AN ANALYTIC MODEL OF AXISYMMETRIC MANTLE

PLUME DUE TO THERMAL AND CHEMICAL DIFFUSION

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ABSTRACT

We present an analytic model of axisymmetric mantle plumes driven by either thermal diffusion or combined diffusion of both heat and chemical species from a point source. The governing equations are solved numerically in cylindrical coordinates for a Newtonian fluid with constant viscosity. Instead of starting from an assumed plume source, we deduce constraints on the source parameters, such as the depth of the source regions and the total heat input from the plume sources, using the geophysical characteristics of mantle plumes inferred from modelling of hotspot swells. The Hawaiian hotspot and the Bermuda hotspot are used as examples. Narrow mantle plumes are expected for likely mantle viscosities. The temperature anomaly and the size of thermal plumes underneath the lithosphere can be sensitive indicators of plume depth. The Hawaiian plume is likely to originate at a much greater depth than the Bermuda plume. One suggestive result puts the Hawaiian plume source at a depth near the core-mantle boundary and the source of the Bermuda plume in the upper mantle, close to the 700 km discontinuity. The total thermal energy input by the source region to the Hawaiian plume is about 5×10^{10} watts. The corresponding diameter of the source region is about 100-150 km. Chemical diffusion from the same source does not affect the thermal structure of the plume. The chemical plume is much thinner than the thermal plume due to the much smaller diffusivities of chemical species in the mantle. Accordingly, the pure chemical signatures of the source region may only be observed near the plume center. Solutions for a two-dimensional thermal plume driven by a line source are also obtained for comparison, and the results show that dimension is more important than the likely rheologic variations for the nature of mantle plumes.

INTRODUCTION

The concept of a "hotspot", initially proposed to account for the Hawaiian volcanic chain [*Deitz and Menard*, 1953; *Wilson*, 1963] and later developed by *Morgan* [1971, 1972] as hot mantle plumes from the deep mantle, has become a very important part of today's understanding of the dynamic earth. Not only do hotspots play a large role in the evolution of oceanic lithosphere, of which up to 30-50% is within hotspot swells [*Crough*, 1983]; but more importantly, hotspots may be one of the few "windows" for us to look into the lower mantle. The apparent fixity of hotspots suggests that they may not be rooted in the near-surface convection system directly associated with plate motions [*Chase*, 1979a]. This is further suggested by the impressive correlation of hotspot distributions with the long-wavelength geoid anomalies [*Chase*, 1979; *Crough and Jurdy*, 1980]. Knowledge of the characteristics of mantle plume source regions, such as their depth, is essential to understand the implication of hotspots for the dynamic and chemical nature of the deep mantle.

Despite much work on mantle plumes, some very basic questions are still undecided. Our knowledge of the plume source regions is largely speculative. This is mainly attributable to the complexity of the problem. Also, observations are limited in number and precision, and their inversion is not obviously unique. Various simplifications have to be introduced to make the problem solvable. *Parmentier et al.* [1975] simulated the formation of mantle plumes in a cylindrical enclosure heated from the base; *Yuen and Schubert* [1976] studied thermal plumes adjacent to a vertical isothermal plate. More recently numerical solutions have been obtained for time dependent two-dimensional convective sheets with various rheology [*Christensen*, 1984; *Boss and Stacks*, 1985]. There is also some experimental work [*Olson*, 1985; *Whitehead*, 1975], but their relevance to mantle plumes is not very clear.

While the previous models are mostly restricted to two dimensions, mantle plumes are clearly three dimensional. We will show later that dimension is more important than rheologic

variations. We also need to put some constraints on the plume source parameters, which are usually taken as *a prior* condition in the earlier models. The depths assumed for plume sources range from 220 km [*Anderson*, 1981] to the core-mantle boundary [*Yuen and Peltier*, 1980; *Stacy and Loper*, 1983] in various models. On the other hand, large differences in the chemical and isotopic features among oceanic island basalts (OIB) indicate that mantle plumes may originate from various source regions [see *White*, 1985], and perhaps at various depths. Clearly, models linking the physical and chemical features observed at hotspots to the plume source regions would be very helpful in understanding the nature of mantle plumes.

We present here a model of steady-state, axisymmetric mantle plumes driven by either thermal diffusion or simultaneous diffusion of both heat and chemical species from a point source. The problem is solved in cylindrical coordinates for a Newtonian fluid with constant viscosity. Instead of starting with an assumed source region, we tried to put some constraints on the source parameters, such as the depth and size, by using the characteristics of plumes underneath oceanic lithosphere deduced from swell modelling [*Liu and Chase*, 1989; *McNutt*, 1987]. Despite the simple rheology used in this model, the results do reveal some important features of mantle plumes. In the following we shall first present our analytic model for axisymmetric mantle plumes. We will then discuss the nature of thermal plumes and see what constraints we may put on the plume source parameters. Finally we will discuss mantle plumes driven by simultaneous diffusion of heat and chemical species, and show that the pure chemical signatures of the plume source regions may only be observed near the plume center.

AXISYMMETRIC MANTLE PLUME MODEL

Mantle plumes, like other natural convective flows, are driven by the buoyancy force due to thermal gradients, or chemical gradients, or a combination of both. When the source region is much smaller than the depth of the plume, which is most likely to be true for

mantle plumes, it can be best approximated as a point source [*Jaluria*, 1982, p. 110]. For Newtonian fluid and with Boussinesq and boundary layer approximations, the governing equations for the axisymmetric plumes resulting from simultaneous diffusion of heat and chemical species from a point source can be written as [*Mollendorf and Gebhart*, 1974]:

Continuity:

$$\frac{\partial(yu)}{\partial x} + \frac{\partial(yv)}{\partial y} = 0 \tag{1}$$

Momentum Conservation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{1}{y} \frac{\partial}{\partial y}(y\frac{\partial u}{\partial y}) + g\alpha(t-t_\infty) + g\alpha^*(c-c_\infty) \tag{2}$$

Energy Conservation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = K\frac{1}{y} \frac{\partial}{\partial y} (y\frac{\partial T}{\partial y}) \tag{3}$$

Mass Conservation:

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{1}{y} \frac{\partial}{\partial y} (y\frac{\partial c}{\partial y}) \tag{4}$$

As shown in Fig.1, x, u and y, v are the coordinates and velocity components in the axial and radial directions, respectively, $\nu = \mu/\rho$ is the kinematic viscosity, g is the gravitational acceleration, T is temperature and c is the concentration of chemical species, α and α^* are the volumetric thermal and chemical expansion coefficients, respectively. K is the thermal diffusivity and D is the chemical diffusivity. The subscript ∞ denotes locations far from the plume center. Nomenclature used in this work are also tabulated in Appendix A. Boundary conditions result from the axial symmetry and asymptotic nature of the plume:

at y=0: $\frac{\partial u}{\partial y} = v = \frac{\partial T}{\partial y} = \frac{\partial c}{\partial y} = 0$; $T = T_0$, $c = c_0$

as y$\rightarrow\infty$: $u\rightarrow 0$; $T\rightarrow T_\infty$; $c\rightarrow c_\infty$ $\tag{5}$

Mollendorf and Gebhart [1974] showed that equations (1) - (5) may be turned into a set of ordinary differential equations by employing the following similarity transformations:

$$\eta = \frac{y}{x} \sqrt[4]{G_{r,t}} \; ; \quad \psi = \nu x f(\eta), \tag{6}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_0 - T_\infty} \; ; \quad C = \frac{c - c_\infty}{c_0 - c_\infty}$$

where η is a nondimensional space variable, $f(\eta)$ is the nondimensional stream function, and

$$G_{r,t} = g\alpha x^3 (T - T_\infty)/\nu^2 \; ; \quad G_{r,c} = g\alpha^* x^3 (c - c_\infty)/\nu^2 \; ;$$

$$T_0 - T_\infty = N_t x^n \; ; \qquad c_0 - c_\infty = N_c x^n \tag{7}$$

$G_{r,t}$ and $G_{r,c}$ are the Grashof numbers, N_t, N_c and n are constants. If there are no sources in the flow field and viscous dissipation is neglected, n is required to have the value of -1 by the law of conservation of energy and mass [*Mellondorf and Gebhart*, 1974]. The stream function ψ defined by

$$yu = \frac{\partial \psi}{\partial y} \; ; \qquad yv = - \frac{\partial \psi}{\partial x} \tag{8}$$

automatically satisfies the continuity equation. Substituting equations (6) - (8) into (1) - (4), we have

$$\frac{f'''}{\eta} + (\frac{1-f}{\eta})(\frac{f'}{\eta})' + \theta + (\frac{Gr,c}{Gr,t})C = 0 \tag{9}$$

$$(\eta\theta')' + Pr(f\theta' + f'\theta) = 0 \tag{10}$$

$$(\eta C')' + Sc(fC' + f'C) = 0 \tag{11}$$

where $Pr = \nu/K$ is the Prandtl number and $Sc = \nu/D$ is the Schmidt number. The boundary conditions after similarity transformations are

$$f(0) = f'(0) = \theta(0) - 1 = \theta'(0) = C(0) - 1 = C'(0) = 0$$

and $f'(\infty)$ is bounded. (12)

The two-point boundary value problem posed in equations (9) - (12) is solved numerically using the shooting method. The validity of the numerical results is checked by comparing

results from both a standard fourth-order Range-Kutta integration scheme with fixed step length and the fifth-order Range-Kutta integration scheme with adaptive step size [*Press et al.*, 1984]. We also reproduced the results of *Mollendorf and Gebhart* [1974] for low Pr and Sc (Pr: 0.7 - 7; Sc: 0.1- 700). For these ranges of low Pr and Sc values, the numerical scheme is quite stable. Fast convergence and good accuracy for velocity, temperature and chemical concentration fields are generally obtained, provided a reasonably close guess of the starting value of f'(0) is used in the shooting scheme. The velocity profiles are generally a few times wider than the temperature profiles. However, when Pr and Sc values increase to those relevant to the mantle, the numerical scheme becomes fragile. One of the reasons is that when Pr and Sc are huge, independent variables become very different in numerical magnitude, thus making numerical errors easy to cumulate and, since they are coupled, easy to propagate. Accuracies of $f(\eta)$ and its derivatives often have to be traded off with the stability of the numerical scheme. As a result the velocity field is generally poorly resolved, and shows noticeable differences for the two different integration schemes. On the other hand, the temperature field and the chemical field are generally well resolved. In the following section we show some results for mantle plumes driven by thermal diffusion alone and discuss the possible constraints we may put on the plume source regions.

THERMAL PLUMES AND CONSTRAINTS ON SOURCE REGIONS

The importance of thermal diffusion in the formation of mantle plumes is explicit in the term "hotspot". Indeed, even though for some hotspots chemical diffusion may play an important role [*Hofmann and White*, 1982; *Presnall and Helsley*, 1982], thermal perturbation will likely still be the dominant driving mechanism. In this section we show some results of mantle plumes driven by thermal diffusion alone. In this case equation (11), the last term in equation (9), and the associated boundary conditions vanish from the governing equations. The physical

behavior of the plume at any location can be obtained in terms of nondimensional variables once the governing equations are solved, provided Q_t, the total thermal energy input by the source region is given:

a.) Temperature anomaly at plume center:

$$\Delta T = T_0 - T_\infty = \frac{N_t}{x} \tag{13}$$

where

$$N_t = \frac{Q_t}{2\pi\mu C_p I_t} ; \qquad I_t = \int_0^\infty f'(\eta)\theta(\eta)d\eta$$

b.) Vertical velocity:

$$u = \frac{\nu}{x} \sqrt{G_{r,t}} \; (\frac{f'}{\eta}) \tag{14}$$

c.) Radius of thermal plume (also called thickness of thermal boundary layer, defined at θ = 0.01):

$$\delta_t = \eta_e \; [\; \frac{2\pi C_p I_t \mu\nu^2}{g\alpha Q_t} \;]^{\frac{1}{4}} \sqrt{x} = \eta_e \; [\; \frac{\nu^2}{g\alpha N_t}]^{\frac{1}{4}} \sqrt{x} \tag{15}$$

where η_e is the value of η corresponding to θ = 0.01.

d.) Mass flow rate:

$$m = 2\pi\mu x f(\infty) \tag{16}$$

Some mantle parameters used in this work are presented in Table 1. Our goal here is to look for inverse solutions. Since the above properties of mantle plumes underneath lithosphere can be constrained directly by the surface observations, such as topography and geoid anomaly, and deduced from models of thermal swells [Liu and Chase, 1989; McNutt, 1987; Yuen and Fleitout, 1985], we would like to use these results to constrain the mantle plume source parameters.

Before we do the inversion, it is helpful to evaluate the plume parameters we are going to use. The temperature anomaly, ΔT, is probably one of the best constrained plume quantities. ΔT of the Hawaiian plume beneath the lithosphere is estimated to be in the range of 200-300

°C from various models [*Yuen and Fleitout*, 1985; *McNutt*, 1987; *Liu and Chase*, 1989]. The radius of the thermal plume, δ_t, underneath the lithosphere is not clear, but at least we may have a good upper limit as indicated by the "size of hotspot", defined by *Jackson et al.* [1972] as the area of simultaneous volcanic activity. The plume velocity is difficult to deduce directly from surface observations in the swell models, but the resolution of the velocity field is poor in this model. Other studies also show that while the thermal structure of mantle plumes does not strongly depend on the chosen mantle rheology, the velocity is very sensitive to rheologic variations [*Li et al*, 1983a]. Similar problems are associated with the mass transfer rate. Therefore, instead of seeking inverse solutions in the "least-square" sense, we think the best approach is to avoid using the unreliable constraints as much as we can under the circumstances.

First we try to estimate the depth of plume source regions. From equation (14) we have $N_t = x\Delta T$, substitute it into (15) and rearrange:

$$d = \frac{\delta_t^4 \, (g\alpha\Delta T)}{\eta_e^4 \, \nu^2} \tag{17}$$

Here we use d to denote the depth of plume source region, in order to avoid confusion with the spatial variable x. Accordingly, δ_t and ΔT in this equation should be the values beneath the lithosphere. This equation shows that the depth of plume source regions may be constrained by the two better-known plume parameters, ΔT and δ_t. Notice that d varies with δ_t^4, thus the predicted depth of the plume source is very sensitive to the size of the thermal plume beneath the lithosphere.

Viscosity also has a strong effect on the plume structure, lower viscosities corresponding to narrower plumes. As shown in Fig. 2, narrow mantle plumes are expected for relevant mantle rheology. The value of $\mu = 10^{22}$ poise from glacial rebound studies [*Cathles*, 1975; *Peltier*, 1976] is commonly used for normal mantle. Studies of 2-D plumes using temperature

dependent rheology shows that the viscosity at plume center can be 2 - 3 orders of magnitude smaller than the ambient mantle [Yuen and Schubert, 1976; Li et al., 1983b].

For a given mean plume viscosity, we may use equation (17) to estimate the depth of plume source regions. Table 2 show one example. The plume quantities, δ_t and ΔT, deduced from modeling of hotspot swells [Liu and Chase, 1989] are used to bracket the range of the depth of the source regions for the Hawaiian and Bermuda plumes. The average plume viscosity is taken to be $\mu = 10^{20}$ poise here, although the actual effects of the temperature- and pressure-dependent viscosity may not be so straight forward. As one can see from Table 2, the source region of the Hawaiian plume appears very deep, close to the core-mantle boundary, while the source of the Bermuda plume is much shallower.

The major problem here is the uncertainties of δ_t. They are undoubtedly much smaller than the half-wavelength of the hotspot swells, since flow is deflected radially as the plume impinges on the base of lithosphere [Sleep, 1986]. The diameter of the Hawaiian hotspot, defined as the area of simultaneous volcanic activity, is about 300 km [Jackson et al., 1972], imposing a closer upper bound. Diameter of the Hawaiian plume below the lithosphere is estimated to be 150 km to 60 km by various authors [Morgan, 1972; Presnall and Helsley, 1982]. We think $\delta_t = 100$ km is probably a safe upper limit for the radius, and this would put the Hawaiian plume source at a depth close to the core-mantle boundary (see Table 2). Assuming δ_t is proportional to the corresponding swell wavelength, comparing the size of the Hawaiian swell and the Bermuda rise suggests that δ_t for the Bermuda plume is about 60 - 80 km. Together with the smaller temperature anomaly of the Bermuda plume deduced from the swell model [Liu and Chase, 1989], the source region of the Bermuda plume is constrained to be generally in the upper mantle, near the 700 km discontinuity.

The estimated depths in Table 2 are only suggestive, since small errors in δ_t will cause large errors in the predicted plume depth. What we may gain here with confidence is the relative depth of these two plumes. It is clear from equation (17) and Table 2 that d is very

sensitive to δ_t and ΔT. The significantly greater values of δ_t and ΔT for Hawaiian plume, as constrained by the surface observations in the swell model [*Liu and Chase*, 1989], clearly indicate that its source region is much deeper than that of the Bermuda plume. One alternative explanation for the lager δ_t of the Hawaiian plume would be that its source region is larger. However, the greater temperature anomaly of the Hawaiian plume still predicts a deeper source region.

Another source parameter of interest is Q_t, the total thermal energy input from the source region. Q_t could be calculated directly from equation (14), once plume depth is determined. Large uncertainties of d will, however, severely overshadow the validity of Q_t obtained this way. Fortunately, there is a better way to do it. Notice in our model of a steady-state plume, there is no heat sources within the flow and viscous dissipation is neglected. Thus by the law of energy conservation, the total thermal energy convected in the plume must not vary down stream [*Mollendorf and Gebhart*, 1974; *Jaluria*, 1982]:

$$Q(x) = \int_0^\infty \rho \, C_p(T - T_\infty)\mu(2\pi y)dy = \text{Constant} = Q_t \qquad (18)$$

where $Q(x)$ is the total thermal energy convected at x. Thus we may use the heat supplied to the lithosphere by mantle plumes to approximate Q_t. In the case of Hawaiian hotspot, heat flux distribution beneath the lithosphere can be constrained from surface observations, such as topography, heat flow anomalies and geoid anomalies [*Liu and Chase*, 1989; *McNutt*, 1987; *Sandwell*, 1982]. Integrating heat flux from the Hawaiian plume constrained from our swell model [*Liu and Chase*, 1989], we find that Q_t of the Hawaiian plume is about 5 x 10^{10} watts. The total heat loss from the core is around 2.5 x 10^{12} watts [*Stacy and Loper*, 1983; *Gubbins et al.*, 1979, *Loper*, 1978], which is approximately the total heat flow carried by mantle plumes as estimated by *Davies* [1988]. The heat flow from the core estimated by *Sleep et al.* [1988] is slightly lower (about 1.1 x 10^{12} watts). Thus the heat input from the source region of the Hawaiian plume is about 1/30 - 1/50 of the total heat loss from the core. This is probably one

of the reasons we do not observe many hotspots as strong as the Hawaiian hotspot, especially if base-heating is the dominant mechanism for plume formation [*Parmentier et al.*, 1975].

Knowing Q_t, we may also estimate the size of the plume source regions. Assuming the heat flux from the source region is a few, say 2 to 4, times stronger than the heat flux from the mantle plume beneath the lithosphere, then the source region of the Hawaiian plume is approximately 100 - 150 km in diameter, not much smaller than the plume beneath the plate. This, however, may be an upper bound for the size of the plume source. The heat flux from the source region must be very strong, at least for the Hawaiian plume, to allow the mantle plume to penetrate a large part of the mantle and not be wiped out by the background convection. With a constant Q_t, a stronger heat flux from the source region implies a smaller size of the source.

We can estimate Q_t and the size of source region for the Bermuda plume in the same way. They are about 1 x 10^{10} watts and 60-80 km, respectively. However, constraints for calculating heat flux to the base of the lithosphere at the Bermuda rise are poorer than those of Hawaiian hotspot [*Liu and Chase*, 1989].

SIMULTANEOUS DIFFUSION OF HEAT AND CHEMICAL SPECIES

Natural convection may also be driven by chemical gradients. Chemical and isotopic studies of ocean island basalts (OIB) have lead various authors to propose that mantle plumes may have originated from fragments of ancient subducted oceanic crust [*Chase*, 1981; *Hofmann and White*, 1982], or from residual peridotite, formed when oceanic basalts are produced and accumulated in the deep mantle after being subducted [*Presnall and Helsley*, 1982]. Although there are still a lot of controversies regarding the origin of OIBs, the fact that their chemical and isotopic features are significantly different from that of the mid-ocean ridge basalts

(MORB) clearly indicate that their source regions are different from that of MORB [e.g., *White*, 1985; *Allegre*, 1982].

Chemical diffusion alone may not be sufficient to produce significant mantle plumes. It is most likely coupled with thermal diffusion. Plumes due to simultaneous diffusion of heat and chemical species are described in equations (1) - (5). Thermal and chemical fields are both coupled with the velocity field, as shown in the equation of momentum conservation, but are decoupled from each other. It can be proved that, for the given conditions in equation (5), the chemical field is related to the thermal field as [*Boura and Gebhart*, 1976]:

$$C = \theta^{Le} \tag{19}$$

where $Le = K/D$ is the Lewis number. Exact values of the diffusivity for various chemical species in the mantle are not clear, but in any case they must be much smaller than the thermal diffusivity.

It is suggested that at the temperature of particular interest for isotopic homogenization between phases (1300°C - 1400°C), chemical diffusion may be characterized by a single diffusion coefficient [*Zindler and Hart*, 1986]. Experimental work suggests the value of 10^{-13} cm^2 s^{-1} for solid phases and 10^{-7} cm^2 s^{-1} for melt phases [*Hofmann and Hart*, 1978]. Thus even in melts, D is still about 5 orders of magnitude smaller than K. Fig. 3 shows one of the resultant thermal and chemical profiles in the model plume. It is not surprising to see that the chemical boundary layers are much thinner than the thermal boundary layer.

Figure 3 has important implications regarding the generation of OIB magma. If pressure-release partial melting occurs in the upwelling mantle plume, a large part of the melts may come from the surrounding mantle, because the thermal plume is wider. Pure chemical signatures of the plume source region may only be observed in a restricted area near the plume center. However, the radial-deflection of the plume head beneath the lithosphere makes the effective chemical radius harder to estimate.

A large range in chemical character is observed in OIBs even from a single volcanic island [*West et al.*, 1987; *Zindler et al.*, 1984]. It has been suggested that the Hawaiian basalts are produced by a mantle source with primitive and enriched components mixing with common MORB source [e.g., *Chen*, 1987; *Chen and Frey*, 1983, *Feigenson*, 1986]. Melts may come from both the mantle plume and the lithosphere, and the evolution may differ from one hotspot to the other since the mechanism of heating the lithosphere may be different for various hotspots [*Liu and Chase*, 1989]. The generation of OIB is obviously a complicated problem and much more work is needed. This model predicts the general fields of temperature and concentration of chemical species in the mantle plumes, thus may be helpful for further investigation of this problem.

DISCUSSION

We have presented a model of axisymmetric mantle plumes driven by diffusion of heat and chemical species from a point source. Unlike most previous models where plume source parameters are used as *a priori* condition, we have tried to put constraints on source parameters using physical quantities that can be directly constrained by the surface observation. We have shown that the size and temperature anomaly of plumes beneath the lithosphere can be sensitive indicators of plume depth. The Hawaiian plume is likely much deeper in origin than the Bermuda plume. Chemical diffusion does not affect the thermal structure of plumes, and the chemical boundary layer is much thinner than the thermal boundary layer. Various simplifications are introduced into this model. We now discuss briefly the effects these simplifications may have on our results.

One of the major simplifications in our model is the constant viscosity. This is a sacrifice in order to obtain analytic expressions for the physical characteristics of plumes, which are needed for the source-parameter inversion. Previous models have suggested that

variable viscosity is important to produce narrow mantle plumes [Parmentier et al., 1975; Yuen and Schubert, 1976]. In this model we have no difficulty in producing narrow mantle plumes using constant viscosities relevant to the mantle, likely a result of the 3-D nature in this model. Nonetheless, our results do show a strong influence of viscosity on the thermal and chemical profiles in the mantle plume. Lower viscosity is associated with narrower plumes, as shown in Fig. 3.

Using temperature- and pressure-dependent viscosity will certainly cause some modification of the predicted thermal and chemical profiles, but not much may be gained as far as our goal here is concerned. Because of the high sensitivity of predicted plume depth to the size of the thermal plume beneath the lithosphere, as shown by the $d - \delta_t^4$ relation in equation (17), uncertainties associated with δ_t may overshadow any improvement of calculating the exact plume depth by using more sophisticated rheology. For the same reason, the conclusion that the Hawaiian plume is much deeper than the Bermuda plume is unlikely to change with more realistic rheology, since δ_t and ΔT of the Hawaiian plume is clearly greater than that of the Bermuda plume [Liu and Chase, 1989; Davies, 1988; Detrick et al., 1986]. Similar arguments may be applied to justify our choice of using Newtonian fluid, although there are some suggestions that cubic-power fluid may be more relevant to the mantle [Schubert, 1976].

Since the temperature anomaly and, especially, the size of thermal plumes beneath the lithosphere can be sensitive indicators of plume depth, they may be used to estimate the relative depth of mantle plumes. Knowing the relative depth of mantle plumes are critical in terms of understanding the dynamics and chemical structures of the mantle. The strong correlation of hotspot distribution with long-wavelength geoid anomalies [Chase, 1979; Crough and Jurdy, 1980] indicate that these mantle plumes may embedded in the lower mantle convective system. Studies of hotspot proper motions show that their direction of motion are consistent with the gradient of the positive long-wavelength geoid anomalies [Chase and Liu,

1985]. Combining these results with relative depth of these mantle plumes may help us to draw a three-dimensional picture of the pattern of convection in the lower mantle.

Another simplification used in this model is the point source approximation, although it is mathematically a proper choice when the source region is much smaller than plume depth. However, the real source regions are, after all, a few tens to over one hundred kilometers in diameter. So we may have overestimated δ_t using the size of hotspots. Correction for this would put all predicted depths in Table 2 to smaller values. But again, the conclusion of relative depth for these two plumes will not be changed.

Other results of this work, such as total thermal energy input from source regions and the relations of the chemical plumes to the thermal plumes, are not strongly dependent on choice of rheology parameters in the model. Notice we also assume steady-state plumes in this model. This is probably adequate as far as our goal of constraining the plume source region is concerned, since here we are only interested in fully developed mantle plumes. The surface observations, such as topography and heat flow anomalies, which are used to deduce constraints for this model, are also long term time-averaged results.

While a lot of efforts have been devoted to bring more sophisticated rheology into plume models, our knowledge of mantle rheology is still at such an early stage that not many conclusive remarks can be made regarding various rheologic parameters. On the other hand, the dimension of plumes and thus the associated boundary conditions are so important for determining plume structure that they may outweigh any effects due to relevant variations of mantle rheology. To illustrate this we have also studied the problem of 2-D plumes driven by a line source, and the results are very different from the axisymmetric plumes. For instance, in the 2-D plumes the depth of the plume source is *proportional* to the size of the thermal plume beneath the lithosphere:

$$d = \frac{\delta_t Q}{4\eta_e(\Delta t)\mu C_p I_t} \tag{20}$$

Here Q is thermal energy input per unit length. This is significantly different from the d-$\delta_t{}^4$ relation for axisymmetric plumes as shown in equation (17). The temperature at the plume center drops with $x^{-3/5}$ in the 2-D plumes, which is slow compared to the $\Delta T \sim x^{-1}$ relationship in the axisymmetric mantle plumes (see equation (14)). Overall, the 2-D plumes tend to be much thicker than the 3-D plumes, as one would expect. One numerical result shows that, if the Hawaiian plume is generated near the core-mantle boundary, then the diameter of the "line source" has to be very small, around 15-20 km.

The nature of plume source regions and how can they last for millions of years, like the Hawaiian plume, are questions that need much more work. However, no matter what the cause would be, as long as the sources are restricted to a relatively small region, this model offers an excellent first order approximation. This or similar models should be the base for further mantle plume modelling to take into consideration the 3-D nature of mantle plumes.

CONCLUSIONS

An analytic model of steady-state, axisymmetric mantle plumes driven by either thermal diffusion or simultaneous diffusion of heat and chemical species is solved in cylindrical coordinates for a Newtonian fluid. We have tried to constrain the plume source parameters using physical characteristics of mantle plumes deduced from swell modelling [*Liu and Chase*, 1989; *McNutt*, 1987]. The following are some conclusions we may draw from this work:

1. Narrow mantle plumes are expected for relevant mantle rheology. Viscosity has a strong influence on plume structures. Lower viscosity is associated with narrower mantle plumes.

2. The depth of plume sources may be estimated from the temperature anomaly at the plume center, ΔT, and especially, radius of thermal plume, δ_t. The Hawaiian plume may originate at great depth, probably near the core-mantle boundary. The source of the

Bermuda plume is likely much shallower, perhaps close to the 700 km discontinuity. The numerical values in the Table 2 are only suggestive due to the uncertainty of the values of δt. However, the significant greater values of δ_t and ΔT of the Hawaiian plume clearly indicate that its source region is much deeper than that of the Bermuda plume.

3. Total thermal energy, Q_t, from the plume sources may be estimated by integrating the heat flux supplied to the lithospheric base by plumes over the hotspot. In the case of the Hawaiian plume, Q_t is about 5×10^{10} watts, which is about 1/30 to 1/50 of the total heat loss from the core. The corresponding size of the source region of the Hawaiian plume is about 100-150 km in diameter.

4. Simultaneous diffusion of chemical species does not affect the thermal structure of mantle plumes. Due to the much smaller chemical diffusivities, the chemical boundary layer is much thinner than the thermal boundary layer. As a result of this, the pure chemical signatures of the source region may only be observed near the plume center. Composition of oceanic island basalts likely varies from pure plume chemistry to mixture with melted oceanic lithosphere or asthenosphere away from the center of the plume.

5. Dimension is probably more important than variation of mantle rheology for the nature of mantle plumes. Two-dimensional plumes tend to be much thicker than the axisymmetric three-dimensional plumes.

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Table. 1. Physical Parameters used in this work

Parameters	Definitions	Values
α	Volumetric thermal expansion coefficient	$3.5 \times 10^{-5}\,°K^{-1}$
K	Thermal diffusivity	$8 \times 10^{-7}\,m^2s^{-1}$ ($0.8 \times 10^2\,cm^2\,s^{-1}$)
p	Mantle density	$3300\,kg\,m^{-3}$ ($3.3\,g\,cm^{-3}$)
g	gravitational acceleration	$10\,m\,s^{-2}$ ($1000\,cm\,s^{-2}$)
C_p	Heat capacity	$1256\,J\,kg^{-1}°K^{-1}$ ($0.3\,cal\,g^{-1}\,°K^{-1}$)

Table 2. Estimation of The Depth of Plume Source Regions

δ_t(km)	Plume Depth (km)	
a. Hawaiian Plume	(ΔT = 250°K)	(ΔT = 300°K)
80	1254.3	1505.2
90	2009.1	2411.0
100	3062.2	3674.7
b. Bermuda plume	(ΔT = 100°K)	(ΔT = 200°K)
60	158.8	317.5
70	294.1	588.2
80	501.7	1003.4

Note: Depth is measured from lithospheric base downward. The assumed mean viscosity of the plume is $\mu = 10^{20}$ poise.
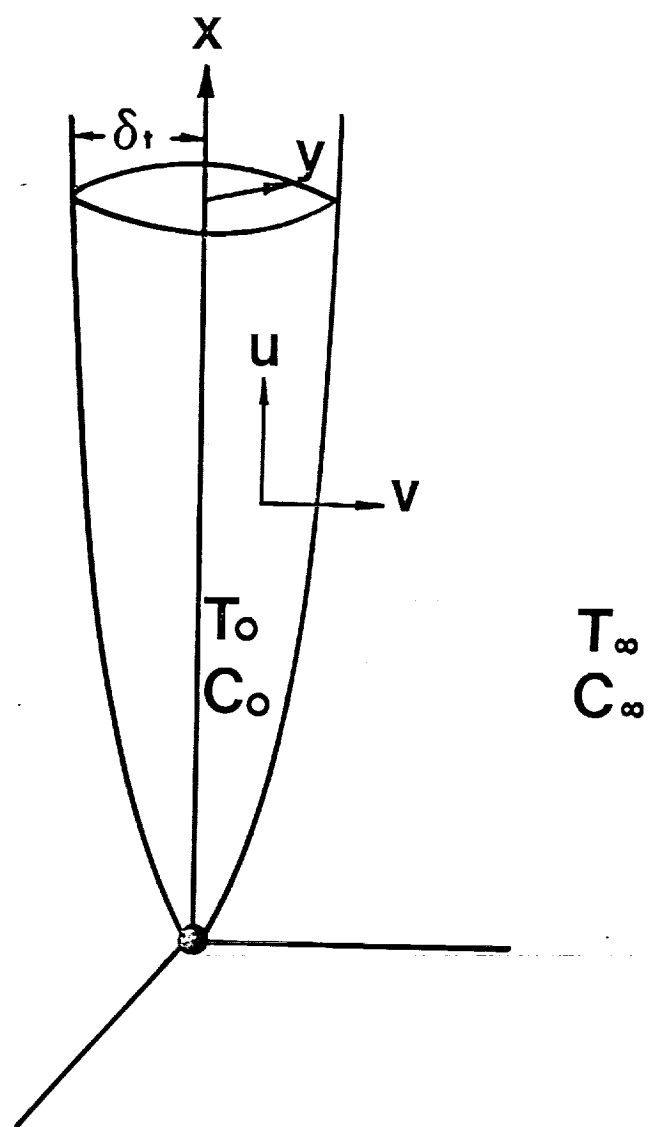
Appendix A. Nomenclature

c	concentration of diffusing chemical species
$C(\eta)$	concentration excess ratio, $(c-c_\infty)/(c_0-c_\infty)$
C_p	heat capacity
D	chemical diffusivity
$f(\eta)$	nondimensional stream function
g	gravitational acceleration
$G_{r,c}$	Grashof number, $g\alpha^* x^3(c_0-c_\infty)/\nu^2$
$G_{r,t}$	Grashof number, $g\alpha x^3(T_0-T_\infty)/\nu^2$
K	thermal diffusivity
Le	Lewis number
Pr	Prandtl number
Qt	total thermal energy input by the point source
Sc	Schmidt number
T	temperature
α	volumetric coefficient of thermal expansion
α^*	volumetric coefficient of expansion with concentration
δ_t	thickness of thermal boundary layer (or radius of the thermal plume)
η	nondimensional spatial variable
$\theta(\eta)$	temperature excess ratio, $(T-T_\infty)/(T_0-T_\infty)$
ρ	normal mantle density
μ	absolute viscosity

ν kinematic viscosity

ψ stream function

0   subscript denoting location on plume axis

∞ subscript denoting location far from plume axis

FIGURE CAPTIONS

Figure 1.   Coordinate system for an axisymmetric mantle plume driven by diffusion of heat and chemical species from a point source. x, u and y, v are the coordinates and velocity components in the axial and radial directions, respectively. T_0 and c_0 denote the temperature and concentration of chemical species on the plume axis. T_∞, c_∞ are the temperature and chemical concentration far from the plume axis. δ_t is the thickness of the thermal plume (defined at $\theta = 0.01$).

Figure 2.   Effects of viscosity on the structure of the thermal plumes. Height of the plume is measured upwards from the plume source. Radius is the thickness of the thermal plumes (= δ_t). Radius = 0 is the plume axis. The numbered curves mark the boundaries of the thermal plumes calculated for different mean plume viscosities. 1: $\mu = 10^{20}$ poise; 2: $\mu = 10^{21}$ poise; 3: $\mu = 10^{22}$ poise. See text for more discussion.

Figure 3.   Temperature and chemical concentration fields in a model plume, calculated for Pr = 3.75 x 10^{23}. θ and C are the nondimensional temperature and chemical concentration, respectively. η is the nondimensional spatial variable ($\eta = 0$ corresponding to the location of the plume axis). C_1 and C_2 corresponding to chemical fields for Le = 10^3 and 10^5, respectively. Notice that the chemical plumes are much thinner than the thermal plume due to the low diffusivities of chemical species in the mantle.
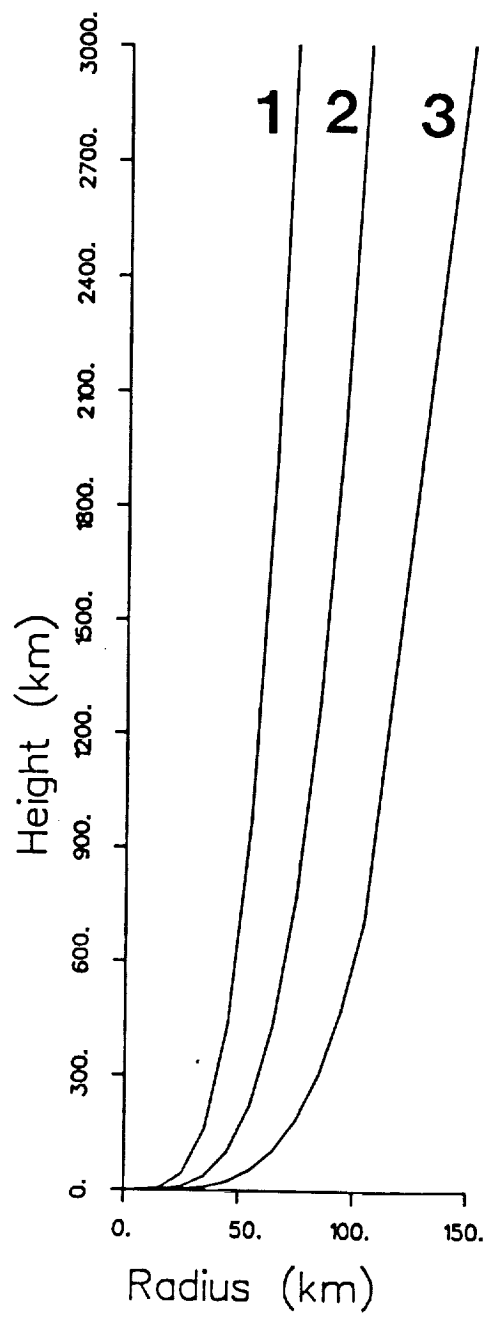
Fig. 3

θ

C_1

C_2

$\eta (\times 10^{-8})$

θ, C

1.00 0.75 0.50 0.25 0.00

0.00 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10 0.11 0.12 0.13 0.14