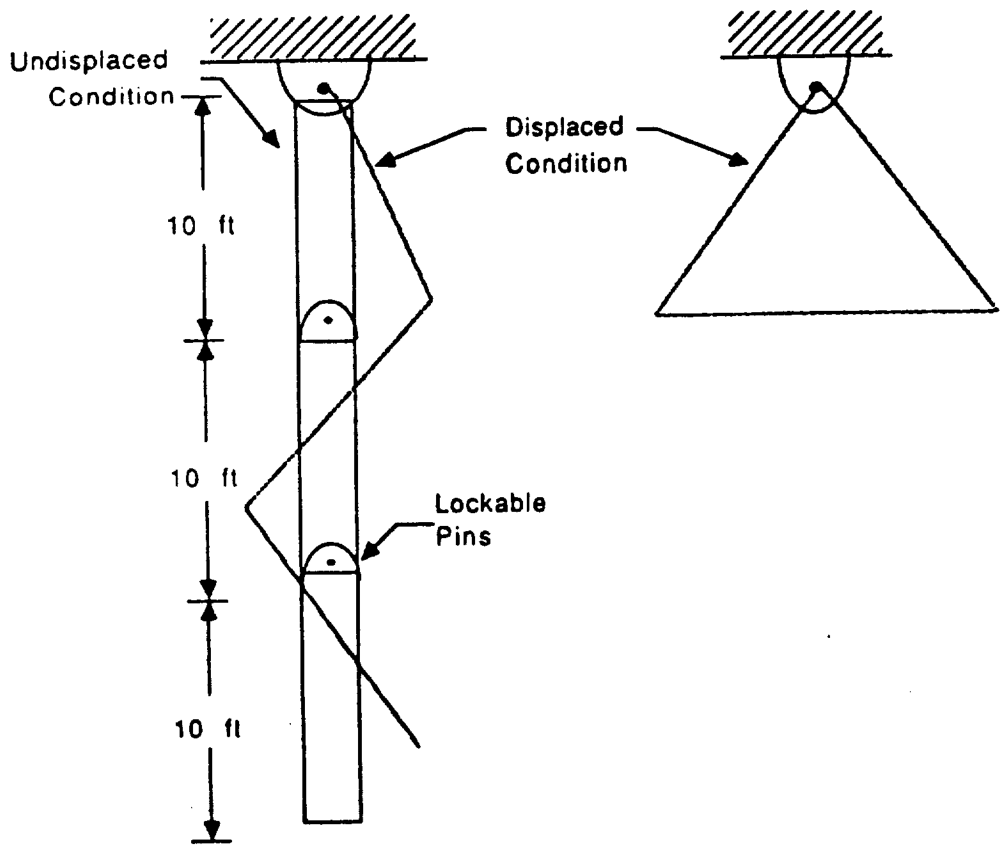

Figure 7. Sample Open and Closed Tree Topologies [1]

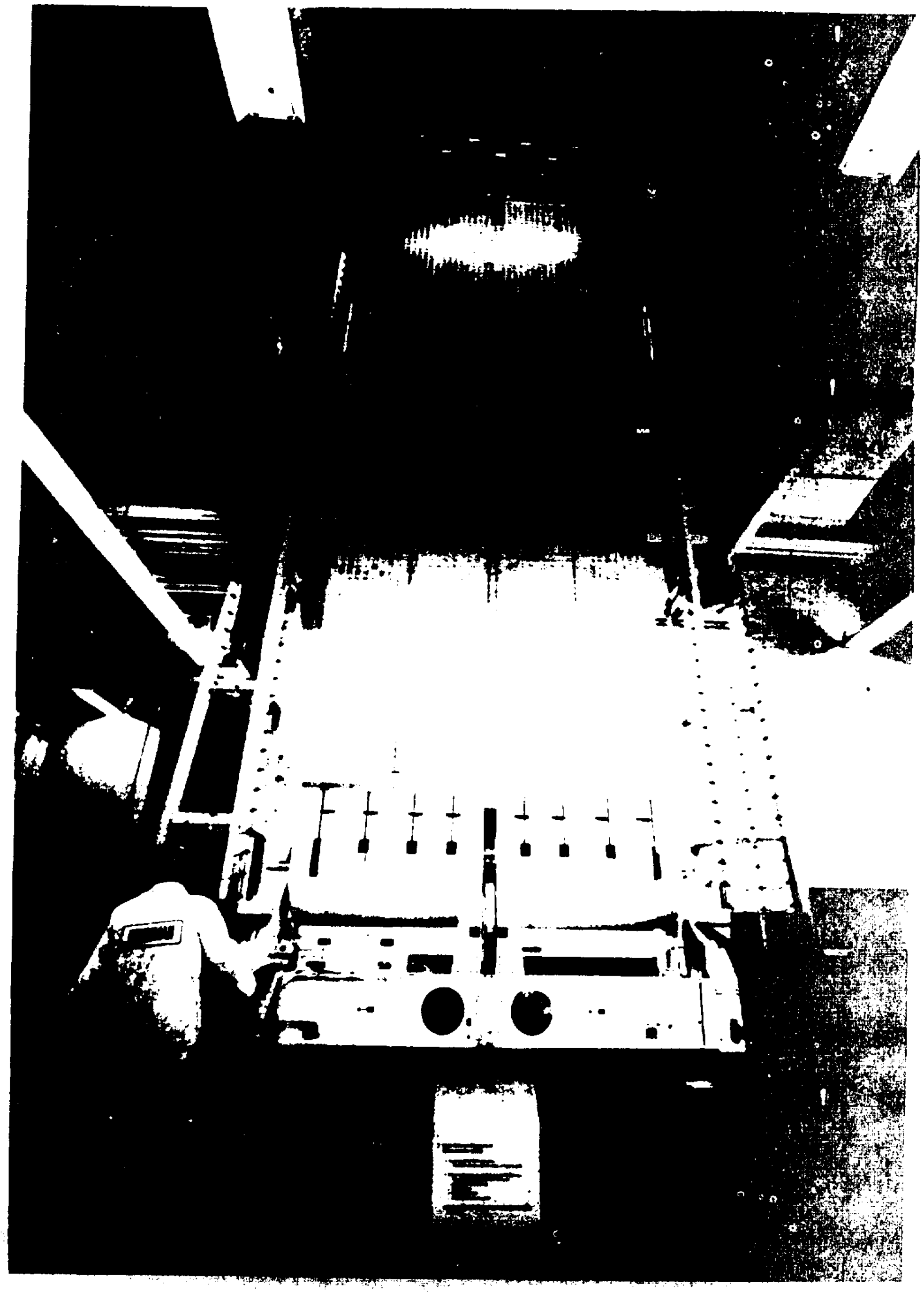

Figure 8. Lens Antenna Deployment Demonstration (LADD) Hardware

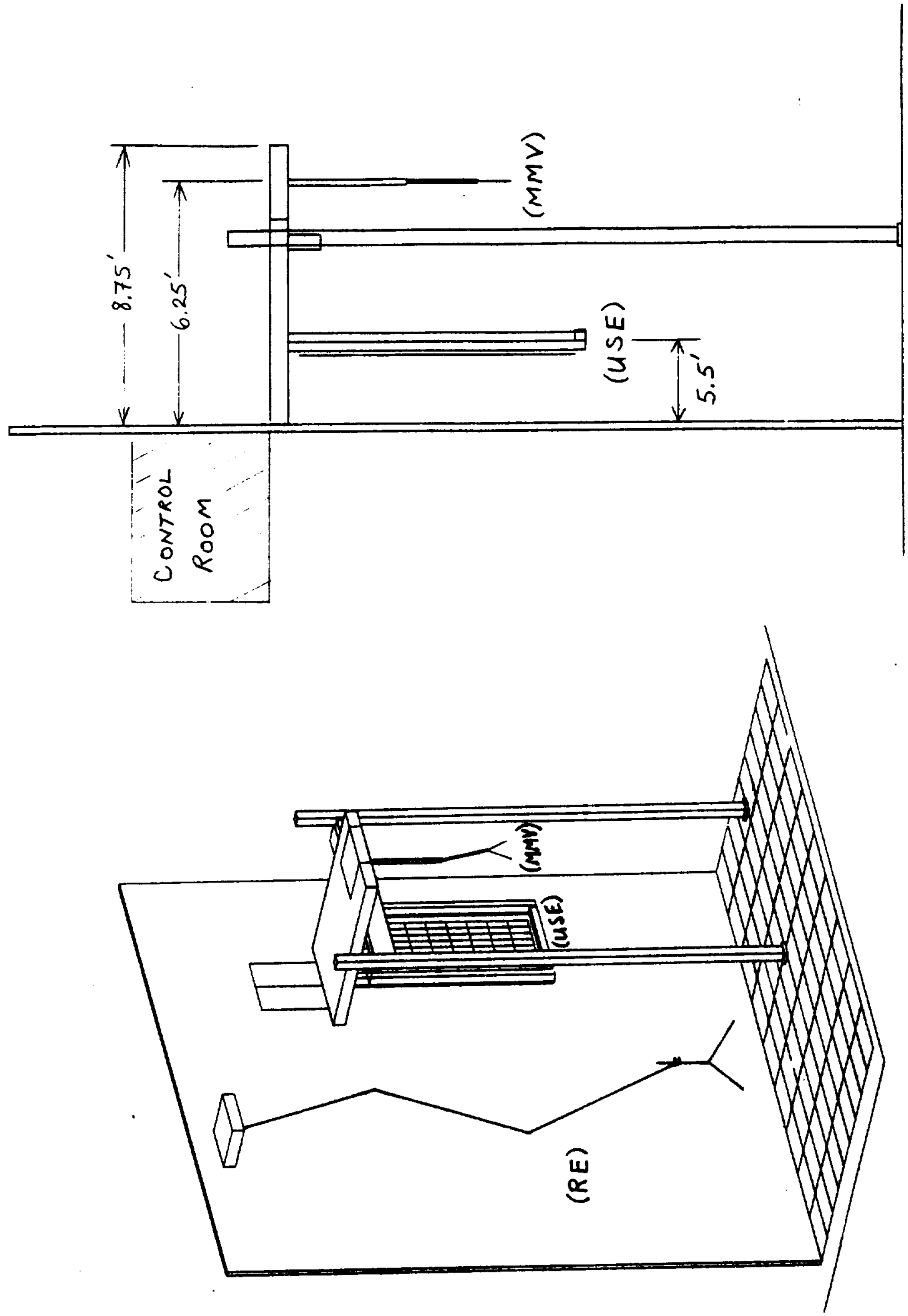

Figure 9. General Facility Layout for MMV, USE and RE

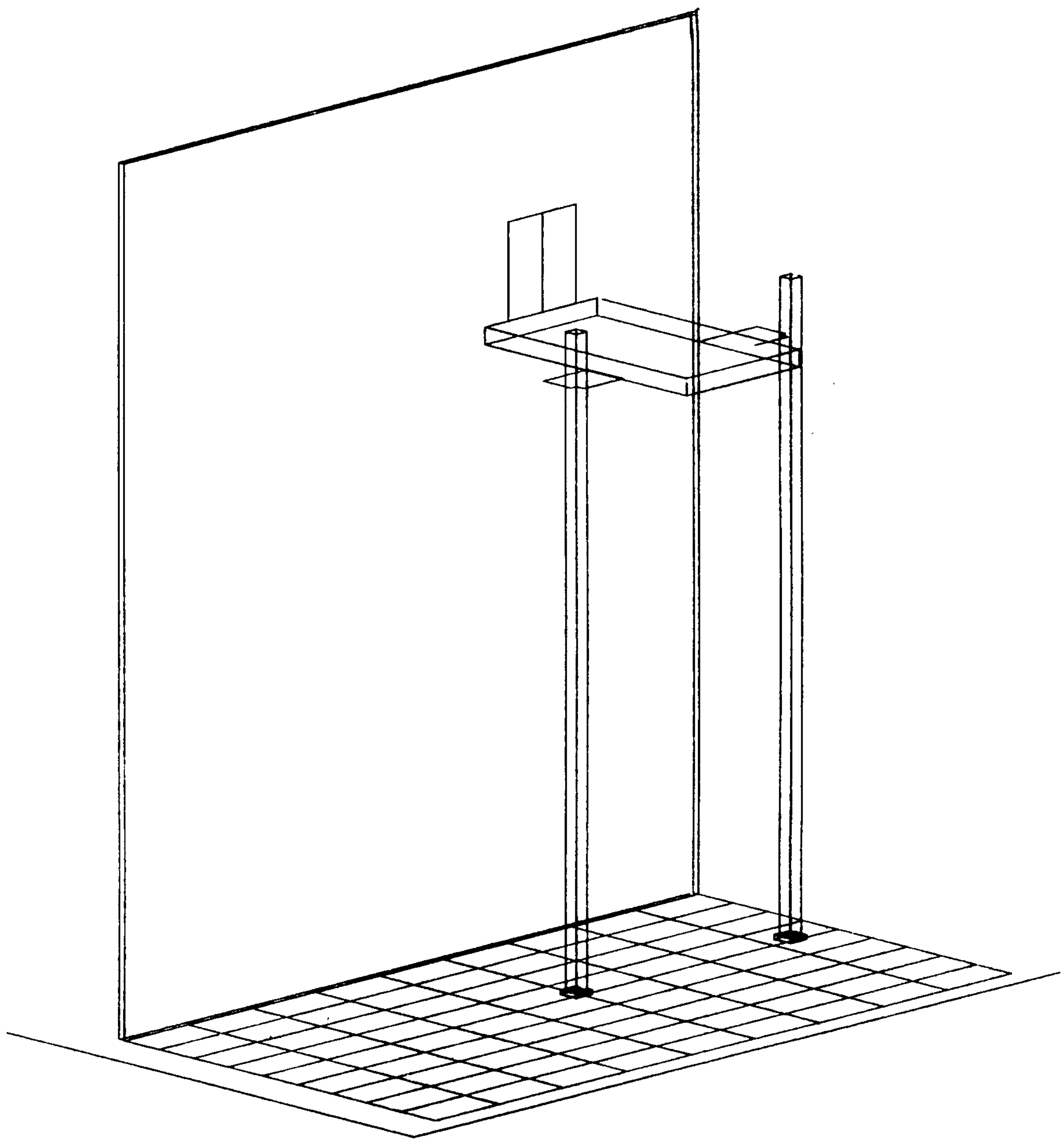

Figure 10. Current Major Structural Layout

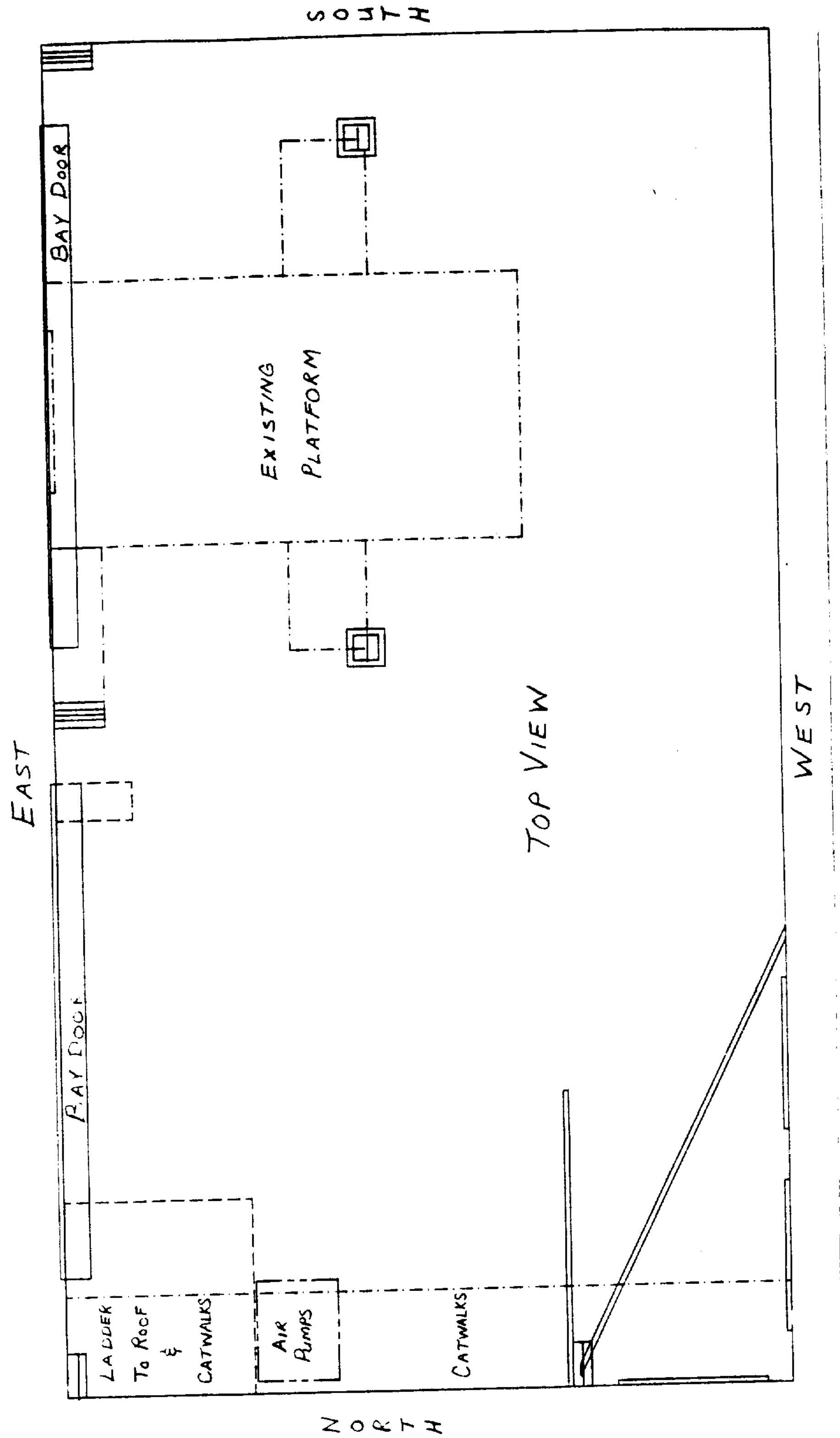

Figure 11. Existing Facility Floor Plan with Unused Air-Handling Units Removed

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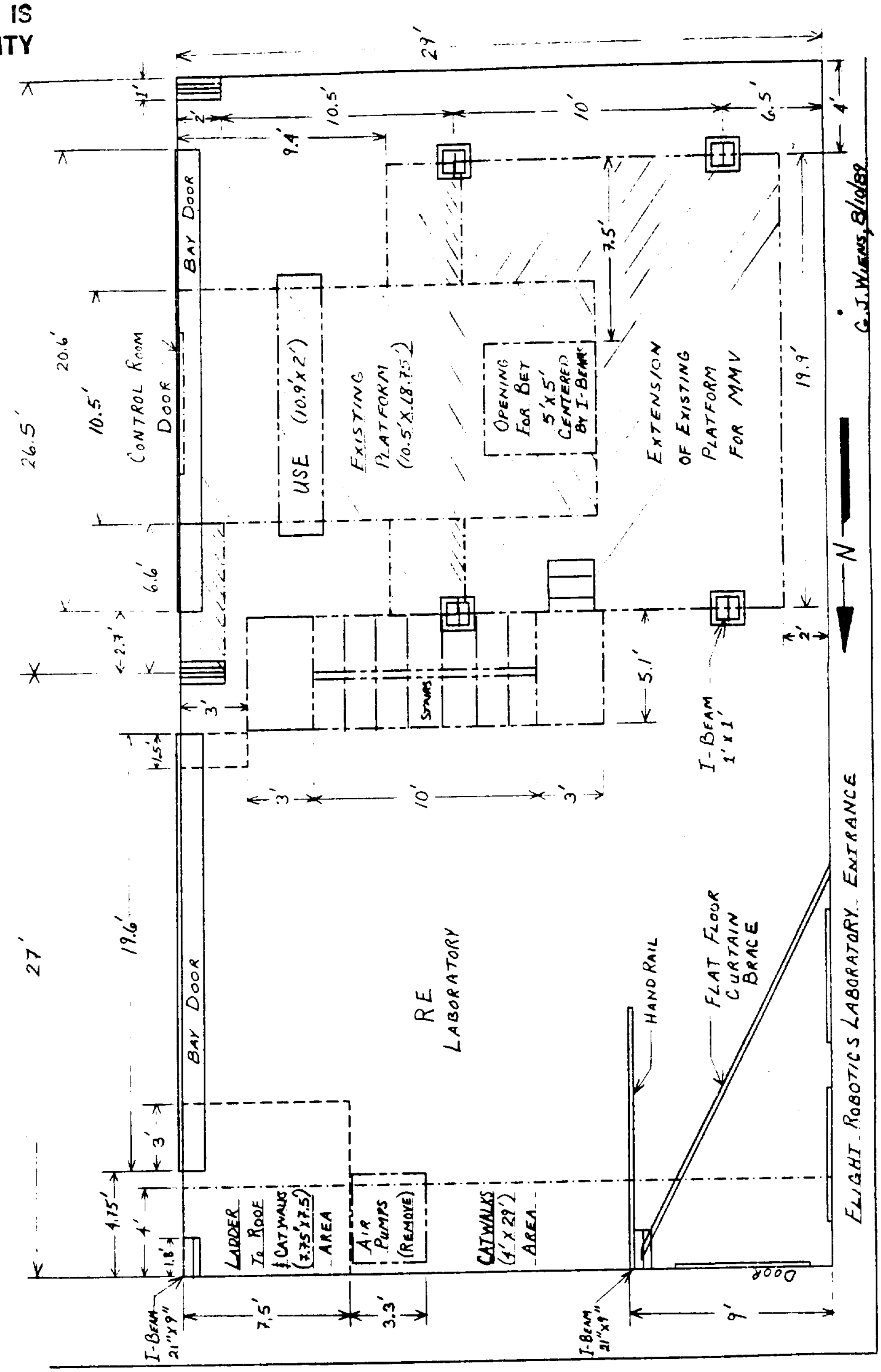

Figure 12. Proposed Facility Layout, Top View

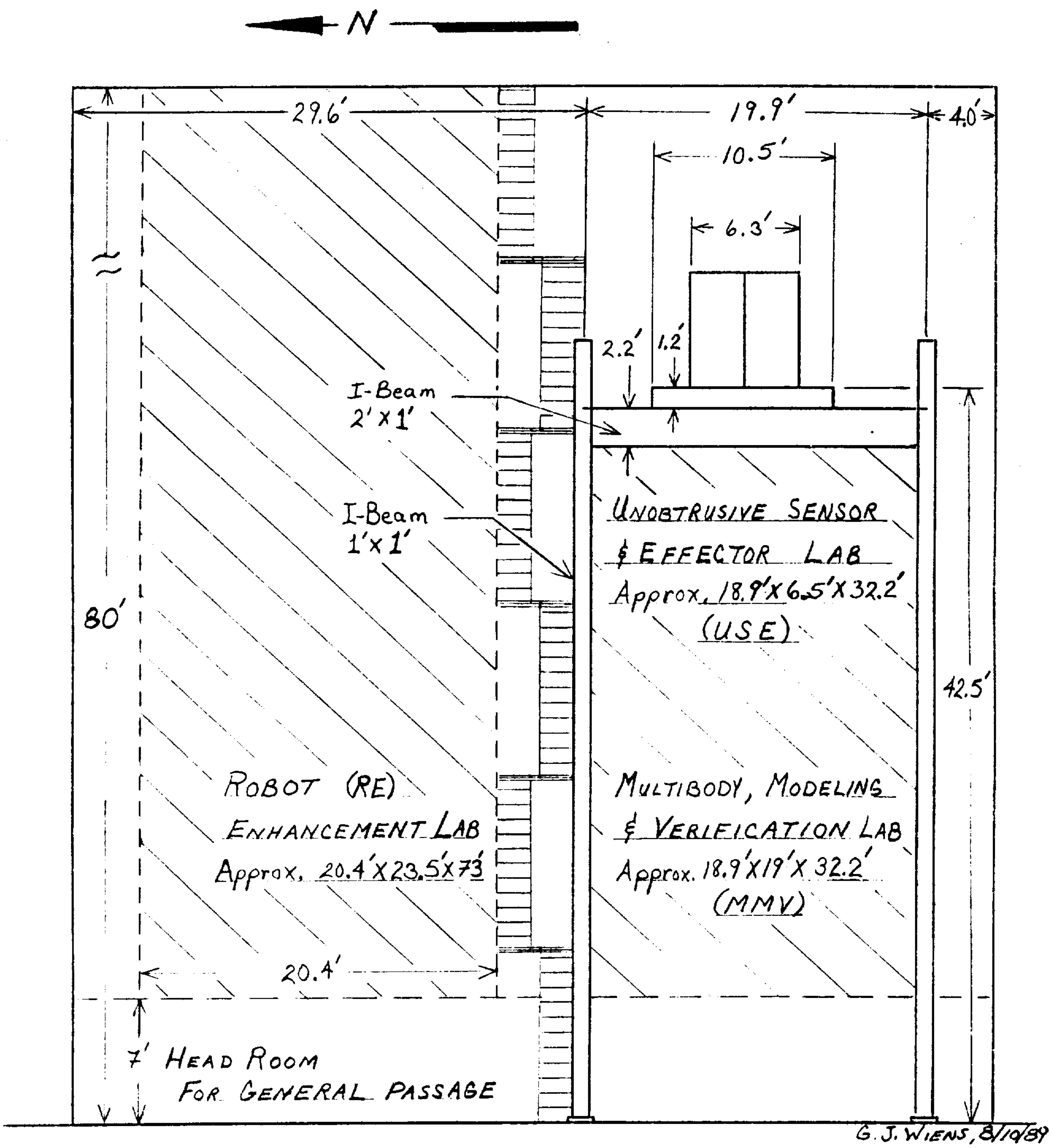

Figure 13. Proposed Facility Layout, West View Facing East

Table 1.

MMV Test Article Material and Physical Properties.

Link	R(cm)	b(cm)	h(cm)	t(cm)	A(m2)	I(m4)
1	-----	7.62	2.54	0.340	6.45e-4	6.69e-8
2	-----	3.81	1.27	-----	4.84e-4	6.50e-9
3 (each branch)	0.127	----	----	-----	5.07e-6	2.04e-12

Link	(kg/m3)	E(N/m2)	L(m)	mg(N)	Torquer wt.(N)
1	1.65e3	41.4e9	2.44	25.5	578
2	1.65e3	41.4e9	1.52	11.9	45.8
3 (each branch)	2.11e3	61.0e9	0.762	0.0799	0.862

Material: Link 1 -- Carbon/Carbon Composite
Link 2 -- Carbon/Carbon Composite
Link 3 -- Epoxy 70% S-Glass

A = b*h or A = 2*t*(b + h) - 4*t*t

I = b*h*h*h/12
or I = [b*h*h*h - (b - 2*t)*((h - 2*t)**3)]/12

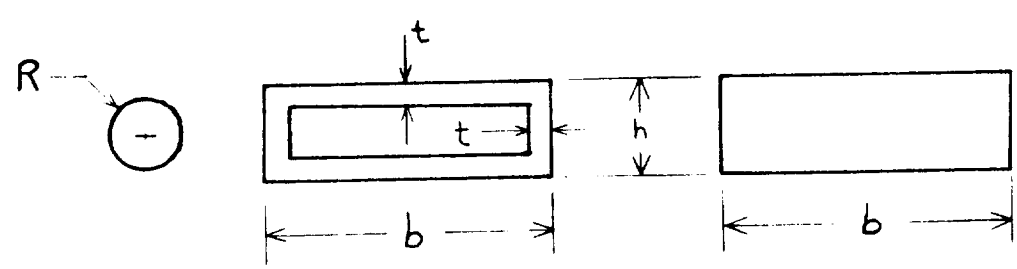

Table 2.

Component Mode Shapes and Frequencies

LINK #1 MS= 1.7984 KS= 0.00000E+00 IS= 0.00000E+00 TS= 0.00000E+00

B2FREQ= 1.3637 TMASS= 7.2660

MODE	FREQ (Hz)	BETA	An	Bn	Cn	Dn	KA	KB	KC	KD	ALPHA	NMODE
1 1.656	1.102	0.5213	-.5478	-.5213	0.5478	0.9516	-1.000	-.9516	1.000	0.3381	5.717	
2 21.69	3.988	0.6186	-.6161	-.6186	0.6161	1.004	-1.000	-1.004	1.000	0.1100	0.4980	
3 68.87	7.106	0.6185	-.6186	-.6185	0.6186	0.9999	-1.000	-.9999	1.000	0.6221E-01	0.1553	
4 142.9	10.24	0.6355	-.6355	-.6355	0.6355	1.000	-1.000	-1.000	1.000	0.4435E-01	0.7660E-01	

LINK #2 MS= 0.78321E-01 KS= 0.00000E+00 IS= 0.00000E+00 TS= 0.00000E+00

B2FREQ= 1.3143 TMASS= 1.2116

MODE	FREQ (Hz)	BETA	An	Bn	Cn	Dn	KA	KB	KC	KD	ALPHA	NMODE
1 4.029	1.751	0.6750	-.9145	-.6750	0.9145	0.7381	-1.000	-.7381	1.000	0.7150	1.019	
2 25.94	4.443	0.9409	-.9251	-.9409	0.9251	1.017	-1.000	-1.017	1.000	0.3928	0.1601	
3 73.94	7.500	0.9309	-.9315	-.9309	0.9315	0.9993	-1.000	-.9993	1.000	0.2302	0.5656E-01	
4 146.9	10.57	0.9568	-.9568	-.9568	0.9568	1.000	-1.000	-1.000	1.000	0.1679	0.2925E-01	

LINK #3 MS= 0.00000E+00 KS= 0.00000E+00 IS= 0.00000E+00 TS= 0.00000E+00

B2FREQ= 0.93486 TMASS= 0.81516E-02

MODE	FREQ (Hz)	BETA	An	Bn	Cn	Dn	KA	KB	KC	KD	ALPHA	NMODE
1 3.287	1.875	8.131	-11.00	-8.131	11.00	0.7341	-1.000	-.7341	1.000	8.672	0.3913E-01	
2 20.60	4.694	11.28	-11.00	-11.28	11.00	1.018	-1.000	-1.018	1.000	4.806	0.6244E-02	
3 57.68	7.855	11.07	-11.00	-11.07	11.00	0.9992	-1.000	-.9992	1.000	2.019	0.2231E-02	
4 113.0	11.00	11.34	-11.34	-11.34	11.34	1.000	-1.000	-1.000	1.000	2.062	0.1165E-02	

[EOB]

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Table 3

[illegible]

[illegible]087	0.265535595[illegible]	
[illegible]	0.[illegible]47[illegible]	
0.0791[illegible]224	0.0126031[illegible]02	
0.270024212611220	0.042975688191573	4 → Link 2
0.202093293470986	0.032164146621629	2 → Link 3
1.302302799500687	0.207267927942948	3 → Link 3
0.206526434400293	0.032869702913951	
1.294278973489974	0.205990896370834	

$$h_{mode} = \int_0^L \rho A x \phi(x)\, dx + L\phi(L) M_{TIP} \;;\; M_{TIP} = \text{mass [illegible] end of Link}$$

$$\phi_n(x) = A_n \sin \beta_n x + B_n \cos \beta_n x + C_n \sinh \beta_n x \; D_n \cosh \beta_n x$$

$$1 = \int_0^L \rho A \phi_n^2(x)\, dx = \rho A P^2 \int_0^L (K_A \sin \beta_n x + K_B \cos \beta_n x + K_C \sinh \beta_n x + K_D \cosh \beta_n x)^2 dx$$

$$A_n = PK_A \;;\; B_n = PK_B ,\; C_n = PK_C ,\; D_n = PK_D$$

$$\alpha = \left[\left(\int_0^L \rho A \phi(x)\, dx + M_{TIP}\, \phi(L) \right) \Big/ \left(\int_0^L \rho A\, dx + M_{TIP} \right) \right]$$

REFERENCES

1. Waites, H. B., Jones, V. L., and Seltzer, S. M., "Cost Effective Development of a National Test Bed", NASA TM-100321, February 1988.

2. Likins, P. W., "Multibody Dynamics-An Historical Perspective", JPL D-5190, Vol. I, Proc. of the Workshop on Multibody Simulation, April 1988, pp. 10-24.

3. Man, G., Laskin, R. A., and Tolivar, A. F., "Computational Controls for Aerospace Systems", NASA TM-101578, Part One, "Workshop on Computational Aspects in the Control of Flexible Systems", Williamsburg, Virginia, July 1988, pp. 33-48.

4. Oakley, C. M., and Cannon, Jr., R. H., "Initial Experiments on the Control of a Two-Link Manipulator with a Very Flexible Forearm", Proc. of the American Control Conference, June 1988, pp. 996-1002.

5. Crawley, E. F., de Luis, J., and Miller, D. W., "Middeck Active Control Experiment (MACE)", SSL # 7-89, Final Report for NASA Grant NAG-1-915, June 1989.

6. Chace, M. A., and Smith, D. A., "DAMN-A Digital Computer Program for the Dynamic Analysis of Generalized Mechanical Systems", SAE paper 710244, January 1971.

7. Gregory, C. Z. "Reduction of Large Flexible Spacecraft Models Using Internal Balancing Theory", AIAA Paper 83-2292, G&C Conference, Gatlinburg, TN, August 1983.

8. Shabana, A. A., Dynamics of Multibody Systems, John Wiley & Sons, 1989.

9. Craig, Jr., R. R., Structural Dynamics, An Introduction to Computer Methods, John Wiley & Sons, 1981.

10. Craig, R. R., and Bampton, M. C., "Coupling of Substructures for Dynamic Analyses", AIAA Journal, Vol. 6, No. 7, July 1968, pp. 1313-1319.

11. Ho, J. Y., "Direct Path Method for Flexible Multibody Spacecraft Dynamics", Jr. Spacecraft, Vol. 14, No. 2, Feb. 1977, pp. 102-110.

12. Jerkovsky, W., "The Structure of Multibody Dynamics Equations", Jr. Guidance and Control, Vol. 1, No. 3, May/June 1978, pp. 173-182.

13. Singh, R. P., VanderVoort, R. J., and Likins, P. W., "Dynamics of Flexible Bodies in Tree Topology-A Computer Oriented Approach", Proc. of the AIAA Dynamics Specialist Conference, Palm Springs, CA, May 1984, pp. 327-385.

14. Shabana, A. A., "Substructure Synthesis Methods for Dynamic Analysis of Multi-Body Systems", Computers and Structures, Vol. 20, No. 4, 1985, pp. 737-744.

15. Khulief, Y. A., Shabana, A. A., "Dynamics of Multibody Systems with Variable Kinematic Structure", ASME Jr. of Mechanisms, Transmissions, and Automation in Design, Vol. 108, June 1986, pp. 167-175.

16. Fang, L. Y., Shabana, A. A., Agrawal, O. P., "Application of Perturbation Techniques to Flexible Multibody System Dynamics", Computers and Structures, Vol. 27, No. 5, 1987, pp. 631-637.

17. Benfield, W. A., and Hruda, R. F., "Vibration Analysis of Structures by Component Mode Substitution" AIAA Journal, Vol. 9, No. 7, July 1971, pp. 1255-1261.

18. Yoo, W. S., and Haug, E. J., "Dynamics of Flexible Mechanical Systems Using Vibration and Static Correction Modes", ASME Jr. of Mechanisms, Transmissions, and Automation in Design, Vol. 108, Sept. 1986, pp. 315-322.

19. Huckelbridge, A. A., and Lawrence, C., "Identification of Structural Interface Characteristics Using Component Mode Synthesis", ASME Jr. of Vibration, Acoustics, Stress, and Reliability in Design, Vol. 111, April 1989, pp. 140-147.

20. Chang, L.-W., and Gannon, K. K., "A Dynamic Model on a Single-Link Flexible Manipulator", ASME Modal Testing and Analysis, DE-Vol. 3, 1987, pp. 23-28.

21. Padovan, J., and Kazempour, A., "Multibody Instantly Centered Moving Lagrangian Observer Schemes-Part I. Formulation", Computers and Structures, Vol. 32, No. 1, 1989, pp. 93-100.

22. Lee, J. D., and Wang, B.-L., "Dynamic Equations for a Two-Link Flexible Robot Arm", Computers and Structures, Vol. 29, No. 3, 1988, pp. 469-477.

23. Bucher, C. U., "A Modal Synthesis Method Employing Physical Coordinates, Free Component Modes, and Residual Flexibilities", Computers and Structures, Vol. 22, No. 4, 1986, pp. 559-564.

24. Rauh, J., and Schiehlen, W., "Various Approaches for the Modeling of Flexible Robot Arms", Lecture Notes in Engineering: Refined Dynamics Theories of Beams, Plates and Shells and Their Applications, No. 28, Springer-Verlag, 1986, pp. 420-429.

25. Panossian, H. V., "Uncertainty Management in Modeling and Control of Large Flexible Structures", Structural Dynamics Testing and Analysis, SP-596, Aerospace Congress & Exposition, Long Beach, CA, Oct. 1984, pp. 55-58.

26. Kim, S. S., and Vanderploeg, M. J., "A General and Efficient Method for Dynamic Analysis of Mechanical Systems Using Velocity Transformations", ASME Jr. of Mechanisms, Transmissions, and Automation in Design, Vol. 108, June 1986, pp. 176-182.

27. Wittenburg, J., "Dynamics of Multibody Systems - A Brief Review", Acta Astronautica, Vol. 20, 1989, pp. 89-92.

28. Simo, J. C., and Vu-Quoc, L., "On the Dynamics of Flexible Beams Under Large Overall Motions--The Plane Case: Part I & II", ASME Jr. of Applied Mechanics, Vol. 53, Dec. 1986, pp. 849-863.

29. Simo, J. C., and Vu-Quoc, L., "Dynamics of Earth-Orbiting Flexible Satellites with Multibody Components", Jr. Guidance, Vol. 10, No. 6, 1987, pp. 549-558.

30. Li, D., and Likins, P. W., "Dynamics of a Multibody System with Relative Translation on Curved, Flexible Tracks", Jr. Guidance, Vol. 10, No. 3, 1987, pp. 299-306.

31. Thomas, M., and Tesar, D., "Dynamic Modeling of Serial Manipulator Arms", ASME Jr. of Dynamic Systems, Measurement, and Control, Vol. 104, Sept. 1982, pp. 218-228.

32. Engels, R. C., "A Solution to the Craig/Bampton Eigenvalue Problem for Multi-Component Structures", IMAC Proc. of the 3rd International Modal Analysis Conference, Orlando, FL, 1985, Vol. I, pp. 299-304.

33. Amirouche, F. M., and Jia, T., "Modelling of Clearances and Joint Flexibility Effects in Multibody Systems Dynamics", Computers and Structures, Vol. 29, No. 6, 1988, pp. 983-991.

34. Amirouche, F. M., and Jia, T., "Pseudouptriangular Decomposition Method for Constrained Multibody Systems

Using Kane's Equations", Jr. Guidance, Vol. 11, No. 1, 1988, pp. 39-46.

35. Ryan, R. R., "Flexible Multibody Dynamics: Problems and Solutions", JPL D-5190, Vol. I, Proc. of the Workshop on Multibody Simulation, April 1988, pp. 103-190.

36. Banerjee, A. K., and Lemak, M. E., "Large Motion Dynamics of Systems of Rigid Bodies, Beams and Plates", JPL D-5190, Vol. I, Proc. of the Workshop on Multibody Simulation, April 1988, pp. 219-234.

37. Jones, R. E., "Multi Flexbody Dynamics for Control Design", JPL D-5190, Vol. I, Proc. of the Workshop on Multibody Simulation, April 1988, pp. 354-382.

38. Ho, J. Y., "The Direct Path Method for Flexible Multibody Dynamics", JPL D-5190, Vol. I, Proc. of the Workshop on Multibody Simulation, April 1988, pp. 383-417.

39. Huston, R. L., "Redundant and Constrained Multibody Systems: Modelling and Computational Methods", JPL D-5190, Vol. I, Proc. of the Workshop on Multibody Simulation, April 1988, pp. 443-454.

40. Taylor, Jr., L. W., "A Comparison of Software for the Modeling and Control of Flexible Systems", NASA TM-101578, Part One, "Workshop on Computational Aspects in the Control of Flexible Systems", Williamsburg, Virginia, July 1988, pp. 33-48.

41. Steiber, M. E., "Flexible Structure Control Experiments Using a Real-Time Workstation for Computer-Aided Control Engineering", NASA TM-101578, Part One, "Workshop on Computational Aspects in the Control of Flexible Systems", Williamsburg, Virginia, July 1988, pp. 67-88.

42. Shabana, A., "Dynamics of Inertia-Variant Flexible Systems Using Experimentally Identified Parameters", ASME Jr. of Mechanisms, Transmissions, and Automation in Design, Vol. 108, Sept. 1986, pp. 358-366.

43. Oppenheim, I. J., and Shimoyama, I., "Flexible Robot Control: Modeling and Experiments", NASA TM-101578, Part Two, "Workshop on Computational Aspects in the Control of Flexible Systems", Williamsburg, Virginia, July 1988, pp. 549-579.

44. Ulsoy, A. G., "Experimental Validation of Flexible Robot Arm Modeling and Control", NASA TM-101578, Part Two, "Workshop on Computational Aspects in the Control

of Flexible Systems", Williamsburg, Virginia, July 1988, pp. 745-777.

45. Huggins, J. D., Kwon, D.-S., Lee, J. W., and Book, W. J., "Alternative Modeling and Verification Techniques for a Large Flexible Arm", 1987.

46. LaRC CSI Office and Draper Lab, "RMS-Based Controls-Structures Interaction (CSI) Flight Experiment", JSC Briefing, April 1989.

47. Demeo, M., and Turnbull, J., "Remote Manipulator System (RMS)-Based Controls Structures Interaction (CSI) Flight Experiment Feasibility Study", NASA Headquarters Briefing, July 1989.

48. Wang, W.-J., Lu, S.-S., and Hsu, C.-F., "Experiments on the Position Control of a One-Link Flexible Robot Arm", IEEE Trans. on Robotics and Automation, Vol. 5, No. 3, June 1989.

49. Turcic, D. A., Midha, A., and Bosnik, J. R. "Dynamic Analysis of Elastic Mechanism Systems. Part II: Experimental Results", ASME Jr. of Dynamic Systems, Measurement, and Control, Vol. 106, pp. 255-260.

50. Chen, J.-C., Garba, J. A., and Demsetz, L. A., "Verification of Large Space Structures using Scale Modelling Laws", Proc. of the 3rd International Modal Analysis Conference, Orlando, Florida, 1985, Vol. I, pp. 31-36.

51. Niedbal, N., "Experimental System Identification for Experimental/Analytical Correlation and Modelling", ASME AMD-Vol. 67, 1985, pp. 195-204.

52. Hasselman, T. K., and Chrostowski, J. D., "Dynamic Model Verification of a Multi-Component System", "Aerospace Congress & Exposition", Long Beach, CA, Oct. 1984, SP-596, pp. 89-96.

53. Popov, E. P., Mechanics of Materials, Prentice-Hall, 1976.

54. Hill, H., Johnston, D., and Frauenberger, H., "Development of the Lens Antenna Deployment Demonstration (LADD) Shuttle-Attached Flight Experiment", "First NASA/DOD CSI Technology Conference, Nov. 1986, pp. 125-144.