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The Effects of Sinusoidal Interference on the Second-Order Carrier Tracking Loop Preceded by a Bandpass Limiter in the Block IV Receiver

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Drop-lock relationships for the second-order phase-locked loop are derived when the carrier and a sinusoidal interference signal lie within the predetection filter bandwidth of the Block IV receiver. Limiter suppression factors are calculated when a bandpass hard limiter is used to maintain constant total power at the loop. The parameters of interest are the interference-to-signal power ratio (ISR), the input signal-to-noise power ratio (SNR), and the interference signal frequency offset from carrier Δf. Limiter suppression caused by the combined effects of the noise and the interference signal accounts for the variability in the drop-lock threshold for given values of the input SNR and ISR parameters. This article goes beyond earlier published work that focused on the limiter's effect on the drop-lock threshold; it accounts for the limiter action in the interference mode and provides an overall improvement in the prediction accuracy of the drop-lock model.

I. Introduction

One major application of the phase-locked loop in a DSN receiver is tracking the carrier of the received signals [1]. The receiver phase-locks to the carrier and loses lock when the carrier margin drops below the lock threshold, or when an interfering signal is received at the critical amplitude and frequency offset from the carrier. Although telecommunications links are designed with sufficient margins to ensure performance requirements for the lifetime of the mission, interference can occur at any time. If the interfering signal power and frequency exceed the threshold limit, the carrier tracking loop drops the weaker carrier signal and locks up to the stronger interference signal. This jump phenomenon is due to the inherent nonlinearities present in the phase-locked loop for conditions when the interference-to-noise power ratio (INR) is sufficiently high. As the INR decreases, the signal-to-noise ratio (SNR) becomes the dominant factor, which can cause the loop to lose lock when it decreases below the noise threshold level. This article investigates the effect of the bandpass limiter when drop-lock of this type occurs.

The carrier tracking loop employed in the Block IV receiver consists mainly of a second-order phase-locked loop preceded by a bandpass limiter. The hard limiter provides constant power at the input to the loop and effectively minimizes the total mean-square error of the loop over a wide range of the SNR. If, in addition to the noise, a sinusoidal interference signal is inserted into the limiter along with the carrier signal, the interference signal will also contribute to the limiter suppression. The limiter's effect on the drop-lock threshold becomes evident from its impact on the loop gain and loop interference-to-signal power ratio (ISR) loop input. Limiter suppression factors for these parameters are calculated and incorporated into the basic drop-lock model to improve its prediction accuracy for large variations in the loop SNR.

II. Carrier Drop-Lock Model

Figure 1 shows a representative second-order phase-locked loop preceded by a bandpass limiter. Bruno [2] derived the loop equations for the case of a strong signal and a sinusoidal interferer, without the limiter. The voltage-controlled oscillator (VCO) output is equal to

$$2\cos\left[\omega_1 t + \phi_0(t)\right]$$

where ω_1 is the VCO frequency (rad/sec) and $\phi_0(t)$ is the phase modulation due to the input through the loop action. The phase detector is assumed to be a perfect multiplier, and the loop filter has a transfer characteristic described as $F(s)$.

Ignoring the effects of narrow-band Gaussian noise, the input to the loop consists of the sum of two sinusoidal components:

$$A\ \sin(\omega_c t) + B\ \sin(\omega_c + \Delta\omega)t \tag{1}$$

where the first term of Eq. (1) is the wanted signal component with frequency ω_c having constant amplitude A volts when the limiter reaches a constant output. The interference component has an amplitude equal to B volts and is offset in frequency from the signal component by an amount equal to $\Delta\omega$. Defining $\sqrt{\mathrm{ISR}}$ as B/A, Eq. (1) can be rewritten as

$$A\ \sin(\omega_c t) + \sqrt{\mathrm{ISR}}\, A\ \sin(\omega_c t + \Delta\omega)t$$

The output modulation $\phi_0(t)$ is given by

$$\phi_0(t) = \frac{\mathrm{KF}(p)}{p}\left[-\sin\phi_0(t) + \mathrm{ISR}\ \sin\left(\Delta\omega t - \phi_0(t)\right)\right]$$

where p represents the operator d/dt, $F(p)$ is the loop filter, and K (1/sec) is the open-loop gain, which includes the VCO and the phase-detector loop gain. This nonlinear differential equation cannot be solved analytically; however, using the method of perturbations, solutions with best-approximation trial functions can be obtained. A steady-state trial solution is assumed to be

$$\phi_0(t) = \lambda + \theta\sin(\Delta\omega t + \nu)$$

where λ represents the static phase error, θ is the phase deviation, and ν is the phase shift. Bruno [2] derived the lock constraints as

$$\sin\lambda = \frac{-\theta^2\delta\cos\psi}{2J_0(\theta)} \tag{2}$$

$$\sin(\lambda - \nu) = \frac{-\theta^2\delta\cos\psi}{2\sqrt{\mathrm{ISR}}\, J_1(\theta)}$$

$$\left[\frac{\theta\delta\sin\psi + 2J_1(\theta)\cos\lambda}{J_0(\theta) - J_2(\theta)}\right]^2 + \left[\frac{\theta^2\delta\cos\psi}{2J_1(\theta)}\right]^2 = \mathrm{ISR} \tag{3}$$

where J_0, J_1, and J_2 are Bessel functions of the first kind, ψ is the phase angle of $F(s)$, and δ is the normalized offset frequency

$$\delta = \frac{\Delta\omega}{K|F(s)|}$$

where $s = j\Delta\omega$, and K = the open-loop gain.

Restricting the second-order loop filter with transfer characteristics,

$$F(s) = \frac{1+\tau_2 s}{1+\tau_1 s}$$

where $\tau_1 \gg \tau_2$. With $\Delta\omega \gg 1/\tau_2$, one obtains the reasonable approximation

$$|F(s)| \approx \frac{\tau_2}{\tau_1} \qquad \text{for } \psi \approx 0$$

The loop is expected to drop lock when the static phase error approaches 90 deg. Applying this condition and using Eqs. (2) and (3) with the condition that $J_1(\theta) \approx \theta/2$ and $J_0(\theta) \approx 1$ for small phase deviations, the critical ISR can be given as

$$(\mathrm{ISR})_c = \frac{4\pi\Delta f}{K\,(\tau_2/\tau_1)}$$

This describes the drop-lock threshold for critical values of ISR and offset frequency Δf without the limiter action.

III. Calculation of the Limiter Suppression Factors

The effect of the bandpass limiter also needs to be taken into account, when the carrier, interfering sinusoidal signal, and narrow-band Gaussian noise are present at the input to the limiter. Jones [3] calculated the autocorrelation function of the ideal hard limiter under similar conditions. The interaction of the two signals s_1 and s_2 with noise generates a filtered output with autocorrelation function given by

$$R(\tau) = \sum_{i=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} \sum_{k=|i|,|i|+2}^{\infty} 2\frac{b^2 k|\ell|\left|\ell - |i+1|\right|}{\left(\frac{k+|i|}{2}\right)!\left(\frac{k-|i|}{2}\right)!}\rho^k(\tau)$$

$$\times \cos\left[|i|\omega_c - |i+1|\omega_2 + \ell(\omega_2 - \omega_1)\right]\tau$$

where ω_1 and ω_2 represent the frequencies of s_1 and s_2 respectively, ω_c is the bandpass filter center frequency, and ρk is the normalized noise-power envelope function containing both discrete (due to the period terms) and continuous components (associated with the output noise). The total power contained in these discrete components at the output is then given by

$$R^{(\tau)}_{s_1 x s_2} = \sum_{\ell=-\infty}^{\infty} 2b_0^2|\ell|\,|\ell-1|\cos\left[\ell(\omega_2-\omega_1)-\omega_2\right]\tau$$

and the output signal powers are given as

$$(s_1)_0 = 2b_{010}^2 = \frac{2}{\pi^2}\left(\frac{s_1}{N}\right)$$

$$\left[\sum_{i=0}^{\infty}\frac{(-1)^i(s_1/N)^i}{i!(i+1)!}\Gamma\left(i+\frac{1}{2}\right)\,{}_2F_1\left(-i,i-1;1;\frac{s_2}{s_1}\right)\right]^2$$

and

$$(s_2)_0 = 2b_{001}^2 = \frac{2}{\pi^2}\left(\frac{s_2}{N}\right)$$

$$\left[\sum_{i=0}^{\infty}\frac{(-1)^i(s_1/N)^i}{(i!)^2}\Gamma\left(i+\frac{1}{2}\right)\,{}_2F_1\left(-i,-i;2;\frac{s_2}{s_1}\right)\right]^2$$

where Γ and ${}_2F_1$ are the gamma and confluent hypergeometric function, respectively. For the case where both the carrier and interference power are much greater than the noise power, the convergence properties of these equations become unstable. Then it becomes necessary to use the asymptotic forms

$$(s_1)_0 = \frac{2}{\pi^2}\left(\frac{s_1}{s_2}\right)\left[{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;\frac{s_1}{s_2}\right)\right]^2$$

and

$$(s_2)_0 = \frac{2}{\pi^2}\left(\frac{s_1}{s_2}\right)\left[\frac{\Gamma(1/2)}{\Gamma(3/2)}\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2};1;\frac{s_1}{s_2}\right)\right]^2$$

for

$$\frac{s_2}{N}\to\infty;\ \frac{s_1}{s_2}<1;\ k=0$$

These relationships can be used to calculate the limiter suppression on the carrier power and the power ratio of the interference and carrier signal. For the case where interference is not present, the limiter suppression factor reduces to that calculated by Davenport [4] for a sinusoid and noise only. With interference present, the limiter suppression becomes affected by changes in both the ISR and SNR power ratios. Limiter suppression of the carrier signal from the limited strong signal peak level, which results in a corresponding suppression of the loop gain, is given by

$$\left[\frac{s}{8/\pi^2}\right]^{1/2}$$

Alternately, the limiter suppression of the output ISR with respect to the input ISR becomes

$$\frac{(\mathrm{ISR})_0}{(\mathrm{ISR})_i}$$

Together these factors allow adjustment of the model's critical values of ISR and frequency offset, at which carrier drop-lock occurs. The combined effect of the limiter can now be given as the product of these two ratios. The threshold limiter suppression product is defined as

$$S_t = \left[\frac{s}{8\pi^2}\right]^{1/2}\frac{(\mathrm{ISR})_0}{(\mathrm{ISR})_i}$$

showing that S_t is the suppression of loop gain times the suppression of interference-to-signal power ratio. Figure 2 illustrates the threshold limiter suppression product for varying carrier margin values.

IV. Effect of the Bandpass Limiter on the Drop-Lock Model

The limiter suppression product can now be used to determine the limiting effect caused mainly by the interference signal. The basic drop-lock equation can be written as

$$\mathrm{ISR} = \frac{4\pi\Delta f}{S_t K(\tau_2/\tau_1)}$$

Figures 3 through 8 show the critical ISR and frequency offset for the possible tracking loop modes of the Block IV receiver, with values of input SNR necessary to cause carrier drop-lock. Table 1 lists the various loop mode parameters used to generate the curves. Note that as the interference power increases, the limiter suppression approaches a constant level. The slope of the drop-lock equation is affected only slightly by the increasing interference power for high SNR, and unaffected for low SNR. This observation corroborates well with the trend shown in the measured data and indicates that the limiter action, overall, produces a linear translation of the drop-lock threshold.

V. Experimental Validation

Drop-lock threshold tests were conducted independently at the Telecommunications Development Lab (TDL) and at the Compatibility Test Area (CTA-21). The purpose of the tests was to validate the drop-lock model with the Block IV receiver under the conditions of interference described above. Only one tracking loop mode was tested for $2B_{LO} = 10$ Hz. Figures 9 and 10 show the comparison of the measured data obtained from TDL and CTA-21, respectively, to the theoretical curves for varying levels of SNR at the limiter input. The measurements were restricted to frequency offsets from 10 to 1000 Hz. Initial ISR power ratio was set 5 dB higher than the loop threshold SNR ($C/2B_{LO}N_O$), where C is the carrier power in watts, $2B_{LO}$ is the two-sided loop threshold noise bandwidth in Hz, and N_O is the noise-power density in watts per Hz.

VI. Conclusions

It has been shown that when a bandpass hard limiter precedes the carrier tracking loop, limiter effects on the drop-lock threshold can be calculated for variations in the input SNR and ISR power ratios. Limiter suppression of the loop gain and ISR is evidenced in the responses of the drop-lock threshold for strong input signal variations, with varying combinations of levels in the input SNR and ISR. However, as the input ISR increases, the limiter suppression stabilizes, exhibiting less sensitivity to varying levels in the input SNR.

Drop-lock calculations for the possible loop modes of the Block IV receiver indicate that the loop becomes more susceptible to interference as the loop gain and loop bandwidth increase. The predetection filter noise bandwidth is also a factor. For example, the filter provides no attenuation of the interference signal when its frequency lies within the filter passband; for this particular case, the narrow bandwidth modes only tend to reduce the drop-lock threshold. For larger interference frequency offsets outside the predetection filter passband, the calculations become more conservative due to the approximation of the filter's transfer characteristics in the basic loop model.

Experimental data from tests conducted at TDL and the CTA-21 facility show good agreement with the theoretical calculations. The model shows a tendency to underpredict at the low SNR values, which is inherent in the assumption of negligible noise in the derivation of the basic model. On the other hand, the model overpredicts for higher SNR values between 100- and 1000-Hz frequency offsets, attributable largely to the limiter model which was assumed to be ideal. The model shows the best agreement with the measured data (within 1 dB) for smaller frequency offsets and loop SNR of 10 dB or greater.

Acknowledgments

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References

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[3] J. J. Jones, "Hard-Limiting for Two Signals in Random Noise," *IEEE Transactions on Information Theory*, IT-9, pp. 34–42, January 1963.

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Table 1. Block IV receiver tracking loop modes[a]

$2B_{LO}$, Hz	Mode	K Open-loop gain, 1/sec	$\frac{\tau_2}{\tau_1}$	BW, kHz Predetection Filter
1	Narrow	9.6089×10^5	4.4340×10^{-5}	0.200
3	Narrow	9.6009×10^5	7.7057×10^{-5}	0.200
10	Narrow	9.6089×10^5	4.4343×10^{-4}	2.0
10	Wide	9.6089×10^5	4.4343×10^{-5}	2.0
30	Wide	9.6009×10^6	7.7057×10^{-5}	2.0
100	Wide	9.6089×10^6	4.4343×10^{-4}	20.0
300	Wide	9.6009×10^6	7.7065×10^{-4}	20.0

[a] From "Receiver/Exciter Block IV Equipment, Subsystem Detail Specifications," Doc. ES 505736, Rev. A (internal document), Jet Propulsion Laboratory, Pasadena, California, October 1974.

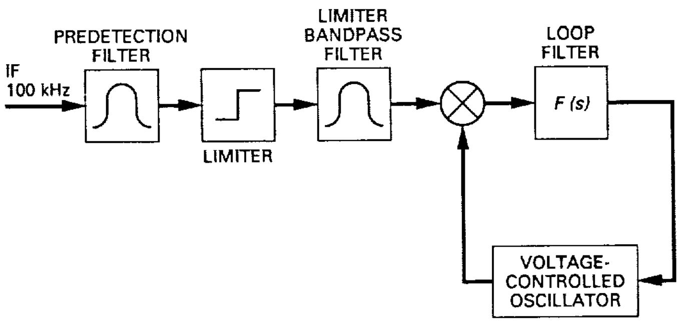

Fig. 1. Carrier tracking loop drop-lock model.

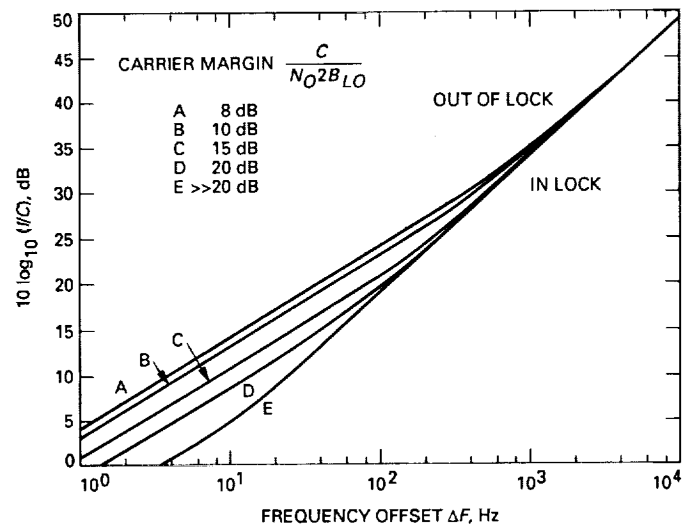

Fig. 3. Critical ISR versus frequency offset with input SNR values causing carrier drop-lock, tracking loop Mode 1, $2B_{LO} = 1$ Hz.

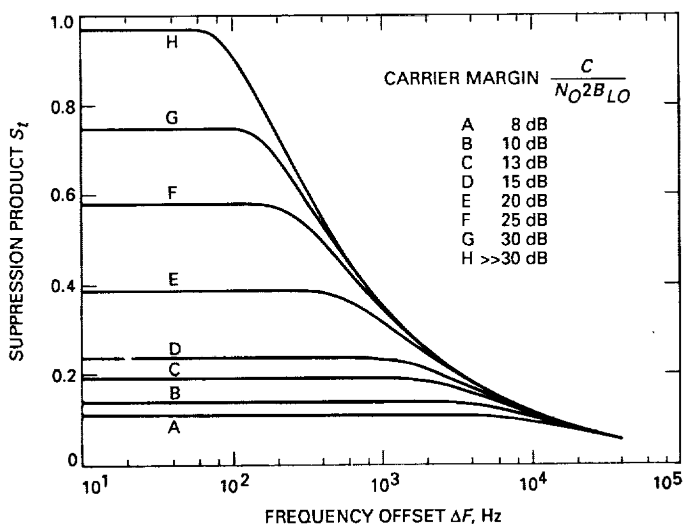

Fig. 2. Threshold limiter suppression product for various carrier margin values.

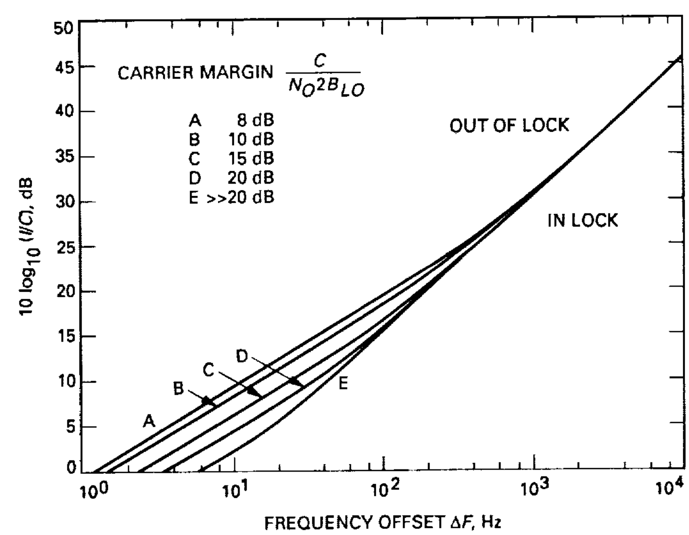

Fig. 4. Critical ISR versus frequency offset with input SNR values causing carrier drop-lock, tracking loop Mode 2, $2B_{LO} = 3$ Hz.

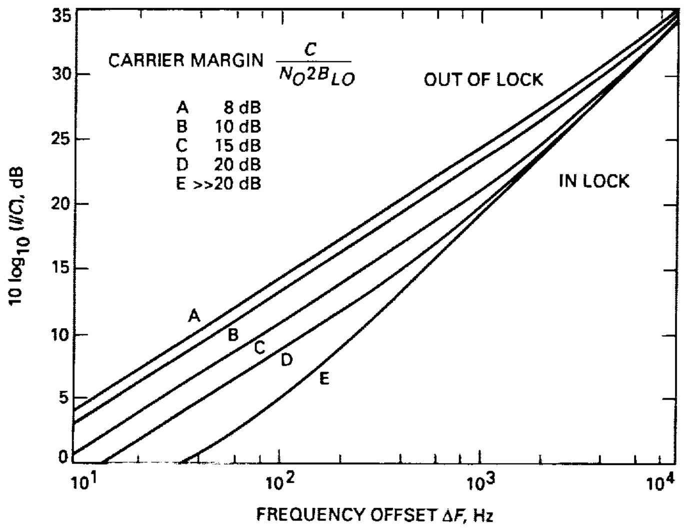

Fig. 5. Critical ISR versus frequency offset with input SNR values causing carrier drop-lock, tracking loop Mode 3, $2B_{LO} = 10$ Hz.

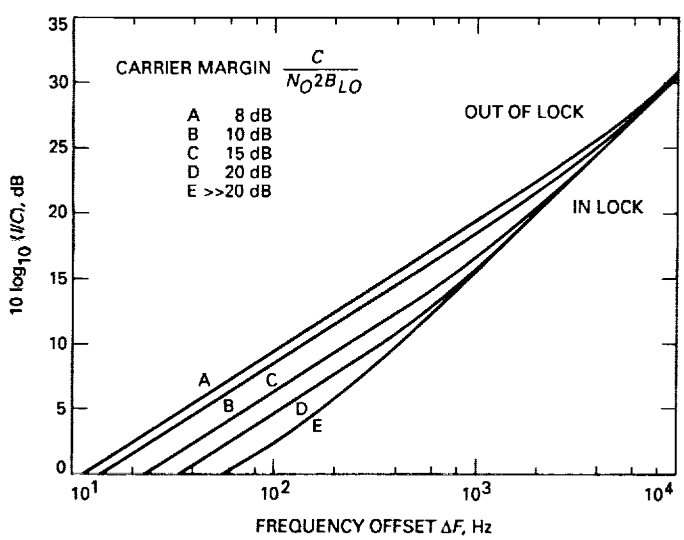

Fig. 6. Critical ISR versus frequency offset with input SNR values causing carrier drop-lock, tracking loop Mode 4, $2B_{LO} = 30$ Hz.

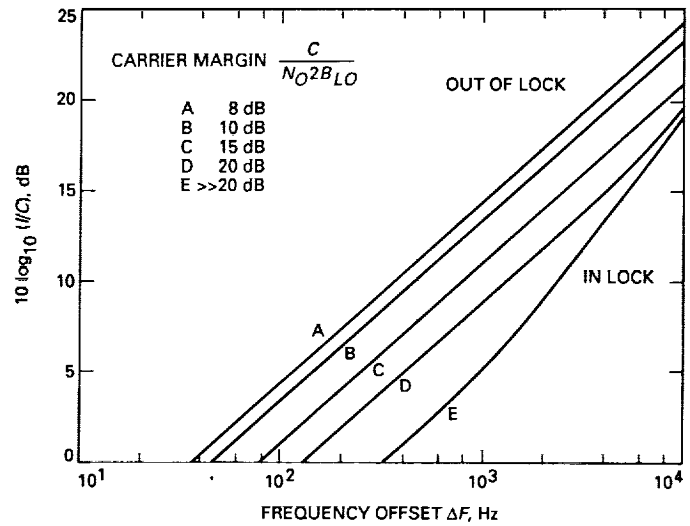

Fig. 7. Critical ISR versus frequency offset with input SNR values causing carrier drop-lock, tracking loop Mode 5, $2B_{LO} = 100$ Hz.

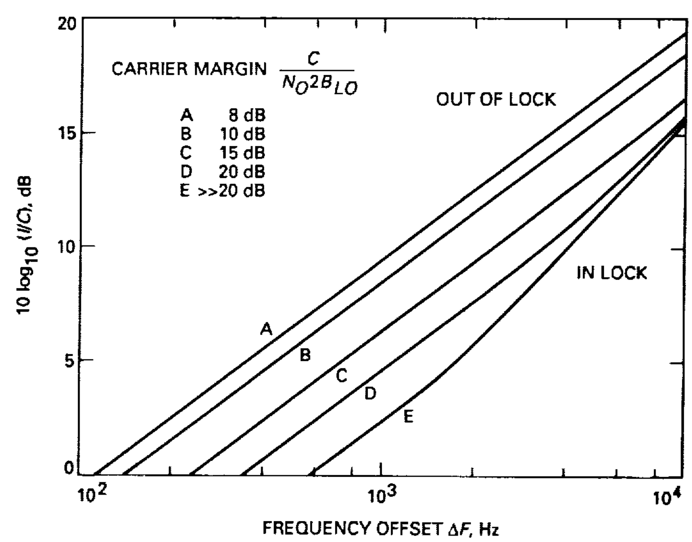

Fig. 8. Critical ISR versus frequency offset with input SNR values causing carrier drop-lock, tracking loop Mode 6, $2B_{LO} = 300$ Hz.

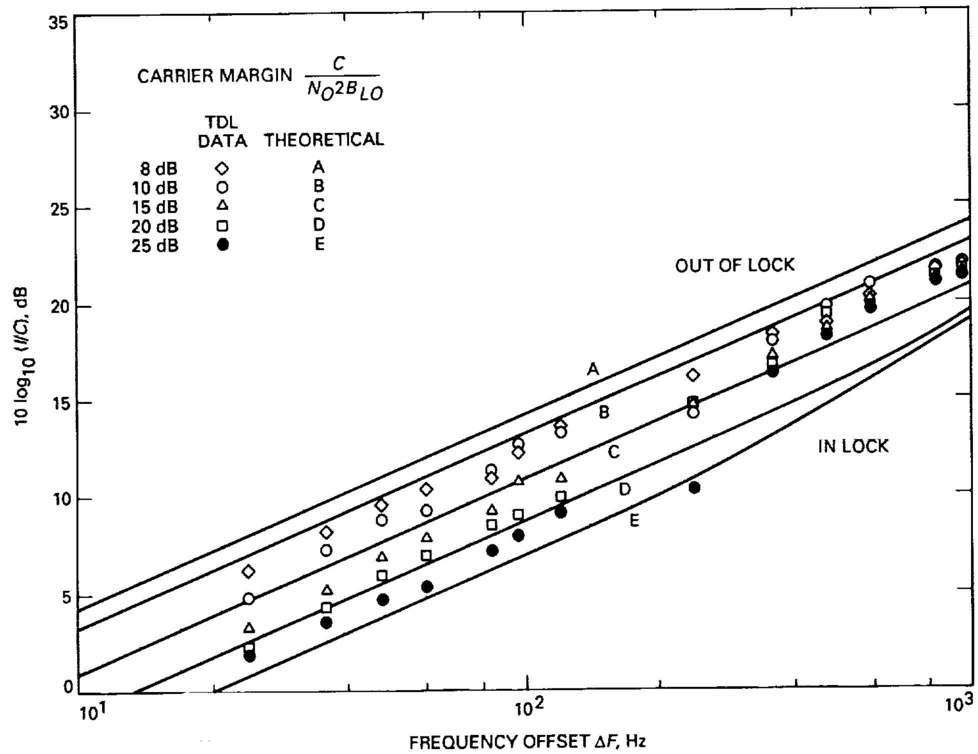

Fig. 9. Comparison of TDL data to theoretical levels of SNR at the limiter input, tracking loop Mode 3, $2B_{LO} = 10$ Hz.

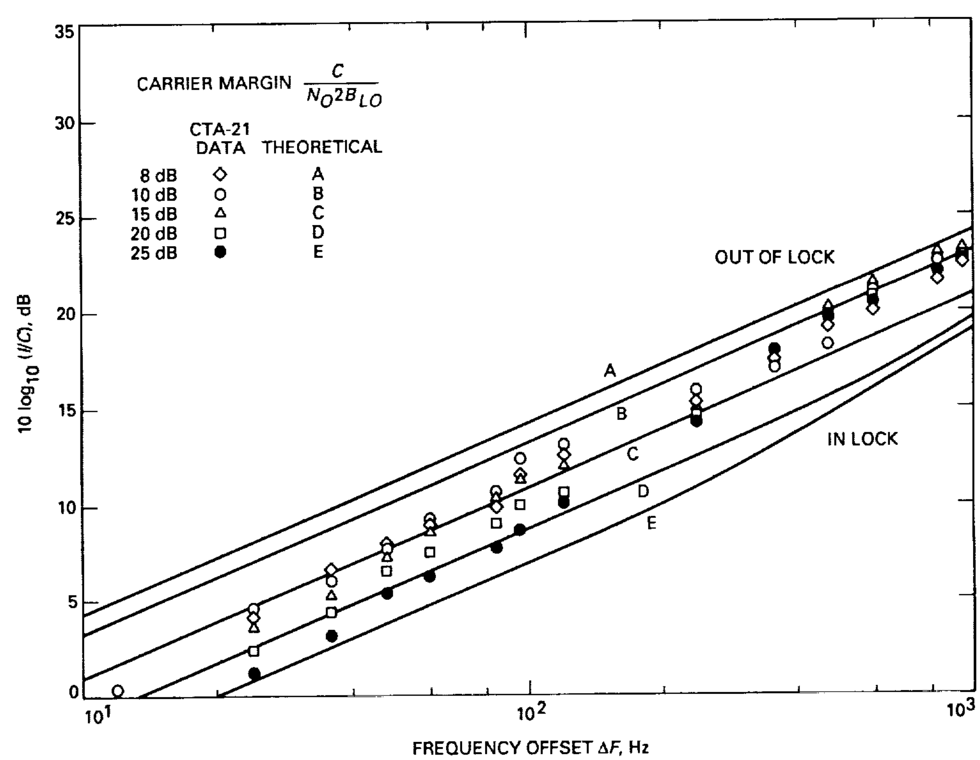

Fig. 10. Comparison of CTA-21 data to theoretical levels of SNR at the limiter input, tracking loop Mode 3, $2B_{LO} = 10$ Hz.