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An Improved Model for the Earth's Gravity Field

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ABSTRACT

An improved model for the Earth's gravity field, TEG-1, has been determined using data sets from fourteen satellites, spanning the inclination ranges from 15° to 115°, and global surface gravity anomaly data. The satellite measurements include laser ranging data, doppler range-rate data, and satellite-to-ocean radar altimeter data measurements, which include the direct height measurement and the differenced measurements at ground track crossings (crossover measurements). Also determined was another gravity field model, TEG-1S, which included all the data sets in TEG-1 with the exception of direct altimeter data. The effort has included an intense scrutiny of the gravity field solution methodology. The estimated parameters included geopotential coefficients complete to degree and order 50 with selected higher order coefficients, ocean and solid Earth tide parameters, doppler tracking station coordinates and the quasi-stationary sea surface topography. Extensive error analysis and calibration of the formal covariance matrix indicate that the gravity field model is a significant improvement over previous models and can be used for general applications in geodesy.

1. INTRODUCTION

Significant progress has been achieved during the last decade in the determination of the spherical harmonic coefficients of the Earth's external gravitational potential. A substantial portion of this progress can be directly attributed to the advent of Earth-orbiting artificial satellites and to the ability to observe their motion from either ground-based or satellite-originated tracking data. While the satellite data primarily resolve the long and intermediate wavelengths (≥ 1500 km), global surface gravity measurements and the altimeter data are capable of recovering the shorter wavelength components of the Earth's gravity field. Recent trends in gravity model improvement have been driven, in part, by requirements for more accurate satellite orbits to achieve the objectives of the Crustal Dynamics Project and the recently approved NASA/CNES Topex/Poseidon mission. A joint effort to develop an improved model for the Earth's gravity field has been undertaken to develop a gravity model to meet the orbit accuracy requirement of the Topex/Poseidon mission. The gravity field solution will represent the first complete reiteration of the historical tracking data used to define the NASA Goddard Space Flight Center (GSFC) Earth Model series. The GSFC GEM-T1 [*Marsh et al.*, 1988] and the University of Texas (UT) TEG-1 fields, described in this paper, are preliminary versions of the Topex gravity field solution.

2. THEORY AND METHOD

The gravitational potential, U, due to the Earth's nonspherical mass distribution can be expressed as follows

$$U = \frac{GM}{r} \sum_{l=0}^{\infty} \sum_{m=0}^{l} \left[\frac{R_e}{r}\right]^l \bar{P}_{lm}(\sin\phi) \left[(\bar{C}_{lm} + \Delta\bar{C}_{lm}) \cos m\lambda + (\bar{S}_{lm} + \Delta\bar{S}_{lm}) \sin m\lambda \right]$$

where GM is the product of the gravitational constant and the total mass of the Earth and the atmosphere; R_e is the mean equatorial radius of the Earth; $\bar{P}_{lm}$ are the normalized Legendre associated function of degree l and order m; $\bar{C}_{lm}, \bar{S}_{lm}$ are the the normalized spherical harmonic coefficients whose values are functions of the mass distribution within the Earth and the atmosphere; $\Delta\bar{C}_{lm}, \Delta\bar{S}_{lm}$ are the time-varying components of $\bar{C}_{lm}$ and $\bar{S}_{lm}$ caused by tides; also are functions of the tidal coefficients, $C_{lm}^{\pm}$ and $S_{lm}^{\pm}$; and r, ϕ, λ are the Earth-fixed spherical coordinate system; r is the radial distance, ϕ is the geocentric latitude and λ is the longitude measured from the Greenwich meridian.

The estimation of $\bar{C}_{lm}, \bar{S}_{lm}$, $C^{\pm}_{lm}, S^{\pm}_{lm}$ and other orbit and geophysical parameters can be accomplished using a modified least-squares estimation procedure. This estimation procedure, which provides adjustments to satellite orbit-dependent parameters and other geophysical and geodetic parameters, is given by *Tapley* [1973] and modified to include the simultaneous estimation of the relative weights for the individual satellite information arrays [*Yuan et al.*, 1988]:

$$\hat{x} = (H^T \hat{R}^{-1} H)^{-1} H^T \hat{R}^{-1} y \ ; \ \hat{R}_i = 1/k_i \, (y_i - H_i \, \hat{x})^{-1} \, (y_i - H_i \, \hat{x}) \cdot I$$

where $\hat{x}$ is the state parameter; $\hat{R}$ is the weighting matrix; I is the identity matrix; H_i is the partial derivative with respect to x for the i th data set; and k_i is the number of observations for i th dataset.

The system of equations given above can be solved iteratively using orthogonal transformation techniques [*Gentleman*, 1973]. The estimation process has been implemented in the University of Texas Orbit Processor (UTOPIA) software system [*Schutz and Tapley*, 1980]. The optimal weighting algorithm to combine satellite and nonsatellite information equations was installed in the Large Linear System Solver (LLISS). Vectorized versions of UTOPIA and LLISS are operational on the University of Texas System Center for High Performance Computing Cray X-MP/24 supercomputer. Reference orbits for each of these satellites were computed using UTOPIA with the best *a priori* gravity model and gravity field information equations were generated for each data set. The combination solution was performed using LLISS.

3. DATA AND MODELS

Fourteen satellites were selected for the current gravity model solution. Their orbital characteristics and data types are summarized in Table 1. The inclinations of these satellite orbits vary from 15° for Peole to 115° for Geos-3. The solution includes data at 90° for Oscar-14 and Nova-1. Data types include laser range, one-way range-rate, altimeter, altimeter crossover and surface gravity data. Detailed descriptions of the gravitational and nongravitational force models, the Earth orientation and time model, laser, doppler, direct altimeter and surface gravity measurement models are summarized in *Tapley et al.* [1987].

4. SOLUTION

The list of parameters which are simultaneously estimated with a relative weighting factor for each data set include: (1) geopotential complete to degree and order 50, plus selected coefficients; (2) GM, (3) ocean tides which include long period tides ($m = 0$, $l = 2,3$): Ssa, Sa, Mm, and Mf; diurnal tides ($m = 1$, $l = 2,3,4,5$): Q1, O1, P1, and K1; semi-diurnal tides ($m = 2$, $l = 2,3,4,5$): N2, M2, S2, K2 and T2 ($l = 2$); (4) quasi-stationary sea surface topography, complete to degree and order 15; (5) equipotential surface, W_o, or altimeter biases; (6) correction to significant wave height, $H_{1/3}$; (7) doppler and low inclination satellite laser station coordinates; (8) arc parameters for satellite orbits, which include position and velocity vectors, drag and solar radiation pressure coefficients, density correction parameters for selected satellites, and pass-dependent frequency biases for doppler satellites. Kaula's constraint equation [*Kaula*, 1966], which was inferred from surface gravity anomaly data, was used as an *a priori* constraint for degrees 19–50 of the geopotential. Two gravity models, TEG-1 and TEG-1S, were generated. TEG-1S did not include direct altimeter data.

5. ACCURACY EVALUATION

Efforts to evaluate and calibrate the accuracy of the UT gravity models were performed. Comparison of orbit fits using different gravity fields for Starlette, Ajisai, Seasat and Geosat were performed. It is shown that using TEG-1, a Starlette five-day orbit fit is at the ~20 cm level, Ajisai five-day orbit fit is at the ~15 cm level, and that a Seasat six-day orbit and a Geosat 17-day orbit have

Table 1. Satellite Data for the University of Texas Gravity Model, TEG-1					
Satellite	Launch Date	Data	Inclination	Eccentricity	Altitude (km)
Vanguard-1	1958	Optical†	34°	0.190	2318
Vanguard-2RB	1959	Optical†	33°	0.183	2318
Courier-1B	1965	Optical†	28°	0.016	1100
Geos-1	1965	Laser	59°	0.072	1600
BE-C	1966	Laser	41°	0.026	1130
DI-C	1967	Laser, Optical†	40°	0.053	1000
DI-D	1967	Laser, Optical†	39°	0.085	1200
Oscar-14	1967	Doppler	89°	0.005	1100
Geos-2	1968	Laser	106°	0.033	1400
Peole	1971	Laser, Optical†	15°	0.015	650
Geos-3	1975	Laser	115°	0.002	830
Starlette	1975	Laser	50°	0.020	900
Lageos	1976	Laser	110°	0.004	5900
Seasat	1978	Laser, Doppler, Altimeter and Crossover	108°	0.002	800
Nova-1	1980	Doppler	90°	0.002	1200
Geosat	1985	Doppler, Altimeter and Crossover	108°	0.000	800
Ajisai	1986	Laser	50°	0.001	1500
† Optical data currently withheld from gravity field solution					

Surface Gravity Data
1° × 1° terrestrial mean gravity anomaly from Ohio State University [*Rapp*, 1986]

crossover residuals at the ~25 cm level. Table 2 shows the summary for the Geosat orbit fits. Gravity field comparison using surface gravity data and a comparison of estimated TEG-1 ocean tidal parameters with solutions derived by other studies were also performed. Covariance matrices for TEG-1 and TEG-1S were calibrated to obtain estimates of errors associated with the gravity field using the consider covariance calibration technique [*Yuan et al.*, 1988]. The predicted radial orbit errors using TEG-1 gravity field covariance matrix for Topex and Geosat are 13 cm and 24 cm, respectively (Table 3).

6. CONCLUSION

In this investigation, two gravity models, TEG-1 and TEG-1S, each complete to degree and order 50 plus resonant coefficients, were generated. Ground-based tracking data collected by 14 satellites, altimeter crossover and surface gravity data were used to determine the TEG-1S gravity field model. TEG-1 contains Seasat and Geosat direct altimeter data in addition to all the data in TEG-1S. The gravity field models were derived simultaneously with orbit, ocean tides, quasi-stationary sea surface topography, and other geophysical parameters as well as the relative weights for each data set. The fields were evaluated using both data included and data withheld from the solution. Formal covariance matrices were calibrated to reflect realistic error estimates of the gravity field. Evaluations based on orbit fits and gravity anomaly residuals indicate that the gravity models have achieved a significant advancement over previously existing gravity models.

Table 2. Gravity Field Accuracy Evaluation Using Geosat Orbit Fits

$\bar{r}_o$, $\bar{v}_o$, C_R, daily C_D, density correction parameters adjusted

Epoch 17-day orbits		TEG-1S (rms)	TEG-1 (rms)
86/12/7	Doppler (cm/sec)	0.67	0.62
	Crossover (cm)†	25	22
	Altimeter (cm)†	180	32
87/01/7	Doppler (cm/sec)	0.62	0.61
	Crossover (cm)†	24	25
	Altimeter (cm)†	180	32

† Data types used for residual prediction only; altimeter data smoothed to represent gravity spectrum to (50×50)

Table 3. Gravity Field Accuracy Evaluation Using Covariance Analysis

Model	Predicted Topex Radial Orbit Error (cm)	Predicted Geosat Radial Orbit Error (cm)
GEM-T1	25	54
TEG-1	13	24

Topex orbit: 65° inclination, 1354 km altitude
Geosat orbit: 108° inclination, 800 km altitude

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