TOWARD HIGHER ORDER TESTS OF THE GRAVITATIONAL INTERACTION

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Analyses and interpretations of experiments which test post-Newtonian gravity are usually done within the assumption that gravity is a metric field phenomenon – a manifestation of space-time geometry. This, however, is unnecessary and one can start at a more primitive level – that there simply exists a phenomenological, gravitational, many-body equation of motion which must be determined by a package of observations. In fact, over the last couple decades, a diverse collection of solar system interbody tracking observations, supplemented by data from the binary pulsar system PSR 1913 + 16, has completely mapped out the first post-Newtonian order (order $\frac{1}{c^2}$ gravitational equations of motion for photons ($\vec{r}_p$) and particles ($\vec{r}_j$)), yielding (in a particular coordinate system):

$$\frac{d^2\vec{r}_p}{dt^2} = G\sum_j \frac{m_j}{r_{pj}^3}\left(2\,\vec{r}_{jp} - 4\,\vec{r}_{jp}\cdot\hat{c}\;\hat{c}\right)$$

$$\frac{d^2\vec{r}_i}{dt^2} = G\sum_j \frac{m_j\,\vec{r}_{ji}}{r_{ij}^3}\left[1 + \frac{1}{c^2}\left(v_i^2 + 2v_j^2 - 4\vec{v}_i\cdot\vec{v}_j - \frac{3}{2}\left(v_j\cdot\hat{r}_{ij}\right)^2\right)\right]$$

$$+\frac{G}{c^2}\sum_j \frac{m_j}{r_{ij}^3}\left[4\vec{r}_{ij}\cdot\vec{v}_i\,\vec{v}_i + 3\vec{r}_{ij}\cdot\vec{v}_j\,\vec{v}_j - 4\vec{r}_{ij}\cdot\vec{v}_i\,\vec{v}_j - 3\vec{r}_{ij}\,\vec{v}_j\,\vec{v}_i\right]$$

$$+\frac{G^2}{c^2}\sum_{jk} m_j m_k\left[\frac{\vec{r}_{ik}}{r_{ik}^3}\left(\frac{4}{r_{ij}} + \frac{1}{r_{jk}}\right) - \frac{(7/2)\vec{r}_{jk} + (1/2)\,\vec{r}_{jk}\cdot\hat{r}_{ij}\,\hat{r}_{ij}}{r_{jk}^3\,r_{ij}}\right]$$

Actually, the photon equation of motion is of Newtonian order: the high speed of photons ($v \sim c$) produces effects which are post-Newtonian — proportional to inverse powers of c. Each of the many observations constrains some linear combination of the numerical coefficients which appear in the equations of motion (including some coefficients which are zero and don't appear above); the coefficients in the end becoming empirically fixed to accuracies which range from a percent to a part in 10^7.

After the fact, using these empirically determined equations of motion, along with some observed properties of nongravitational clocks and rulers and conservation laws for energy, momentum and angular momentum, a post-Newtonian Lagrangian can be constructed, a geometrical space-time metric field conceptual interpretation can be developed, Lorentz invariance of the equations of motion can be shown (the same equations can be used in all cosmic inertial frames), and the equations of motion are found to agree with the predictions of Einstein's gravitational theory, General Relativity, within experimental accuracy.

These fully mapped-out equations of motion include the so-called "gravitomagnetic" terms — those three terms above that are proportional to both $\vec{v_i}$ (velocity of body being accelerated) and $\vec{v_j}$(velocity of other source bodies). Since the gravitomagnetic terms have such a historically interesting and conceptually unique interpretation (that moving matter "drags" our very inertial reference frames), a variety of experiments have been proposed to directly see these components (without participation of other components) of the above gravitational equations of motion (Ciufolini, Everitt, Mashoon, in these proceedings).

Future work in first post-Newtonian order gravity appears to primarily be two-fold: to improve the accuracy of the map of these first post-Newtonian order equations of motion; and to perform new redundant tests of the equations. A failure to confirm these equations in new contexts would require a radical revision of our basic physical assumptions.

At higher levels of precision, second post-Newtonian order $\left(\text{order } \frac{1}{c^4}\right)$ corrections must be made to the gravitational interaction and testing gravitational theory.

An indication that we are beginning to need $2PN$ order gravity, in order to properly interpret solar system phenomena, is the remarkable alignment of the Sun's spin axis (about 5 arc-degrees) with the solar system angular momentum vector after 4.5 billions years of existence. This implies that the Newtonian gravitational interaction is spatially isotropic (directionally independent) to a part in 10^{13} accuracy (Nordtvedt 1987) even in the presence of asymmetries in the solar system's environment — a nearby galaxy with gravitational potential $\left(\frac{GM}{c^2R}\right)_{galaxy} \sim 10^{-6}$ and a speed of the solar system (w) relative to the cosmic rest frame $\left(\frac{w}{c}\right)^2_{solar\ system} \sim 4 \cdot 10^{-6}$. The fact that both the above dimensionless environmental numbers, when squared, exceed 10^{-13} means that the spin axis history of the Sun requires the $2PN$ order gravitational interaction for its proper analysis, and in fact imposes constraints on the structure of $2PN$ order gravity.

Second-order light deflection experiments are being studied (Reasenberg in these proceedings) which will probe $2PN$ order gravity.

Since the binary pulsar system PSR 1913+16 is believed to be a pair of neutron stars in close orbit, and since the internal gravity of neutron stars is very strong, $\left(\frac{GM}{c^2r}\right)_{neutron\ star} \sim 0.1$, it is plausible that second (and higher!) post-Newtonian order gravity would be relevant to understanding the pulse-arrival-time data from such systems. We developed a formalism to examine that question (Nordtvedt 1985) in which internal gravity of celestial bodies was treated nonperturbatively while interbody orbital dynamics was treated at first post-Newtonian order. We found that under simple assumptions, *e.g.*, Lorentz invariance of the gravitational interaction, *etc.*, and accepting the first post-Newtonian order experimental constraint on the parameterized post-Newtonian (PPN) coefficients, $(4\beta - 3 - \gamma)_{exp} \approx 0$, then the equations of motion of the binary pulsar system become identical to the above exhibited $1PN$ order equations of motion for test bodies: *i.e.*, the orbital dynamics of compact celestial bodies with strong internal gravity are not efficient probes of $2PN$ order gravity.

Consequently we have begun development of a theoretical framework for analysis and design of $2PN$ order gravitational experiments and observations. At this initial state, we assume properties of gravity which have strongest empirical support (that there exist conservation laws for energy, momentum, and angular momentum, and that the gravitational interaction is Lorentz invariant), but otherwise we start with the most general possible phenomenological $2PN$ order $\left(\text{order } \frac{1}{c^4}\right)$ gravitational many-body Lagrangian as a supplement to the $1PN$ order equations of motion exhibited above. A main goal of this framework is to discover what new degrees of freedom can exist in the $2PN$ order gravitational interaction under these assumptions, and what types of experiments could measure these new aspects of gravity. This framework assumes no particular theory of gravity; it is a framework for testing gravitational theory generally.

At $2PN$ order the gravitational Lagrangian consists of four, dimensionally speaking, generic classes of terms:

$$L^{2PN} = \frac{1}{16} \sum_i \frac{m_i v_i^{\ 6}}{c^4} + L_2\left(\frac{Gm^2v^4}{c^4 r}\right) + L_3\left(\frac{G^2m^3v^2}{c^4 r^2}\right) + L_4\left(\frac{G^3m^4}{c^4 r^3}\right)$$

with m, v and r representing body masses, body velocities, and interbody distances. $L_{2,3,4}$ can be thought of as being two-body, three-body, and four-body interactions, respectively, although all these terms contribute to systems consisting of only two bodies.

We have found that $L_2\left(\frac{Gm^2v^4}{c^4 r}\right)$ is uniquely determined by $1PN$ order gravity plus the assumption of Lorentz invariance — no new degrees of freedom appear in this part of the $2PN$ order Lagrangian (Benacquista and Nordtvedt 1988). Under the same assumptions, $L_3\left(\frac{G^2m^3v^2}{c^4 r^2}\right)$ has been found to contain only one new degree of freedom. This new parameter could be measured by a second order light deflection experiment. While $L_4\left(\frac{G^3m^4}{c^4 r^3}\right)$ cannot be constrained by Lorentz invariance, consistency with the isotropy observations of the Newtonian gravitational interaction suggests that L_4 will have two new degrees of freedom. The challenge facing the experimental and observational future in gravity is to find ways to measure these new aspects of gravity which will contribute to $2PN$ order body dynamics.

REFERENCES

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DISCUSSION

SHAPIRO: Can you explain in more detail the sense in which you infer post Newtonian isotropy from the (small) inclination of the sun's spin angular momentum vector to the solar system's angular momentum vector?

NORDTVEDT: If the gravitational interaction between the matter in the oblate, rotating Sun is not spatially isotropic to a part in 10^{13}, self-torque would have precessed the Sun's axis by more than its present alignment during the past 4.5×10^{9} years. Since 10^{-13} is a smaller number than the square of the galactic potential or the fourth power of the Sun's speed through the cosmos, second post-Newtonian order gravity is constrained by this observation.