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PULSAR TIMING AND GRAVITATIONAL WAVES

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In the last few years, several researcher Detweiler, 1979, Hellings et al. 1983, Romani, et al, 1983 and Davis, et al., 1985) have used timing data from pulsars to search for ultra-low frequency (ULF) gravitational waves (waves at periods from a few days to a few years), especially for the waves making up the stochastic cosmic background such waves. It is the purpose of this talk to discuss how these limits are obtained and to point out several precautions that must be taken in the analysis of these data.

In pulsar timing, the times of arrival of pulses are measured and compared with a model. The UTC times of arrival τ are transformed to TDB times of arrival τ via well-known algorithms (Hellings, 1986 and Backer, et al., 1986). The t's are related to the TDB times of emission T by

$$ct = cT + \mathbf{k} \cdot (\mathbf{R} - \mathbf{r}) - (1 + \gamma) \sum_p \frac{GM_p}{c^2} \ln \left[\frac{\mathbf{k} \cdot \mathbf{r}_p + r_p}{\mathbf{k} \cdot \mathbf{R}_p + R_p}\right]$$

where $\mathbf{R} = \mathbf{R_0} + \mathbf{V}T$ is the location of the pulsar at time T, $\mathbf{r}$ is the position of the radio observatory on the earth, $\mathbf{k}$ is a unit vector toward the pulsar, and $\mathbf{r_p}$ and $\mathbf{R_p}$ are the position of intervening body p relative to the earth and the pulsar, respectively, at the time when the signal passes closest to the body. The $(1+\gamma)$ term is of course the Shapiro time delay, with PPN parameter γ parametrizing the curvature of space. The position of the observatory may be written as $\mathbf{r} = \mathbf{q} + \xi$, where $\mathbf{q}$ is the position of the center of the earth, determined from numerically-integrated planetary ephemerides, and ξ is the geocentric position vector of the observatory, determined from observatory coordinates and from a model of the physical ephemeris of the earth.

The actual times of arrival may be compared with the predicted times of arrival to give timing residuals, δt. Among the noise sources contributing to these residuals might be the variation of the spacetime metric created by the passage of a gravitational wave. The rate of change of the timing residuals will be proportional to the dimensionless amplitude of the wave

$$\frac{d}{dt}(\delta t) = \frac{\Delta \nu}{\nu} = h(t).$$

If there is only a single pulsar being observed, then the spectral density of cosmic gravitational waves is simply less than or equal to the spectral density of the residuals. However, if there are several pulsars being observed over the same period of time, it is possible to dig into much larger noise to detect the gravitational wave noise source since it will be a common signal in the time series for each pulsar. Thus we may write the frequency residuals from the i^{th} pulsar as (Hellings et al., 1983)

$$\frac{\Delta \nu_i}{\nu} = \alpha_i h(t) + n_i(t),$$

where α_i contains geometrical factors, resulting from the relation of the polarization and propagation vectors of the gravitational wave to the line-of-sight from the earth to the pulsar, and $n_i(t)$ is the independent noise in the data from each pulsar. Cross-correlating the data from pulsars i and j, one finds

$$v^{-2} \langle \Delta v_i \Delta v_j \rangle = \alpha_i \alpha_j \langle h^2 \rangle + \alpha_i \langle h n_j \rangle + \alpha_j \langle h n_i \rangle + \langle n_i n_j \rangle,$$

where the brackets indicate cross-correlation. Since n_i and n_j are independent of each other and of h, all of these terms will tend to zero as the square-root of the number of data points except for $\langle h^2 \rangle$, which is the autocorrelation function of the gravitational wave amplitude.

Using data from the single millisecond pulsar, PSR1937+21, limits have been set[4] for gravitational waves of periods less than one year. Using data from several quiet normal pulsars, limits were set[2] using the cross-correlation technique at periods from a few months to five years. These limits are compared with other direct limits and with possible critical energy densities in Figure 1 (Zimmerman et al., 1980).

There is one caution which must be observed in analysis of pulsar data. This is that in order to reduce the timing residuals to the levels that appear in the literature, several deterministic signals have had to be subtracted away. These signals correspond to unknown (and therefore erroneous) values for the period, period derivatives, position, proper motion, and possible parallax of the pulsar and, as data accumulates for the most precise pulsars, the parameters of the earth's orbit and perturbing solar system parameters. The point of this for gravitational wave analysis is that there might have been enormous gravitational wave signatures in the data originally, but, if there had been, they would have been subtracted away by adjusting one of the adjustable parameters of the model.

The method which must be used to take this process into account is to treat the parameter adjustment process as a data filter and to compute the transfer function of the filter. A transfer function is the function which multiplies the input spectrum, frequency be frequency, to produce the output spectrum. The spectrum of the post-fit residuals must therefore be divided by this transfer function to give the realistic limits that may be inferred on the original gravitational wave noise in the timing data records.

Blandford, et al. (1984) computed the transfer function for a filter that adjusted the pulsar parameters only. We have recently worked out the transfer function for a combined adjustment of the pulsar parameters and adjustment of solar system parameters, consistent with the level at which these parameters are known from other solar system astrometric data. Since the solar system model is based on numerical integration, it was not possible to produce an analytical expression for this transfer function. Rather a Monte Carlo analysis was performed in which twenty years of pulsar timing data were simulated, one point per week, and these data added to the combined set of solar system data while all parameters, pulsar and solar system, were adjusted. Twenty such simulated data sets were analysed and the pre- and post-fit power spectra were compared to get the transfer function for each set. A mean transfer function was found as a average of the twenty transfer functions. The results of this analysis are shown in Figure 2. It should be noted that there has been a noticeable subtraction of power at Mars's orbital period and that other longer period planetary perturbations combine to subtract almost all power at periods longer than about five years. The strong absorbtion line at one year combines period planetary perturbations combine to subtract almost all power at periods longer than about five years. The strong absorbtion line at one year combines uncertainty in Earth orbital parameters and uncertainty in pulsar position and proper motion (the latter $t \sin t$ parameters acting to keep the line relatively broad).

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Figure 1. Limits on the spectrum of cosmic gravitational radiation energy density from several direct gravitational wave experiments. The line labeled "critical densities" represents the locus of peaks of a set of broad-band spectra, each of which would provide a critical energy density. The line labeled "PULSARS" is from the analysis of Hellings and Downs (1983). The line labeled "1937+21" comes from the results of Davis et al (1985).

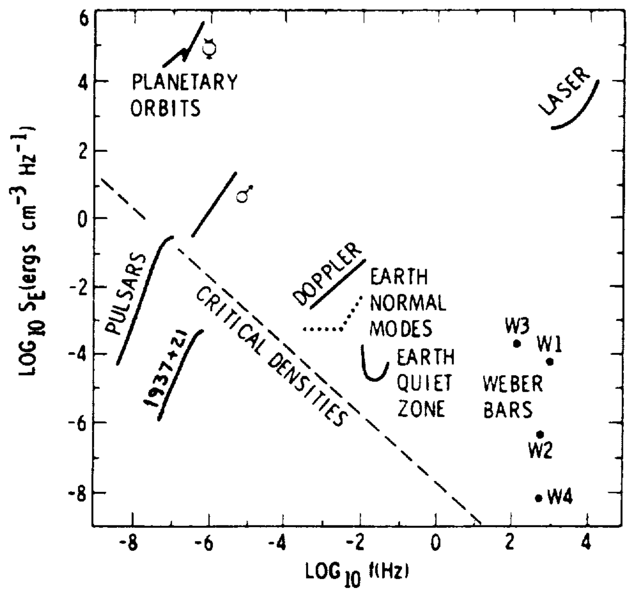
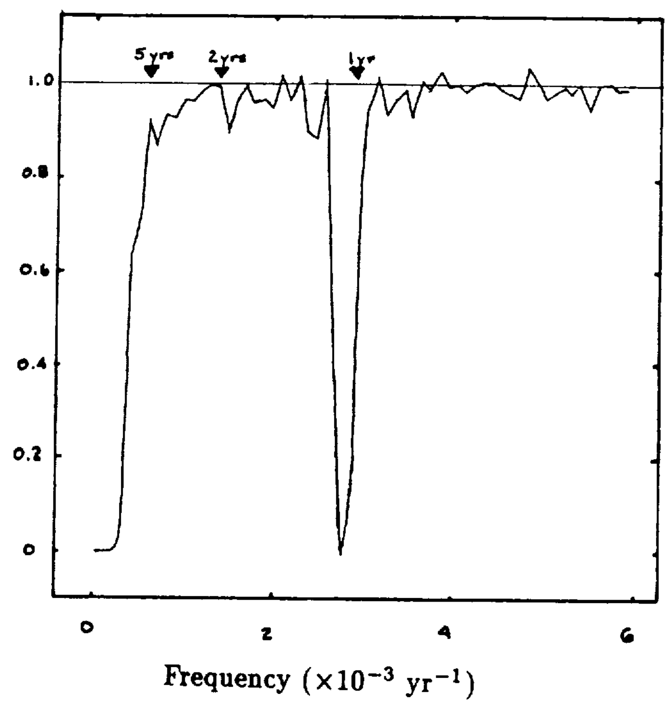


Figure 2. Mean transfer function of the solar system data analysis filter. Ordinate is relative power. Abscissa is frequency in inverse days.



DISCUSSION

SCHUTZ: Could you clarify one point please? Although an increasing sum of data may not lower the minimum frequency at which you can set limits, presumably it does continue to improve limits on the gravitational wave background at higher frequencies?

HELLINGS: Yes.

TREUHAFT: Would VLBI positions of the millisecond pulsar help eliminate parameters from your fit of pulsar data?

HELLINGS: Yes. Roger Linfield at JPL has some data to do that in the can, but it hasn't yet been analyzed. Of course, this assumes a tie between the VLBI reference frame and the planetary ephemeris reference frame, in which the timing positions will be given.