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An Applicational Process for Dynamic Balancing of Turbomachinery Shafting

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SUMMARY

The NASA Lewis Research Center has developed and implemented a time-efficient methodology for dynamically balancing turbomachinery shafting. This methodology minimizes costly facility downtime by using a balancing arbor (mandrel) that simulates the turbomachinery (rig) shafting.

This report discusses in detail the need for precision dynamic balancing of turbomachinery shafting and for a dynamic balancing methodology. It also discusses the inherent problems (and their causes and effects) associated with unbalanced turbomachinery shafting as a function of increasing shaft rotational speeds. Included in this discussion are the design criteria concerning rotor weight differentials for rotors made of different materials that have similar parameters and shafting. The balancing methodology for applications where rotor replaceability is a requirement is also covered. This report is intended for use as a reference when designing, fabricating, and troubleshooting turbomachinery shafting.

INTRODUCTION

The need for a shortened dynamic balancing process results from increased interest in more highly productive turbine and compressor facilities. Testing facilities have been established for evaluating a wide variety of turbine and compressor rotors with similar airflow parameters, diameters, and vane thicknesses. Rotor replaceability without rig disassembly (to obtain rig shafting for rotor balancing) is a key to these highly productive facilities. The methodology for dynamic balancing of turbomachinery shafting described herein allows use of a balancing arbor to identically simulate rig shafting (minus the rotor) with respect to weight, size, and shape. This balancing arbor, when properly balanced, will be identical to the rig shaft and have a maximum dynamic unbalance equal to the accuracies of the dynamic balancing machines. The maximum accuracies or resolutions of the dynamic balancing machines equal 0.000020-in. displacement.

In turbomachinery, where shaft rotational speeds range from 5000 rpm to more than 120 000 rpm, high centrifugal forces intensify with the degree of unbalance. High centrifugal forces cause accelerated bearing wear from nonuniform bearing loading as well as increased coupling fatigue from turbomachinery misalignment. Turbomachinery shafting operating with high vibrations as a result of shaft unbalances can lead to a dangerous condition. If the shaft unbalance is large enough to cause a bearing or coupling failure, control of the turbomachine will be lost. The possible aftermath of such a failure could be total machine destruction. The cost of a turbomachinery bearing or coupling failure is unaffordable, since most turbomachines are estimated to cost $250 000 or more.

When a shaft is rotated, centrifugal forces are developed. These forces are amplified when the rotating shaft is unbalanced. The magnitude of these centrifugal forces is determined by the mass of the shaft (including the rotor), the radius of the unbalance, and the shaft rotational speed. It can be calculated from the rotational form of Newton's second law (centrifugal force equation) (ref. 1).

$$F_c = \frac{mrw^2}{u} \tag{1}$$

In the past, balancing was achieved only through flexible shaft and rotor balancing techniques or field balancing of the turbomachinery shafting. Several techniques for flexible shaft and flexible rotor balancing are readily available. Field balancing techniques are also readily available for various flexible shaft and rotor and rigid turbomachinery shaft unbalance problems. This report discusses, for the first time, the balancing of rigid turbomachinery shafting at turbomachinery shaft rotational speeds by using current shaft balancing methodology. It covers techniques, methodologies, and predictable unbalances that are apparent at higher turbomachinery shaft speeds. Centrifugal loading as a function of the balancing machine's resolution and turbomachinery shaft rotational speeds is predicted. Other turbomachinery design considerations are also predicted. These include centrifugal loading multipliers, acceleration magnification factors, and maximum allowable rotor weight differentials, all of which are functions of turbomachinery shaft rotational speed. In designing a turbomachinery facility all of these design criteria will generally be considered.

SYMBOLS

A_g acceleration magnification factor

a acceleration, in./s^2

C_m centrifugal loading multiplier

D rotor weight differential

F_c centrifugal force, lbf

g gravitational constant, in./s^2

M_o mass offset, oz in.

M_r mass removed (or added) at r_i, oz

m mass, lbm

m_o mass offset, oz

m_s mass graduation setting, oz

m_1 first mass offset, oz

m_2 second mass offset, oz

O_a balancing arbor pilot offset, in.

O_m resolution of balancing machines, in.

O_r rig shaft pilot offset, in.

O_t total offset, in.

r radius of unbalance or mass offset, in.

r_e final material radius setting, in.

r_i initial material radius setting, in.

r_t shaft machining tolerance (full indicator reading), in.

r_1 radius of first mass offset, in.

r_2 radius of second mass offset, in.

T_d total displacement, oz in.

U_m underbalanced mass offset, oz in.

u unit conversion factor

W_i initial checkout rotor mass, lbm

W_n new test rotor mass, lbm

w shaft rotational speed, rpm

BACKGROUND

Origin of Unbalanced Shafting

Major turbomachinery shafting unbalances originate from tolerances associated with the casting and the machining of shafts and the assembly and reassembly of shafts and bearings. An unbalance occurs when the rotational axis is not concentric and coplanar with the principal axis (inertia axis, mass-offset axis, or mass axis). The tolerances for cast shafts are 0.125 in. (full indicator reading) for small castings to 1.000 in. (full indicator reading) for large castings. Ideally, the mass axis should be identical to the rotational axis. Most turbomachinery shafts require machining after being cast. Machining tolerances for turbomachinery shafts range from 0.0001 to 0.0050 in. (full indicator reading) depending on the turbomachinery shaft weight, speed, and application. Most shafts machined below the 0.0010-in. (full indicator reading) tolerance are not cost effective. Shaft machining tolerances above 0.0050 in. (full indicator reading) are considered inadequate for turbomachinery operation because high centrifugal loads develop during operation.

Most turbomachinery rotor shafting requires assembly. After the shaft has been assembled, it is balanced as a unit. Occasionally, the shafts need to be disassembled before they are installed into the turbomachinery rig. Serious vibrations can occur when the shaft is reassembled and installed in the turbomachinery rig without due care in attaining the identical part-to-part realignment (match mark) required to achieve acceptable shaft balance repeatability. Proper attention to match-marked part realignment is critical because a few degrees of misalignment can create detrimental shaft unbalance.

Turbomachinery shaft unbalances are usually responsible for turbomachinery vibrations. However, vibrations are not always caused by unbalanced turbomachinery shafting. Turbomachinery vibrations can also result from worn or insufficient bearings and couplings, worn or damaged gear teeth, inadequate casing and shaft stiffness, shaft and hardware misalignment, critical speeds, damaged facility shafting, loosening and shifting of components at their pilots from centrifugal forces, insufficient tolerances in gear tooth couplings, damaged bearings, and damaged rotor blades. Facility preventive maintenance and health monitoring are also obtainable through the charting of turbomachinery vibrations. Vibration charts are used as a tool in field balancing turbomachinery shafting and in troubleshooting excessive and intolerable facility vibrations. Facility vibrations are usually measured by accelerometers and are represented in the form of displacements, velocities, or accelerations. With the wide frequency range available from accelerometers, exact vibration locations and thus apparent problems can be isolated. The changes in vibration frequency spectrums are ideal analysis tools for troubleshooting and locating excessive turbomachinery vibrations.

When turbomachinery vibrations escalate and become a problem, facility shaft unbalances are usually considered first in resolving the problem. If proper turbomachinery shaft balancing procedures are followed, the turbomachinery vibrations can be attributed to any of the vibration sources previously listed. The cause of turbomachinery vibrations can be isolated by using perfected machine vibration analysis techniques. These techniques have been established and are readily available (ref. 2). They are also used in field balancing and in troubleshooting excessive turbomachinery vibrations.

Classification of Unbalanced Shafting

A perfectly balanced turbomachinery shaft would be ideal but is unrealistic and unattainable. Even after a shaft has been balanced, some unbalance is still apparent in the shaft that the balancing machines cannot isolate. This is the residual (final) unbalance. Before balancing, a shaft can usually be defined as statically unbalanced, dynamically unbalanced, couple unbalanced, or quasi-statically unbalanced. Shafts that are statically (single plane) unbalanced have their central mass axis parallel to the shaft rotational axis. The mass axis is radius r from the rotational axis center of gravity (fig. 1). Statically unbalanced shafts when rotated tend to have equally loaded bearings with bearing loads in identical directions. The dynamically (two plane) unbalanced shaft's mass axis intersects the rotational axis (fig. 2). Rotating dynamically unbalanced shafts causes unequal unidirectional bearing loads. Turbomachinery shafts are generally balanced dynamically, since centrifugal forces are the largest when dynamically unbalanced. A couple unbalance results from the mass axis intersecting the rotational axis at the shaft axis center of gravity (fig. 3). Shafts rotated with a couple unbalance generate a couple force that tends to turn the shaft end over end. The bearing loads are equal but in opposite directions. Quasi-statically unbalanced shafts have characteristics of static, dynamic, and couple unbalanced shafts. In quasi-statically unbalanced shafts, the mass axis intersects the shaft rotational axis at a point other than the shaft axis center of gravity (fig. 4). During rotation the unbalanced centrifugal forces create a couple reaction that tends to turn the shaft end over end. The bearings are loaded in opposite directions with unequal forces. Shafts balanced quasi-statically usually support thin rotors or disks because they are difficult to balance dynamically.

Dynamic Balancing

The NASA Lewis Research Center presently has three balancing machines available to balance rotors and turbomachinery shafting. Since a typical rotor-shaft assembly weighs from 1 to more than 200 lbm, these balancing machines along with existing field balancing equipment and techniques suit the Center's needs.

The largest capacity balancing machine is the Schenck balancer (fig. 5), which can handle shaft weights from 20 to 5000 lbm. The operational shaft rotational speed range for this balancer is 600 to 1400 rpm with a maximum speed of 3500 rpm. The midrange-capacity balancing machine is a Hoffman balancer (fig. 6) with a 3- to 1000-lbm shaft weight range, an operating range of 1000 to 1500 rpm, and a maximum shaft speed of 2000 rpm. The smallest balancing machine is also a Hoffman balancer (fig. 7). It has a shaft weight range of 1 to 250 lbm, an operating range of 1500 to 2000 rpm, and a maximum shaft speed of 4000 rpm. Shaft balancing speeds below 300 rpm are generally not recommended because of the balancing machine's sensitivities.

Two balancing operational settings are available, dynamic (two plane) balancing or quasi-static force couple balancing (ref. 3). Turbomachinery shafts are typically balanced dynamically (in two planes), although some applications require quasi-static force couple balancing (for thin rotors or disks).

The following measuring equipment is used in conjunction with the balancing machines: angle indicator, angle reference generator, angle datum marks, vector measuring device, and component measuring device. The angle of the unbalance is specified by an angle indicator. The angle reference generator produces a signal that defines the angular position of the shaft. Shafts are marked to denote an angle reference called an angle datum mark. Vector and component measuring devices gage and display unbalance in terms of angle and mass offsets.

Turbomachinery shafts are balanced by removing or adding material in correspondence with readings obtained from polar mass-offset maps and mass-offset angle indications on the balancing machine operator's display. Mass graduation settings and material radius settings are jointly used to minimize shaft unbalance. Initially, the mass graduation is set on the highest scale and lowered whenever the mass-offset angle indicator can no longer register on an isolated mass offset and respective angle of displacement. The initial material radius setting is the radius desired for material removal or addition. The material radius setting will be adjusted in concurrence with the mass graduation settings, although shaft material will always be removed or added at the initial material setting. The quantity of material removed from the material radius setting is equal to the product of the mass offset, the mass graduation setting, and the ratio of final material radius setting to initial material radius setting. The mass removal equation is therefore stated as

$$M_r = \frac{m_o m_s r_e}{r_i} \tag{2}$$

where $r_e = r_i$ for the initial reading. Material is removed at the angle specified by the angle reference generator or added 180° from this angle. This procedure continues for repeatedly smaller mass graduation and material radius settings.

A balancing machine reaches its limit when it is unable to detect and indicate a minimum amount of unbalance. This limit is called the minimum response, or resolution. The resolution is equivalent to the detectable difference between the axis of rotation and the mass axis and is typically approximately 0.000020-in. displacement (fig. 8). The unbalance remaining in the shaft is called the minimum residual unbalance.

Turbomachinery shafts normally are balanced without their bearings. Instead, bearing spacers are employed. The bearing spacers are precision-ground sleeves that are similar in size and weight to rig bearings. Bearing spacers have outside diameters comparable to and inside diameters identical to the turbomachinery bearing's inner race diameters. These diameters must also be true running to achieve an effective shaft balance. Using bearing spacers alleviates possible turbomachinery bearing damage and balancing errors attributed to bearing inaccuracies. Before balancing, the shaft parameters must be entered into the balancing machine, and the balancing machine instrumentation must be installed and adjusted. The shaft is then balanced to the balancing machine's accuracies. A shaft is considered optimumly balanced when the detectable unbalance is below or equal to the allowable total displacement. The allowable total displacement for a shaft is the product of the balancing machine's resolution and the shaft weight.

$$T_d = mO_m \tag{3}$$

Balancing Machine Accuracies

New technology requirements for faster turbomachinery shaft speeds result in newly uncovered turbomachinery balancing problems. As the shaft speeds on these new turbomachines increase, so will the centrifugal shaft loads due to minimal offsets and unbalances. As stated previously, the present balancing machines have accuracies (resolutions) of 0.000020-in. displacement. Thus, the maximum or residual offset that a rotor-shaft assembly balanced to balancing machine accuracies could have would equal the 0.000020-in. displacement. When these turbomachinery shafts are rotated, centrifugal forces will develop from the mass-axis-to-axis-of-rotation offset. Centrifugal loading at high shaft rotational speeds may exceed the manufacturer's suggested bearing loads and could lead to bearing failure and turbomachinery rig destruction. The centrifugal loading due to the balancing machine's resolution is obtained by letting r equal the balancing machine's resolution of 0.000020-in. displacement and substituting relevant parameters into the centrifugal force equation:

$$F_c = \frac{mw^2}{1\ 759\ 584\ 204}\ \text{lbf} \tag{4}$$

where m is in pound mass and w is in revolutions per minute.

A shaft is usually balanced to the limits of the balancing machine, which include the balancing machine's resolution of 0.000020-in. displacement and the maximum balancing shaft rotational speed. Shaft rotational speeds at the lower end of the turbomachinery regime and with the present accuracies and limits of the balancing machines may not be a problem when evaluating bearing limitation due to centrifugal loading. At higher shaft rotational speeds centrifugal loading caused by the limits of the balancing machine's resolution and the shaft rotational speed may become a problem. These centrifugal loads are a factor in turbomachinery hardware design and should be considered (ref. 4).

Unbalances Due to Shaft Machining Tolerances

Balancing arbors with size, weight, and shape comparable to those of the facility rig shafting are preferred but are not always necessary. However, the balancing arbor's pilot, bearing pilot diameters, and bearing locations must be identical to those of the actual rig shaft in order to minimize shaft unbalances. The balancing arbor must be machined with dimensions and tolerances the same as or more precise than those of the rig shaft in order to ensure comparable shaft pilots and bearing locations.

The typical allowable offset for machining turbomachinery shafting and pilots is 0.0001 to 0.0050 in. (full indicator reading). Problems arise when turbomachinery shafts are rotated without proper balancing. The problems result from shaft loading that is directly attributed to rotor offset mass rotation. If turbomachinery shafting were not balanced prior to operation, a mass-axis-to-axis-of-rotation offset would exist and be equal to the turbomachinery shaft machining tolerance. Centrifugal bearing loading under these conditions can be calculated from the centrifugal force equation by letting r equal the shaft machining tolerance r_t:

$$F_c = \frac{mr_t w^2}{35\ 191.64807}\ \text{lbf} \tag{5}$$

where r_t is in inches. Centrifugal loading as a function of unbalanced turbomachinery shafting can now be calculated when given shaft machining tolerances, rotor weights, and shaft rotational speeds.

The maximum centrifugal bearing loads determine the maximum acceptable shaft rotational speeds at a given rotor weight and shaft machining tolerance--

but are only applicable when shaft or rotor assembly prebalancing is neglected. It becomes obvious that centrifugal loading accelerates rapidly if the rig shaft machining tolerances are allowed to be greater than 0.0010 in. (full indicator reading). Therefore, the maximum allowable centrifugal loading should be checked before determining shaft machining tolerances if turbomachinery shaft prebalancing is going to be neglected.

DYNAMIC BALANCING METHODOLOGY

Balancing arbors accommodate turbomachinery test rigs where frequent rotor replaceability is desired. These balancing arbors also need to be precision balanced to prevent turbomachinery vibrations. The following procedure for balancing turbomachinery shafting when using a balancing arbor assumes the use of a second checkout rotor, which is convenient for rechecking the balancing arbor at a later date:

(1) Assemble the facility rig shaft without the rotor and with all of the associated shaft rotational hardware and precision bearing spacers.

(2) Match mark the assembled parts of the facility rig shaft's rotational hardware with respect to each other and an analogous 0° angle location.

(3) Balance the assembled facility rig shaft to the balancing machine's accuracies. Balancing rotation will be done on the precision bearing spacers. Remove or add material according to the assembled facility rig shaft balancing specifications.

(4) Assemble the checkout rotor onto the balanced, assembled facility rig shaft.

(5) Match mark the checkout rotor with respect to the balanced, assembled facility rig shaft and the analogous 0° angle location.

(6) Balance the checkout rotor and facility rig shaft assembly to the balancing machine's accuracies. Remove from or add material to the rotor according to the checkout rotor's balancing specifications.

(7) Remove the checkout rotor from the facility rig shaft.

(8) Remove the precision bearing spacers from the facility rig shaft.

(9) Install the precision bearing spacers onto the balancing arbor if required.

(10) Assemble the checkout rotor onto the arbor.

(11) Match mark the arbor with respect to the checkout rotor and an analogous 0° angle location.

(12) Balance the arbor and the checkout rotor to the balancing machine's accuracies. Remove material from or add it to the arbor according to the arbor's balancing specifications. The balancing arbor is now calibrated to the assembled facility rig shaft. The above procedure will compensate for the

facility rig shaft and balancing arbor shaft machining tolerances. Balance new rotors for facility operation by the following procedure:

(13) Assemble the new test rotor onto the balanced arbor.

(14) Match mark the new test rotor with respect to the arbor and an analogous 0° location.

(15) Balance the new test rotor to the balancing machine's accuracies by using the arbor. Remove material from or add it to the rotor according to the new rotor's balancing specifications.

(16) Remove the balanced new test rotor from the arbor.

(17) Install and align (according to the match-marked analogous 0° angle location) the balanced new test rotor onto the facility rig shaft.

The new test rotor is now balanced and installed for turbomachinery facility operation. If a balancing arbor is not required, the new test rotor can be balanced by following steps (1) to (8).

This is typically the procedure for balancing turbomachinery shafting and calibrating balancing arbors. Even though this balancing procedure is followed, problems may still develop that will require field balancing.

BALANCING ARBORS

The balancing arbor methodology was devised to fill the need for rotor replaceability without rig disassembly. Some facilities are structured and built for multiple-rotor testing. Different rotors use the same rig shaft but may vary in size, shape, or weight. Rotor-to-shaft alignment for multiple-rotor testing facilities is usually accomplished by using rotor-to-shaft pilot interferences, alignment pins, P-3 polygons, or curvic couplings. The best results are obtained with rotor-to-shaft interferences or alignment pins. All four alignment processes have machining tolerances that are referenced to the shaft rotational axis. These machining tolerances are also referred to as shaft or pilot full indicator readings.

Unbalanced shaft problems, which are associated with multiple-rotor facilities, occur when the rig shaft pilot and the balancing arbor pilot have machining tolerances of 0.0001 to 0.0050 in. (full indicator reading). The worst case of shaft unbalance could exist when both pilots are 0.0050 in. (full indicator reading) and 180° apart.

For example, a rig shaft pilot offset is equal to 0.0050 in. This offset, in conjunction with the 0.000020-in. balancing machine tolerance, will be induced into the checkout rotor during step (12) of the balancing process. The balancing arbor pilot is also offset 0.0050 in. The worst condition occurs when the rig pilot offset is 180° from the balancing arbor pilot offset. In step (15) of the balancing process a total of 0.010040-in. offset will be compensated for in the balancing arbor.

$$O_t = O_r + O_m + O_a + O_m \tag{6}$$

$$\begin{aligned} O_r &= 0.0050 \text{ in.} \\ O_m &= 0.000020 \text{ in.} \\ O_a &= 0.0050 \text{ in.} \\ \underline{O_m} &\underline{= 0.000020 \text{ in.}} \\ O_t &= 0.010040 \text{ in.} \end{aligned}$$

If a checkout rotor weight of 50 lbm (800 oz) is used during the balancing process, a 4.0160-oz in. mass offset will be induced into the checkout rotor after balancing.

$$M_o = m(O_r + O_m) = 4.0160 \text{ oz in.} \tag{7}$$

When the balancing arbor is balanced to the calibrated checkout rotor, a 8.0320-oz in. mass offset will be induced into the balancing arbor.

$$M_o = mO_t = 8.0320 \text{ oz in.} \tag{8}$$

The problem is compensating for these large mass offsets occurs when the calibrated balancing arbor is used to balance new test rotors. The new test rotors will be balanced only to the tolerances of the balancing machines O_m and not to the inaccuracies that exist in the calibrated balancing arbor $(O_r + O_m)$. These inaccuracies could cause excessive test rig operating vibrations resulting from unbalanced shafting. This potential problem is compensated for by using the theory of rotor weight differentials.

ROTOR WEIGHT DIFFERENTIALS

Turbomachinery test rigs that are designed to accommodate multiple rotors generally all have identical test rig shafting and related hardware. These rotors are typically similar in size and shape unless a new test rotor casing is installed on the test rig.

Rotor weight can easily vary with different types of rotor material (e.g., titanium and stainless steel). Typical rotors range from 6 to 22 in. in diameter with thicknesses of 0.75 to 2.50 in., respectively. A new balancing arbor is required every time a new test rotor differs in weight. In order to balance the new arbor, the original rig shaft is needed to balance the new test rotor. The reason for separate balancing arbors is that underbalancing occurs when only one balancing arbor is used.

For example, in the previous section, the balancing arbor was calibrated with a 0.010040-in. offset induced into it by using a 800-oz checkout rotor. If a new test rotor weighs 2400 oz, then 24.0960-oz in. mass offset will require compensation when balancing.

$$M_o = mO_t = 24.0960 \text{ oz in.}$$

However, when the new test rotor is balanced to the balancing arbor (step (15) of the balancing procedure), only the original 8.0320 oz in. will be compensated for and not the desired 24.0960 oz in. When the new test rotor is assembled to the rig shaft and rotated, an unbalance will be apparent. This unbalance originates from underbalancing the new test rotor. The new test rotor needs to be compensated for 24.0960 oz in. when balanced on the rig shaft. The new test rotor was underbalanced 8.0320 oz in.

$$U_m = W_n(O_r + O_a + 2O_m) - W_i(O_r + O_a + 2O_m) - (W_n - W_i)(O_a + O_m) \tag{9}$$

$$U_m = (W_n - W_i)(O_r + O_m) \tag{10}$$

where

$$O_r + O_m = O_t \tag{11}$$

Substituting equation (11) into equation (10) gives

$$U_m = (W_n - W_i)(O_t) = 8.0320 \text{ oz in.} \tag{12}$$

In order to determine if this underbalance requires a new balancing arbor, the concept of rotor weight differentials is introduced.

Rotor weight differentials are the weight increase (in percent) from the initial rotor weight to the new rotor weight. Rotor weight differentials may be caused by switching from bladeless rotors to bladed rotors, switching rotor materials, and changing rotor vane thicknesses. Since balancing arbors are costly and many factors can change weight differentials, loading should be calculated for various rotor weight differentials from 0 to 500 percent. Rotor weight differentials will be considered acceptable if centrifugal loads do not exceed turbomachinery bearing load limits or exceed the maximum acceleration magnification factor. These centrifugal loads can be directly associated with rotor weight differentials and referred to as centrifugal loading multipliers.

An acceleration magnification factor is equal to the gravitational constant g divided by the constant of mass acceleration a (described later in this report). Typical acceptable maximum acceleration magnification factors vary from 0.40 to 3. Any centrifugal loading that exceeds its turbomachinery bearing load limits or its acceleration magnification factor limits will be a primary candidate for a new balancing arbor.

Centrifugal Loading Multiplier

The centrifugal loading due to a rotor weight differential is equal to the product of the rotor weight and the centrifugal loading multiplier. Therefore, the centrifugal loading multiplier reflects the new rotor weight as a percentage of the initial rotor weight. The centrifugal loading multiplier can be acquired through the modification of the centrifugal force equation and used to determine the need for additional balancing arbors. The underbalance due to rotor weight differential is obtained from the underbalance weight differential equation, where the radius of unbalance equals the total offset. Now the centrifugal force equation becomes

$$F_c = \frac{(W_n - W_i)O_t w^2}{u} \tag{13}$$

where the rotor weight differential equals

$$D = \frac{W_n}{W_i} - 1 \tag{14}$$

Rearranging equation (14) for W_n gives

$$W_n = (1 + D)W_i \tag{15}$$

Substituting equation (15) into equation (13) yields

$$F_c = \frac{W_i D O_t w^2}{u} \tag{16}$$

Centrifugal force is also equal to the product of the initial rotor weight and the centrifugal loading multiplier.

$$F_c = C_m W_i \tag{17}$$

Simplifying equations (16) and (17) gives

$$C_m W_i = \frac{W_i D O_t w^2}{u} \tag{18}$$

Initial rotor mass cancels out, and the centrifugal loading multiplier equation becomes

$$C_m = \frac{D O_t w^2}{u} \tag{19}$$

Substituting known constants into equation (19) yields

$$C_m = \frac{D O_t w^2}{35\ 191.68407} \tag{20}$$

where O_t is in inches and w is in revolutions per minute.

The centrifugal loading multiplier can now be calculated for a particular rotor weight differential by using equation (20) or can be interpolated from the following graphs. Figure 9 shows the centrifugal loading multiplier for rotor weight differentials from 50 to 500 percent, rotor shaft speeds from 0 to 120 000 rpm, and shaft machining tolerances from 0.0001 to 0.0050 in. Centrifugal loading multipliers as high as 10 000 are shown.

After the centrifugal loading multiplier has been determined, the centrifugal force can be calculated from equation (17). This centrifugal load can be compared with the maximum turbomachinery bearing and turbomachinery rig load limits for acceptability. If the load is unacceptable, a new balancing arbor is required for the new weight rotor.

Acceleration Magnification Factor

For all rotor weight differentials an acceleration magnification factor can be predicted. This is possible by setting the centrifugal force equal to the force resulting from the absolute linear acceleration of the new test rotor weight acting on the turbomachinery shaft's bearings as shown in equation (21).

$$F_c = \frac{W_n a}{g} \tag{21}$$

The mass acceleration is equal to the product of the acceleration magnification factor and the gravitational constant g.

$$a = A_g g \tag{22}$$

Substituting equation (22) into equation (21) and simplifying gives

$$F_c = A_g W_n \tag{23}$$

By substituting equation (15) into equation (23), centrifugal force as a result of the absolute linear acceleration of the new test rotor becomes

$$F_c = A_g(1 + D)W_i \tag{24}$$

The acceleration magnification factor can now be found by equating equations (16) and (24).

$$A_g = \frac{DO_t w^2}{(1 + D)u} \tag{25}$$

Equation (25) becomes

$$A_g = \frac{DO_t w^2}{(1 + D)(35\ 191.68407)} \tag{26}$$

where O_t is in inches and w is in revolutions per minute.

The acceleration magnification factor can be interpolated from the following graphs or can be calculated from the acceleration magnification factor equation (26). Figure 10 shows the acceleration magnification factor for rotor weight differentials from 50 to 500 percent, rotor shaft speeds from 0 to 120 000 rpm, and shaft machining tolerances from 0.0001 to 0.0050 in. Acceleration magnification factors as high as 1700 are shown.

Often the maximum acceleration magnification factor is known along with the desired shaft rotational speed, but the maximum allowable rotor weight differential for various rig shaft machining full-indicator-reading tolerances is unknown. Therefore, after rearranging and solving for rotor weight differential, equation (25) becomes

$$D = \frac{A_g u}{A_g u + O_t w^2} \tag{27}$$

Equation (27) becomes

$$D = \frac{(A_g)(35\ 191.68407)}{(A_g)(35\ 191.68407) + O_t w^2} \tag{28}$$

where O_t is in inches and w is in revolutions per minute.

Maximum rotor weight differential can now be interpolated from the following graphs or calculated if the acceleration multiplication factor along with the desired shaft rotational speed and rig shaft machining tolerance are known. Figure 11 shows the rotor weight differential for acceleration magnification factors from 0.5 to 18.0, rotor shaft speeds from 0 to 120 000 rpm, and shaft machining tolerances from 0.0001 to 0.0050 in. Rotor weight differentials as high as 5 (500 percent) are shown.

A tradeoff exists between rotor weight differential, shaft rotational speed, and shaft loading. Shaft loading can be in the form of the centrifugal loading multiplier or the acceleration magnification factor. As shaft rotational speed increases, the acceptable shaft loading requires the allowable rotor weight differential to be lowered to stay within the test rig loading limits. The allowable rotor weight differential will be lowered further as shaft machining tolerances increase. Most multiple-rotor test rigs consider rotor weight differentials of 0.10 (10 percent) or less to be acceptable before a new balancing arbor is required. Going from a typical bladeless rotor to a bladed rotor produces a rotor weight differential equivalent to 0.10. Multiple-rotor test rigs usually have maximum shaft machining tolerances of 0.0010 in. (full indicator reading) or less. Rotor weight differentials of 10 percent or less are considered acceptable for these tolerances. Also, rotor weight differentials are considered tolerable if shaft loading is within an acceptable range for a given range of test rig shaft rotational speeds. The acceptable rotor weight differentials will decrease as shaft rotational speeds increase for a particular shaft machining tolerance to a limit where the acceptable rotor weight differential becomes equivalent to zero. In instances such as these, the rotor weights for multiple-rotor facilities must be equal or unbalancing problems will exist. Since rotor weight differentials equivalently equal to zero are favorable for multiple-rotor facilities, some design changes are possible. Tightening the shaft machining tolerance along with decreasing shaft rotational speeds or increasing allowable shaft loading limits will increase rotor weight differential.

CONCLUDING REMARKS

Predicting and presolving turbomachinery test rig problems is important. Underdesign of turbomachinery test rigs may result in total test rig destruction. Overdesign of turbomachinery test rigs may make their funding and machining unrealistic and unattainable. In the design of turbomachinery a safety factor is usually used. Often this safety factor is based on the worst-case scenarios that the turbomachinery test rig can encounter. In this report it was assumed that the rig shaft machining tolerance equaled the balancing arbor machining tolerance and that these tolerances were offset 180° from one another. The resulting safety factor would be that the rig shaft pilot and the balancing arbor pilot have identical 180° offset machining tolerances. Therefore, the entire turbomachine should be designed with consideration of this condition when determining maximum allowable bearing loads, turbomachinery casing and supports, and facility life.

Determining maximum allowables for turbomachinery shafting is a function of many unconstrained variables. Turbomachines, such as steam turbines in hydroelectric generating stations can be designed to run continuously with long maintenance intervals. Most research turbomachinery is designed less conservatively because of their shorter run requirements and shorter maintenance intervals. Material strength and equipment life expectancies change dramatically from one turbomachinery facility to another. Furthermore, a turbomachinery facility designed with a shorter life or test expectancy can normally tolerate higher loading, since continuous long-term operation of the facility is not required. The same high loading on a turbomachinery facility designed with a longer life expectancy would greatly shorten its operational life. Therefore, determining maximum allowables as a factor of shaft rotational speed alone would be counterproductive and futile, since maximum allowables are a function of many different, unconstrained variables that change for dissimilar circumstances. Turbomachinery centrifugal loading for individual situations can now be interpolated from the graphs or calculated from the equations described in this report.

The contents of this report can be used as a tool in designing, fabricating, modifying, and troubleshooting turbomachinery unbalanced-shafting problems. The equations developed herein can be used to determine safety factors and tolerable maximums that turbomachinery facilities can withstand under worst-case scenarios. Most turbomachinery shaft loadings are below those prescribed herein. Real-life turbomachinery shaft loadings should always be lower than those predicted from the equations developed herein, or insufficient safety factors were used in designing the turbomachinery facility. As a result of the higher turbomachinery shaft rotational speed and resulting higher shaft loading, maximum design parameters became apparent. Shaft machining tolerances above 0.0010 in. (full indicator reading) should be considered undesirable for turbomachinery design and operation. Additionally, turbomachinery test rigs that are designed for rotor interchangeability should have different balancing arbors when rotor weight differentials are above 10 percent. These assumptions apply for typical turbomachinery facilities and take precedence when shaft loading exceeds or is equivalent to turbomachinery test rig vibration and bearing loading limits (ref. 4).

ACKNOWLEDGMENT

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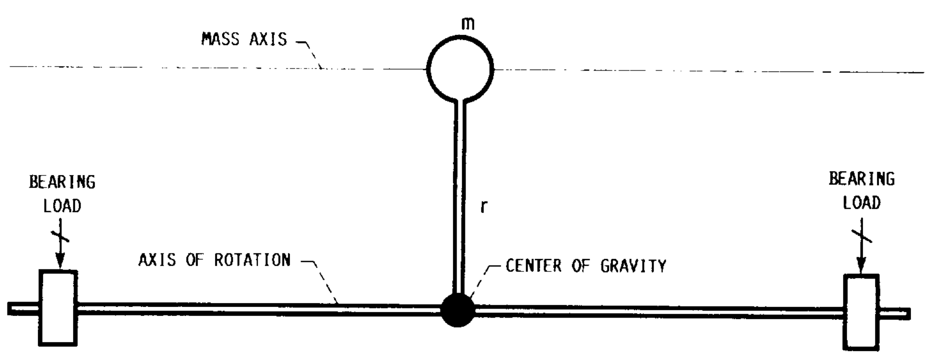

FIGURE 1. - STATIC UNBALANCE.

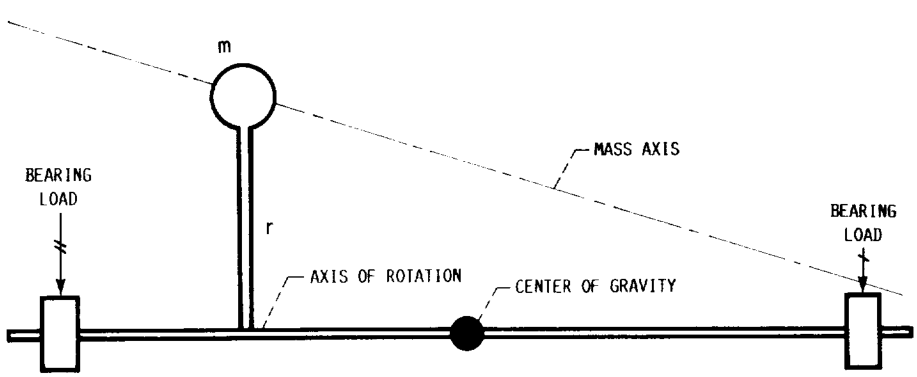

FIGURE 2. - DYNAMIC UNBALANCE.

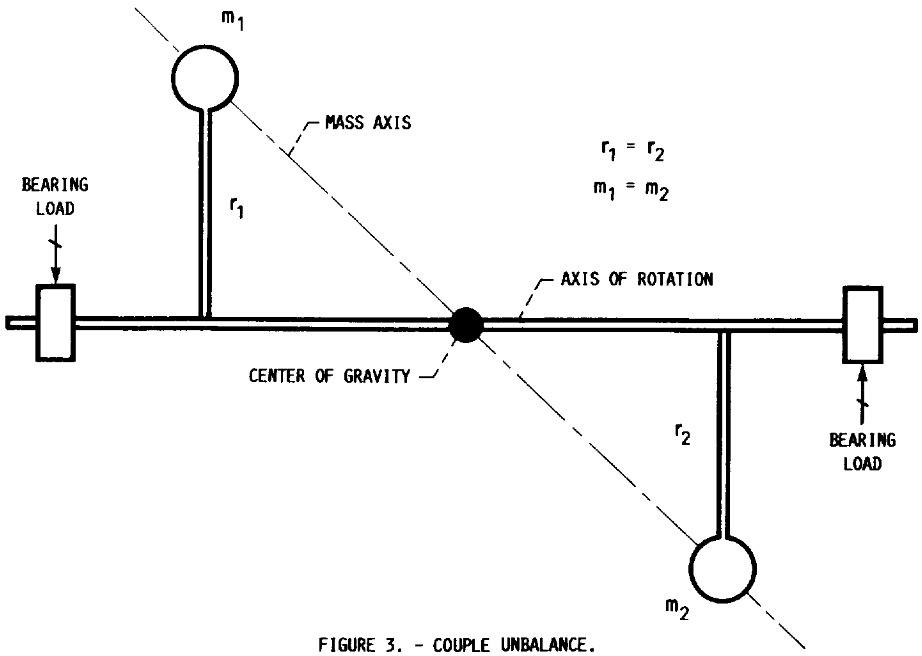

FIGURE 3. - COUPLE UNBALANCE.

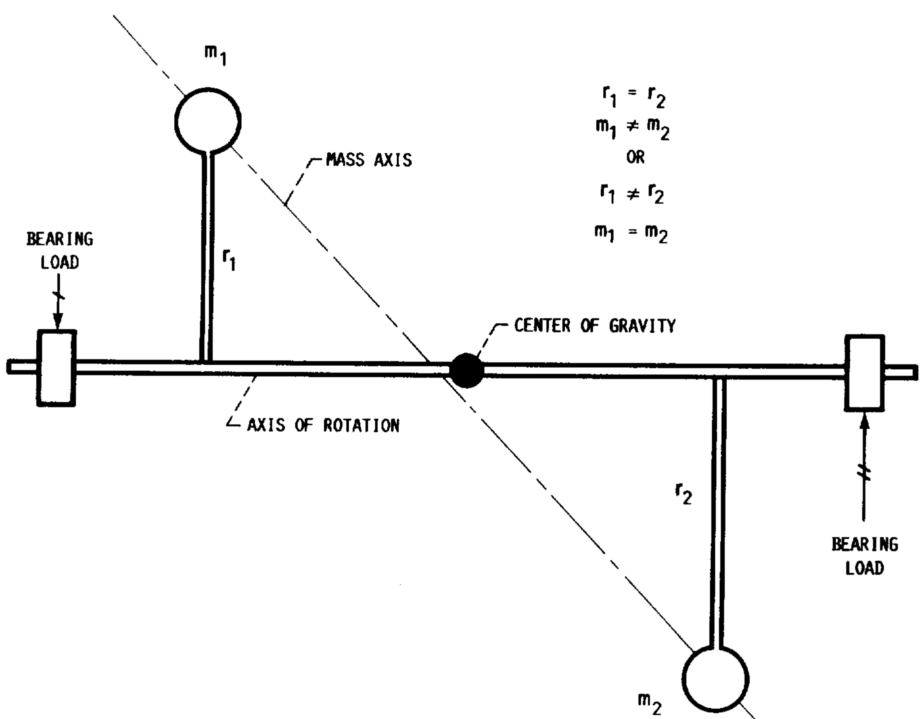

FIGURE 4. - QUASI-STATIC UNBALANCE.

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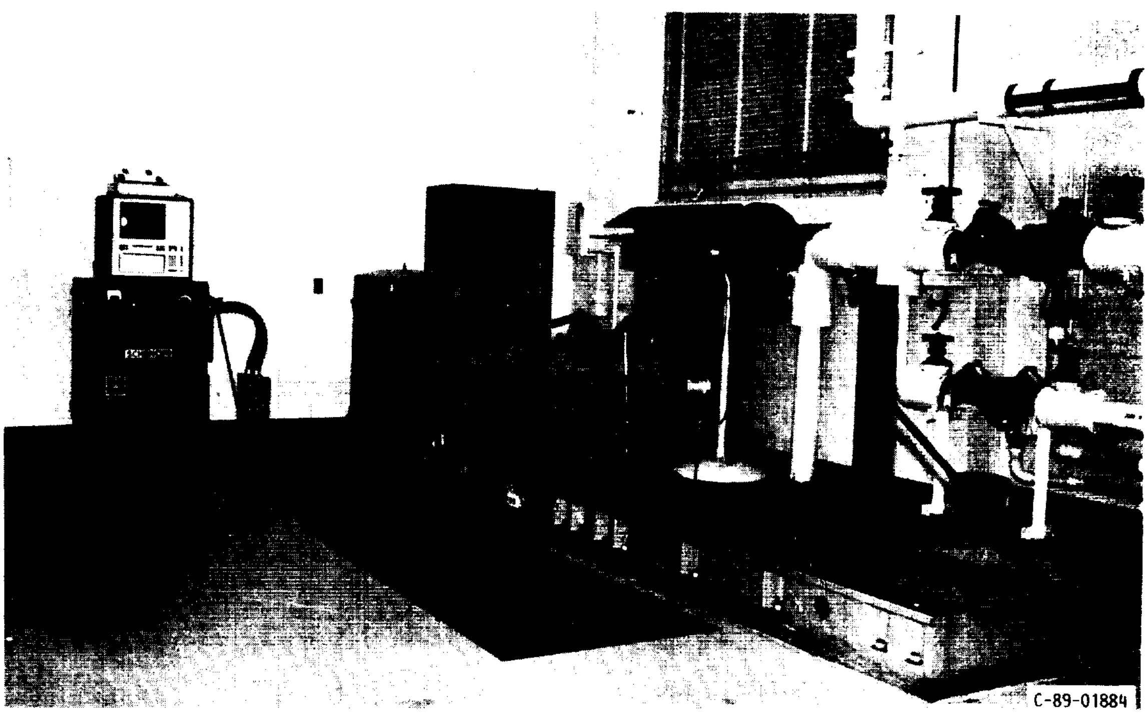


FIGURE 5. - SCHENCK BALANCER (20 TO 5000 lbm).

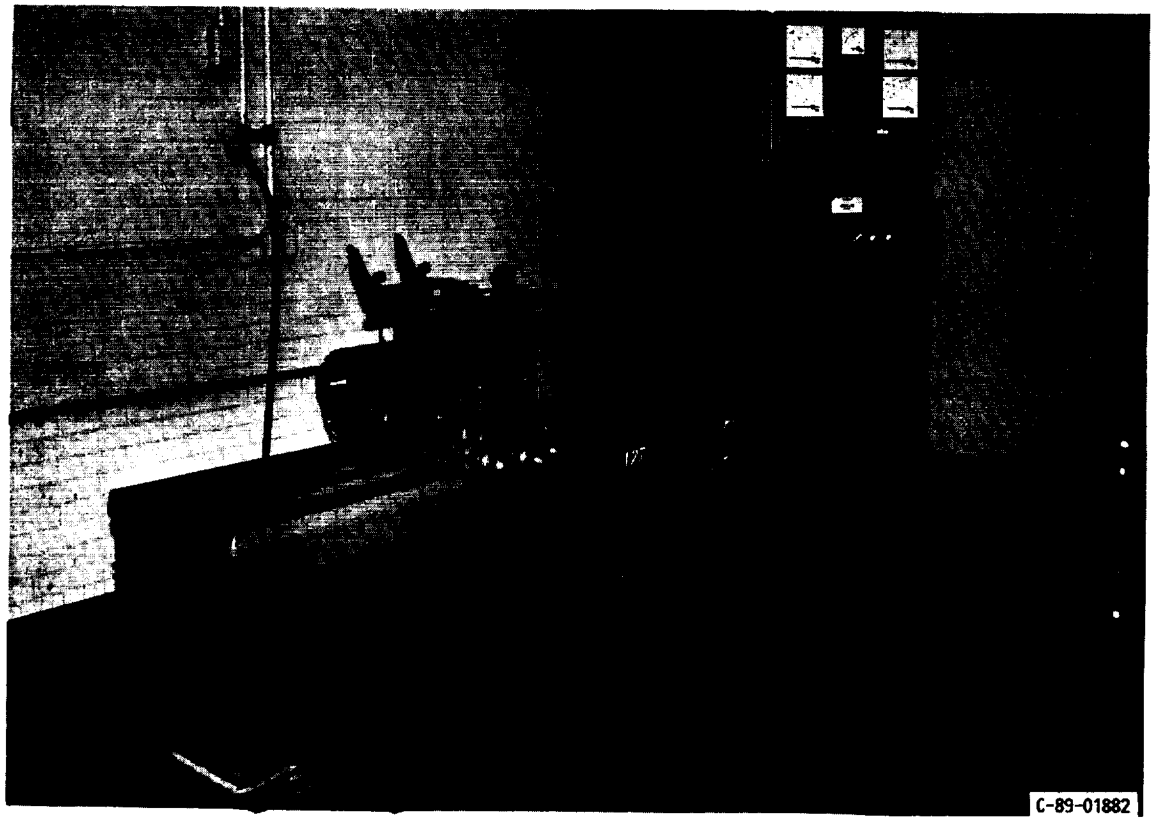


FIGURE 6. - HOFFMAN BALANCER (3 TO 1000 lbm).

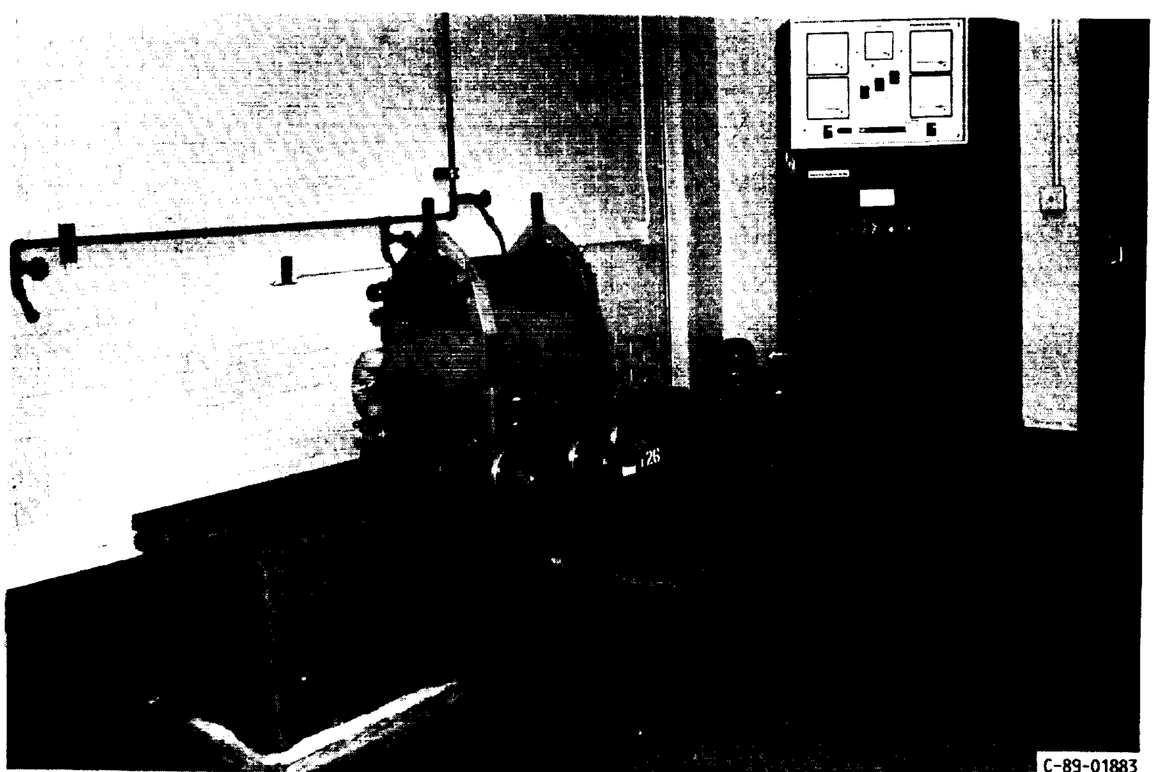


FIGURE 7. - HOFFMAN BALANCER (1 TO 250 lbm).

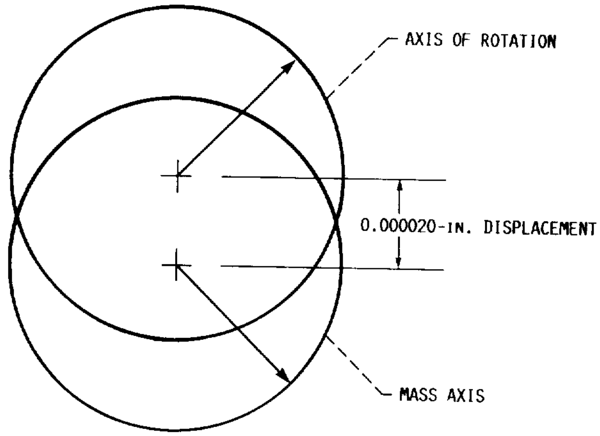
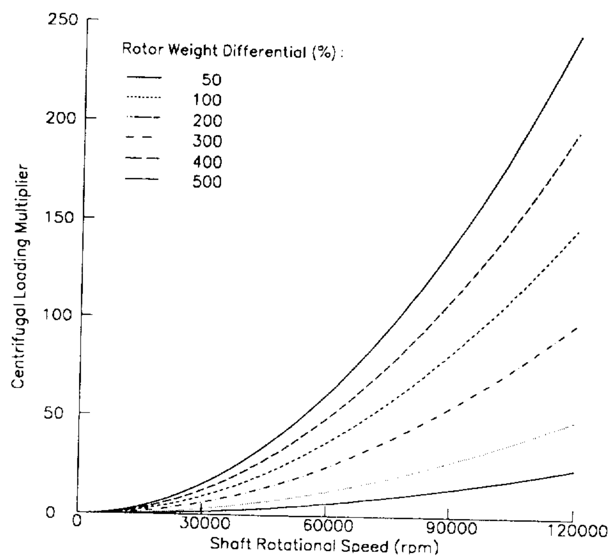
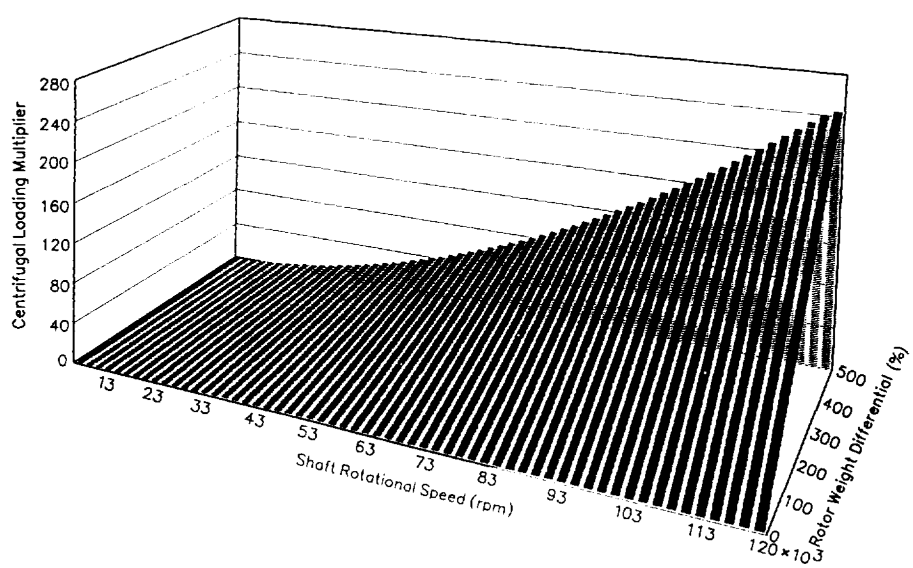


FIGURE 8. - BALANCING MACHINE RESOLUTION





(a) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0001 IN.

FIGURE 9. - CENTRIFUGAL LOADING MULTIPLIER VERSUS SHAFT ROTATIONAL SPEED AND ROTOR WEIGHT DIFFERENTIAL (0 TO 500%) FOR RIG SHAFT MACHINING TOLERANCES (FULL INDICATOR READING) FROM 0.0001 TO 0.0050 IN.

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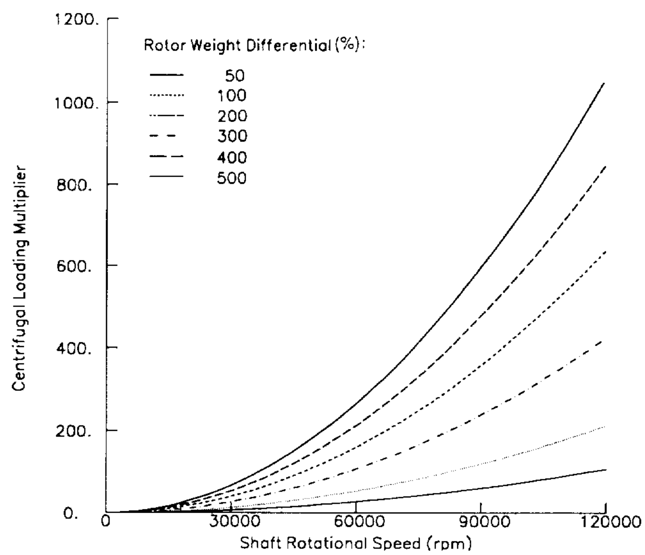
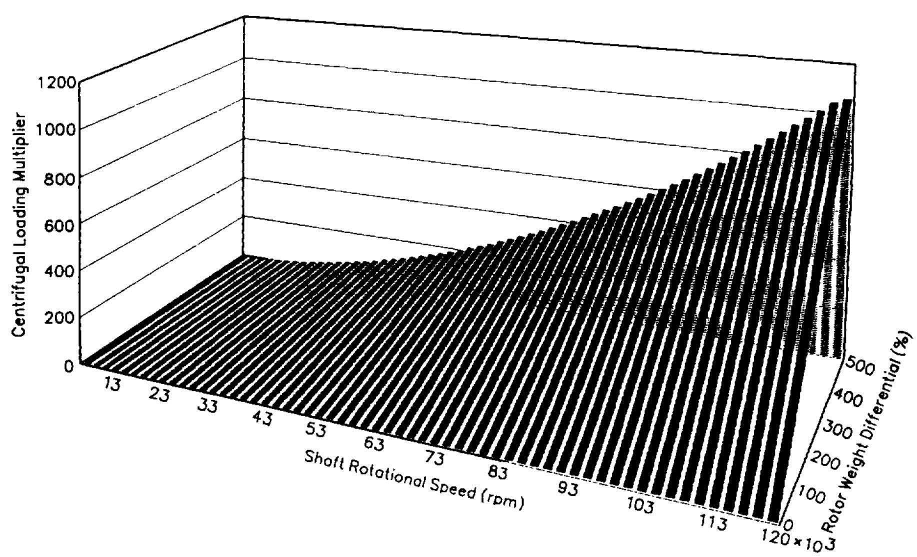
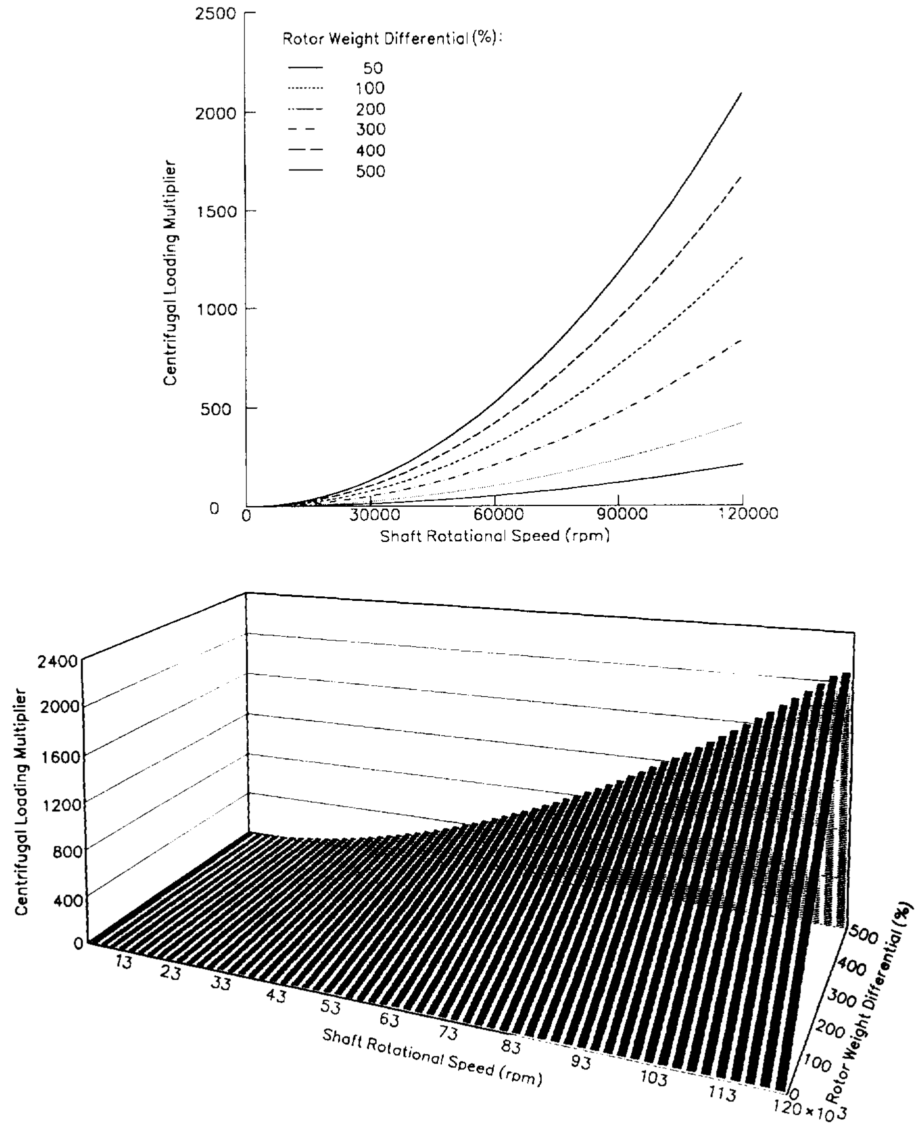




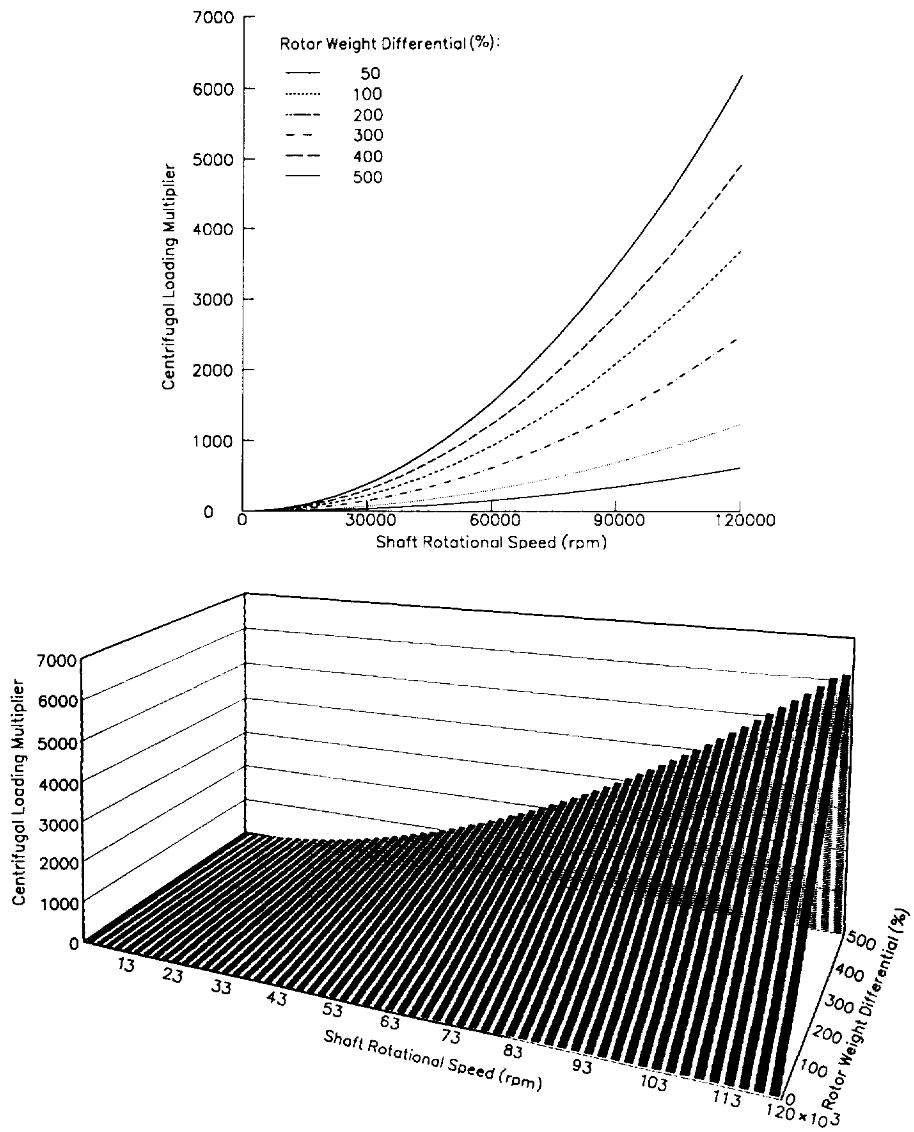
(b) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0005 IN.

FIGURE 9. - CONTINUED.



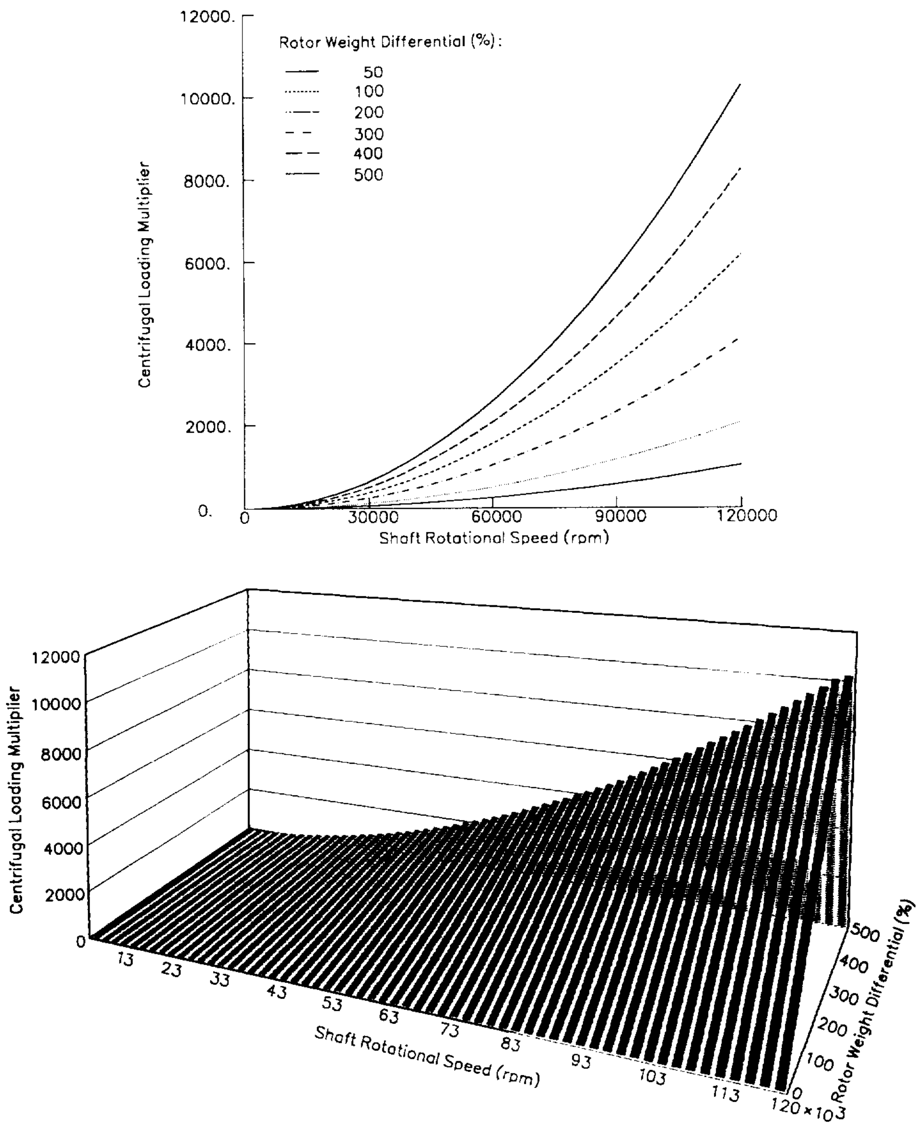
(c) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0010 IN.

FIGURE 9. - CONTINUED.



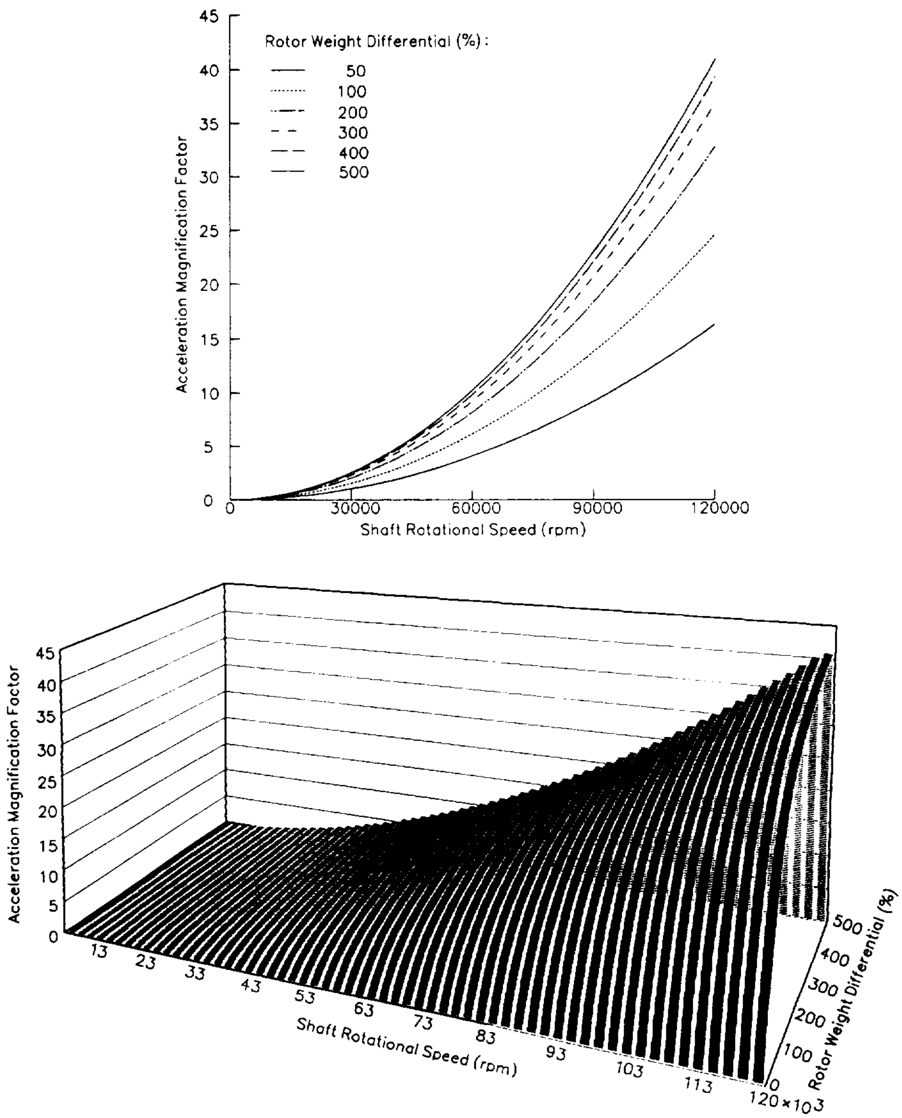
(d) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0030 IN.

FIGURE 9. - CONTINUED.



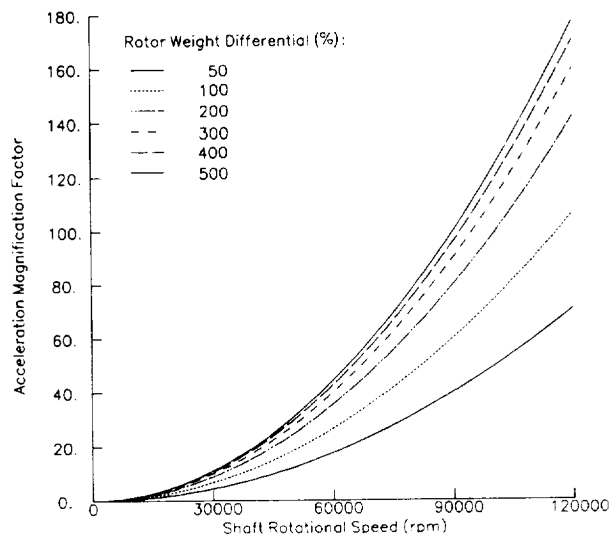
(e) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0050 IN.

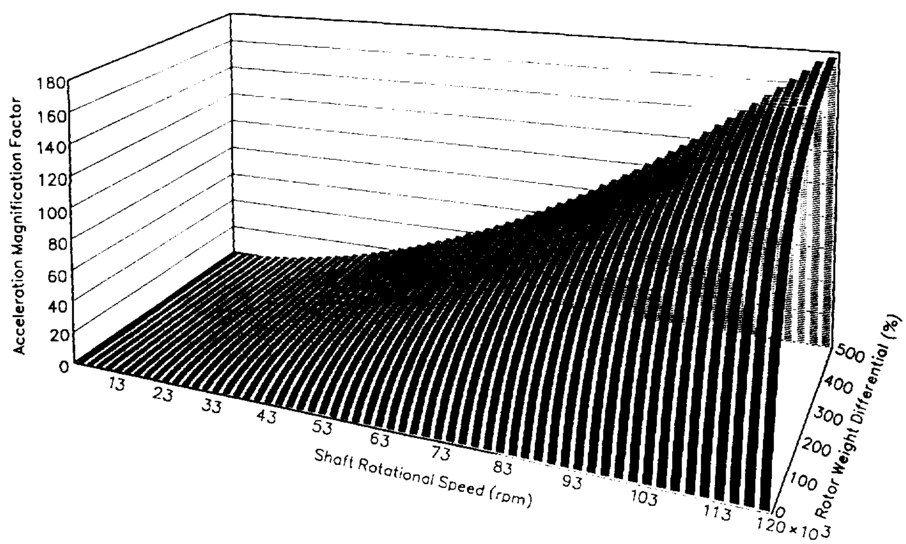
FIGURE 9. - CONCLUDED.



(a) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0001 IN.

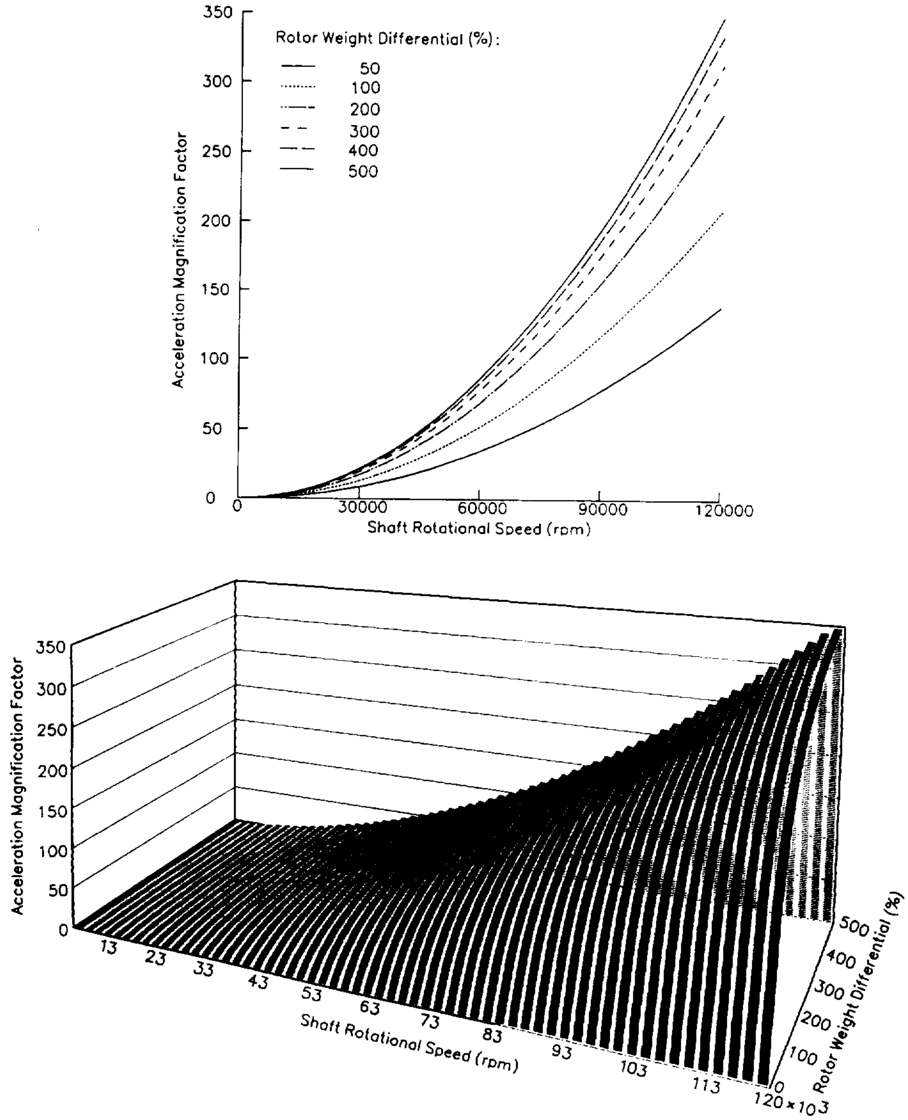
FIGURE 10. - ACCELERATION MAGNIFICATION FACTOR VERSUS SHAFT ROTATIONAL SPEED AND ROTOR WEIGHT DIFFERENTIAL (0 TO 500%) FOR RIG SHAFT MACHINING TOLERANCES (FULL INDICATOR READING) FROM 0.0001 TO 0.0050.





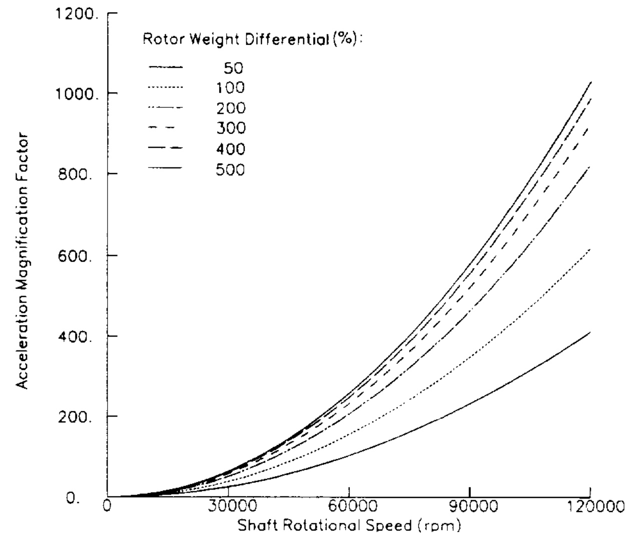
(b) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0005 IN.

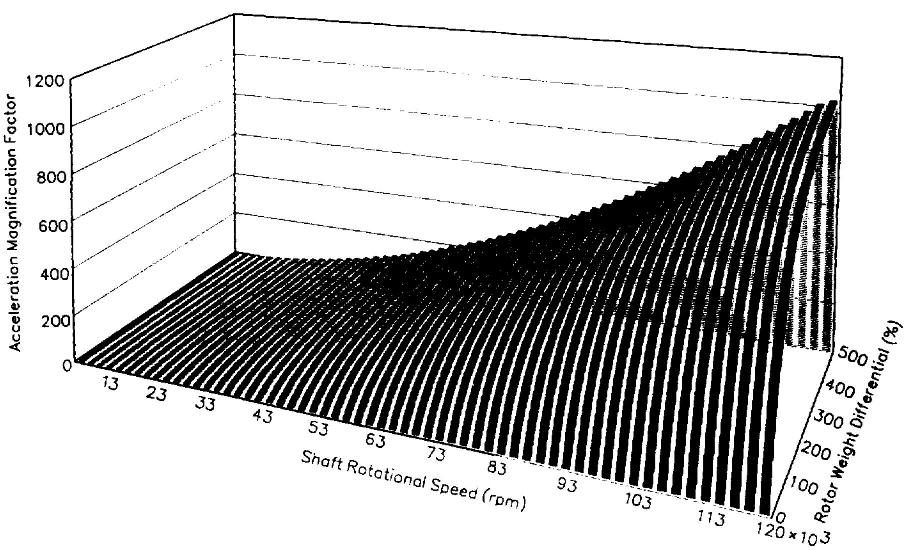
FIGURE 10. - CONTINUED.



(c) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0010 IN.

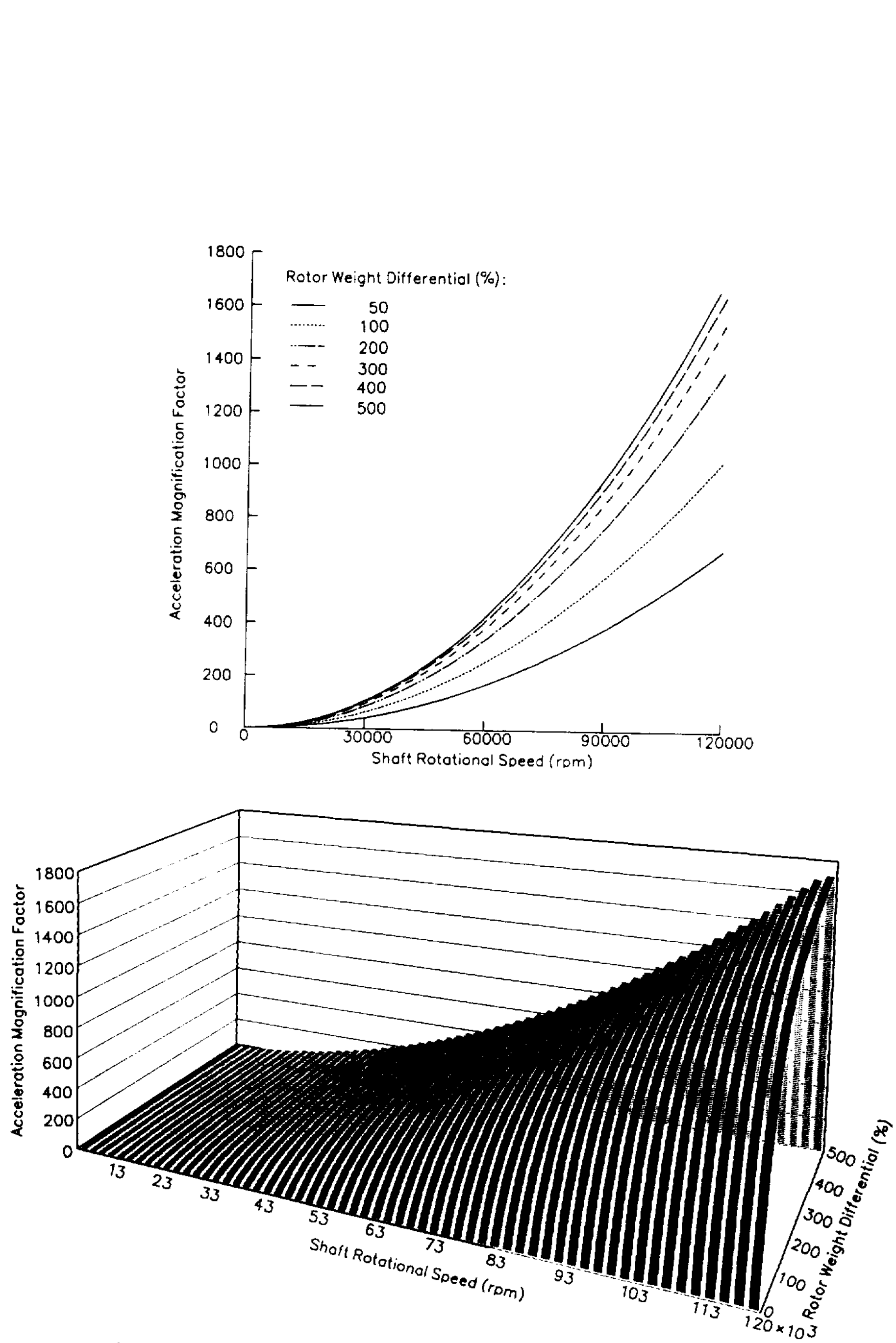
FIGURE 10. - CONTINUED.





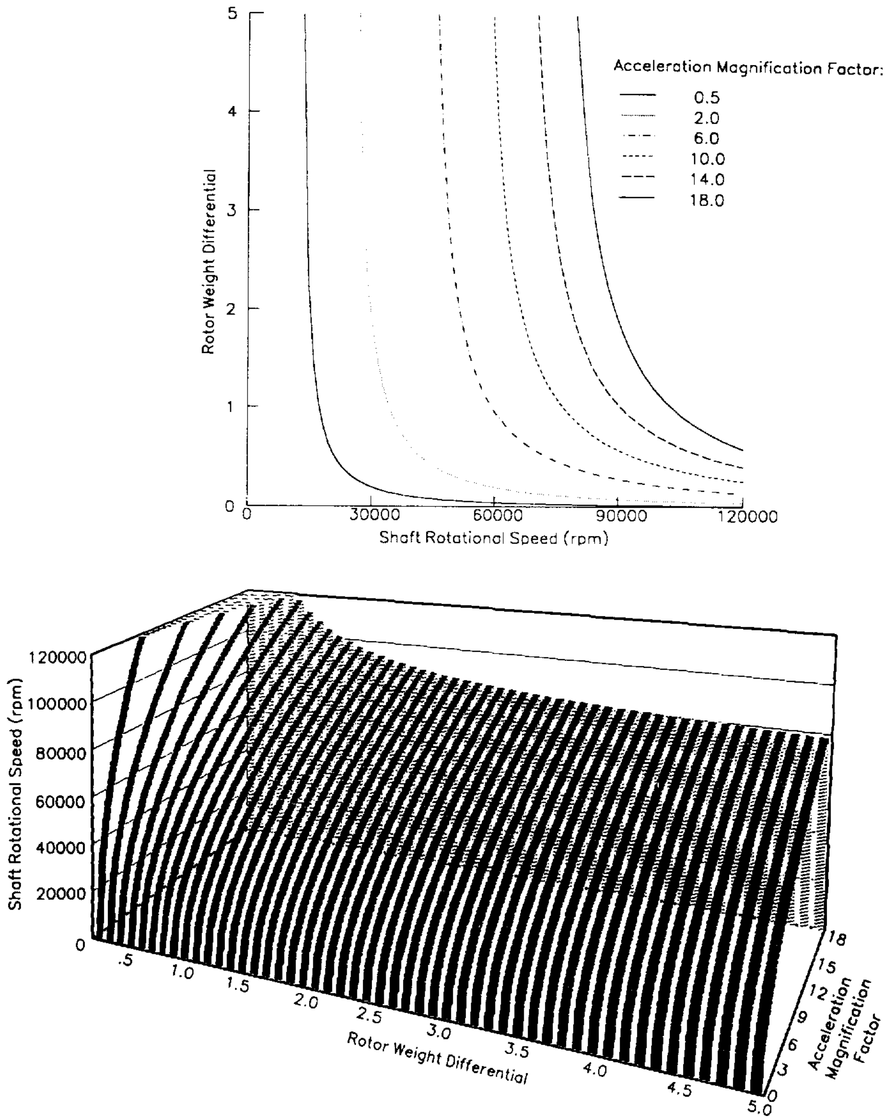
(d) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0030 IN.

FIGURE 10. - CONTINUED.



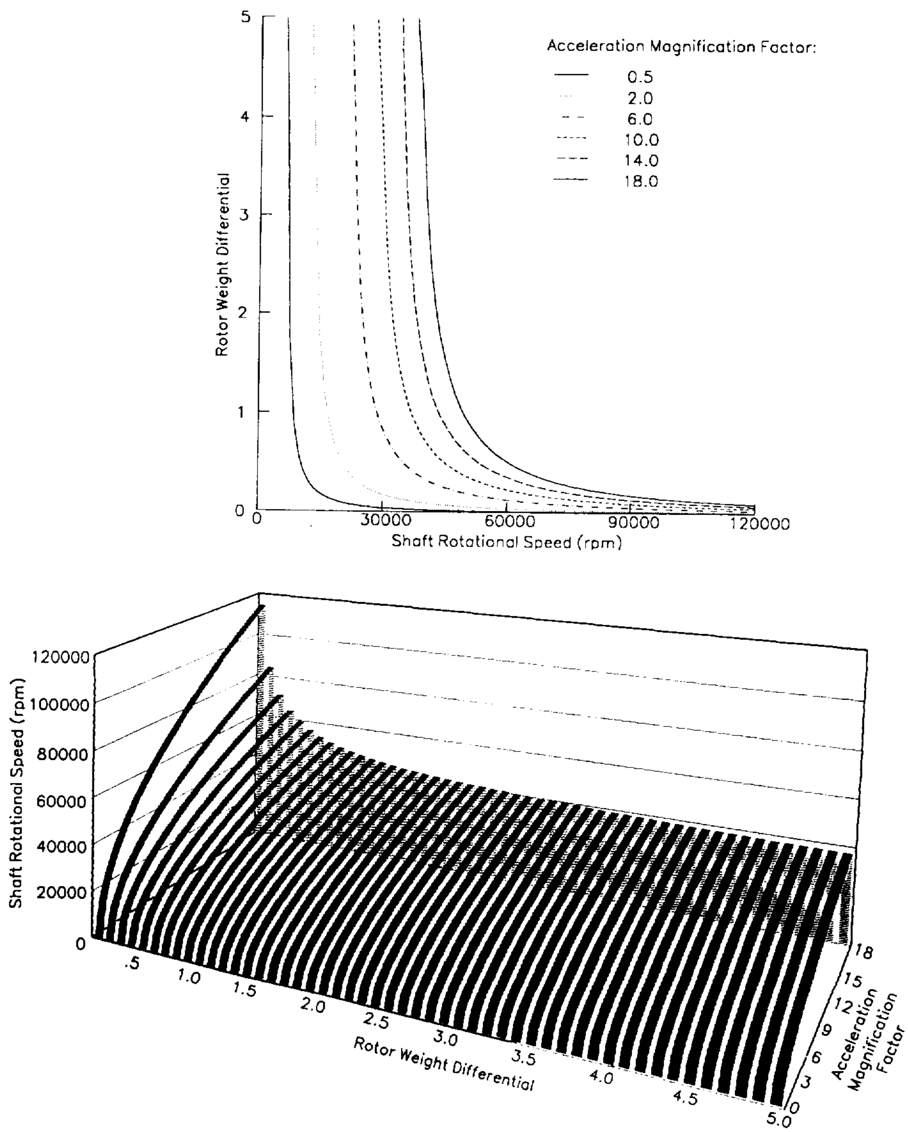
(e) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0050 IN.

FIGURE 10. - CONCLUDED.



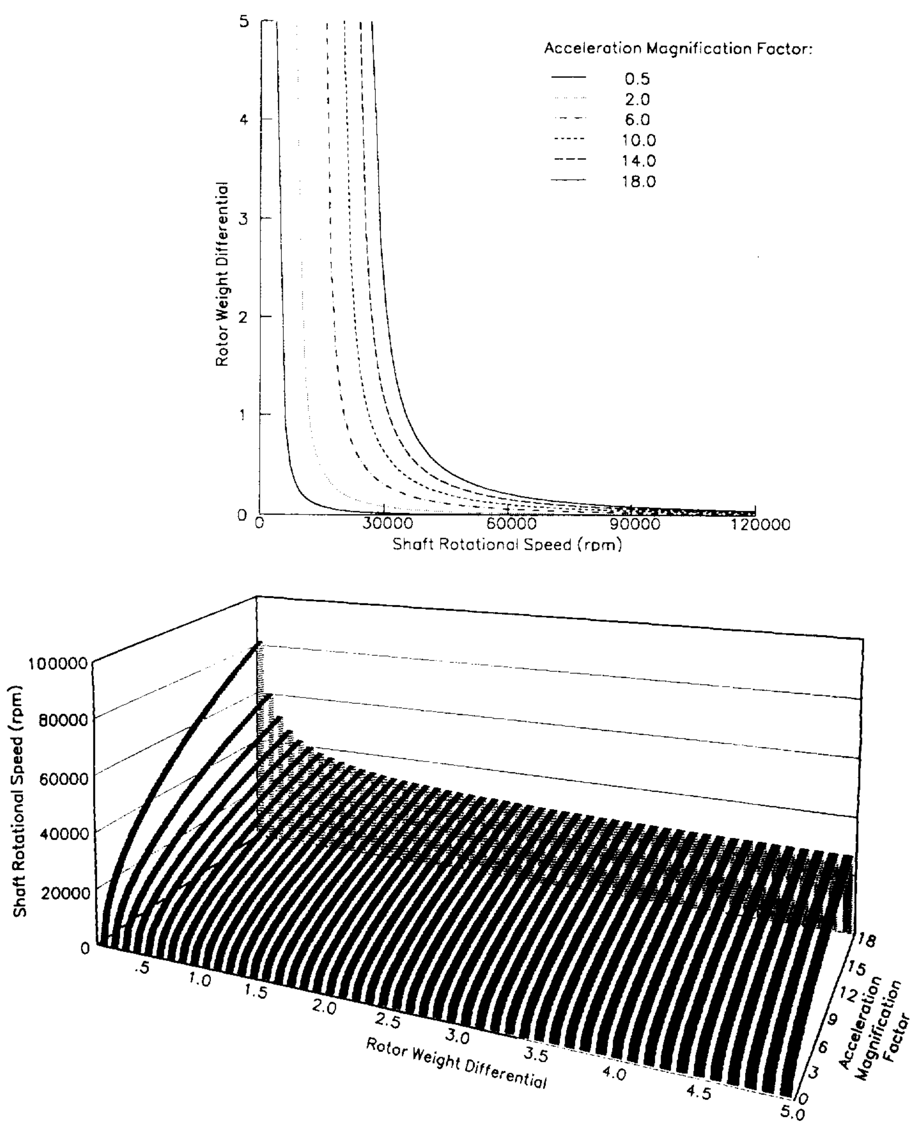
(a) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0001 IN.

FIGURE 11. - ROTOR WEIGHT DIFFERENTIAL VERSUS SHAFT ROTATIONAL SPEED AND ACCELERATION MAGNIFICATION FACTOR FOR RIG SHAFT MACHINING TOLERANCES (FULL INDICATOR READING) FROM 0.0001 TO 0.0050 IN.



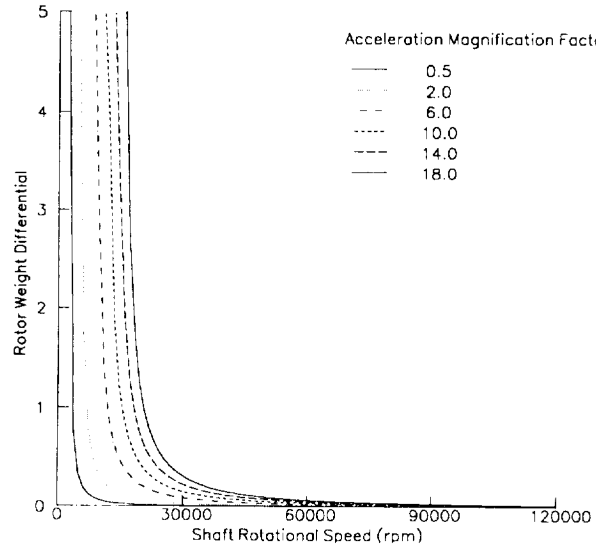
(b) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0005 IN.

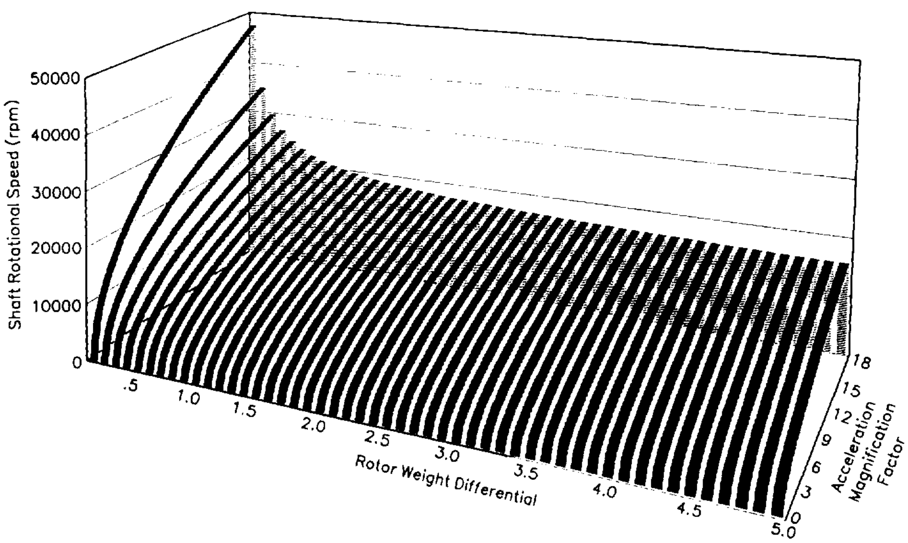
FIGURE 11. - CONTINUED.



(c) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0010 IN.

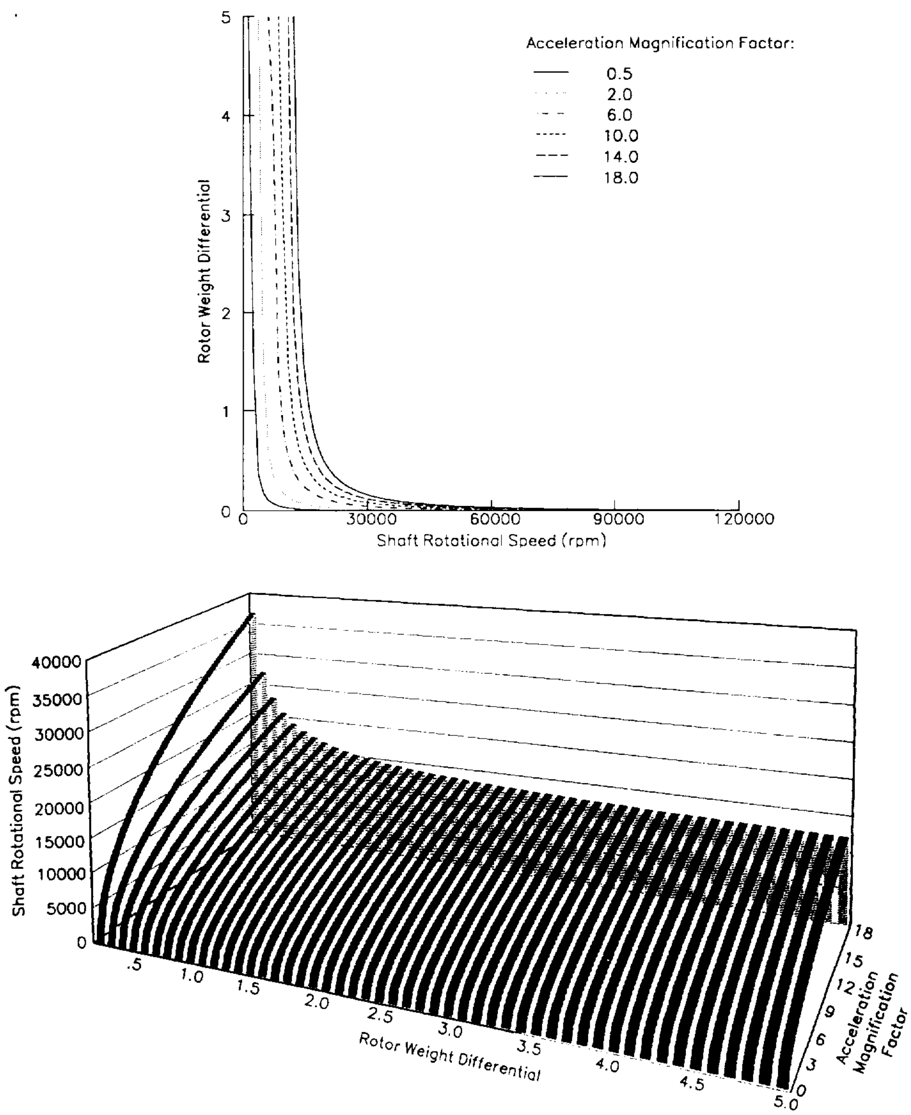
FIGURE 11. - CONTINUED.





(d) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0030 IN.

FIGURE 11. - CONTINUED.



(e) RIG SHAFT MACHINING TOLERANCE (FULL INDICATOR READING), 0.0050 IN.

FIGURE 11. - CONCLUDED.

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15. Supplementary Notes

16. Abstract

The NASA Lewis Research Center has developed and implemented a time-efficient methodology for dynamically balancing turbomachinery shafting. This methodology minimizes costly facility downtime by using a balancing arbor (mandrel) that simulates the turbomachinery (rig) shafting. This report discusses in detail the need for precision dynamic balancing of turbomachinery shafting and for a dynamic balancing methodology. Additionally, it discusses the inherent problems (and their causes and effects) associated with unbalanced turbomachinery shafting as a function of increasing shaft rotational speeds. Included in this discussion are the design criteria concerning rotor weight differentials for rotors made of different materials that have similar parameters and shafting. The balancing methodology for applications where rotor replaceability is a requirement is also covered. This report is intended for use as a reference when designing, fabricating, and troubleshooting turbomachinery shafting.

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